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RESEARCH ARTICLE

Theory of negative consumption externalities with applications to the economics of happiness

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Abstract This paper investigates the problem of obtaining Pareto efficient allocations in the presence of negative consumption externalities. In contrast to the conventional wisdom, we show that even if consumers' preferences are monotonically increasing in their own consumption, one may have to dispose of resources to achieve Pareto efficiency when negative consumption externalities exist. We provide characterization results on destruction both for pure exchange economies and for production economies. As an application, our results provide an explanation to Easterlin's paradox: average happiness levels do not increase as countries grow wealthier.

Keywords Negative consumption externalities · Pareto efficiency · Happiness economics

JEL Classification D61 · D62 · H23

1 Introduction

Does an increase in gross domestic product necessarily improve people's happiness? More specifically, should resources be completely exhausted to achieve Pareto

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efficiency in an economy with externalities even though consumers' preferences are strictly increasing in their own consumption of goods? Most economists would say yes. Indeed, most standard textbooks such as Laffont (1988); Varian (1992), and Salanie (2000) offer a positive answer by providing characterizations for economies with positive consumption externalities. However, this paper shows that as long as there are negative consumption externalities the answer may be different, and one may have to destroy some resources to achieve Pareto efficient allocations. Neither an institution nor a rule can mitigate the destruction of resources to achieve efficient allocations without changing the economic environment such as the endowments of the other goods. As an application of our results to the economics of happiness, we give a formal economic explanation for Easterlin's empirical findings: an increase in income may not increase the happiness of a human being (Easterlin 1974, 1995, 2000, 2001, 2003, 2005).

Pareto efficiency and externalities are two important concepts in economics. Pareto efficiency is a highly desirable property to achieve when allocating resources. The importance and wide use of Pareto efficiency lies in its ability to offer us a minimal and uncontroversial test, which any social optimal economic outcome should pass. Implicit in every Pareto efficient outcome is the condition that all possible improvements to society have been exhausted. In addition, Pareto efficiency conveniently separates economic efficiency from the more controversial (and political) questions regarding the ideal distribution of wealth across individuals.

The most important result obtained in general equilibrium theory is the first fundamental theorem of welfare economics. It is a formal expression of Adam Smith's claim of the existence of an "invisible hand" at work in markets. The first welfare theorem provides a set of conditions under which a market economy will achieve a Pareto optimal outcome. Thus, any inefficiency which arises in a market economy, and hence any role for Pareto-improving market intervention, must be traceable to a violation of at least one assumption in the first welfare theorem.

One typical violation of these assumptions is the existence of externalities in an economy. This violation leads to nonPareto optimal outcomes and market failures. Various suggestions have been proposed in the economic literature to allocate resources in alternative ways in order to obtain Pareto efficiency. The conventional wisdom is that resources must be exhausted to reach a Pareto efficient allocation, provided that commodities are divisible and the preferences of individuals are strongly monotone. As a result, the destruction of resources in order to achieve Pareto efficient allocations is unwarranted and, in fact, counterintuitive. However, in the presence of consumption externalities, this conventional wisdom may fail, and thus there may be quite different implications.

This paper investigates the problem of achieving Pareto efficient allocations for both exchange and general production economies with consumption and production externalities. In contrast to conventional wisdom, we show that for economies with negative consumption externalities, even if consumers' preferences are strictly increasing in their own consumption, information is complete, and transaction costs are zero, one may still have to destroy some units of the goods that have negative consumption externalities, when endowments of these goods are sufficiently large. Intuitively, the consumption of the good with negative externalities has two effects: a positive effect, which benefits the agent who directly consumes the good, and a negative effect, which harms another agent through the channel of the externality. The tradeoff between these two effects determines whether destruction is needed to achieve efficient allocations. If the negative effect dominates the positive one, then destruction is necessary.

