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RESEARCH ARTICLE

Love thy children or money

Reflections on debt neutrality and estate taxation

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Abstract This paper presents a simple OLG model which is consistent with observed consumer behavior, capital accumulation and wealth distribution, and yields some new conclusions about fiscal policy. By considering a society in which individuals are distinguished according to two characteristics, altruism and wealth preference, we show that those who in the long run hold the bulk of private capital are not so much motivated by dynastic altruism as by preference for wealth. In this setting, estate taxation is a questionable instrument of redistribution: it penalizes the wealthy, but favors the top wealthy. On the other hand, even though Ricardian equivalence holds, both public debt and PAYG pensions lead to a transfer of resources from the top wealthy to the other individuals.

Keywords Altruism · Preference for wealth · Capital accumulation · Wealth distribution · Estate taxation · Ricardian equivalence

JEL Classification D64 · H55 · H63

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"The love of wealth is therefore to be traced, as either a principal or accessory motive, at the bottom of all that the Americans do"

Alexis de Tocqueville 1841.

1 Introduction

The main goal of this paper is to present a simple overlapping generations model which is consistent with observed consumer behavior, capital accumulation and wealth distribution, and yields some new and quite surprising conclusions about fiscal policy.

Recently, Mankiw (2000) presented some evidence suggesting that we need a new macroeconomic model of fiscal policy. Accordingly, the two canonical macrodynamic models—namely, the Barro–Ramsey model with infinite horizon and the standard OLG model (due to Samuelson 1958 and Diamond 1965) with finite horizon—are inconsistent with the empirical finding that consumption tracks current income and with the numerous households with near zero wealth. In addition, the Diamond–Samuelson model does not acknowledge the importance of bequests in aggregate wealth accumulation. Thus according to Mankiw (2000, p. 121): "A new model of fiscal policy needs a particular sort of heterogeneity. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year but also from generation to generation. That is, we need a model in which some consumers plan ahead for themselves and their descendants, while others live paycheck to paycheck."

From these observations, macroeconomists have introduced a new distinction to segment society, between spenders and savers, which echoes that introduced some time ago by Ramsey (1928) between people with high and low impatience and more recently between altruists and non-altruists.[1] The gist of these distinctions is that savers, patient or altruistic households, end up accumulating wealth for the sake of bequeathing their children, whereas spenders, impatient or non-altruistic households, do not save at all and if they do so, they do it only for their own future consumption. Such a result holds as soon as there exist some agents with infinite horizon (infinitely lived agents or unconstrained altruistic agents).[2]

[1] Among the models with "savers" and "spenders" such as presented by Mankiw (2000), let us mention Michel and Pestieau (1998, 1999, 2005) who analyze the effects of alternative fiscal policies (debt, PAYG, estate taxation) in settings where the only heterogeneity is the degree of altruism with fixed or endogenous labor supply or where people vary according to altruism and productivity. Using the same dichotomy "savers–spenders", Smetters (1999) analyzes the robustness of the Ricardian equivalence; Nourry and Venditti (2001) study the stability and the determination of the long-run equilibrium; Laitner (2001) tries to explain secular changes in wealth inequality and inheritance in the US and UK data; and Thibault (2005) tries to explain the emergence of rentiers (i.e., of capitalists). Alternatively, Lambrecht et al. (2006), Rapoport and Vidal (2007), and Alonso-Carrera et al. (2008) have introduced new forms of intergenerational altruism in the agents' utility function in order to explain some empirical facts that cannot be reconciled with the traditional models.

[2] According to Muller and Woodford (1988), Nourry and Venditti (2001) or Thibault (2005), in societies with at least an agent *à la* Barro–Ramsey, the steady state is always Pareto optimal and the long-run capital stock, driven by the degree of patience (or altruism) of the most patient (or altruistic) agents, is only held by these agents.

The question one can raise at this point is whether such a simple representation of society bears any resemblance to reality. In others words, is the degree of altruism the key parameter of wealth accumulation? It is undoubtedly an important parameter as without bequest motive dynastic wealth is not sustainable. However, even if individuals differ in many respects including altruism, we observe at the same time in real life societies that those who control capital accumulation are not particularly altruistic. For instance, Arrondel and Lafferère (1998) who distinguish very wealthy and just wealthy in France, show that for the former altruism plays a much smaller role than for the latter. More recently, De Nardi (2004) and Reiter (2004) developed general equilibrium models calibrated on US and/or Swedish data. They show that altruism cannot explain the top tail of the wealth distribution. To do so, they use a growth model based on Max Weber's theory of "the spirit of capitalism" generalized by Kurz (1968): capitalists accumulate wealth for the sake of wealth. To cite Weber (1958, p. 53): "Man is dominated by making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal relationship, so irrational from a naive point of view, is a leading principle of capitalism." This view has been shared by many contemporary and past economists including[3] A. Smith, J.S. Mill, J. Schumpeter or J.M. Keynes. It has been used by many authors who have tried to explain growth and savings (see recently, De Nardi 2004 or Galor and Moav 2006). To summarize, as argued by Carroll (2000): "the saving behavior of the (American) richest households cannot be explained by models in which the only purpose of wealth accumulation is to finance future consumption, either their own or that of heirs." Then, to explain such a behavior, one has to assume that some consumers regard accumulation as an end in itself or as channel leading to power which is equivalent to assume that wealth is intrinsically desirable,[4] what we call here "preference for wealth".

In a nutshell, it seems that those who hold the bulk of private wealth are not so much motivated by dynastic altruism as by a preference for wealth. In other words, the key source of heterogeneity would not only be impatience or altruism but also preference for holding wealth.[5] In this paper, we look at this issue by considering a society in which individuals or rather dynasties are distinguished according to these two characteristics, altruism and wealth preference.

Let us now describe the main results that we obtain in this paper by considering an OLG economy with individuals differing in altruism and in preference for wealth. Whatever their degree or altruism (nil or positive), those with preference for wealth are labeled "hoarders". Those without such preference are either weak or strong pure

[3] See Zou (1994, 1995).

[4] The idea here is that wealth itself provides benefits to the individual: e.g., it can be a measure of success or the goal in itself. According to the recent survey of Kopczuk (2009), "this kind of (*animal spirit*) motive is the best candidate for modeling large bequests". The finding of Kopczuk (2007) that wealth continues to increase with age among the elderly wealthy is consistent with this motive. It is also consistent with another finding in the same paper that much of estate planning happens following the onset of terminal illness, suggesting the desire for holding on to wealth prior to the time when death becomes inevitable.

[5] The title of this paper is a take on that by Weil (1987), who focuses on love for children and not love for money.

altruists. To summarize, our society can be viewed as a society with a mix of agents *à la Diamond*, agents *à la Barro* and hoarders, i.e., agents with preference for wealth and with or without bequest motive.

As it will become clear below instead of wealth preference we could have used the bequest motive known as the "joy of giving" or the "warm glow" effect, which was originally introduced by Andreoni (1989, 1990). Under this latter interpretation, individuals obtain utility not so much from holding wealth but from giving it away to their heirs. There would then be a double bequest motive: pure altruism la Barro and joy of giving. Formally, either interpretation yields the same result. Empirically, there is some supportive evidence for both as shown by Arrondel et al. (1997) and Arrondel and Masson (2006). The presence of children is generally considered as a condition for any bequest motive. In this paper, we stick to the preference for wealth interpretation, and thus the only bequest motive considered is pure altruism.

Only the weak pure altruists do not bequeath at the equilibrium and, as in the Barro–Ramsey model and the "savers–spenders" model, the stock of capital is the ruled by the Modified Golden Rule (hereafter MGR).

Our findings are particularly interesting with regard to wealth distribution. The bequeathing dynasties hold a strong stand as to the way aggregate wealth is shared. The strong pure altruists determine the MGR equilibrium but they are not the only ones to hold wealth. The hoarders also hold some wealth.

In a recent survey of what he calls the "Ramsey's steady state conjecture", Robert Becker (2006) discusses the history of the idea according to which when individuals have different degrees of impatience the most patient ends up with all the wealth at the MGR equilibrium. The survey goes from even before Ramsey's work to the savers/spenders model of Mankiw (2000). Then, with our second source of heterogeneity, we enrich Mankiw's story and we show that this enrichment leads to the invalidation of the "Ramsey's steady state conjecture". To the best of our knowledge, this is the first model with logarithmic (and non-recursive) utility function and without taxation yielding a non-degenerate distribution of wealth at the standard MGR capital stock equilibrium.[6] In accordance with reality, it is possible that few hoarders with a low degree of altruism hold more capital than a large number of savers with strong altruism.

The relation between wealth holding and altruism is more complex than in earlier models. We obtain new results concerning the incidence of fiscal policy. In the traditional "savers–spenders" models, public debt and PAYG pensions seem to be regressive as they favor the savers at the expenses of the spenders. Introducing hoarders mitigates

[6] Epstein and Hynes (1983) or Lucas and Stokey (1984) show that there may exist stationary equilibria in which all households own positive amounts of capital when preferences are described by recursive utility (see also Becker and Boyd 1992). Sarte (1997) establishes that progressive taxation is another reason for the existence of stationary equilibria with a non-degenerate wealth distribution. A non-degenerate wealth distribution is also obtained by Dutta and Michel (1998) in a setting with imperfect altruism and linear price. Recently, Sorger (2002) shows, in the case where a government levies a progressive income tax, there exist infinitely many stationary equilibria in which all households own positive capital stocks. Note that Sorger (2008) exhibits a mechanism (based on the observation that price-taking behavior is no longer justified when all wealth is owned by a single household) that prevents the long-run distribution of wealth from becoming degenerate when households have different time-preference rates. Borissov and Lambrecht (2009) also focus on growth and distribution using an AK-model with endogenous impatience.

the above finding. Granted that the wealthiest are hoarders, either the PAYG pension system or public borrowing is shown to benefit the savers and to penalize not only the spenders but also the hoarders. In other words, these policies are not anymore unambiguously regressive.

At this point, a remark is in order. The idea that an OLG model with warm-glow type intergenerational linkages leads to a non-degenerate long-run wealth distribution is known. But our model can be viewed as complementary to those studies of wealth accumulation that consist of theoretical models with human capital considerations (as in Benabou 1996) or imperfect credit markets. This line of research has generally three main ingredients: imperfect capital market, exogenous prices and "pure joy of giving". The spirit of our paper is different; as mentioned above, it wants to deal with the micro-foundations of the "Ramsey's steady state conjecture". It also differs from this other literature by analyzing fiscal policy issues in a setting with double individual heterogeneity. On this point, we have findings that are quite at variance with those obtained by Mankiw (2000) with just savers and spenders.

