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RESEARCH ARTICLE

Optimal debt contracts under costly enforcement

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Abstract We consider a financing game where monitoring is costly, non-contractible, and allowed to be stochastic. The optimal contract, which is debt, induces creditor leniency and strategic defaults on the equilibrium path, consistent with empirical evidence on repayment and monitoring behavior in credit markets. Our paper is the first where the optimal contract is debt and default is not synonymous with bankruptcy.

Keywords Costly state verification · Debt contract · Priority violation · Strategic defaults

JEL Classification D02 · D82 · G21 · G3

1 Introduction

A celebrated framework that has been used to explain the prevalence of debt contracts is the Costly State Verification model. In this model, the entrepreneur has superior

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information compared to investors about the true cash flow of the firm and may attempt to divert cash rather than repay investors. As a protection against the diversion problem, investors may request a verification of the firm's income, but this is costly. Townsend (1979) and Gale and Hellwig (1985) derive debt as the optimal contract under such circumstances.[1] Under the optimal contract, the entrepreneur pays in full what is owed if the firm has sufficient cash to do so. Otherwise, the entrepreneur defaults and the investor–creditor verifies with probability one. In other words, a default leads automatically to a costly verification, or bankruptcy.

We derive the optimal contract under costly state verification, but unlike Townsend (1979) and Gale and Hellwig (1985) we do not assume that verification is contractible and we allow for random verification. The optimal contract that we derive contains a fixed (debt) payment obligation, and verification (monitoring) under the optimal contract is stochastic. Furthermore, equilibrium repayment behavior yields both strategic defaults (debt is not repaid in full even if the firm has enough cash do so) and absolute priority violations (equity gets a positive payout even if the creditor is not repaid in full). The implication that debt incurs strategic defaults is consistent with a large theoretical literature building on Hart and Moore (1998) that considers optimal contracting under symmetric and unverifiable information.[2] It is also consistent with an emerging empirical corporate finance literature on repayment behavior in credit markets such as Brown et al. (2006), Carlier and Renou (2006) and Davydenko (2008). For example, in a broad sample of firms, Davydenko (2008) finds that about 70% of defaulting firms are not liquidated. To our knowledge, our paper is the first where the optimal contract is debt and default is not synonymous with bankruptcy.

Border and Sobel (1987), Mookherjee and Png (1989), and Krasa and Villamil (1994) retain the assumption of Townsend (1979) and Gale and Hellwig (1985) that verification is contractible but allow for stochastic verification, and show that stochastic verification without a fixed payment is optimal. In other words, they show that debt is not optimal under contractible stochastic verification.[3] We do not restrict the contracting space to deterministic schemes and we do not assume that verification is contractible; yet, we show that the optimal contract is debt and that verification under the optimal contract is stochastic.

Gale and Hellwig (1989) studied repayment behavior in a setting similar to ours in which verification is non-contractible, and derive necessary conditions for the existence of the same type of separating equilibrium as considered in the present paper. They do not deal with optimal contracting, which is the central issue of the present paper. Persons (1997), Khalil (1997), and Khalil and Parigi (1998) consider optimal contracting under costly state verification under the assumption that verification is

[1] See also Diamond (1984), Williamson (1987), and Winton (1995).

[2] For example, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) use strategic defaults to explain why observed risk premia on debt exceeds that implied by the Merton (1974) debt valuation model.

[3] Carlier and Renou (2005, 2006) assume contractible and deterministic monitoring and show that (standard) debt may not necessarily survive as the optimal contract when the borrower and the lender have heterogeneous beliefs.

not contractible, as we do. While they consider a two-state model, we assume a continuous state space, which allows for a more meaningful distinction between debt and other types of contracts (such as outside equity). In addition, our more general formulation allows us to solve for a general monitoring probability function.

Krasa and Villamil (2000) derive debt as the optimal contract in a setting similar to ours in which monitoring is non-contractible. In the equilibrium that they develop, the borrower offers the creditor either full repayment (if the firm has sufficient liquidity to do so) or the borrower defaults by offering a zero repayment (if the firm has insufficient liquidity to satisfy the full repayment). As a result, a default is uninformative (beyond informing the creditor that the borrower has insufficient cash to avoid a default) and thus the expected payoff from verifying will be positive (given appropriate parameter restrictions), which in turn implies that verification will be optimal ex post. While both papers derive debt as the optimal contract, they make very different predictions regarding repayment behavior and contract enforcement. In particular, while our setup predicts both absolute priority violations and strategic defaults, in their setup a default leads automatically to bankruptcy.

Although the extensive form considered in our paper and theirs are closely related, there are important differences. First, we do not require "time consistency" (we rule out interim renegotiation of contracts) and as a result stochastic monitoring is made possible. Second, we require the equilibrium payment function to be absolutely continuous in the underlying cash flow. This assumption means that we can employ differentiation techniques to solve our problem and, more importantly, it implies that pooling equilibria of the Krasa–Villamil type will be eliminated (since it is discontinuous in the point where the cash flow equals the debt obligation).

Krasa et al. (2008) extend Krasa and Villamil (2000) to study the effect of bankruptcy law on the incentives of firms to default. In their model, a default always leads to bankruptcy, and strategic defaults arise to the extent that bankruptcy law allows the debtor a positive payoff in formal bankruptcy. In our model, a default need not lead to bankruptcy, and strategic defaults arise from the possibility that the creditor may accept repayment offers from the borrower that are below the amount owed.

The rest of the paper is structured as follows. In Sect. 2, we present the model. Section 3 contains the results. Section 4 concludes the paper. All proofs are found in Appendix.

2 Model

There are two risk-neutral agents, an entrepreneur and an investor. The entrepreneur is endowed with a project that requires I units of funding to yield the cash flow x, which is stochastic with a strictly positive and differentiable density $h(.)$ defined on $X = [x_L, x_H]$. The entrepreneur has no funds on his own and hence must obtain I units of funding from the investor, who is operating in a perfectly competitive financial market. The risk-less interest rate is zero.