This result helps to explain a well-known puzzle of the happiness-income relationship in the economic and psychology literatures: happiness rises with income up to a point, but not beyond it (Graham 2005, p. 4). For example, well-being has declined over the last quarter of a century in the U.S., and life satisfaction has run approximately flat across the same time in Britain (Blanchflower and Oswald 2004). If we interpret income as a good, and if consumers envy each other in terms of consumption levels, by our result, when the income reaches a certain level, one may have to freely dispose of wealth to achieve Pareto efficient allocations; otherwise the resulting allocations will be Pareto inefficient. Therefore, economic growth does not raise well-being indexed by any social welfare function once the critical income level is achieved.

In fact, the presence of negative income externalities is actually a basic assumption of aspiration theory (or reference group theory), developed in the psychology literature to study the economics of happiness (Easterlin 1995, 2001, 2005; Frey and Stutzer 2002). Our results provide a possibility of building a formal foundation for aspiration theory, which is essentially a variant of social comparison theory[1] or a variant of interpreferences theory (Pollak 1976), both of which are built upon consumption externalities.

The most related results of negative consumption externalities are in the context of models of jealousy and "keeping up with the Joneses." In this literature, agents care about their social status, and their relative positions in society are determined by either relative wealth (Cole et al. 1992; Corneo and Jeanne 1997; Futagami and Shibata 1998) or relative consumption (Rauscher 1997; Fisher and Hof 2000; Grossmann 1998; Dupor and Liu 2003; Liu and Turnovsky 2005; García-Peñalosa and Turnovsky 2007). Specifically, the utility function of a typical agent depends not only on his own consumption, but also on the per capita aggregate consumption. Increasing aggregate consumption has two consequences: one directly decreases individual utility level, and is termed jealousy; the other changes the individual's incentive to supply labor, and is called "keeping up with the Joneses." (Dupor and Liu 2003) These models investigate the impact of status-seeking orientation on asset pricing (Abel 1990; Campbell and Cochrane 1999), saving behavior, and economic growth and explore the optimal tax policies to correct for the potential "over-consumption" or "too much" work effort relative to a social planner's solution (Rauscher 1997; Grossmann 1998; Fisher and Hof 2000; Liu and Turnovsky 2005).

Our approach and issue of interest are quite different. First, most of the above studies are conducted in a representative agent framework,[2] in which all agents are

[1] Social comparison here means that people compare themselves to others. The effects of social comparisons on consumption and savings behavior are analyzed in the classic works of Veblen (1899) and Duesenberry (1949) in economics.

[2] Exceptions are Grossmann (1998) who allows for heterogeneity in factor endowments when discussing voting behavior with respect to tax policies, and García-Peñalosa and Turnovsky (2007) who consider different initial wealth endowments and different reference consumption levels to analyze the distributions of income and wealth in a growth model.

identical along the equilibrium paths. None of them focuses on Pareto efficiency, let alone characterizes the threshold values of income destruction. Hockman and Rodgers (1969) argued that in the presence of interdependent preferences, redistributive activities can be justified by Pareto efficiency only, without invoking any social welfare functions. Therefore, Pareto efficiency is an appropriate concept when analyzing the welfare implications of negative consumption externalities. Second, the "keeping up with the Joneses" models typically assume that "utility will increase if everyone's consumption (including this consumer's own consumption) is equal and goes up in tandem." (Dupor and Liu 2003, p. 424) This assumption directly rules out the possibility of income destruction, because in this scenario, the direct positive effect always dominates the indirect negative effect from the representative agent's additional unit of consumption (also see Remark 3 in Sect. 2). The destruction issue is not only important in theory, but also relevant to reality. It perfectly explains Easterlin paradox, without resorting to any other factors like the environmental disruption effect in other work (Ng and Wang 1993; Ng and Ng 2001; Ng 2003). As such, we believe our paper complements this literature.

The paper is organized as follows. Section 2 considers the destruction problem for the simple case of pure exchange economies. To set the stage, we present a numerical example to illustrate the main results of the paper. At the same time, this example provides a formal explanation of the happiness–income puzzle. We then provide characterization results for general utility functions. In Sect. 3, we consider more general production economies with various types of externalities and obtain similar results. We present concluding remarks in Sect. 4. All proofs are carried out in the Appendix.