As to estate taxation, our results differ also from those obtained in the "savers–spenders" setting.[7] Here, estate taxation is clearly a questionable instrument of redistribution: it penalizes the wealthy, but may favor the top wealthy. Indeed, estate taxation worsens the welfare of both spenders and savers but increases (decreases) the one of the hoarders if the degree of altruism of the savers is sufficiently high (low).

To summarize, if one wants to burden the savers, estate taxation is good and if one wants to favor them, either PAYG pension or public debt becomes the appropriate instrument. We also find here a new reason to deal with estate taxation with caution. If the objective of such a tax is to fight top wealth holding, we show that society might be better off using another tool.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 is devoted to the long-run capital accumulation and the long-run wealth distribution. Section 4 studies the incidence of social security, public debt and estate taxation. A final section concludes. Proofs are gathered in Appendices.

2 The economy

Consider a perfectly competitive economy which extends over infinite discrete time periods. The economy consists of $N \geq 1$ dynasties denoted by $h \in \{1, \ldots, N\}$. In each period t, the size of each dynasty h is denoted by N_t^h and grows at rate n. Total population size is N_t. We denote by p^h the positive (assumed time invariant) relative size of each dynasty h. Hence, we have $\sum_{h=1}^{h=N} p^h = 1$.

2.1 The consumers

Individuals of a dynasty h are identical within as well as across generations and live for two periods. We adopt Barro's (1974) definition of altruism: parents can care about

7 See Michel and Pestieau (1998) for an analysis of the effect of estate taxation in a Cobb-Douglas setting.

their children welfare by including their children's utility in their own utility function and possibly leaving them a bequest. When young, altruists of dynasty h, born at time t, receive a bequest x_t^h, work during their first period (inelastic labor supply), receive the market wage w_t, consume c_t^h and save s_t^h. When old, they consume d_{t+1}^h a part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}^h$ to their $1+n$ children. They perfectly foresee the interest factor R_{t+1}. Bequest is restricted to be non-negative, which is an important assumption. We denote by V_t^h the utility of an altruist of dynasty h:

$$V_t^h(x_t^h) = \max_{c_t^h, s_t^h, d_{t+1}^h, x_{t+1}^h} \ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h + \gamma^h \, V_{t+1}^h(x_{t+1}^h)$$

$$s.t \; w_t + x_t^h = c_t^h + s_t^h \tag{1}$$

$$R_{t+1} s_t^h = d_{t+1}^h + (1+n)x_{t+1}^h \tag{2}$$

$$x_{t+1}^h \geq 0$$

where $V_{t+1}^h(x_{t+1}^h)$ denotes the utility of a representative child who inherits x_{t+1}^h. The parameter $\delta^h \geq 0$ measures the preference for wealth, $\gamma^h \in [0, 1)$ is the intergenerational degree of altruism of the dynasty h and $\beta \in (0, 1]$ is the factor of time preference.

Contrary to Barro (1974), our log-linear life-cycle utility is not restricted to depend only on life-cycle consumption. Indeed, the agent enjoys accumulating wealth for itself when $\delta^h > 0$. The reasons why wealth directly enters in the utility function have been discussed above. Such a specification is old. For example, before[8] the well-known article of Kurz (1968), Yaari (1964) focused "on the notion that consumer preferences depend not only on the rate of consumption but also on terminal wealth (or bequests). Much earlier, Marshall (1920, p. 228), for instance, referred to family affections as the chief motive of saving ".

As mentioned above, δ^h can receive another interpretation, that of joy of giving. In other words, an individual with $\delta^h > 0$ obtains some utility from the mere act of giving x^h and not from holding it. Actually, this specification fits well our model as the argument of the joy of giving function is x, the wealth transmitted, and not Rs, the wealth held. However, to keep the presentation simple, we have chosen to restrict ourselves to one specification, that of preference of wealth, to which an increasing amount of work is devoted.

In our economy, the heterogeneity comes from two parameters triggering saving (besides old age consumption): the preference for wealth δ^h and the degree of altruism γ^h. Then, each dynasty h can be characterized by a pair $(\delta^h, \gamma^h) \in \mathbb{R}_+ \times [0, 1)$. From each pair (δ^h, γ^h), we can define the key parameter $\bar{\gamma}^h$ as follows:

$$\bar{\gamma}^h = \frac{\gamma^h(1+\beta) + \delta^h}{1+\beta+\delta^h} \geqslant \gamma^h.$$

[8] A theoretical discussion on this issue can also be found in the Tobin's unpublished dissertation (1947).

This parameter represents a "modified degree of altruism" which is larger than or equal to γ^h. Indeed, when $\delta^h = 0$ we have $\bar{\gamma}^h = \gamma^h$ whereas $\bar{\gamma}^h > \gamma^h$ as soon as $\delta^h > 0$. This degree of altruism modified by the wealth preference allows us to segment, without any loss of generality, the $N > 0$ dynasties:

- *M dynasties of pure altruists indexed from $h = 1$ to M.*
  We first have M dynasties ($M \in \{1, 2, \ldots, N-1\}$) of pure altruists which have no preference for wealth (i.e., $\delta^h = 0$). They are labeled from $h = 1$ to M and we assume that M is the most altruistic dynasty among the dynasties which do not have any preference for accumulating wealth. Then, by convention:
  - Dynasties 1 to $M - 1$ consist of weak pure altruists (i.e., $\delta^h = 0$ and $\gamma^h \in [0, \gamma^M)$ for $h \in \{1, \ldots, M-1\}$).
  - Dynasty M is the dynasty of strong pure altruists ($\delta^M = 0$ and $\gamma^M > \gamma^h$ for $h \in \{1, \ldots, M-1\}$).
- *$N - M$ dynasties of hoarders indexed from $h = M + 1$ to N.*
  We then have $N - M$ dynasties of hoarders which have some preference for wealth (i.e., $\delta^h > 0$). We assume that $\gamma^h \in [0, 1)$ and (if $N > M + 1$) $\bar{\gamma}^h \in [0, \bar{\gamma}^N)$ for $h \in \{M, \ldots, N-1\}$.
  - Dynasties $M + 1$ to $N - 1$ consist of hoarders (i.e., $\delta^h > 0$ and $\bar{\gamma}^h \in (0, \bar{\gamma}^N)$ for $h \in \{M+1, \ldots, N-1\}$).
  - Dynasty N is the dynasty of hoarders with the higher modified degree of altruism ($\delta^N > 0$ and $\bar{\gamma}^N \geq \bar{\gamma}^h$ for $h \in \{M+1, \ldots, N-1\}$).

Given a dynasty h, maximizing $V_t^h(x_t^h)$ subject to (1) and (2) gives the following first-order conditions:

$$\forall h \in \{0, N\} \quad d_{t+1}^h = \beta R_{t+1} c_t^h \tag{3}$$

$$\forall h \leq M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} \leq 0 \quad (= \text{ if } x_{t+1}^h > 0) \tag{4}$$

$$\forall h > M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} = 0. \tag{5}$$

Unlike the M dynasties of pure altruists, the optimal bequests of the $N - M$ dynasties of hoarders are necessarily positive at all date, and hence, in the long run.

Using (1) and (3), (2) can be rewritten as:

$$R_{t+1}s_t^h - (1+n)x_{t+1}^h = \beta R_{t+1}\left[w_t + x_t^h - s_t^h\right]$$

Then we can also write the saving of a given dynasty h:

$$s_t^h = \frac{1}{1+\beta}\left[\beta(w_t + x_t^h) + \phi(R_{t+1})x_{t+1}^h\right] \tag{6}$$

where $\phi(R) = (1+n)/R$ can be interpreted as a dynastic discount factor. Thus, ceteris paribus, the higher the inheritance x_t^h and earning w_t, the higher is saving. Saving increases also with intended bequest x_{t+1}^h.

2.2 The firms

Let us now turn to the production side. Production technology is represented by a Cobb-Douglas function with two inputs, capital K_t and labor L_t, i.e., $Y_t = F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0, 1)$ and $A > 0$. Homogeneity of degree one allows us to write output per worker as a function of the capital/labor ratio $k_t = K_t/L_t$, $Y_t/L_t = F(k_t, 1) = f(k_t) = Ak_t^\alpha$. Markets are perfectly competitive. Assuming, without loss of generality, that capital fully depreciates after one period, each factor is paid its marginal product:

$$w_t = f(k_t) - k_t f'(k_t) = A(1-\alpha)k_t^\alpha \quad \text{and} \quad R_t = f'(k_t) = A\alpha k_t^{\alpha-1}. \tag{7}$$

In each period, labor market clears, i.e., $L_t = N_t$ and the capital stock at time $t+1$ is financed by the savings of the young generation born in t. Hence,, we have $K_{t+1} = N_t s_t$ with $s_t = \sum_{h=1}^{h=N} p^h s_t^h$. Therefore, in intensive form:

$$(1+n)k_{t+1} = \sum_{h=1}^{h=N} p^h s_t^h. \tag{8}$$

3 Capital accumulation and wealth distribution

3.1 Steady-state solutions

In this section, as in the rest of the paper, we restrict our analysis to the steady state. The dynamics of the problem at hand is important but out of the scope of this work. We first study the long-run behavior of hoarders. We have seen that their bequests x_t^h are positive at each date t. Then, according to (3) and (5) we obtain in the steady state denoted by subscript $\star$:

$$\forall h > M \quad c_\star^h = \frac{\phi(R_\star) - \gamma^h}{\delta^h} x_\star^h \quad \text{and} \quad d_\star^h = \beta R_\star c_\star^h. \tag{9}$$

Merging (1), (2) and (9) we obtain the (positive) level of stationary bequest of a dynasty h of hoarders:

$$\forall h > M \quad x_\star^h = \frac{\bar{\delta}^h w_\star}{\phi(R_\star) - \bar{\gamma}^h} \tag{10}$$

where $\bar{\delta}^h \equiv \delta^h/(1+\beta+\delta^h)$ is the relative weight of wealth in the life-cycle utility $\ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h$.