In return for providing I, the investor gets a claim on x. This claim is a function $f : X \to \Re$. After being funded, x is generated and is observed only by the entrepreneur. Upon observing x, the entrepreneur makes a take-it-or-leave-it payment offer r

to the investor.[4] The payment function $r(x)$ is a mapping $r : X \to \Re$ with the liquidity restriction $r \leq x$. We consider payment functions $r(x)$ that are deterministic and absolutely continuous (the role played by the latter assumption is discussed below).[5] The set of payment functions satisfying these criteria is denoted by **R**. The investor accepts or rejects the offer r based on his posterior beliefs $\hat{h}$. If the investor accepts, he receives r, and the entrepreneur gets the residual $x - r$. If the investor rejects the offer, and hence monitors, a verification cost c is incurred. Upon verification, the investor gets a payoff $f(x)$ according to the written contract. We let $f(x)$ be written on the net payoff of the investor. Since the entrepreneur does not have funds, c must be borne by the investor or by the firm (our results do not depend upon this). This implies the feasibility restriction $f(x) \leq x - c$, $\forall x \in X$. The set of contracts that satisfy this condition is denoted by **F**. Hence, upon verification, the payoff of the investor is $f(x)$ and the payoff of the entrepreneur is $x - f(x) - c$. Debt is defined by

$$f^D(x) = \min(x - c, d), \tag{1}$$

which gives the investor a claim to the minimum of the cash flow and a fixed payment d.[6]

The investor's accept probability function is a mapping $P{:}\Re \to [0, 1]$. To ensure sufficient liquidity to cover the monitoring cost, we assume that $c \leq x_L$. Finally, we assume that $x_L - c < I$ so that risk-less debt cannot be used to fund the project.

Let e be an indicator variable that takes the value zero if the investor accepts the borrower's repayment offer and one if the investor rejects/monitors. The payoff functions π_i, where $i = I, E$ are then given by,

$$\begin{aligned} \pi_E &= (1 - e)(x - r) + e(x - c - f) \\ \pi_I &= (1 - e)r + ef \end{aligned} \tag{2}$$

for, respectively, the entrepreneur and the investor. For a given $\langle r(x), P(r)\rangle$ the expected payoffs are given by,

[4] The restriction to take-it-or-leave-it offers is important because it rules out renegotiation once the true cash flow is revealed. A similar restriction is implicit in Gale and Hellwig (1989). An alternative would be to consider a sequential game of the type considered by Grossman and Perry (1986). They study a sequential bargaining game with an informed and an uninformed player in which the uninformed player's information improves over time as offers and counter-offers are made but in which the informed player's information is not fully revealed at the payment stage. A second alternative would be to allow for interim renegotiation of contracts as in Krasa and Villamil (2000), which would rule out stochastic monitoring.

[5] There are technical problems in defining mixed strategies for a continuous type space. Barring such problems, we conjecture that a mixed repayment strategy is not consistent with equilibrium (in contrast to in Persons 1997, who operates with a finite type space). The intuition is that a continuous X pins down a unique accept probability function $P(.)$, which in turn makes only one repayment offer optimal for given $\langle f(x), x\rangle$. Martimort and Stole (2001) make a similar observation in a different context.

[6] This definition of debt is not identical to that of the standard/simple debt contract (SDC) as used in the literature, since the SDC includes a clause that default will be followed by monitoring with probability one. The definition in (1) is wider as it makes no assumptions on the monitoring behavior of the investor.

$$E\pi_E = \int_X [P(r(x))(x-r) + (1-P(r(x)))(x-c-f)]\mathrm{d}H$$
$$E\pi_I = \int_X [P(r(x))r + (1-P(r(x)))f]\mathrm{d}H \tag{3}$$

The investor's participation constraint emerges from setting $E\pi_I \geq I$. The basic trade-offs are as follows. The entrepreneur makes a payment offer to the investor trading off the gains from diverting cash against higher expected verification costs. The investor follows a monitoring strategy that balances off the cost of monitoring against the possible gain from detecting a diversion attempt by the entrepreneur. We focus on perfect Bayesian equilibria (PBE) of the payment game. A tuple $\langle r(x), P(r), h, \hat{h}\rangle$ is a PBE if (a) $P(r)$ is optimal play by the investor given his posterior beliefs $\hat{h}$, (b) The entrepreneur anticipates the investor's behavior and chooses r to maximize his payoff, and (c) The investor's posterior beliefs are formed using Bayes' rule whenever possible.

The implementation problem can be formulated as,

$$\begin{aligned} &\text{Problem 1} \\ &\underset{\langle r(.),P(.)\rangle}{\text{Max}} \; E\pi_E \\ &\text{s.t. } E\pi_I \geq I \\ &r(x) \in \mathbf{R} \\ &f(x) \in \mathbf{F} \\ &\text{Strategies and beliefs are PBE} \end{aligned} \tag{4}$$

Problem 1 amounts to find the payment function and monitor probability function that maximize the expected utility of the entrepreneur given the incentive and feasibility constraints. Problem 1 is equivalent to finding a contract $f(x)$ that minimizes the expected monitoring (verification) cost $V = \int_X (1-P(.))cdH$ subject to the incentive and feasibility constraints.

3 Analysis

The main result of the paper is the following.

Theorem 1 *The optimal contract is debt. Under the optimal contract, the borrower's repayment function equals* $r^*(x) = \min[x-c, d]$ *and the creditor accepts offers along* $r^*(x)$ *with probability* $P^*(r) = \min[1, \exp[(r-d)/c]]$.

The optimal repayment function is given by $r^*(x) = \min[x-c, d]$, which can be implemented by issuing a debt contract $f^D(x) = \min(x-c, d)$. Under this contract, the borrower repays d if $x \geq d + c$ and defaults whenever $x < d + c$ by offering the creditor $x - c$. While defaults for $x \in [d, d + c)$ are purely strategic, defaults for $x < d$ are liquidity based, but also strategic since the borrower offers $x - c$ rather

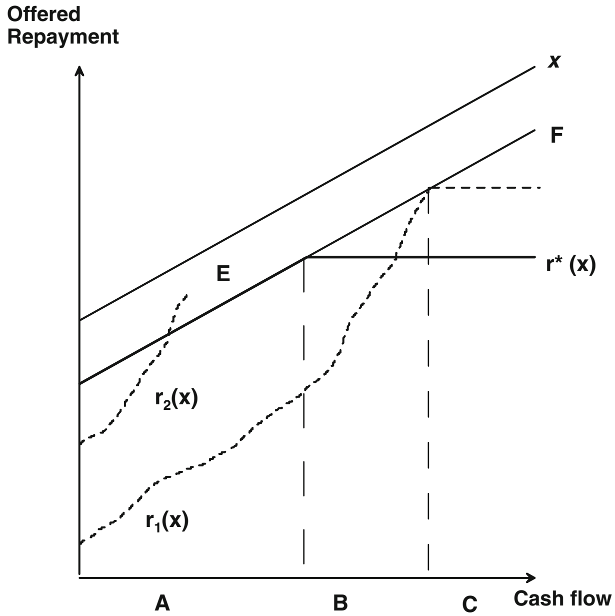

Fig. 1 The optimal repayment function $r^*(x)$ and alternative, non-optimal, repayment functions, $r_1(x)$ and $r_2(x)$, in terms of the realized cash flow x

than the amount x that the creditor is entitled to under his contract. In equilibrium, the creditor is indifferent between accepting and rejecting offers $r < d$. A default leads to bankruptcy with a probability, $1 - P(r)$, that is increasing in the size of the default.