2 Consumption externalities and destruction of resources

In this section we provide a characterization of results for the destruction of resources in pure exchange economies with general utility functions and negative externalities. We first specify pure exchange economies in the presence of externalities, and then illustrate the destruction issue with an example of the happiness–income puzzle. The example shows that, even if utility functions are strictly monotonic in consumers' own consumption, one may have to destroy some resources to achieve Pareto optimal allocations when the level of resources is greater than some critical point.

2.1 Economic environments with consumption externalities

Consider pure exchange economies with consumption externalities. Suppose there are two goods and two consumers. Consumer i's consumption of the two goods is denoted by $x_i \equiv (x_{i1}, x_{i2})$, $i = 1, 2$, where the first subscript indexes agents and the second one indexes goods. Assume that good 1's consumption exhibits a negative externality, which means that the utility of consumer i is adversely affected by consumer j's good 1 consumption x_{j1} for $j \neq i$. Examples of goods with negative consumption externalities are tobacco, loud music, and alcohol. Another typical example of negative consumption externalities could be found in a society in which some people envy another's living standard. Consumer i's utility function is then denoted by $u_i(x_{11}, x_{21}, x_{i2})$ for $i = 1, 2$. Initially, there are w_1 units of good 1 and w_2 units of good 2.

An allocation $x \equiv (x_{11}, x_{12}, x_{21}, x_{22})$ is *feasible* if $x \in \mathbb{R}^4_{++}$,[3] $x_{11} + x_{21} \leq w_1$, and $x_{12} + x_{22} \leq w_2$. An allocation is said to be *balanced* if $x_{11} + x_{21} = w_1$ and $x_{12} + x_{22} = w_2$. An allocation x is *Pareto optimal (efficient)* if it is feasible, and there is no other feasible allocation, x', such that $u_i(x'_{11}, x'_{21}, x'_{i2}) \geq u_i(x_{11}, x_{21}, x_{i2})$ for all $i = 1, 2$ and $u_i(x'_{11}, x'_{21}, x'_{i2}) > u_i(x_{11}, x_{21}, x_{i2})$ for some i. Denote the set of all Pareto optimal allocations by PO.

2.2 An example: happiness–income puzzle

To illustrate the necessity of destroying resources to achieve Pareto optimal allocations in the presence of negative consumption externalities, let us consider the following numerical example constructed to address the happiness–income puzzle. Suppose the two goods are interpreted as income and nonincome. Good 1 stands for income which can be used to purchase material goods, and good 2 represents nonincome factors, which are considered in the literature by psychologists to explain the subjective well-being differences across countries, such as health, marriage, mental status, family life, and employment status.

As Easterlin (1995, p. 36) argued, aspiration theory says an individual's well-being varies directly with one's own income and inversely with the incomes of others. For simplicity, assume that consumers' preferences are given by the following specific utility function[4]

$$u_i(x_{11}, x_{21}, x_{i2}) = \sqrt{x_{i1}x_{i2}} - x_{j1}, \quad i \in \{1, 2\}, \quad j \in \{1, 2\}, \quad j \neq i.$$

According to Varian (1992, p. 330), the Pareto efficient allocations are completely characterized by the first-order conditions of the following problem:

$$\max_{x \in R^4_{++}} \sqrt{x_{21}x_{22}} - x_{11}$$

subject to

$$x_{11} + x_{21} \leq w_1, x_{12} + x_{22} \leq w_2, \quad \sqrt{x_{11}x_{12}} - x_{21} \geq u_1^*,$$

where $u_1^* = \sqrt{x_{11}^* x_{12}^*} - x_{21}^*$, i.e., consumer 1's utility level evaluated at the Pareto efficient allocation x^*. Setting up the Lagrangian function with multipliers λ_1, λ_2, and μ, we have the following first-order conditions:

[3] Here, we implicitly assume the consumption sets of both consumers are open sets, in order to apply the Kuhn-Tucker theorem easily.