Since, the bequest of hoarders is positive, the steady-state $\phi(R_\star)$ necessarily satisfies $\bar{\gamma}^N < \phi(R_\star)$ and, according to (6) and (10), their stationary saving $s_\star^h = x_\star^h + w_\star - c_\star^h$ is given by:

$$\forall h > M \quad s_\star^h = \frac{1}{1+\beta}\left(\beta + \frac{\bar{\delta}^h\left[\beta + \phi(R_\star)\right]}{\phi(R_\star) - \bar{\gamma}^h}\right) w_\star. \tag{11}$$

We now turn to the dynasties of pure altruists. According to (3) and (4), the long-term behavior of each of them must satisfy:

$$\forall h \le M \qquad \gamma^h \le \phi(R_\star) \quad (= \text{ if } x_\star^h > 0). \tag{12}$$

Hence, among these M dynasties, only the strong pure altruists, i.e., the dynasty M, that with the highest degree of altruism, has the possibility to leave a bequest. Indeed, if there exists a dynasty $m \in \{1, \ldots, M-1\}$ such that $x_\star^m > 0$ then (12) is not satisfied for dynasties h where $h \in \{m+1, \ldots, M\}$. Then, the weak pure altruists are constrained in the long run and their saving $s_\star^h$ is such that:

$$\forall h \le M-1 \qquad s_\star^h = \frac{\beta}{1+\beta} w_\star \tag{13}$$

The behavior of strong pure altruists is more complicated. According to (12), if $x_\star^M$ is positive then the steady-state capital stock $k_\star$ is equal to that of the MGR capital stock (i.e., $k_\star = f'^{-1}[(1+n)/\gamma^M]$) and we have:

$$k = \left[\frac{\alpha A \gamma^M}{1+n}\right]^{\frac{1}{1-\alpha}} \equiv k_\star^M, \;\; R = \frac{1+n}{\gamma^M} \equiv R_\star^M, \;\; w = A(1-\alpha)\left[\frac{\alpha A \gamma^M}{1+n}\right]^{\frac{\alpha}{1-\alpha}} \equiv w_\star^M.$$

When $x_\star^M$ is positive, we have $\phi(R^M) = \gamma^M$. Using $\varepsilon = \beta(1-\alpha)/[\alpha(1+\beta)]$ and according to (6), (11) and (13), the equilibrium condition (8) gives:

$$x_\star^M = F(\gamma^M) \times \frac{w_\star^M}{p^M} \quad \text{with} \quad F(\rho) = \frac{\beta(\rho - \varepsilon)}{\varepsilon(\rho + \beta)} - \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h}{\rho - \bar{\gamma}^h}. \tag{14}$$

This expression determines the bequest left by the strong pure altruists in the steady state. This bequest is positive if and only if $\gamma^M > \bar{\gamma}^N$ and $F(\gamma^M) > 0$. As $F'(.) > 0$, the strong pure altruists leave positive bequest as soon as

$$\gamma^M > \tilde{\gamma} \tag{15}$$

where $\tilde{\gamma} > \bar{\gamma}^N$ is the unique solution of $F(\tilde{\gamma}) = 0$.

If there were no hoarders, we would have the standard result (see Weil 1987; Michel and Pestieau 1998, or Thibault 2000):

$$\gamma^M > \varepsilon \equiv \frac{\beta(1-\alpha)}{\alpha(1+\beta)}$$

which guarantees that with our logarithmic specification Barro's 1974 model exhibits positive (operative) bequests. To compare our results with those of the Barro–Ramsey model and those of Mankiw's savers–spenders theory, we assume in the rest of the paper that (15) holds;[9] in other words, the strong altruists are savers and non-spenders.[10]

Note that the value of $\tilde{\gamma}$, which is larger than or equal to ε, depends neither on the degree of altruism nor on the proportion of spenders. On the contrary, as F' is positive and for all $h > M+1$, F'_{p^h}, $F'_{\bar{\delta}^h}$ and $F'_{\bar{\gamma}^h}$ are negative, this threshold value $\tilde{\gamma}$ depends on the proportion and the degree of altruism of the $N - M$ dynasties of hoarders. All things being equal the higher the proportion, the degree of altruism and the wealth preference of hoarders, the higher is the threshold $\tilde{\gamma}$.

3.2 The long-run accumulation

We have shown that the weak pure altruists are spenders and, assuming that (15) holds, strong pure altruists are savers. Since, whatever their positive preference for wealth, the hoarders leave positive bequests, we then obtain a (long-run) social segmentation which depends on the behavior of strong pure altruists. Concerning the long-run accumulation, we have also established the following result:

Proposition 1 (The long-run capital accumulation)
The stationary capital stock is equal to that of the MGR capital stock $k_\star^M$. It increases with the degree of altruism of the savers but is independent of the proportion of each dynasty, and of the degree of altruism of the spenders and the hoarders.

At the macroeconomic level, the economy is at the MGR steady state which depends on the degree of altruism γ^M of savers, but not on their proportion. Note that this result is similar to those of Kaldorian models but it is obtained in an endogenous way. As well known since Ramsey (1928) and Becker (1980), the most patients (or altruistic individuals) impose their view on the long-run capital accumulation, whatever their size. Then, our savers–spenders–hoarders equilibrium seems to be equivalent to the equilibrium obtained in the "savers–spenders" models with heterogenous pure altruists. It is, however, noteworthy that unlike the "savers–spenders" models and contrary

9 Our threshold $\tilde{\gamma}$ is higher than the ε of Weil (1987). However, the degree of altruism γ^M that has to be superior to that threshold does not receive the same meaning. In a setting of homogenous altruistic individuals (Weil 1987 or Mankiw 2000), γ^M represents the average level of altruism or that of a representative agent. In our heterogenous setting, γ^M is the degree of altruism of the most altruistic dynasty. Then, our setting makes the existence of savers more likely than the setting of Weil (1987) or Mankiw (2000).

10 If condition (15) does not hold strong altruists could become spenders if their degrees of altruism were sufficiently low. Two types of equilibrium would then be possible: a two-class equilibrium with spenders and hoarders and a three-class equilibrium with spenders, hoarders and savers, that is studied here. It is possible to observe switches from one equilibrium to the other depending on the variation of $\bar{\gamma}$. Switches can also occur following changes in fiscal policy. For a complete analysis, see Pestieau and Thibault (2007).

to Ramsey's intuition,[11] the long-run equilibrium has not two categories of individuals but three: savers, spenders and hoarders.

At the microeconomic level, the introduction of hoarders has an important implication. Even though the dynasty of strong altruists imposes its view on capital accumulation, it is not the only one to bequeath and hold wealth. The hoarders are also bequeathing even though they can be non-altruistic.

3.3 The long-run wealth distribution

We now turn to the way capital is held by the different agents. So doing we obtain a better grasp of the long-run distribution of wealth. One of the motivations of this paper is to show that top wealth is not necessarily held by the strong pure altruists. Compared to the "savers–spenders" literature, our model seems to be most relevant to study the long-term distribution of wealth. In the earlier models relying on the single characteristic of either patience or altruism, the equilibrium wealth distribution was reduced to two points: positive wealth for the most patient or altruistic, and zero wealth for the others. By introducing some preference for wealth and thus the category of hoarders, we now have a more complex and realistic distribution of wealth. We now have $N - M + 1$ types of wealth holders. Indeed, it is straightforward that the dynasties which held long-run wealth are the dynasties of bequeathers. We could get the share of wealth held by each of the N dynasties. However to keep the analysis simple, we now focus on the share ν of each of the three types of dynasties: savers ($\nu^{\mathrm{SA}} = p^M s^M/[(1+n)k_\star]$), spenders ($\nu^{\mathrm{SP}} = \sum_{h=1}^{M-1}[p^h s^h/((1+n)k_\star)]$) and hoarders ($\nu^{\mathrm{HO}} = \sum_{h=M+1}^{N}[p^h s^h/((1+n)k_\star)]$). With this simple presentation, we now study the comparative statics of wealth holding.

Proposition 2 (The long-run wealth distribution)
The higher the degree of altruism of the savers, the higher is the fraction ν^{SA} of the stationary capital $k_\star^M$ held by the savers and the lower are ν^{HO} and ν^{SP} the fractions, respectively, held by the hoarders and the spenders. The higher the proportion of hoarders, their degree of altruism and their preference for wealth, the higher is ν^{HO}.

Proof See Appendix A. □

The spenders hold some capital that is related to saving for retirement.[12] Wealth holding at the MGR equilibrium depends on the degree of altruism of both savers and hoarders. The more altruistic the savers, the higher is the capital stock $k_\star^M$, and the higher is their share. When the degree of altruism of the savers increases, $\nu^{\mathrm{HO}}/\nu^{\mathrm{SP}}$ decreases and, thus, even if the shares of both savers and hoarders diminish, the hoarders lose more than the spenders. When the factor of altruism of hoarders increases, then the share and the amount of capital held by the hoarders increase at the only

[11] Even though optimal growth theorists and Ramsey in particular are not concerned by intragenerational issue, he notes in his 1928s seminal paper that if agents differ in their time preference society will be split in two classes: "the thrifty enjoying bliss and the improvident at the subsistence level".

[12] This fact implies that, even if the spenders save, we can consider that they do not hold wealth. Indeed, in a successive generations model, i.e., a model without retirement period, the spenders do not hold any wealth.

expenses of savers. In other words, variation in the degree of altruism of hoarders has no impact on the share of capital held by spenders.

The bequeathing dynasties control the way capital is shared: both hoarders and savers impose their view, whereas the degree of altruism of spenders has no impact on the distribution of wealth. It is important to see that the share of capital held by the hoarders increases with their preference for wealth. Then, in accordance with reality, it is possible that few hoarders with little altruism hold a large share of capital.

4 Alternative fiscal policies

We now turn to alternative fiscal policies such as PAYG pension system, public debt and estate taxation. We want to see how the results obtained in optimal growth models with spenders and/or savers change or can be extended with the introduction of hoarders.

4.1 Pay-as-you-go pensions and public debt

Our PAYG pension system consists of a payroll levy, τ_t, paid in period t by workers and a pension benefit, θ_t, paid to the retirees of generation $t-1$ so that $\theta_t = (1+n)\,\tau_t$. Then, the budget constraints (1) and (2) become:

$$w_t + x_t^h - \tau_t = c_t^h + s_t^h \quad \text{and} \quad R_{t+1}s_t^h + \theta_{t+1} = d_{t+1}^h + (1+n)\,x_{t+1}^h. \tag{16}$$

We begin our study of the incidence of PAYG pension system by focusing both on the saving and the bequeathing behavior of each type of individuals.