The result that the creditor accepts offers below d with a positive probability implies a priority violation in the sense that it gives the borrower a positive payoff even though he fails to repay his debt in full. In other words, the optimal contract yields absolute priority violations as well as strategic defaults, both features of repayment behavior observed in real credit markets.

The intuition behind our result can be understood from Fig. 1. The figure depicts the realized cash flow x on the horizontal axis and the amount offered for repayment $r(x)$ on the vertical axis. The bold line depicts the optimal repayment function $r^*(x) = \min[d, x - c]$, which follows the feasibility barrier **F** for x in region **A** and gives a constant payout d in regions **B** and **C**. We will now argue that $r^*(x)$ must be better than alternative payment functions, such as $r_1(x)$, by having lower monitoring costs, where the constant payout associated with $r_1(x)$ is an amount $\hat{d}$ such that $\hat{d} > d$. Assume that $r^*(x)$ and $r_1(x)$ both satisfy the investor's participation constraint. First note that to induce any non-constant $r(x)$, the investor must be more likely to monitor the lower is the payment. At the level of the maximal payment, d and $\hat{d}$, the monitoring probability is zero.

Given these observations, let us compare the monitoring costs for $r^*(x)$ and for $r_1(x)$ in the regions **A**, **B**, **C**. In region **C**, the investor receives his maximal payout under both $r^*(x)$ and $r_1(x)$ and does not have incentives to monitor in either case.

In region **C**, therefore, $r^*(x)$ and $r_1(x)$ are equally good. In region **B**, $r^*(x)$ offers the maximal payout, and hence incurs no monitoring, while $r_1(x)$ pays less than its maximal payout and therefore must imply some monitoring by the investor (if not, the entrepreneur would never offer the maximal payout). Therefore, $r^*(x)$ yields lower monitoring costs compared to $r_1(x)$ in region **B**. This must also be the case in region **A**, because the size of the default (i.e., the difference between the actual payment offer and the face value of debt) is larger under $r_1(x)$ than under $r^*(x)$. Thus $r^*(x)$ dominates $r_1(x)$ in all regions **A**, **B**, and **C**, and must therefore yield lower monitoring costs than $r_1(x)$.

Now consider a payment scheme $r_2(x)$ that crosses the line $\mathbf{F} = x - c$ and enters the area **E**. Payments in **E**, however, are not feasible. If the entrepreneur pays $r_2(x)$, it would be strictly optimal for the investor not to monitor following repayment offers in **E** since by monitoring he gets at most $x - c$, while by accepting the payment offer he gets more. But then an equilibrium with $r_2(x)$ would unravel and therefore cannot exist.

Townsend (1979); Gale and Hellwig (1985), and Krasa and Villamil (2000) obtain debt as the optimal contract in settings related to ours. While we also obtain debt as the optimal contract, the predicted repayment behavior under the optimal contract differs markedly. In their setting, the borrower defaults only if $x < d$ and receives a zero payment in this case. In our case, the borrower defaults whenever $x < d + c$, and gets a positive expected payoff even after default. This difference in repayment behavior is mirrored by the differences in monitoring strategy by the investor: in their setting the investor monitors with probability one whenever $r < d$, while in our setting the investor is lenient by monitoring with a probability that is less than one upon default.

Example 1 Let $c = 1$ and let x be uniformly distributed on $[x_L, x_H] = [1, 2]$.

The debt contract is given by $f^D(x) = \min(x - c, d)$. This contract implies that the entrepreneur offers to pay $r^*(x) = \min(x - c, d)$. The creditor monitors according to $P(r^*(x)) = \min[1, \exp[(x - d - c)/c]]$. Given $r^*(x)$, the investor's participation constraint simplifies to $\int_{x_L}^{d+c}(x - c)\mathrm{d}H + \mathrm{d}\int_{d+c}^{x_H}\mathrm{d}H = I$. Substituting in for $c = 1$, $x_L = 1$, and $x_H = 2$, and solving with respect to d gives $d = 1 - \sqrt{1 - 2I}$. The maximum fundable amount is obtained for $d = x_H - c = 1$, in which case the investor's payoff becomes $\int_X (x - c)\mathrm{d}H = \int_1^2 (x - 1)\mathrm{d}x = 1/2$, and hence any $I \in [0, 1/2]$ is obtainable from the investor.

A key assumption of our analysis is that $r(x)$ is absolutely continuous. As shown by Krasa and Villamil (2000) in a related setting, under discontinuous $r(.)$ there can exist pooling equilibria of the Townsend (1979) type, where the entrepreneur plays $r = d$ when $x \geq d$ and $r = 0$ when $x < d$. The creditor accepts $r = d$ with probability one and $r < d$ with probability zero. Strategic defaults, therefore, do not occur.[7] The assumption that $r(x)$ is absolutely continuous eliminates this pooling equilibrium,

[7] The pooling equilibrium is supported by the creditor having "optimistic" off-equilibrium-path beliefs about deviating entrepreneur types. For example, an offer $\tilde{D} - \varepsilon$, where ε is small, will be rejected due to beliefs assigning this offer to types with sufficient funds to pay D in full. There is nothing in the definition of a PBE that excludes these beliefs since only $\tilde{D} = D$ and $\tilde{D} = 0$ are observed on the equilibrium path.

since $r(x)$ then is discontinuous at $x = d$. We next show with an example that $r^*(x)$ is not necessarily optimal if we allow for discontinuous $r(x)$.