[4] Of course, in this example, the utility function itself does not require an open consumption set. However, if we use the closed first quadrant as the consumption set, the result would not change significantly, which will be seen in Remark 1 below.

$$x_{11}: \frac{\partial u_2}{\partial x_{11}} - \lambda_1 + \mu \frac{\partial u_1}{\partial x_{11}} = -1 - \lambda_1 + \frac{\mu}{2}\sqrt{\frac{x_{12}}{x_{11}}} = 0, \tag{1}$$

$$x_{12}: -\lambda_2 + \mu \frac{\partial u_1}{\partial x_{12}} = -\lambda_2 + \frac{\mu}{2}\sqrt{\frac{x_{11}}{x_{12}}} = 0, \tag{2}$$

$$x_{21}: \frac{\partial u_2}{\partial x_{21}} - \lambda_1 + \mu \frac{\partial u_1}{\partial x_{21}} = \frac{1}{2}\sqrt{\frac{x_{22}}{x_{21}}} - \lambda_1 - \mu = 0, \tag{3}$$

$$x_{22}: \frac{\partial u_2}{\partial x_{22}} - \lambda_2 = \frac{1}{2}\sqrt{\frac{x_{21}}{x_{22}}} - \lambda_2 = 0, \tag{4}$$

$$\lambda_1: w_1 - x_{11} - x_{21} \geq 0, \quad \lambda_1 \geq 0, \; \lambda_1 (w_1 - x_{11} - x_{21}) = 0, \tag{5}$$

$$\lambda_2: w_2 - x_{12} - x_{22} \geq 0, \quad \lambda_2 \geq 0, \; \lambda_2 (w_2 - x_{12} - x_{22}) = 0, \tag{6}$$

$$\mu: \sqrt{x_{11}x_{12}} - x_{21} - u_1^* \geq 0, \quad \mu \geq 0, \; \mu \left(\sqrt{x_{11}x_{12}} - x_{21} - u_1^*\right) = 0.$$

By (4), we have $\lambda_2 > 0$ and thus, by (6),

$$x_{12} + x_{22} = w_2. \tag{7}$$

Using (1)–(4), we can cancel out x and λ_2, which leads to

$$(1 - \mu)\left[\lambda_1 (1 + \mu) + 1 + \mu + \mu^2\right] = 0$$

which implies that $\mu = 1$.

Using the fact that $\mu = 1$ and $\lambda_2 > 0$, from (1) and (2)

$$\lambda_1 = \frac{1}{4\lambda_2} - 1. \tag{8}$$

By (2), (4), $\mu = 1$, and $\lambda_2 > 0$, we have

$$x_{12} = \frac{1}{4\lambda_2^2} x_{11}, \tag{9}$$

$$x_{22} = \frac{1}{4\lambda_2^2} x_{21}. \tag{10}$$

Summing up (9) and (10) and using (7),

$$\lambda_2 = \frac{1}{2}\sqrt{\frac{x_{11} + x_{21}}{w_2}}, \tag{11}$$

which implies that,

$$x_{12} = \frac{x_{11} w_2}{x_{11} + x_{21}}, \tag{12}$$

$$x_{22} = \frac{x_{21} w_2}{x_{11} + x_{21}}. \tag{13}$$

By substituting (11) into (8), we have

$$\lambda_1 = \frac{\sqrt{w_2}}{2\sqrt{x_{11} + x_{21}}} - 1 \tag{14}$$

which will be used to determine the critical income level beyond which there will be destruction. Since $\lambda_1 \geq 0$ at equilibrium, there are two cases to consider.

Case 1 $\lambda_1 > 0$. In this case, we have $x_{11} + x_{21} < w_2/4$ by (14). By (5),

$$x_{11} + x_{21} = w_1. \tag{15}$$

Therefore, if the income endowment $w_1 < w_2/4$, there is no destruction, and by (12) and (13)

$$x_{12} = \frac{x_{11} w_2}{w_1}, \tag{16}$$

$$x_{22} = \frac{x_{21} w_2}{w_1}. \tag{17}$$

Case 2 $\lambda_1 = 0$. Then, by (14), we have $x_{11} + x_{21} = w_2/4$ which is true for any $w_1 \geq w_2/4$.