Obviously, the spenders do not leave bequest. Then, we obtain:

$$\forall h < M \quad s_\star^h(\tau) = \frac{\beta w_\star(\tau) - [\beta + \phi_\star(\tau)]\tau}{1+\beta} \qquad \text{and} \qquad x_\star^h(\tau) = 0. \tag{17}$$

As to the hoarders, they always bequeath and the equilibrium conditions (9) hold. Using $\tilde{\omega}_\star(\tau) = w_\star(\tau) - [1-\phi_\star(\tau)]\tau$ the part of the life-cycle income $\Omega_\star^h(\tau)$ that is independent of h, their savings and bequests are such that:

$$\forall h > M \quad s_\star^h(\tau) = \frac{\zeta_\star^h(\tau)w_\star(\tau)}{1+\beta} - \frac{\tau[\beta+\phi_\star(\tau)]\chi_\star^h(\tau)}{1+\beta} \quad \text{and} \quad x_\star^h(\tau) = \frac{\bar{\delta}^h\tilde{\omega}_\star(\tau)}{\phi_\star(\tau) - \bar{\gamma}^h} \tag{18}$$

where

$$\zeta_\star^h(.) = \beta + \frac{\bar{\delta}^h[\beta + \phi_\star(.)]}{\phi_\star(.) - \bar{\gamma}^h} \qquad \text{and} \qquad \chi_\star^h(.) = 1 + \frac{\bar{\delta}^h[1-\phi_\star(.)]}{\phi_\star(.) - \bar{\gamma}^h}$$

Finally, due to the presence of savers, the economy follows the MGR. Indeed, (12) holds because the first-order conditions (3) and (4) are not modified by the PAYG pension system. Then, equality (8) allows us to obtain, after some computations, the bequests of savers:

$$x_\star^M(\tau) = \digamma(\gamma^M) \times \frac{w^M}{p^M} + \frac{\tau}{p^M} \times \vartheta(\gamma^M) \quad \text{with}$$
$$\vartheta(\rho) = 1 + \sum_{h=M+1}^{N} \frac{p^h \bar{\delta}^h [1-\rho]}{\rho - \bar{\gamma}^h}. \tag{19}$$

Note that, when $\tau = 0$, we recoup (14). According to (19), savers exist (i.e., $x_\star^M(\tau) > 0$) if and only if $\tau > \tau^M \equiv -w^M \digamma(\gamma^M)/\vartheta(\gamma^M)$. Even if ρ can be greater than one, the function $\vartheta(.)$ is always positive. Indeed, $\vartheta(\rho)$ is a decreasing function such that $\lim_{\rho \to +\infty} \vartheta(\rho) = 1 - \sum_{h=M+1}^{N} p^h \bar{\delta}^h > 0$. Consequently, as (15) implies $\digamma(\gamma^M) > 0$, our savers–spenders–hoarders equilibrium remains so with PAYG regardless of the size of τ. From (19), we can now look at the impact of a PAYG pension system on long-run capital accumulation, the redistribution across the dynasties and the welfare of each dynasty.

Proposition 3 (The effects of a PAYG pension system)

At the MGR equilibrium, PAYG pensions have no effect on the accumulation of capital. The decrease in saving by both spenders and hoarders is compensated by an increase in saving by savers. PAYG pensions reduce the bequests of hoarders but increase those of the savers. They also increase the share of capital held by savers but decrease the share of capital held, respectively, by spenders and hoarders.

PAYG pensions improve the welfare of savers but it lowers that of both spenders and hoarders.

Proof See Appendix B. □

At the macroeconomic level, the PAYG pension system is thus neutral given that the stationary stock of capital remains equal to the MGR capital stock. The neutrality property obtained in the "savers–spenders" literature resists to the introduction of hoarders.

At the microeconomic level, PAYG pensions modify the long-run wealth distribution because saving by the three classes is affected. Saving by both spenders and hoarders decreases and that by savers increases. Saving by hoarders and spenders does not equally react to a change in the payroll tax because their bequests do not move in the same way. PAYG pensions increase bequests by savers while reducing those by hoarders. Intuitively, the hoarders have a bequest motive that is related to wealth accumulation. Their bequests are thus a constant proportion of income $w_\star^M - (1-\gamma^M)\tau$, which decreases with τ. Things are different for the savers as their bequests increase with τ, given that $x_\star^M(\tau) = x_\star^M(0) + \tau\vartheta(\gamma^M)/p^M$. Following (8), the sum of all the bequests (i.e., $\sum_{h=1}^{N} p^h x_\star^h$) is constant at the MGR equilibrium. Thus, as τ is raised, increased bequests by the savers fully compensate the drop in bequests by the hoarders.

The fact that, contrary to the capital stock, wealth distribution is modified is already a result obtained by Michel and Pestieau (1998) or Mankiw (2000): they show that a PAYG pension system implies wealth redistribution from the spenders to the savers. Introducing hoarders extends those results: the fraction of capital they hold $\nu^{\text{HO}}(\tau)$ decreases with τ. There is thus some redistribution from the hoarders to the savers. Then, PAYG pensions, which seemed unfair in "savers–spenders" models, here penalize the top wealthy. The direction of redistribution between spenders and hoarders is ambiguous. It depends on the proportion of hoarders; it has to be high enough so that $\nu^{\text{HO}}(\tau) - \nu^{\text{SP}}(\tau)$ increases with τ.

Let us now turn to the effect of a PAYG pension system on individual welfare. As the stock of capital $k_\star^M$, and thus $R_\star^M$, do not depend on τ, both the welfare of the spenders and the hoarders are function of $\ln[w_\star^M - (1-\gamma^M)\tau]$. Hence, τ has a depressive effect on the welfare of these two types of dynasties. The welfare of savers is a function of $\ln[w_\star^M + (1-\gamma^M)(x_\star^M(\tau) - \tau)]$. Since $x_\star^M(\tau) - \tau - x_\star^M(0) = \tau[\vartheta(\gamma^M)/p^M - 1]$ is positive, one clearly sees that τ has a positive effect on the welfare of savers.

We find here an extension of the key result of Michel and Pestieau (1998) and Mankiw (2000) that PAYG pensions improve the welfare of the savers while decreasing the welfare of the spenders. Their result resists to the introduction of hoarders but it is mitigated by the fact that PAYG pension system increases the welfare of savers but decreases that of hoarders, even if the latter have an infinitesimal preference for wealth.

We now turn to the question of whether or not debt policy can be steady-state welfare improving. In each period, the government faces the budget constraint $B_t = R_t B_{t-1} - L_t T_t$, where B_t is the total level of debt in t and T_t is a lump-sum tax paid by the working generation. We assume that the debt was used at time 0 to the benefits of the retirees. There is no other government spending. We write $b_t = B_{t-1}/N_t$ and assume that $b_t = b$ is constant. This yields $T_t = [R_t - (1+n)]b$. With this public debt scheme, only two equations are changed. The first period budget constraint (1) is now:

$$w_t + x_t^h - T_t = c_t^h + s_t^h \tag{20}$$

and the relation linking capital and savings (8) becomes:

$$(1+n)(k_{t+1} + b) = \sum_{h=1}^{h=N} p^h s_t^h. \tag{21}$$

We begin our study of the incidence of public debt by focusing on the savings and the bequeathing behavior of each type of individuals. Obviously, the spenders do not leave bequest. Then, after calculation and using the first period income net of tax b, $\tilde{\omega}_\star(b) = w_\star(b) - [R_\star(b) - (1+n)]b$, their savings and their bequests satisfy:

$$\forall h < M \qquad s_\star^h(b) = \frac{\beta}{1+\beta}\tilde{\omega}_\star(b) \quad \text{and} \quad x_\star^h(b) = 0 \tag{22}$$

Concerning the hoarders, they always bequeath and the equilibrium condition (9) always holds. Thus, after some computations, their saving and their bequest are such that:

$$\forall h > M \qquad s_\star^h(b) = \frac{\zeta_\star^h(b)\tilde{\omega}_\star(b)}{1+\beta} \quad \text{and} \quad x_\star^h(b) = \frac{\bar{\delta}^h \tilde{\omega}_\star(b)}{\phi_\star(b) - \bar{\gamma}^h} \tag{23}$$

where $\zeta_\star^h(.)$ is defined in (18).

Due to the presence of savers, the economy is at the MGR capital stock $k_\star^M$. Indeed, (12) holds because the first-order conditions (3) and (4) are not modified by the public debt scheme. Then, the equation of capital accumulation (21) allows us to obtain, after some tedious manipulations, the bequests of savers:

$$x_\star^M(b) = F(\gamma^M) \times \frac{w_\star^M}{p^M} + \frac{(1+n)b}{\gamma^M p^M} \times \vartheta(\gamma^M). \tag{24}$$

Note that, when $b = 0$, we recoup (14). According to (24) savers exist (i.e., $x_\star^M(b) > 0$) if and only if $b > b^M \equiv -F(\gamma^M)w^M\gamma^M/[(1+n)\vartheta(\gamma^M)]$. Consequently, as (15) implies $F(\gamma^M) > 0$ and as $\vartheta(.)$ is always positive, our savers–spenders-hoarders equilibrium remains so with public debt regardless of the level of b. From (24), we can now look at the impact of the public debt on long-run capital accumulation, the redistribution across the dynasties and the welfare of each dynasties. We can now state the following proposition.

Proposition 4 (The effects of public debt)
At the MGR equilibrium, public debt has no effect on capital accumulation. It leads to a decrease in saving by spenders and hoarders, which is compensated by an increase in saving by savers. Public debt reduces bequests of hoarders and increases that of savers. It increases the share of wealth (capital plus bonds) held by savers but decreases the share of wealth held, respectively, by spenders and hoarders.

Public debt improves the welfare of savers but it lowers that of both spenders and hoarders.

Proof See Appendix C. □

Public debt has no macroeconomic effect on the MGR equilibrium. This neutrality result at the aggregate level strengthens the intuition that just one saver is enough to obtain Ricardian equivalence. Michel and Pestieau (1998) and Mankiw (2000) show that this result keeps holding with the introduction of spenders. Here, we show that it resists to the further introduction of hoarders.

If the public debt is neutral in aggregate terms, it modifies wealth distribution because saving by our three classes of individuals changes. Saving by spenders and hoarders is reduced and that by savers is increased. Saving by hoarders and spenders does not react equally to public borrowing because of bequests. Public debt increases bequests of savers and reduces those of hoarders. Hoarders leave a bequest that is a proportion of the income $\tilde{\omega}_\star^M(b) = w_\star^M - (1+n)(1/\gamma^M - 1)b$ which declines as b increases. As to the savers, their bequests increase with b as $x_\star^M(b) = x_\star^M(0) + (1+n)$

$b\vartheta(\gamma^M)/(\gamma^M p^M)$. However, the sum of all the bequests is an increasing function of b. Hence, in contrast with the PAYG pension system case, savers necessarily increase their bequests by an amount higher than what is necessary to compensate the decrease of bequests by the hoarders when b increases.

The share of capital held by savers increases with b, whereas the share held by both spenders and hoarders decreases. The direction of redistribution between the hoarders and the savers is ambiguous; it depends on the proportion of hoarders in society.

Concerning individual utilities, the welfare of spenders and hoarders is a function of $\ln[w_\star^M - (1/\gamma^M - 1)(1+n)b]$. Thus, the higher the public debt, the lower is the welfare of both spenders and hoarders is. Turning to the savers, their welfare is a function of $\ln\left[w_\star^M + (1-\gamma^M)\left(x_\star^M(b) - \frac{b(1+n)}{\gamma^M}\right)\right]$. Since $x_\star^M(b) - b(1+n)/\gamma^M - x_\star^M(0) = b(1+n)[\vartheta(\gamma^M)/p^M - 1]/\gamma^M > 0$ and $\vartheta(\gamma^M) > 1$, public borrowing increases their welfare.