Consider the example again, but now allow for discontinuous $r(x)$. Let the contract be given by $f^P(x) = \min(x, m)$, where m is the face value of debt, and consider a possible pooling PBE where the entrepreneur plays $r^P(x) = m$ if $x \geq m$ and 0 otherwise.[8] The investor monitors if the entrepreneur defaults. Given $r^P(x)$, the investor's participation constraint simplifies to $\int_{x_L}^{m}(x - c)\mathrm{d}H + \int_{m}^{x_H} m\mathrm{d}H = I$. Solving with respect to m and substituting in for $c = 1$, $x_L = 1$, and $x_H = 2$ gives $m = 1 - \sqrt{2 - 2I}$. The expected verification cost equals 0 if $m \leq 1$ (since a default never occurs) and $\int_{x_L}^{m} c\mathrm{d}H = \int_{1}^{m} 1\mathrm{d}H = m - 1$ if $m > 1$. In the separating equilibrium, the maximum fundable amount is $\frac{1}{2}$, and the expected verification cost is positive for any raised amount. The pooling equilibrium, on the other hand, has a zero expected verification cost for amounts raised equal to $1/2$ or less. Clearly, therefore, the pooling equilibrium dominates the separating equilibrium.

4 Conclusion

Townsend (1979); Gale and Hellwig (1985), and others derive standard debt as the optimal contract under costly state verification, relying on a restriction to deterministic verification and on the ability of the investor to commit to the optimal ex ante verification rule. In contrast, we consider a setting without commitment, and allow for stochastic verification. The optimal contract is still debt. Under the optimal contract, the borrower defaults strategically and the creditor is lenient towards defaults by accepting offers from the borrower below the full debt payment with a positive probability. This is unlike the standard debt contract derived by Townsend (1979), Gale and Hellwig (1985), and Krasa and Villamil (2000) which does not distinguish between default and bankruptcy and hence rules out strategic defaults and priority violations.

Our result that the optimal contract implies strategic defaults and absolute priority violations is consistent with the empirical corporate finance literature on repayment behavior of debt in real financial markets. It would be of interest to see whether our closed-form solution for the relation between cash flow, defaults, and bankruptcy probability could be embedded in a structural econometric model.

Appendix

We prove Theorem 1 in several steps. Since the investor cannot precommit to a monitoring strategy, the revelation principle does not apply, and we need to use a more indirect method of proof.[9] In particular, we first solve a simplified version of Problem 1, and then show that the solution to this simplified problem also solves Problem 1.

[8] To construct beliefs supporting the equilibrium is straightforward and omitted.

[9] Bester and Strausz (2001) show that a modified version of the revelation principle holds under limited commitment. Since we operate in a setting with a continuous type space, their results do not immediately apply.

Denote the class of payment functions satisfying $r(x) \leq x - c$ by **M**, and define Problem 1′ as Problem 1 except that $r(x) \in \mathbf{R}$ in the third line of (4) is replaced by $r(x) \in \mathbf{M}$. We start out by solving Problem 1′ in Lemmas 1–2, and then we solve Problem 1 in Lemma 3.

Fix arbitrary $r(x) \in \mathbf{R}$ and denote the set of offers made with positive density under $r(x)$ by R. Denote the minimal element in R by $r_{\min}$, and the maximal element by $r_{\max}$. By the assumption that a constant payout on X will not satisfy the investor's participation constraint we can confine attention to $r(x)$ with $r_{\max} > r_{\min}$. By the continuity of $r(x)$ and since $h(x)$ has positive density everywhere on X, it follows that $R = [r_{\min}, r_{\max}]$. Define the corresponding open set by $R_I = (r_{\min}, r_{\max})$. Further, let $\Gamma(f)$ be the set of PBE induced by a contract $f(x)$. We say that the contract $f(x)$ *induces* (implements) $r(x)$ if $r(x)$ is contained in $\Gamma(f)$. For an arbitrary set $S \subset X$, denote the complement set by S^c, i.e., $S^c = X/S$.

Lemma 1 *For any inducible $r(x) \in \mathbf{M}$: (i) $f^*(x) = r(x)$ induces $r(x)$. (ii) The associated monitoring probability is $P(r) = \exp[(r - r_{\max})/c]$. (iii) $f^*(x)$ is the unique contract that induces $r(x)$.*

Proof (i) and (ii). Fix $r(x) \in \mathbf{M}$ and suppose that the contract is $f^*(x)$. Given that the entrepreneur adheres to $r(x)$, the investor is indifferent between monitoring and not monitoring, since he gets $r(x)$ in both cases. Any accept probability function $P(r)$ is therefore consistent with optimal play by the investor (for investor posterior belief $\hat{h}$ appropriately defined). We need to find the $P(r)$ that makes adhering to $r(x)$ incentive compatible for the entrepreneur.

The expected payoff for the entrepreneur from offering r equals,

$$U_E(r) = P(r)(x - r) + (1 - P(r))(x - f^*(x) - c) \tag{5}$$

The first term is the entrepreneur's payoff if the investor accepts the offer r, and the second term is his payoff if the investor rejects the offer. Assuming that $P(r)$ is differentiable,

$$\begin{aligned} U'_E(r) &= P'(r)(x - r) - P(r) - P'(r)(x - f^*(x) - c) \\ &= P'(r)(f^*(x) + c - r) - P(r) \end{aligned} \tag{6}$$

Given an interior solution, the optimal offer r solves the first order condition obtained from setting (6) equal to zero. We now find the $P(r)$ that makes $r(x)$ optimal play by the entrepreneur under $f^*(x)$. Substitute in for $f^*(x) = r(x)$ in (6). Adhering to $r(x)$, i.e., setting $r = r(x)$, is optimal for the entrepreneur if,

$$P'(r)c - P(r) = 0 \tag{7}$$

The unique solution to this differential equation (barring the trivial solution $P(r) = 0$) is $P(r) = K \exp(r/c)$, where K is an integration constant. Invoking the boundary condition $P(r_{\max}) = 1$, i.e., that the investor accepts the maximal offer with probability 1, pins down K. Solving for K and substituting back in, we get

$$P(r) = \exp[(r - r_{\max})/c] \tag{8}$$

as stated in part (ii) of the lemma. $P(r)$ lies between 0 and 1 and is increasing and convex in r.