Thus, when income endowment $w_1 > w_2/4$, there is a required destruction in income to attain Pareto optimality. When $w_1 = w_2/4$, there is no destruction even though $\lambda_1 = 0$. Thus, the critical income level for no destruction is equal to $w_2/4$.

Finally, by (12) and (13) and by $x_{11} + x_{21} = w_2/4$, we have

$$x_{12} = 4x_{11}, \tag{18}$$

$$x_{22} = 4x_{21}, \tag{19}$$

and $\lambda_2 = 1/4$.

Summarizing the above discussion, we have the following proposition:

Proposition 1 *For a pure exchange economy with the above specific utility functions, it is necessary to destroy some income in order to achieve Pareto efficient outcomes if and only if* $w_1 > w_2/4$. *Specially,*

(1) *When the income endowment* $w_1 > \frac{w_2}{4}$, *income destruction is necessary to achieve Pareto efficient allocations. Furthermore, the amount of destruction is*

equal to $w_1 - w_2/4$*, and the set of Pareto optimal allocations is characterized by*

$$\mathrm{PO} = \Big\{x \in R^4_{++} : x_{12} = 4x_{11},\ x_{22} = 4x_{21},\\ x_{12} + x_{22} = w_2,\ w_2/4 = x_{11} + x_{21} < w_1\Big\}.$$

(2) *When* $w_1 \leq \frac{w_2}{4}$*, destruction is unnecessary to achieve any Pareto efficient allocation. Furthermore, the set of Pareto optimal allocations is characterized by*

$$\mathrm{PO} = \left\{x \in R^4_{++} : x_{12}=\frac{2x_{11}}{w_1},\ x_{22}=\frac{2x_{21}}{w_1},\ x_{12}+x_{22}=w_2,\ x_{11}+x_{21}=w_1\right\}.$$

Remark 1 If we allow boundary points, i.e., $x \in R^4_+$, then the two statements in Proposition 1 would change to

(1) When the income endowment $w_1 > w_2/4$, income destruction is necessary to achieve Pareto efficient allocations except for the case in which some agent does not consume any good. Furthermore, the set of Pareto optimal allocations is characterized by

$$\mathrm{PO} = \Big\{x \in R^4_{++} : x_{12}=4x_{11},\ x_{22}=4x_{21},\ x_{12}+x_{22}=w_2,\ w_2/4=x_{11}+x_{21}<w_1\Big\}\\ \cup \{x_{11} = x_{12} = 0, x_{22} = w_2,\ w_2/4 \leq x_{21} \leq w_1\}\\ \cup \{x_{21} = x_{22} = 0,\ x_{12} = w_2,\ w_2/4 \leq x_{11} \leq w_1\}.$$

(2) When $w_1 \leq w_2/4$, destruction is unnecessary to achieve any Pareto efficient allocation. The set of Pareto optimal allocations is characterized by

$$\mathrm{PO} = \left\{x \in R^4_{+} : x_{12}=\frac{x_{11}w_2}{w_1},\ x_{22}=\frac{x_{21}w_2}{w_1},\ x_{12}+x_{22}=w_2,\ x_{11}+x_{21}=w_1\right\}.$$

The proof of the above remark is much more complicated and is given in the Appendix.

Remark 2 The above example provides a formal answer to the happiness–income puzzle and explains why "raising the incomes of all does not increase the happiness of all." (Easterlin 1995, p. 34) According to Proposition 1, in less developed countries, it is more likely that incomes are lower than the critical level, i.e., $w_1 < w_2/4$, and that economic growth will make individuals happier by making a Pareto improvement. However, once an economy surpasses the critical income level, an increase in wealth will inevitably hurt some individuals if all the income is used up such that the resulting allocation is Pareto inefficient. Therefore, this example offers an explanation for the phenomenon documented by Blanchflower and Oswald (2004): well-being has declined over the last quarter of a century in the U.S., and life satisfaction has run approximately flat across the same time in Britain. Following this paper, we further provide a more comprehensive study, both theoretically and empirically, of the reference group theory and the economics of happiness in Tian and Yang (2008).