To sum up, we have the same type of results as for the PAYG pension. Savers benefit from national debt, whereas welfare of both hoarders and spenders decreases. Consequently, if the government wants to favor the savers over the hoarders (i.e., the top wealthy), it can use either a PAYG pension or a public debt.

4.2 Estate taxation

We now turn to another instrument that is natural in a setting where bequests play such an important role: estate taxation. Again, we focus on the steady-state solution. The tax scheme is simple: an estate tax of fixed rate $\kappa \in [0, 1)$, which is redistributed in each period t as a lump-sum amount θ_t, the same for all. Hence, the revenue constraint is simply equivalent to:

$$\theta_t = \kappa \sum_{h=1}^{N} p^h x_t^h.$$

Then, the first budget constraint (1) of an agent of dynasty h becomes:

$$w_t + (1-\kappa)x_t^h + \theta_t = c_t^h + s_t^h. \tag{25}$$

Moreover, given θ_t, the optimal condition for saving (3) is unchanged but that for bequests of both savers and hoarders (4) and (5) is now distorted:

$$\forall h \le M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} \le 0 \quad (= \text{ if } x_{t+1}^h > 0) \tag{26}$$

$$\forall h > M \qquad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} = 0. \tag{27}$$

We now focus both on the savings and the bequeathing behavior of each type of individuals. Obviously, the spenders do not leave bequest. Then, after some calculations and using the notation $\tilde{\omega}_\star(\kappa) = w_\star(\kappa) + \theta_\star(\kappa)$, we obtain:

$$\forall h < M \qquad s_\star^h(\kappa) = \frac{\beta}{1+\beta}\tilde{\omega}_\star(\kappa) \quad \text{and} \quad x_\star^h(\kappa) = 0. \tag{28}$$

Concerning the hoarders, they always bequeath and (9) always holds. Using the above definition of $\tilde{\omega}_\star(\kappa)$, we get:

$$\forall h > M \qquad s_\star^h(\kappa) = \frac{\hat{\zeta}_\star^h(\kappa)\tilde{\omega}_\star(\kappa)}{1+\beta} \quad \text{and} \quad x_\star^h(\kappa) = \frac{\bar{\delta}^h \tilde{\omega}_\star(\kappa)}{\phi_\star(\kappa) - \bar{\gamma}^h(1-\kappa)} \tag{29}$$

where $\hat{\zeta}_\star^h(\kappa) = \frac{\bar{\delta}^h[\beta(1-\kappa)+\phi_\star(\kappa)]}{\phi_\star(\kappa)-\bar{\gamma}^h(1-\kappa)} + \beta$.

We note that bequests by hoarders increase with $\theta_\star$ and decrease with κ.

According to (26), the presence of savers implies $\phi_\star(\kappa) = \gamma^M(1-\kappa)$. Then, contrary to the public debt scheme or the PAYG pension system, an estate tax modifies the long-run capital stock since we have in long run:

$$k_\star^M(\kappa) = \left[\frac{\alpha A \gamma^M (1-\kappa)}{1+n}\right]^{\frac{1}{1-\alpha}}$$

Thus, the higher the tax rate κ, the lower is the long-run capital stock $k_\star^M(\kappa)$. This capital stock does not depend on the proportion of savers, spenders or hoarders. In the no tax case, this capital stock is obviously the one of the MGR. Using the fact that $\phi_\star(\kappa) = \gamma^M(1-\kappa)$, (8) allows us to obtain, after some computations, the bequests of the savers:

$$\begin{aligned} x_\star^M(\kappa) = \Bigg\{ \frac{\beta}{\varepsilon}\left[\frac{\gamma^M(1-\kappa)-\varepsilon}{\gamma^M+\beta+\kappa\gamma^M}\right] - \left[\frac{\gamma^M+\beta+\kappa\beta\gamma^M/\varepsilon}{\gamma^M+\beta+\kappa\gamma^M}\right] \\ \times \sum_{h=M+1}^{N} \frac{p^h\bar{\delta}^h}{\gamma^M-\bar{\gamma}^h}\Bigg\} \frac{w_\star^M(\kappa)}{p^M} \end{aligned} \tag{30}$$

where $w_\star^M(\kappa) = A(1-\alpha)(k_\star^M(\kappa))^\alpha$.

According to (30), savers exist as such (i.e., $x_\star^M(\kappa) > 0$), in other words type M dynasty leaves positive bequests if and only if $\kappa < \kappa^M \equiv \varepsilon(\gamma^M+\beta)\digamma(\gamma^M)/[1+\sum_{h=M+1}^{N} p^h\bar{\delta}^h/(\gamma^M-\bar{\gamma}^h)]$. Thus, as (15) implies $\digamma(\gamma^M) > 0$, our savers–spenders–hoarders equilibrium remains so with estate taxation if this tax is sufficiently low (i.e., lower than κ^M). Intuitively, contrary to public debt or PAYG pension which induces the strong altruists to increase their bequests and hence reinforces the portion of savers, estate taxation discourages bequeathing by the savers and may lead to their disappearance. Consequently, given the complexity of the problem at hand, we make two simplifications. First, we take a tax reform viewpoint by focusing on an infinitesimal change in the tax rate at zero level. Second, we assume that $N = M+1$; in other words, there is only one dynasty of hoarders. Since the hoarders are homogenous we denote hereafter by $h = \text{HO}$ the parameters relative to the dynasty of hoarders.

From (30), we can now study the impact of estate taxation on the long-run capital accumulation and the redistribution across dynasties and this leads to the following proposition.

Proposition 5 (The incidence of estate taxation)
At the MGR equilibrium, estate taxation reduces capital accumulation. It reduces the savings of all agents. It also depresses the bequest of savers but increases (decreases) that of hoarders if the degree of altruism of the savers is sufficiently high (low). Moreover, estate taxation decreases the share of wealth held by savers but increases that held by spenders and hoarders.

Estate taxation worsens the welfare of both spenders and savers but increases (decreases) that of the hoarders if the degree of altruism of the savers or their preference for wealth are sufficiently high (low).

Proof See Appendix D. □

First of all, estate taxation has a depressive incidence on capital accumulation. This is an intuitive result that is consistent with that obtained in the "savers–spenders" literature. Importantly, $\tilde{\omega}_\star(\kappa)$ decreases with κ when the rate of estate taxation is low and $\hat{\zeta}^h_\star(\kappa) = \beta + \bar{\delta}^h(\beta + \gamma^M)/(\gamma^M - \bar{\gamma}^h)$ does not depend on κ. Thus, according to (28) and (29), savings of both spenders and hoarders decrease with κ. As to the influence of κ on saving by the savers, consider first the effect of κ on their bequests described by (30). It is negative. From $(1+\beta)s^M(\kappa) = \beta\tilde{\omega}_\star(\kappa) + (1-\kappa)(\gamma^M + \beta)x^M_\star(\kappa)$, this negative effect has an impact on saving. The first term of the RHS decreases with κ (for low κ) and the second term always decreases with κ. Consequently, estate taxation depresses saving by the savers. To sum up, for low tax rates, saving by the three types decreases and capital accumulation goes down unambiguously. This is in contrast with the neutral effect of either public debt or unfunded pensions.

Turning to the bequests of the hoarders, according to (29), there exist two opposite effects and calculations presented in Appendix D lead to:

$$\forall h > M \quad \left.\frac{\partial x^{\mathrm{HO}}_\star(\kappa)}{\partial \kappa}\right|_{\kappa=0} \gtrless 0 \quad \text{if and only if} \quad \gamma^M \gtrless \frac{\alpha\beta}{\alpha\beta + 1 - \alpha}$$

Thus, contrary to the bequests of the savers, the bequests of the hoarders do not necessarily decrease as a result of estate taxation. The sign of the variation of hoarders' bequests depends on savers' characteristics (degree of altruism) and not on their own characteristics (degree of altruism and preference for wealth). This is typically a general equilibrium result. The key economic variable is disposable income of the hoarders in the second period: $R_\star(\kappa)s^{\mathrm{HO}}_\star(\kappa)$. We know that for the hoarders, $s^{\mathrm{HO}}_\star(\kappa)$ decreases for low value of κ and that $R_\star(\kappa) = (1+n)/[\gamma^M(1-\kappa)]$ increases with κ. We show that for low κ, $x^{\mathrm{HO}'}_\star(0)$ and $R'_\star(0)s^{\mathrm{HO}}_\star(0) + R_\star(0)s^{HO'}_\star(0)$ have the same sign. The lower saving by the hoarders is more than compensated by the increase in the interest factor. This increase is particularly important when the degree of altruism of the savers is high enough.

This result is surprising in several respects. First, it does not concern the savers whose bequests always decrease. It applies to the hoarders even if they have a negligible degree of altruism. Altruism and preference for wealth determine the level of bequests of the hoarders, but not how these bequests react to estate taxation.

Estate taxation affects also wealth distribution. It increases the share of capital held by the spenders and the hoarders at the expense of the savers. If the government wants to penalize the savers, it can introduce estate taxation. Estate taxation is clearly a questionable instrument of redistribution: it redistributes resources from the wealthy towards the top wealthy and the poor.

We now turn to the incidence of estate taxation on welfare.[13] The welfare of the spenders is:

$$V_\star^{\mathrm{SP}}(\kappa) = (1+\beta)\ln\tilde{\omega}_\star(\kappa) + \beta\ln R_\star(\kappa) + \mathrm{cst}$$

where $\tilde{\omega}_\star(\kappa)$ decreases with κ whereas $R_\star(\kappa)$ increases with low κ. According to Appendix D, this second effect is always dominated by the first at the capital stock $k_\star^M(\kappa)$. Then, a (small) increase of (low) κ worsens the welfare of the spenders. As to the savers, we have:

$$V_\star^{\mathrm{SA}}(\kappa) = (1+\beta)\ln\Omega_\star^{\mathrm{SA}}(\kappa) + \beta\ln R_\star(\kappa) + \mathrm{cst}$$

and we can show that a (small) increase of (low) κ worsens the welfare of savers. Note that the decrease in the welfare of savers is larger than the decrease of the welfare of spenders. We find here one of the main results of Michel and Pestieau (1998): estate taxation worsens the welfare of both spenders and savers. Do we find the same result for the hoarders? From (29), their welfare can be written as:

$$V_\star^{\mathrm{HO}}(\kappa) = (1+\beta+\delta^{\mathrm{HO}})\ln\tilde{\omega}_\star^{\mathrm{HO}}(\kappa) + \beta\ln R_\star(\kappa) - \delta^{\mathrm{HO}}\ln(1-\kappa) + \mathrm{cst}$$

and we can show that:

$$\left.\frac{\partial V_\star^{\mathrm{HO}}(\kappa)}{\partial\kappa}\right|_{\kappa=0} \gtrless 0 \quad \text{if and only if} \quad \gamma^M \gtrless \frac{\beta[1+\alpha(\beta+\delta^{\mathrm{HO}})]}{\beta[1+\alpha(\beta+\delta^{\mathrm{HO}})] + \delta^{\mathrm{HO}}(1-\alpha)}.$$

Thus, if the altruism of the savers is sufficiently strong, estate taxation can have a positive effect on the hoarders, namely on bequeathers who expectedly should be penalized. This surprising result can be explained by the general equilibrium mechanism described previously that leads to increased bequests by the hoarders. However, note that the degree of altruism of the savers that is needed to increase the welfare of the hoarders is higher than that needed to increase their bequests. These increases occur when the reduction of saving by the hoarders is more than compensated by the increase in interest rate. We have seen that an increase in bequests by the hoarders depends neither on their altruism nor on their preference for wealth. The increase in utility depends (positively) on δ^{HO}.