To show that $r(x)$ maximizes the entrepreneur's utility under $\langle f^*(x), P(r)\rangle$, observe first that $P'(r) = P(r)/c$. Substituting into the entrepreneur's marginal utility in (6),

$$\begin{aligned} U'_E(r) &= P'(r)(f^*(x) + c - r) - P(r) \\ &= P(r)[f^*(x) - r]/c \\ &= P(r)[r(x) - r]/c \end{aligned} \tag{9}$$

This expression is negative for $r > r(x)$ and positive for $r < r(x)$. Hence, adhering to $r(x)$ is a global maximum for the entrepreneur under $\langle f^*(x), P(r)\rangle$.

We now construct supporting beliefs. The prior of the investor is that x follows $h(x)$, and he forms posterior beliefs after observing an offer r. For an arbitrary offer $\bar{r} \in R$, let $X(\bar{r}) = \{x : r(x) = \bar{r}\}$, i.e., the values of x consistent with $\bar{r}$ given that the entrepreneur adheres to $r(x)$. The investor's posterior beliefs are then $\hat{h}(x) = \frac{h(x)}{\int_{x \in X(\bar{r})} h(x)}$ for $x \in X(\bar{r})$ and zero elsewhere. These posterior beliefs are clearly consistent with the entrepreneur's strategy. For an off-equilibrium path offer, we do not need to restrict the investor beliefs; since $f^*(x) = r(x)$, monitoring will give at least $r_{\min}$ and at most $r_{\max}$ in net payoff for the investor, and it will be optimal for the investor to accept $r \geq r_{\max}$ and optimal to reject $r < r_{\min}$.

To complete the proof of (i) and (ii), we show that $P(r)$ must be continuous (and hence differentiable almost everywhere). Let the contract be $f(x) \in \mathbf{F}$ and consider $\hat{x} \in X$. Denote $r(\hat{x})$ by $\hat{r}$ and suppose that $P(r)$ jumps upwards in the point $\hat{r}$ (if $P(r)$ jumps downwards, the proof is analogous), so that $\lim_{r \to \hat{r}^-} P(r) < \lim_{r \to \hat{r}^+} P(r)$. Denote $\lim_{r \to \hat{r}^-} P(r)$ by P^- by and $\lim_{r \to \hat{r}^+} P(r)$ by P^+. If $\hat{x}$ is realized, sticking to $r(x)$ would give expected payoff $U_E(\hat{r}) = P^-(\hat{x} - \hat{r}) + (1 - P^-)(\hat{x} - f(\hat{x}) - c)$. Deviating by offering $\hat{r} + \varepsilon$, where $\varepsilon > 0$, would give expected payoff equal to $U_E(\hat{r} + \varepsilon) = P(\hat{r} + \varepsilon)(\hat{x} - \hat{r} - \varepsilon) + (1 - P^+)(\hat{x} - f(\hat{x}) - c)$. In the limit, as ε goes to zero from above, the utility from deviating approaches $P^+(\hat{x} - \hat{r}) + (1 - P^+)(\hat{x} - f(\hat{x}) - c)$. Simplifying, to stick with $\hat{r}$ is incentive compatible only if $\hat{x} - \hat{r} \leq \hat{x} - f(\hat{x}) - c$, or in other words if $\hat{r} \geq f(\hat{x}) + c$. If $\hat{r} \geq f(\hat{x}) + c$ then the investor is offered more than he gets from monitoring, and we must have that $P^- = 1$. But that is inconsistent with $P^- < P^+ \leq 1$. Hence $r(x)$ cannot be incentive compatible unless $P(r)$ is continuous. [10]

[10] Left-continuity of $P(r)$ in the point $\hat{r}$ ensures that there exists $\delta > 0$ such that deviations from $r(x)$ will be profitable for x on the interval $[\hat{x} - \delta, \hat{x}]$.

We now show (iii) that only $f^*(x)$ induces $r(x)$ for $r \in R_I$. In Steps 1–2 we prove (iii) under the assumption $P'(r) > 0$. In Step 3 we prove (iii) under the assumption that $P'(r) \leq 0$ for some values of r.

Step 1. Denote a candidate contract inducing $r(x)$ by $\hat{f}(x) \in \mathbf{F}$. First note that since R is an interval, $\hat{f}(x)$ must induce stochastic monitoring for arbitrary $\bar{r} \in R_I$. Suppose, on the contrary, that it is optimal for the investor to accept $\bar{r}$ with probability 1. But that contradicts the assumption $P'(r) > 0$ since $\bar{r} < r_{\max}$. Suppose, on the other hand, that it is optimal for the investor to accept $\bar{r}$ with probability 0. But since $\bar{r} > r_{\min}$ and $P(r_{\min}) \geq 0$ that also contradicts the assumption $P'(r) > 0$.

Step 2. Analogous to (6), the entrepreneur's incentive compatibility constraint under $\hat{f}(x)$ equals,

$$U_E'(r) = P'(r)(\hat{f}(x) + c - r) - P(r) = 0 \tag{10}$$

For $\bar{r} \in R_I$, let $X(\bar{r})$ be the values of x that gives $\bar{r}$ as the solution to (10). $X(\bar{r})$ is non-empty by construction. Equation (10) implies that for any pair $x_1, x_2 \in X(\bar{r})$, then $\hat{f}(x_1) = \hat{f}(x_2)$. For stochastic monitoring to occur we must therefore have that,

$$\bar{r} = \hat{f}(x), \quad x \in X(\bar{r}) \tag{11}$$

On the left hand side is what the investor gets if he accepts the offer $\bar{r}$, and on the right hand side is what he gets if he monitors. But (11) implies immediately that $\hat{f}(x)$ cannot induce $r(x)$ unless $\hat{f}(x) = f^*(x)$.