Remark 3 In the "keeping up with the Joneses" models (e.g., Fisher and Hof 2000; Dupor and Liu 2003; Liu and Turnovsky 2005), the welfare analysis is conducted by maximizing the utility level of a representative agent in the symmetric equilibria in which each agent receives the same consumption bundle. This means that, in these models, the only relevant allocations require that $\mu = 1$, $x_{11} = x_{21}$, and $x_{12} = x_{22}$. Moreover, those models assume the direct positive effect of good 1 consumption dominates its indirect negative externality effect, i.e., $\frac{\partial u_1}{\partial x_{11}} + \frac{\partial u_2}{\partial x_{11}} = \frac{\partial u_2}{\partial x_{21}} + \frac{\partial u_1}{\partial x_{21}} > 0$, when evaluated at $x_{11} = x_{21}$ and $x_{12} = x_{22}$. Combining $\mu = 1$ with (1) or (3), $\lambda_1 = \frac{\partial u_1}{\partial x_{11}} + \frac{\partial u_2}{\partial x_{11}} = \frac{\partial u_2}{\partial x_{21}} + \frac{\partial u_1}{\partial x_{21}} > 0$ always hold in their interested allocations. That is, no destruction issue is involved in those models.

Remark 4 The possibility of destruction of good 1 in Pareto efficient allocations stems from the following two sources: first, the marginal utility (disutility) of good 1 consumption weakly decreases (increases) with its total endowment; second, the endowment of good 1 is relatively large such that it exceeds some critical level. In the context of the example, the first concave term in the utility function describes good 1's positive effect, and the second linear term captures its negative effect. Therefore, the tradeoff between the two effects favors the negative one as the endowment of good 1 (w_1) increases. In particular, when w_1 exceeds $w_2/4$, the negative effect is so large that Pareto efficiency demands disposal of some units of good 1.

2.3 Characterizations on destruction of resources

We now consider pure exchange economies with general utility functions and provide characterization results about the destruction of resources in achieving Pareto efficient allocations.

Assume that utility functions $u_i(x_{11}, x_{21}, x_{i2})$ are continuously differentiable, strictly quasi-concave, and differentiably increasing in $x_{il} > 0$, $l = 1, 2$. Further assume that Slater's condition[5] is satisfied, and the gradient of $u_i(\cdot)$ is nonzero at the Pareto efficient allocations. Thus Pareto efficient allocations x^* can be completely determined by the first-order conditions of the following problem:

$$\max_{x \in \mathbb{R}^4_{++}} u_2(x_{11}, x_{21}, x_{22})$$

subject to

$$x_{11} + x_{21} \leq w_1, x_{12} + x_{22} \leq w_2, u_1(x_{11}, x_{21}, x_{12}) \geq u_1\left(x^*_{11}, x^*_{21}, x^*_{12}\right).$$

Let λ_1, λ_2 and μ be the Kuhn–Tucker multipliers associated with the two resource constraints and the utility constraint. Consumers prefer more of good 2 to less, and no externality occurs in the consumption of good 2. Therefore, good 2 is always of "social value"; specifically, λ_2 is always positive when evaluated at Pareto optimal allocations. Therefore,

[5] Slater's condition states that there is a point $\hat{x} \in R^4_{++}$ such that all constraints hold with strict inequality.

$$x_{12} + x_{22} = w_2 \tag{20}$$

which means there is never destruction of the good which does not exhibit a negative externality. Manipulating the first-order conditions,[6] we obtain the conventional marginal equality condition given in standard textbooks such as Varian (1992, p. 438):

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}}, \tag{21}$$

which expresses the equality of the *social marginal rates of substitution* (SMRS henceforth) for the two consumers at Pareto efficient points. Let $\text{SMRS}_i = \frac{\partial u_i/\partial x_{i1}}{\partial u_i/\partial x_{i2}} + \frac{\partial u_j/\partial x_{i1}}{\partial u_j/\partial x_{j2}}$ denote consumer i's social marginal rate of substitution of good 1 for good 2. Clearly, the two terms in the expression of SMRS_i, capture the direct and indirect effects of consumption activities in the presence of externalities, respectively.