[13] Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h/R_\star$, the long-run welfare of a dynasty h can be rewritten as $V_\star^h = (1+\beta)\ln\Omega_\star^h + \beta\ln R_\star + \delta^h\ln x_\star^h + cst$ where $\Omega_\star^h(\kappa) = \tilde{\omega}_\star(\kappa) + [1-\kappa-\phi_\star(\kappa)]x_\star^h(\kappa)$.

To sum up, we have found a new reason to deal with estate taxation with caution. If the objective of the government is to fight top wealth holding, estate taxation may be ill advised. Note that with a non-linear estate tax such a non-monotone outcome could be avoided.

5 Conclusion

Traditional macroeconomic models rest on the assumption that agents are either altruistic or not and look at the shape of wealth distribution and at the effect of alternative fiscal policies on both capital accumulation and wealth distribution. Though very insightful, these models fail to reflect some real life features, particularly the fact that wealth is not predominantly held by altruistic agents. Empirical studies point out to the fact that wealth accumulation, specially top wealth accumulation, is not motivated by the presence of children but by some type of preference for wealth or for the power and the prestige that wealth conveys. To incorporate this relevant and important fact, this paper considers agents who are characterized not only by some degree of altruism, but also by some preference for wealth. It appears with this double heterogeneity that the properties of the "modified golden rule" obtained with the sole heterogeneity in altruism do not hold true.

First, the most altruistic individuals impose their own rate of time preference, but they do not necessarily hold the bulk of wealth. The top tail of the distribution of wealth is not only due to altruism, as usually stated, but also mainly due to some dynastic taste for wealth. Our finding is in the line of recent papers of Carroll (2000), De Nardi (2004) or Reiter (2004) who explain the top tail of wealth distribution in the US and in Sweden by a capitalist spirit motive according to which capital provides utility services directly, but not just through consumption. This is what we do here with our preference for wealth (see Kopczuk 2007, 2009). Second, our double heterogeneity yields some new and surprising conclusions about fiscal policy. Indeed, if the government wants to penalize the hoarders (i.e., the top wealthy) and to favor the savers, it can use a PAYG pension or a public debt. Estate tax depresses both capital accumulation and bequests of altruists with no preference for wealth but not necessarily the one of altruists with preference for wealth. Thus, it is here clearly a questionable instrument of redistribution: it penalizes the wealthy, but not the top wealthy.

Appendix A

Proof of Proposition 2 At the MGR equilibrium, we have $\phi(R_\star) = \gamma^M$. Then, using the fact that $\alpha\phi(R_\star)w_\star = (1-\alpha)(1+n)k_\star$ and since the saving of the hoarders is given by (11), we have:

$$\nu^{\text{HO}} = \sum_{h=M+1}^{N} \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\gamma^M} \sum_{h=M+1}^{N} p^h \left[1 + \frac{\bar{\delta}^h(\beta+\gamma^M)}{\beta(\gamma^M - \bar{\gamma}^h)}\right].$$

Using the fact that $\alpha\gamma^M w_\star = (1-\alpha)(1+n)k_\star$ and since the saving of the spenders is given by (13), we have:

$$\nu^{\text{SP}} = \sum_{h=1}^{M-1} \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\gamma^M} \sum_{h=1}^{M-1} p^h.$$

Given $\nu^{\text{HO}} + \nu^{\text{SP}} + \nu^{\text{SA}} = 1$ we have:

$$\nu^{\text{SA}} = 1 - \nu^{\text{HO}} - \nu^{\text{SP}} = 1 - \frac{\varepsilon}{\gamma^M}\left[1 - p^M + \sum_{h=M+1}^{h=N} \frac{p^h \bar{\delta}^h (\beta + \gamma^M)}{\beta(\gamma^M - \bar{\gamma}^h)}\right].$$

The higher γ^M, the higher is ν^{SA} and the lower are ν^{HO} and ν^{SP}. The higher the p^h, γ^h, δ^h of a dynasty h of hoarders, the higher is ν^{HO}. □

Appendix B

Proof of Proposition 3 Since the FOC (4) and (5) are not modified by the PAYG pension system, (12) holds and, consequently, the economy is at the MGR capital stock $k_\star^M$. Then, the PAYG pension system has no effects on the stationary capital stock of the savers–spenders–hoarders equilibrium.

Concerning the spenders, their bequests remain nil since (12) is unchanged. Using (17), we obtain for the saving of a dynasty h of spenders: $(1+\beta)s_\star^h(\tau) = \beta w^M - (\beta + \gamma^M)\tau$. Then, the saving of the spenders decreases with τ. Using $s_\star^h(\tau)$, we can focus on the share of capital held by the spenders. After computation we obtain:

$$\nu^{\text{SP}}(\tau) = \nu^{\text{SP}}(0) - \frac{(\beta+\gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \sum_{h=1}^{M-1} p^h.$$

Thus the share of capital $\nu^{\text{SP}}(\tau)$ held by the spenders decreases with τ.

According to (18), the bequest of a dynasty h of hoarders satisfies: $(\gamma^M - \bar{\gamma}^h)x_\star^h(\tau) = \bar{\delta}^h[w^M - (1-\gamma^M)\tau]$. The bequest of hoarders decreases with τ. Using this value, we also obtain the saving of a dynasty h of hoarders: $(1+\beta)(\gamma^M - \bar{\gamma}^h)s_\star^h(\tau) = [\beta(\gamma^M - \bar{\gamma}^h) + (\beta+\gamma^M)\bar{\delta}^h] - (\beta+\gamma^M)[\gamma^M - \bar{\gamma}^h + \bar{\delta}^h(1-\gamma^M)]\tau$. Then, the saving of the hoarders decreases with τ. Using $s_\star^h(\tau)$ we can focus on the share of capital held by the spenders. After computation we obtain:

$$\nu^{\text{HO}}(\tau) = \nu^{\text{HO}}(0) - \frac{(\beta+\gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \sum_{h=M+1}^{N} p^h \left[1 + \frac{\bar{\delta}^h(1-\gamma^M)}{\gamma^M - \bar{\gamma}^h}\right].$$

Hence, the share of capital $\nu^{\text{HO}}(\tau)$ held by the hoarders decreases with τ.

According to (19), the bequest $x_\star^M(\tau)$ of savers increases with τ. As $p^M s_\star^M(\tau) = (1+n)k_\star^M - \sum_{h=1}^{h=M-1} p^h s_\star^h(\tau) - \sum_{h=M+1}^{h=N} p^h s_\star^h(\tau)$ and since both the saving of spenders and hoarders decreases with τ, $s_\star^M(\tau)$ is an increasing function of τ. As the share of capital held by the savers is such that $\nu^{\mathrm{SA}}(\tau) = 1 - \nu^{\mathrm{SP}}(\tau) - \nu^{\mathrm{HO}}(\tau)$ and as both $\nu^{\mathrm{SP}}(\tau)$ and $\nu^{\mathrm{HO}}(\tau)$ decrease with τ, the share $\nu^{\mathrm{SA}}(\tau)$ increases with τ.

Finally, to compare the spenders and the savers, we can use the fact that:

$$\nu^{\mathrm{HO}}(\tau) - \nu^{\mathrm{SP}}(\tau) = \nu^{\mathrm{HO}}(0) - \nu^{\mathrm{SP}}(0) + \frac{(\beta + \gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \times \left(\sum_{h=1}^{M-1} p^h - \sum_{h=M+1}^{N} p^h \left(1 + \frac{\bar{\delta}^h(1-\gamma^M)}{\gamma^M - \bar{\gamma}^h} \right) \right).$$

Consequently, the PAYG pension system reduces (resp., increases) $\nu^{\mathrm{HO}}(\tau) - \nu^{\mathrm{SP}}(\tau)$ if the hoarders are (resp., are not) sufficiently numerous.

Concerning the individual welfare[14] incidence of a PAYG system it is important to note that the stock of capital $k_\star^M$, and thus $R_\star^M$, do not depend on τ. Thus, both the welfare of the spenders and the hoarders can be rewritten as:

$$V_\star^{\mathrm{SP}}(\tau) = (1+\beta)\ln[w_\star^M - (1-\gamma^M)\tau] + \mathrm{cst} \quad \text{and}$$
$$V_\star^{\mathrm{HO}}(\tau) = (1+\beta+\delta^h)\ln[w_\star^M - (1-\gamma^M)\tau] + \mathrm{cst}.$$

Hence, one clearly sees that τ has a depressive effect on the welfare of these two types of dynasties. Turning to the savers, we have:

$$V_\star^{\mathrm{SA}}(\tau) = \ln[w_\star^M + (1-\gamma^M)(x_\star^M(\tau) - \tau)] + \mathrm{cst}$$

Since $x_\star^M(\tau) - \tau - x_\star^M(0) = \tau[\vartheta(\gamma^M)/p^M - 1] > 0$, one clearly sees that τ has a positive effect on the welfare of the savers. □

Appendix C

Proof of Proposition 4 Since the FOC (4) and (5) are not modified by public debt, (12) holds and, consequently, the economy is at the MGR capital stock $k_\star^M$. The public debt has no effects on the capital stock of the savers–spenders–hoarders equilibrium.

Concerning the spenders, their bequests remain nil since (12) is unchanged. Using (22), the saving of a dynasty h of spenders is such that $(1+\beta)s_\star^h(b) = \beta[w^M - (1+n)(1/\gamma^M - 1)b]$. Then, the saving of the spenders decreases with b. From this saving, we can focus on the share of wealth (capital plus bonds) held by the spenders. After computation:

$$\nu^{\mathrm{SP}}(b) = \sum_{h=1}^{M-1} \frac{p^h s_\star^h(b)}{(1+n)(k_\star^M + b)} = \frac{\beta\left[w^M - (1+n)(1/\gamma^M - 1)b\right]}{(1+n)(k_\star^M + b)(1+\beta)} \sum_{h=1}^{M-1} p^h$$

[14] Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h/R_\star$, the long-run welfare of a dynasty h can be rewritten as $V_\star^h = (1+\beta)\ln\Omega_\star^h + \beta\ln R_\star + \delta^h \ln x_\star^h + cst$ where $\Omega_\star^h(\tau) = \tilde{\omega}_\star(\tau) + [1 - \phi_\star(\tau)]x_\star^h(\tau)$.