Step 3. Now consider the case where $P'(r) \leq 0$ for some values of r. We first exclude the case $P'(r) < 0$. Suppose that there exists $\bar{r} \in R_I$ such that $P'(\bar{r}) < 0$. Then it follows that $\bar{r} > \hat{f}(x)+c$ for (6) to hold with equality. If $r > \hat{f}(x)+c$ then the investor is offered more than he gets from monitoring, and we must have that $P(r) = 1$. But by the continuity of $P(r)$, then $P'(r)$ must be negative on an interval around $\bar{r}$. This is inconsistent with the requirement $P(r) = 1$ for any $P'(r) < 0$. Thus it follows that $P'(r)$ cannot be strictly negative (on any interval). Next suppose that there exists $\bar{r} \in R_I$ such that $P'(\bar{r}) = 0$. Then it follows that $P(r) = 0$ for (6) to hold with equality. Since $P'(r)$ cannot be negative, the values of r where $P(r) = 0$ must be an interval starting at $r_{\min}$. Denote the upper endpoint of this interval for r_u. Since $P(r) > 0$ for $r \in (r_u, r_{\max}]$ we must have that $P'(r) > 0$ for $(r_u, r_{\max}]$. It follows that $P(r) \in (0, 1)$ for $r \in (r_u, r_{\max})$, i.e., monitoring is stochastic. For stochastic monitoring to take place on $(r_u, r_{\max}]$, we must have that $\hat{f}(x) = r(x)$ for the underlying values of x (if this condition does not hold, the investor would either strictly prefer to accept the offer, or strictly prefer to reject the offer), and hence $\hat{f}(x) = f^*(x)$. By part (ii), this implies that $P(r) = \exp[(r - r_{\max})/c]$ for offers on $(r_u, r_{\max}]$. But since $\exp[(r - r_{\max})/c]$ is bounded away from zero this is inconsistent with $P(r_u) = 0$ unless $P(r)$ is discontinuous in the point r_u. But we showed above that $P(r)$ must be continuous to be part of an equilibrium. □

Equipped with Lemma 1 we can replace Problem 1′ with Problem 1″.

Problem 1″

$$\underset{\langle r(x)\rangle}{\text{Max}} \int \exp\left[\frac{r(x) - r_{\max}}{c}\right] \mathrm{d}H$$

s.t. $E\pi_I \geq I$

$r(x) \in \mathbf{M}$

$f(x) \in \mathbf{F}$

Strategies and beliefs are PBE

To obtain Problem 1″ from Problem 1′, note first that maximizing the acceptance probability is equivalent to maximizing the objective of Problem 1′. We have substituted in for $P(r) = \exp[\frac{r - r_{\max}}{c}]$ by Lemma 1, and since Lemma 1 enables us to map $r(x)$ into $P(r)$, we now maximize over only $r(x)$ instead of over $\langle r(x), P(r)\rangle$. Define $X_A = [x_L, d + c)$ and $X_B = [d + c, x_H]$.

Lemma 2 (i) *The solution to Problem* 1″ *is* $r^*(x)$*, where*

$$r^*(x) = \begin{cases} x - c & \text{if} \quad x \in X_A \\ d & \text{if} \quad x \in X_B \end{cases} \tag{12}$$

and d is determined from the investor's participation constraint.

(ii) *The debt contract* $f^D(x)$ *is the unique contract that induces* $r^*(x)$.

(iii) *The investor's participation constraint is binding.*

Proof From Lemma 1, part (iii), we know that when implementing arbitrary $r(x) \in \mathbf{M}$, we can restrict attention to the contract $f^*(x)$. From Lemma 1, part (ii) we know that the associated monitoring function is $\exp\left[\frac{r(x) - r_{\max}}{c}\right]$. Since the investor on the equilibrium path is indifferent between accepting and rejecting all offers on R, investor utility is simply $\int_X r(x)\mathrm{d}H$. The expected verification cost equals $\int_X c[1 - P(r(x))]\mathrm{d}H$. Let $\hat{r}(x) \in \mathbf{M}$ be an arbitrary alternative repayment function that raises the same amount as $r^*(x)$, i.e., $\int_X \hat{r}(x)\mathrm{d}H = \int_X r^*(x)\mathrm{d}H$. Let $\hat{V}$ be the expected monitoring cost of $\hat{r}(x)$ and V^* be the expected monitoring cost of $r^*(x)$. To prove (i) that $r^*(x)$ solves Problem1″, we first prove that $\hat{V} \geq V^*$ and then show that the inequality must be strict.

(i) Denote the expected monitoring cost of $r^*(x)$ on X_A (X_B) by V_A^* (V_B^*) and the expected monitoring cost of $\hat{r}(x)$ on X_A (X_B) by $\hat{V}_A$ ($\hat{V}_B$). By definition, $\hat{V}_A + \hat{V}_B = \hat{V}$ and $V_A^* + V_B^* = V^*$. Note that $r^*(x) \geq \hat{r}(x)$ on X_A since $r^*(x)$ follows the upper barrier in $\mathbf{M}$. Since $r^*(x)$ is flat on X_B it follows that $\hat{r}_{\max} \geq r^*_{\max}$. If this condition does not hold, $\hat{r}(x)$ and $r^*(x)$ cannot raise the same amount. For an arbitrary $r(x)$ we have $P(r(x)) = \exp\left[\frac{r(x) - r_{\max}}{c}\right]$. It follows immediately that $\hat{V}_B \geq V_B^*(= 0)$ and $\hat{V}_A \geq V_A^*$. Therefore $\hat{V} \geq V^*$.

To show that $\hat{V} > V^*$, first suppose that $\hat{V}_B = V_B^*(= 0)$. In this case, $\hat{r}(x) = \hat{r}_{\max}$ for $x \in X_B$ by Lemma 1, part (ii). For $r^*(x)$ and $\hat{r}(x)$ to be non-identical, $r^*(x) > \hat{r}(x)$

for at least one interval $X' \in X_A$. But in this case $\hat{r}(x)$ must raise less than $r^*(x)$ on X_A. It follows that $\hat{r}(x)$ must raise more than $r^*(x)$ on X_B, and hence that $\hat{r}_{\max} > r^*_{\max}$. But in this case it follows from Lemma 1, part (ii), that $\hat{V}_A > V^*_A$, and hence that $\hat{V} > V^*$. Now suppose that $\hat{V}_B > V^*_B$. In this case, $\hat{r}_{\max} > r^*_{\max}$. Since $r^*(x) \geq \hat{r}(x)$ it follows from Lemma 1, part (ii), that $\hat{V}_A \geq V^*_A$. Therefore $\hat{V} > V^*$.

Part (ii), that $f^D(x)$, as defined in Eq. (1), is the unique contract that induces $r^*(x)$, follows directly from Lemma 1, part (iii). To prove (iii), note that if $E\pi_I > I$ then we can decrease d in (12) and still make the investor willing to participate. But this would enable a decrease of $r_{\max}$ and hence decrease expected verification costs, since $P(r)$ decreases in $r_{\max}$ by Lemma 1, part (ii). Hence $E\pi_I = I$ under the solution.