Consequently, the share of wealth $\nu^{\mathrm{SP}}(b)$ held by the spenders decreases with b.

According to (23), the bequest of a dynasty h of hoarders satisfies: $(\gamma^M - \bar{\gamma}^h)x_\star^h(b) = \bar{\delta}^h[w^M - (1+n)(1/\gamma^M - 1)b]$. Consequently, the bequest of the hoarders decreases with b. Using (23) the saving of the hoarders is given by $(1+\beta)(\gamma^M - \bar{\gamma}^h)s_\star^h(b) = [\beta(\gamma^M - \bar{\gamma}^h) + (\beta + \gamma^M)\bar{\delta}^h][w^M - (1+n)(1/\gamma^M - 1)b]$. Then, the saving of hoarders decreases with b. Consequently, we can focus on the share of wealth held by hoarders. After computation we obtain:

$$\begin{aligned}\nu^{\mathrm{HO}}(b) &= \sum_{h=M+1}^{N} \frac{p^h s_\star^h(b)}{(1+n)(k_\star^M + b)} \\ &= \frac{\beta\left[w^M - (1+n)(1/\gamma^M - 1)b\right]}{(1+n)(k_\star^M + b)(1+\beta)} \sum_{h=M+1}^{N} p^h \left[1 + \frac{(\beta + \gamma^M)\bar{\delta}^h}{\beta(\gamma^M - \bar{\gamma}^h)}\right].\end{aligned}$$

Hence, the share of wealth $\nu^{\mathrm{HO}}(b)$ held by the hoarders decreases with b.

According to (24), the bequest $x_\star^M(b)$ of savers increases with b. Since $p^M s_\star^M(\tau) = (1+n)(k_\star^M + b) - \sum_{h=1}^{h=M-1} p^h s_\star^h(b) - \sum_{h=M+1}^{h=N} p^h s_\star^h(b)$ and since the saving both of the spenders and the hoarders decrease with b, $s_\star^M(b)$ is an increasing function of b. As the share of wealth held by the savers is such that $\nu^{\mathrm{SA}}(b) = 1 - \nu^{\mathrm{SP}}(b) - \nu^{\mathrm{HO}}(b)$ and as both $\nu^{\mathrm{SP}}(b)$ and $\nu^{\mathrm{HO}}(b)$ decrease with b, the share $\nu^{\mathrm{SA}}(b)$ increases with b.

Finally to compare the spenders and the savers we can use the fact that:

$$\begin{aligned}\nu^{\mathrm{HO}}(b) - \nu^{\mathrm{SP}}(b) &= \frac{\beta\left[w^M - (1+n)(1/\gamma^M - 1)b\right]}{(1+n)(k_\star^M + b)(1+\beta)} \\ &\quad \times \left(\sum_{h=M+1}^{N} p^h \left[1 + \frac{(\beta + \gamma^M)\bar{\delta}^h}{\beta(\gamma^M - \bar{\gamma}^h)}\right] - \sum_{h=1}^{M-1} p^h\right)\end{aligned}$$

As a result the public debt reduces (increases) $\nu^{\mathrm{HO}}(b) - \nu^{\mathrm{SP}}(b)$ if the hoarders are (are not) sufficiently numerous.

Concerning individual welfare,[15] one writes the welfare of spenders as:

$$V_\star^{\mathrm{SP}}(b) = (1+\beta)\ln[w_\star^M - (1/\gamma^M - 1)(1+n)b] + \mathrm{cst}.$$

As to the hoarders, their welfare is:

$$V_\star^{\mathrm{HO}}(b) = (1+\beta+\delta^h)\ln[w_\star^M - (1/\gamma^M - 1)(1+n)b] + \mathrm{cst}$$

15 Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h / R_\star$, the long-run welfare of a dynasty h can be rewritten as $V_\star^h = (1+\beta)\ln \Omega_\star^h + \beta \ln R_\star + \delta^h \ln x_\star^h + cst$ where $\Omega_\star^h(b) = \tilde{\omega}_\star(b) + [1 - \phi_\star(b)]x_\star^h(b)$.

Thus, the higher the public debt, the lower is the welfare of both spenders and hoarders is. Turning to the savers, we have:

$$V_\star^{\mathrm{SA}}(b) = \ln\left[w_\star^M + (1-\gamma^M)\left(x_\star^M(b) - \frac{b(1+n)}{\gamma^M}\right)\right] + \mathrm{cst}$$

Since $x_\star^M(b) - b(1+n)/\gamma^M - x_\star^M(0) = b(1+n)[\vartheta(\gamma^M)/p^M - 1]/\gamma^M > 0$ and $\vartheta(\gamma^M) > 1$, introducing public borrowing increases the welfare of the savers. $\square$

Appendix D

Proof of Proposition 5 According to (26) we have $\phi_\star(\kappa) = \gamma^M(1-\kappa)$. Then, the MGR capital stock is affected by the estate taxation since we have $k_\star = k_\star^M(\kappa) = [\alpha A\gamma^M(1-\kappa)/(1+n)]^{1/(1-\alpha)}$. Then, the larger the tax rate κ, the lower is the long-run capital stock $k_\star^M(\kappa)$.

Using this capital stock we can show after some tedious computations that $\tilde{\omega}_\star(\kappa) = (1-\kappa)^{\frac{1}{1-\alpha}}[\mathscr{A}\kappa^2 + \mathscr{B}\kappa + \mathscr{C}]\Pi/[\mathscr{A}'\kappa^2 + \mathscr{B}'\kappa + \mathscr{C}]$ where $\Pi = A(1-\alpha)[\alpha A\gamma^M/(1+n)]^{1/(1-\alpha)}$, $\mathscr{A} = -\beta\gamma^M(1+\mathscr{E})$, $\mathscr{A}' = -\varepsilon\gamma^M(1+\mathscr{E})$, $\mathscr{B} = \mathscr{B}' + \gamma^M(\beta+\varepsilon)$, $\mathscr{B}' = \varepsilon[\gamma^M - (\beta+\gamma^M)(1+\mathscr{E})]$, $\mathscr{C} = \varepsilon(\beta+\gamma^M)$ and $\mathscr{E} = \sum_{h=M+1}^N [p^h\bar{\delta}^h/(\gamma^M - \bar{\gamma}^h)]$. Then, one can show that, $\tilde{\omega}_\star'(0)$ has the sign of $\mathscr{B} - \mathscr{B}' - \mathscr{C}/(1-\alpha)$ i.e., the sign of $\gamma^M(\beta+\varepsilon) - \varepsilon(\beta+\gamma^M)/(1-\alpha)$, i.e., the sign of $\alpha\gamma^M - 1$. Consequently, $\tilde{\omega}_\star'(0)$ is negative and the introduction of an estate taxation reduces the income $\tilde{\omega}_\star$.

Note that $\hat{\zeta}_\star^h(\kappa) = \bar{\delta}^h[\beta + \gamma^M]/[\gamma^M - \bar{\gamma}^h] + \beta \equiv o_M^h$ is independent of κ. According to (28), and (29), the savings of a dynasty h of spenders are such that $s_\star'^h(0)$ has the sign of $\tilde{\omega}_\star'(0)$ whereas the savings of a dynasty h of hoarders has the sign of $\hat{\zeta}_\star'^h(0)\tilde{\omega}_\star(0) + \hat{\zeta}_\star^h(0)\tilde{\omega}_\star'(0)$. Then, as $\tilde{\omega}_\star'(0)$ is negative and $\hat{\zeta}_\star'^h(\kappa)$ is nil, the introduction of an estate taxation reduces the savings of all dynasties of spenders and of hoarders.

Note that the saving of a dynasty h in the estate taxation case is given by:

$$\forall h \;\; s_\star^h = \frac{1}{1+\beta}\left[\beta(w_\star + \theta_\star) + [\phi_\star(\kappa) + \beta(1-\kappa)]\,x_\star^h\right] \tag{31}$$

To analyze the effect of κ on the savings of the savers we begin by focusing on the effect of κ on the bequests of the savers. According to (30), this bequest $x_\star^M(\kappa)$ is such that $x_\star^M(\kappa) = G(\kappa)w_\star^M(\kappa)/H(\kappa)$ where $G(\kappa) = \beta[\gamma^M(1-\kappa) - \varepsilon] - [(\gamma^M + \beta)\varepsilon + \kappa\beta\gamma^M]\sum_{h=M+1}^N [p^h\bar{\delta}^h/(\gamma^M - \bar{\gamma}^h)]$ and $H(\kappa) = \varepsilon(\gamma^M + \beta + \kappa\gamma^M)$. Since $G'(\kappa) = -\beta\gamma^M(1 + \sum_{h=M+1}^N [p^h\bar{\delta}^h/(\gamma^M - \bar{\gamma}^h)])$ is negative, $H'(\kappa) = \varepsilon\gamma^M$ is positive and $w_\star'^M(\kappa) = \alpha(1-\alpha)A(k_\star^M)^{\alpha-1}(\kappa)k_\star'^M(\kappa)$ is negative, $x_\star^M(\kappa)$ decreases when κ increases.

Concerning the savings of savers, according to (31) we have $(1+\beta)s^M(\kappa) = \beta\tilde{\omega}_\star(\kappa) + (1-\kappa)(\gamma^M + \beta)x_\star^M(\kappa)$. Then, as the first term of the RHS decreases for low κ and the second term always decreases with κ, the introduction of an estate taxation depresses saving by the savers.