□

We have proved that $r^*(x)$ is optimal in $\mathbf{M}$ and that the debt contract $f^D(x)$ is unique in inducing $r^*(x)$. We have therefore proven Theorem 1 under the limitation $r(x) \in \mathbf{M}$. Defining $\mathbf{M}^c = \mathbf{R}/\mathbf{M}$, we complete the proof of Theorem 1 by proving that,

Lemma 3 *Any inducible $r(x) \in \mathbf{M}^c$ must be dominated by $r^*(x)$.*

Proof Denote a candidate payment function in $\mathbf{M}^c$ by $\hat{r}(x)$. The strategy of the proof is to first show that $r^*(x)$ has lower expected monitoring cost than any weakly increasing $\hat{r}(x)$ that raises the same amount. We then show that $\hat{r}(x)$ that is not weakly increasing cannot be optimal.

Part A: $\hat{r}(x)$ weakly increasing.

Step 1. Define $X_F = \{x : \hat{r}(x) > x - c\}$. Since $r(x) \in \mathbf{M}^c$, then X_F is non-empty. If $\hat{r}'(x) > 0$ in a region on X_F then the investor would strictly prefer to accept the corresponding offers, and the equilibrium would unravel. Therefore $\hat{r}(x)$ must be flat on X_F. By the continuity of $\hat{r}(x)$, X_F must be an interval that starts out at x_L.

Step 2. Denote the constant offer made on X_F by q and suppose that q is played on X', where $X_F \subset X'$. Since $\hat{r}(x)$ is weakly increasing, X' must be an interval. Denote the maximal element in X' by t, so that $X' = [x_L, t]$. Then, by the same type of dominance argument as in Lemma 2, for $\hat{r}(x)$ to not be dominated it must be a continuous approximation to (the role of continuity is explained in Step 10),

$$\hat{\rho}(x) = \begin{cases} q & \text{if } x \in [x_L, t] \\ x - c & \text{if } x \in [t, k + c] \\ k & \text{if } x \in [k + c, x_H] \end{cases} \tag{13}$$

where k is set so that the investor's participation constraint is satisfied. It follows directly from Lemma 1, part (ii), that $\hat{P}(r) = \exp\left[\hat{r}(x) - \hat{r}_{\max} c\right]$.[11]

Step 3. We now prove that for $\hat{r}(x)$ as described in Step 2 not to be dominated, it must induce stochastic monitoring following the offer q. If it is strictly optimal for the investor to monitor following q, then $\hat{r}(x)$ must raise less than

[11] This is where the requirement that $\hat{r}(x)$ is continuous bites. See Step 10.

$r^*(x)$ on X'. The reason is that $\hat{r}(x)$ can at most raise $\int_{X'}(x-c)\mathrm{d}H$ on X' (which is obtained for $f(x) = x - c$, as shown in Step 5), while $r^*(x)$ raises $\int_{X'}(x-c)\mathrm{d}H$ on X'. Since $\hat{r}_{\max} \geq r^*_{\max}$, the expected monitoring cost of $\hat{r}(x)$ would be strictly higher than $r^*(x)$ on both X' and X'^c. If it is strictly optimal for the investor to accept q, this either violates the condition $\hat{P}'(r) > 0$ or the investor's participation constraint, and hence $\hat{r}(x)$ would be dominated.[12]

Step 4. Since $\hat{r}(x)$ must induce the investor to monitor q stochastically,

$$q = \int_{X'} f(x)\mathrm{d}H/H(t) \tag{14}$$

On the left hand side is what the investor gets if he accepts the offer q, and on the right hand side is the expected payoff if he monitors. For any choice of contract $f(x)$, equation (14) generates a function $q(t, k)$.

Step 5. We have shown that for $\hat{r}(x)$ to not be dominated, it must have $\hat{r}(x) = q$ on X' and follow $r^*(x)$ on X'^c. We now show that for $\hat{r}(x)$ to not be dominated, it must have $f(x) = x - c$ on X'. Fix t. Observe that for any $q \in [0, \int_{X'}(x-c)\mathrm{d}H/H(t)]$ there exists a contract $f(x)$ that induces q. To induce $q = 0$, set $f(x) = 0$ on X'. To induce $q = \int_{X'}(x-c)\mathrm{d}H/H(t)$, set $f(x) = x - c$. Intermediate values of q can be induced by choosing intermediate $f(x)$. But to minimize the monitoring probability on X', we should maximize q on X'.[13] From (14) it follows that to maximize q on X', we maximize $\int_{X'} f(x)\mathrm{d}H$ with respect to $f(x)$. This implies the solution $f(x) = x - c$ on X'.

Step 6. Substitute $f(x) = x - c$ back into (14). We then have that for $\hat{r}(x)$ not to be dominated, it must satisfy

$$q = \int_{X'} (x-c)\mathrm{d}H/H(t) \tag{15}$$

Note that $\int_{X'} x\mathrm{d}H/H(t) = E(x|x \in X')$, where $E(x|x \in X')$ is the conditional mean of x on X'. Equation (15) implies that $E(x|x \in X') = q + c$, a fact that will be used in Step 9.

Step 7. Since $q = \int_{X'}(x-c)\mathrm{d}H/H(t) = \int_{X'} r^*(x)\mathrm{d}H/H(t)$, it follows that $\hat{r}(x)$ and $r^*(x)$ gives the same investor payoff on X'. For $\hat{r}(x)$ and $r^*(x)$ to give the same investor payoff overall, they must also give identical payoff on X'^c. It follows from Step 2 that $k = d$ and that $\hat{r}(x) = r^*(x)$ for $x \in X'^c$. By Lemma 1, the expected monitoring cost of $\hat{r}(x)$ and $r^*(x)$ are therefore the same on X'^c. It is therefore necessary and sufficient to show that $r^*(x)$ has

[12] To see why, recall the assumption that the function $r(x) = q$ for $x \in X$ does not satisfy the investor's participation constraint. Therefore, $\hat{r}(x)$ with constant payout q for $x \in X'$ must have an average payout for $x \in X'^c$ that is higher than q. But if $P(q) = 1$, the manager would offer q also for $x \in X'^c$. But then the investor's participation constraint would not be satisfied.

[13] Maximizing q on X' minimizes the expected verification cost on X', since $P(q)$ will be maximized. Maximizing q on X' also minimizes the expected verification cost on X'_c, since $\hat{r}_{\max}$ is minimized.

a lower monitoring cost than $\hat{r}(x)$ on X'. This is equivalent to showing that $r^*(x)$ has a higher average accept probability than $\hat{r}(x)$ on X'. This follows from the convexity of $P^*(r)$ and is shown in Steps 8–9.

Step 8. The average accept probability for $r^*(x)$ on X' equals $\int_{X'} P^*(x)\mathrm{d}H/H(t)$, where $P^*(x) = \exp\left[\frac{x-c-r^*_{\max}}{c}\right]$. Since $\hat{r}(x)$ has a constant payout on X', its average accept probability simply equals $\hat{P}(q) = \exp\left[\frac{q-r^*_{\max}}{c}\right]$. We therefore need to show that

$$\exp\left[\frac{q-r^*_{\max}}{c}\right] < \int_{X'} \exp\left[\frac{x-c-r^*_{\max}}{c}\right]\mathrm{d}H/H(t) \tag{16}$$

Step 9. We now prove that (16) holds. Fix $E(x|x \in X')$ and observe by (15) that q is independent of $h(x)$, i.e., q is constant across mean-preserving shifts of $h(x)$. Since $P^*(x)$ is convex in x, the right-hand side of (16) is minimized by minimizing "risk", that is by putting an atom of the size $H(t)$ at the point $x = q + c$. In this case, the right-hand side of (16) reduces to simply $\exp[\frac{q-r^*_{\max}}{c}]$, and the left-hand side and right-hand side of (16) are equal. But since $h(x)$ has full support, the right-hand side of (16) must be larger than $\exp[\frac{q-r^*_{\max}}{c}]$. We have therefore shown that $r^*(x)$ must strictly dominate any $\hat{r}(x)$.

Step 10. In Step 2 we assumed that $\hat{r}(x)$ is a continuous approximation to $\hat{\rho}(x)$. $\hat{\rho}(x)$ will be discontinuous for $t = q + c$, and is therefore not contained in $\mathbf{R}$. The continuous approximation of $\hat{\rho}(x)$ we have in mind is of the following form,

$$\hat{r}(x) = \begin{cases} q & \text{if } x \in [x_L, t] \\ v(x,\varepsilon) & \text{if } x \in [t, t+\varepsilon] \\ x-c & \text{if } x \in [t+\varepsilon, k+c] \\ k & \text{if } x \in [k+c, x_H], \end{cases} \tag{17}$$

where $v(x;\varepsilon)$ is any continuous and strictly increasing function that connects the points (t, q) and $(t+\varepsilon, x-c)$. For $\varepsilon > 0$, $\hat{r}(x)$ is clearly contained in $\mathbf{R}$, and by letting ε become small, the expected verification cost on the interval $[t, t+\varepsilon]$ will become arbitrarily small.

Comment 1: One may wonder why it is important that $\hat{r}(x)$ is continuous. Steps 2 and 8 apply the fact that for continuous $\hat{r}(x)$ then incentive compatibility implies that $\hat{P}(q) = P^*(q)$. Without requiring continuity of $\hat{r}(x)$, then $\hat{P}(r)$ can be constructed without having to make payouts on the interval $[v(t), v(t+\varepsilon)]$ incentive compatible for the entrepreneur. Hence a higher accept probability for the offer q would be inducible under $\hat{\rho}(x)$ than under $\hat{r}(x)$.

Comment 2: Recall the example from Sect. 4 where the (discontinuous) payment function $r^P(x)$, where $r^P(x) = m$ if $x \geq m$ and 0 otherwise, were shown to dominate $r^*(x)$. A question is whether there could exist a continuous approximation to r^P that beats $r^*(x)$. To see why such an approximation is ruled out, note that since $r^P(x) = x$

for $x = m$, such approximations would imply a region where $r(x) > x - c$ and $r'(x) > 0$. This is ruled out in Step 1.

Part B: $\hat{r}(x)$ not weakly increasing.

First, $\hat{r}(x)$ that is monotonically decreasing cannot generate enough payoff to the investor to satisfy the participation constraint. Second, $\hat{r}(x)$ that is constant on X_F and non-monotonic on X_F^c would be eliminated by the same type of dominance argument as in Lemma 1, and cannot be optimal. Let us therefore consider $\hat{r}(x)$ that is non-constant on X_F and show that it is not inducible. The argument of proof is similar to that in Lemma 1, part (iii).

Step 1. Recall that $X_F = \{x : \hat{r}(x) > x - c\}$. First observe that $\hat{r}(x)$ must induce the investor to monitor stochastically on X_F: if it is strictly optimal for the investor to accept then $\hat{r}(x)$ cannot be optimal (by the same argument as in Part A, Step 3) and if it is strictly optimal for the investor to monitor then $\hat{r}(x)$ cannot beat $r^*(x)$.

Step 2. Now pick an arbitrary value in R_I and denote it by $\bar{r}$. Assume for simplicity that there is only one value of x on X_F that gives the payout $\bar{r}$ (the proof easily extends, as in the proof of Lemma 1) and denote the corresponding value of x by x_1. Since $\bar{r} > x_1 - c$ and the investor is required to be indifferent between accepting and rejecting $\bar{r}$, there must exist $x \in X_F$ such that $\hat{r}(x) = \bar{r}$. Assume for simplicity that $\hat{r}(x) = \bar{r}$ holds only for one value of $x \in X_F^c$ (again the proof easily extends) and denote this value by x_2. For $\bar{r}$ to be incentive compatible for the entrepreneur, we must by equation (6) have that,

$$U_E'(\bar{r}) = P'(r)(f(x) + c - \bar{r}) - P = 0; \quad x = x_1, x_2 \tag{18}$$

But then it follows trivially that $f(x_1) = f(x_2)$.

Step 3. For stochastic monitoring following $\bar{r}$ to be incentive compatible for the entrepreneur, we must therefore have that

$$E[f(x)|r = \bar{r}] = \bar{r} \tag{19}$$

On the left-hand side is the entrepreneur's expected payoff if he verifies and on the right-hand side is the entrepreneur's payoff if he accepts. Since $f(x_1) = f(x_2)$, this implies that,

$$f(x_1) = \bar{r} \tag{20}$$

But since $f(x_1) \leq x_1 - c$ this contradicts the assumption that $\bar{r} > x_1 - c$. Thus $\hat{r}(x)$ is not inducible. □

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