Concerning the bequests of the dynasty of hoarders $x_\star^{\text{HO}}(\kappa)$, using (29) it is obvious that $x_\star'^{\text{HO}}(\kappa)$ has the sign of $\ell'(\kappa)$ where $\ell(\kappa) = \tilde{\omega}_\star(\kappa)/(1-\kappa)$. Then, as $\ell'(0) = \tilde{\omega}_\star'(0) + \tilde{\omega}_\star(0)$ and $\tilde{\omega}_\star'(0) = \tilde{\omega}_\star(0)[\gamma^M(\beta+\varepsilon)/\mathscr{C} - 1/(1-\alpha)]$, $x_\star'^{\text{HO}}(0)$ is positive if and only if γ^M is larger than $\alpha\beta/[\alpha\beta + 1 - \alpha]$

We now focus on the impact of κ on the wealth distribution. Using (28) we can focus on the share of wealth held by spenders. After computation we obtain:

$$\nu^{\text{SP}}(\kappa) = \sum_{h=1}^{M-1} \frac{p^h s_\star^h(\kappa)}{(1+n)k_\star(\kappa)} = \frac{\tilde{\omega}_\star(\kappa)}{k_\star^M(\kappa)} \sum_{h=1}^{M-1} \frac{\beta p^h}{(1+n)(1+\beta)}.$$

Using (29) we can also focus on the share of wealth held by the hoarders. We obtain:

$$\nu^{\text{HO}}(\kappa) = \frac{p^{\text{HO}} s_\star^{\text{HO}}(\kappa)}{(1+n)k_\star(\kappa)} = \frac{\tilde{\omega}_\star(\kappa)}{k_\star^M(\kappa)} \frac{o_M^{\text{HO}} p^{\text{HO}}}{(1+n)(1+\beta)}$$

To determine how $\nu^{\text{SP}}(\kappa)$ and $\nu^{\text{HO}}(\kappa)$ vary when κ increases we must study how vary $\psi(\kappa) = \tilde{\omega}_\star(\kappa)/k_\star^M(\kappa)$. As we have $\psi(\kappa) = e \times [\mathscr{A}\kappa^2 + \mathscr{B}\kappa + \mathscr{C}]/[\mathscr{A}'\kappa^2 + \mathscr{B}'\kappa + \mathscr{C}]$ with $e > 0$, $\psi'(0)$ has the sign of $\mathscr{B} - \mathscr{B}' = \gamma^M(\beta+\varepsilon)$. Then, $\psi'(0)$ is positive and consequently, the introduction of an estate taxation increases the shares of wealth ν^{SP} and ν^{HO} held respectively by the spenders and the hoarders. Conversely, the fact that $\nu^{\text{SA}} = 1 - (\nu^{\text{SP}} + \nu^{\text{HO}})$ implies that the introduction of an estate taxation decreases the share of wealth ν^{SA} held by the savers.

At the MGR equilibrium, the stock of capital is given by $k_\star^M(\kappa)$ from which we obtain:

$$\mathscr{W}'(0) \equiv \left.\frac{\tilde{\omega}_\star'(\kappa)}{\tilde{\omega}_\star(\kappa)}\right|_{\kappa=0} = \frac{-\beta(1-\alpha\gamma^M)}{(1-\alpha)(\beta+\gamma^M)} \quad \text{and} \quad \mathscr{R}'(0) \equiv \left.\frac{R_\star'(\kappa)}{R_\star(\kappa)}\right|_{\kappa=0} = 1. \quad (32)$$

From these two equalities, we look for the effect of κ on the welfare of our three types of individuals. From the welfare of spenders and using (32), we have: $V_\star'^{\text{SP}}(0) = (1+\beta)\mathscr{W}'(0) + \beta\mathscr{R}'(0) < 0$.

Turning to the savers, we know that $x_\star'^M(0) < 0$ and $x_\star^M(0) > 0$. Then, $\Omega_\star^M(0) > \tilde{\omega}(0)$ and $\Omega_\star'^M(0) < \tilde{\omega}'(0)$. Consequently, $\Omega_\star'^{\text{SA}}(0)/\Omega_\star^{\text{SA}}(0) < \mathscr{W}'(0)$ which implies: $V_\star'^{\text{SA}}(0) < V_\star'^{\text{SP}}(0)$.

Concerning the hoarders, from $V_\star^{\text{HO}}(\kappa)$ we obtain: $V_\star'^{\text{HO}}(0) = (1+\beta+\delta^{\text{HO}})\mathscr{W}'(0) + \beta\mathscr{R}'(0) + \delta^{\text{HO}}$. Then, using (32), $V_\star'^{\text{HO}}(0)$ is positive if and only if γ^M is larger than $\beta[1+\alpha(\beta+\delta^{\text{HO}})]/\{\beta[1+\alpha(\beta+\delta^{\text{HO}})]+\delta^{\text{HO}}(1-\alpha)\}$. Finally, the proofs concerning results on welfare are given in the main text (end of Sect. 4.2). □

References

Alonso-Carrera, J., Caballé, J., Raurich, X.: Estate taxes, consumption externalities, and altruism. J Public Econ **92**, 1751–1764 (2008)

Andreoni, J.: Giving with impure altruism: applications to charity and Ricardian equivalence. J Political Econ **97**, 1447–1458 (1989)

Andreoni, J.: Impure altruism and donations to public goods: a theory of 'warm glow' giving. Econ J **100**, 464–477 (1990)

Arrondel, L., Lafferère, A.: Succession capitaliste et succession familiale: un modèle économétrique à deux régimes endogènes. Ann Econ Stat **51**, 187–208 (1998)

Arrondel, L., Masson, A.: Altruism, exchange or indirect reciprocity: what do the data on family transfers show? In: Kolm, S.C., Mercier Ythier, J. (eds.) Handbook of the Economics of Giving, Reciprocity and Altruism, Chapter 14, vol. 2, pp. 971–1053. North Holland: Elsevier (2006)

Arrondel, L., Masson, A., Pestieau, P.: Bequests and inheritance: empirical issues and French–US comparison. In: Erreygers, G., Vandervelde, T. (eds.) Is Inheritance Justified? Economic Aspects of Wealth Transfers. Berlin: Springer (1997)

Barro, R.: Are government bonds net wealth? J Political Econ **82**, 1095–1117 (1974)

Becker, R.: On the long run steady state in a simple dynamic model of equilibrium with heterogeneous households. Q J Econ **95**, 375–382 (1980)

Becker, R.: Equilibrium dynamics with many agents. In: Dana, R.A., Le Van, C., Mitra, T., Nishimura, K. (eds.) Handbook on Optimal Growth. Berlin: Springer (2006)

Becker, R., Boyd, J.H.: Recursive utility and optimal capital accumulation II: sensitivity and duality theory. Econ Theory **2**, 547–563 (1992)

Borissov, K., Lambrecht, S.: Growth and distribution in an AK-model with endogenous impatience. Econ Theory **39**, 93–112 (2009)

Carroll, C.D.: Why do the rich save so much? In: Slemrod, J. (ed.) Does Atlas Shrug? The Economic Consequences of Taxing the Rich, pp. 463–485. London: Harvard Press (2000)

De Nardi, M.: Wealth inequality and intergenerational links. Rev Econ Stud **71**, 743–768 (2004)

Diamond, P.A.: National debt in a neoclassical growth model. Am Econ Rev **55**, 1126–1150 (1965)

Dutta, J., Michel, Ph.: The distribution of wealth with imperfect altruism. J Econ Theory **82**, 379–404 (1998)

Epstein, L.G., Hynes, J.A.: The rate of time preference and dynamic economic analysis. J Political Econ **91**, 611–635 (1983)

Galor, O., Moav, O.: Das human kapital: a theory of the demise of the class structure. Rev Econ Stud **73**, 85–117 (2006)

Kopczuk, W.: Bequest and tax planning: evidence from estate tax returns. Q J Econ **122**, 1801–1854 (2007)

Kopczuk, W.: Economics of Estate Taxation: Review of Theory and Evidence. Mimeo (2009)

Kurz, M.: Optimal economic growth and wealth effects. Int Econ Rev **9**, 348–357 (1968)

Laitner, J.: Secular changes in wealth inequality and inheritance. Econ J **111**, 691–721 (2001)

Lambrecht, S., Michel, Ph., Thibault, E.: Capital accumulation and fiscal policy in an OLG model with family altruism. J Public Econ Theory **6**, 465–486 (2006)

Lucas, R. Jr., Stokey, N.: Optimal growth with many consumers. J Econ Theory **32**, 139–171 (1984)

Mankiw, G.: The savers–spenders theory of fiscal policy. Am Econ Rev **90**, 120–125 (2000)

Marshall, A.: Principles of Economics, 8th ed. London: Macmillan (1920)

Michel, Ph., Pestieau, P.: Fiscal policy in a growth model with both altruistic and nonaltruistic agents. South Econ J **64**, 682–697 (1998)

Michel, Ph., Pestieau, P.: Fiscal policy when individuals differ regarding altruism and labor supply. J Public Econ Theory **1**, 187–203 (1999)

Michel, Ph., Pestieau, P.: Fiscal policy with agents differing in altruism and ability. Economica **72**, 121–135 (2005)

Muller, W.J., Woodford, M.: Determinacy of equilibrium in stationary economies with both finite and infinite lived consumers. J Econ Theory **46**, 255–290 (1988)

Nourry, C., Venditti, A.: Determinacy of equilibrium in an overlapping generations model with heterogeneous agents. J Econ Theory **96**, 230–255 (2001)

Pestieau, P., Thibault, E.: The spenders-hoarders theory of capital accumulation, wealth distribution and fiscal policy. CORE Discussion Paper, 2007/40 (2007)

Ramsey, F.P.: A mathematical theory of saving. Econ J **38**, 543–559 (1928)

Rapoport, H., Vidal, J.P.: Economic growth and endogenous intergenerational altruism. J Public Econ **91**, 1231–1246 (2007)

Reiter, M.: Do the rich save too much? How to explain the top tail of the wealth distribution. Pompeu Fabra, unpublished (2004)

Samuelson, P.A.: An exact consumption-loan model of interest with and without the social contrivance of money. J Political Econ **66**, 467–482 (1958)

Sarte, P.D.: Progressive taxation and income inequality in dynamic competitive equilibrium. J Public Econ **66**, 145–171 (1997)

Smetters, K.: Ricardian equivalence: long-run leviathan. J Public Econ **73**, 395–421 (1999)

Sorger, G.: On the long-run distribution of capital in the Ramsey model. J Econ Theory **105**, 226–243 (2002)

Sorger, G.: Strategic saving decisions in the infinite-horizon model. Econ Theory **36**, 353–377 (2008)

Thibault, E.: Existence of equilibrium in an OLG model with production and altruistic preferences. Econ Theory **15**, 709–716 (2000)

Thibault, E.: Existence and specific characters of rentiers. A savers–spenders theory approach. Econ Theory **25**, 401–419 (2005)

Tobin, J.: A theoretical and statistical analysis of consumer saving. Ph.D. dissertation. Harvard University (1947)

Weber, M.: The protestant ethic and the spirit of capitalism. New York: Charles Scribner's Sons (1958)

Weil, P.: Love thy children. Reflections on the Barro debt neutrality theorem. J Monetary Econ **19**, 377–391 (1987)

Yaari, M.: On the consumer's lifetime allocation process. Int Econ Rev **5**, 304–317 (1964)

Zou, H.: The spirit of capitalism and long-run growth. Eur J Political Econ **10**, 279–293 (1994)

Zou, H.: The spirit of capitalism and savings behavior. J Econ Behav Organ **28**, 131–143 (1995)