When the consumption externality is positive, from (21), we can easily see that λ_1 is always positive since $\lambda_2 > 0$. Also, when no externality or a one-sided externality[7] exists, by (21), λ_1 is positive. Thus, the marginal equality condition (21) and the balanced conditions completely determine all Pareto efficient allocations for these cases. However, when there are negative externalities for both consumers, the Kuhn–Tucker multiplier λ_1 indirectly given by (21) is the sum of a negative and a positive term, and thus the sign of λ_1 may be indeterminate. In addition, unlike the claim in some textbooks such as Varian (1992, p. 438), the marginal equality condition, (21), and the balanced conditions may not guarantee finding Pareto efficient allocations correctly.

To guarantee that an allocation is Pareto efficient in the presence of negative externalities, it must be the case that $\lambda_1 \geq 0$ at efficient points, which in turn requires that social marginal rates of substitution be nonnegative; that is,

$$\text{SMRS}_1 = \text{SMRS}_2 \geq 0. \tag{22}$$

Equivalently, we must have that[8]

$$\underbrace{(\partial u_1/\partial x_{11}) \cdot (\partial u_2/\partial x_{21})}_{\text{(joint marginal benefit)}} \geq \underbrace{(\partial u_1/\partial x_{21}) \cdot (\partial u_2/\partial x_{11})}_{\text{(joint marginal cost)}} \tag{23}$$

for all Pareto efficient points. We can interpret the term in the left-hand side of (23), $\frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}}$, as the joint marginal benefit from consuming good 1 and the term in the right-hand side, $\frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}$, as the joint marginal cost of consuming good 1 because the negative externality hurts the consumers. To consume the goods efficiently, a necessary

6 By (2) and (4), $\mu = \frac{\partial u_2/\partial x_{22}}{\partial u_1/\partial x_{12}}$. Then, by (1) and (2), $\frac{\lambda_1}{\lambda_2} = \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}}$. By (3) and (4), $\frac{\lambda_1}{\lambda_2} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}}$.

7 Only one consumer imposes an externality on the other consumer.

8 To see this, by (1) and (3), we could cancel out μ and obtain $\lambda_1 = \frac{(\partial u_1/\partial x_{11})\cdot(\partial u_2/\partial x_{21})-(\partial u_1/\partial x_{21})\cdot(\partial u_2/\partial x_{11})}{\partial u_1/\partial x_{11}-\partial u_1/\partial x_{21}}$. The denominator is positive.

condition is that the joint marginal benefit should not be less than the joint marginal cost.

Thus, the marginal equality condition, (22), $\text{SMRS}_1 = \text{SMRS}_2 \geq 0$; the balanced condition for good 2, (20), $x_{12} + x_{22} = w_2$; the resource constraint for good 1, $x_{11} + x_{21} \leq w_1$; together with a complementary slackness condition for good 1, $\left(\frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}} - \frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}\right)(w_1 - x_{11} - x_{21}) = 0$, constitute a system (PO) from which all Pareto efficient allocations can be obtained. If $\frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}} > \frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}$ at some Pareto efficient allocation, then $\lambda_1 > 0$ and thus $x_{11} + x_{21} = w_1$, i.e., there is no destruction of good 1 for this particular Pareto efficient allocation. If $\frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}} < \frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}$ for any allocations that satisfy $x_{11} + x_{21} = w_1$, $x_{12} + x_{22} = w_2$, and the marginal equality condition (21), the social marginal rates of substitution must be negative. Hence, the allocation will not be Pareto efficient. In this case, there must be destruction in good 1 to achieve Pareto efficiency.

Summarizing, we have the following proposition that provides two categories of sufficiency conditions for characterizing whether or not there should be destruction of endowment w_1 in achieving Pareto efficient allocations:

Proposition 2 *For 2×2 pure exchange economies, suppose that utility functions $u_i(x_{11}, x_{21}, x_{i2})$ are continuously differentiable, strictly quasi-concave, and $\frac{\partial u_i(x_{11}, x_{21}, x_{i2})}{\partial x_{il}} > 0$ for $l = 1, 2$.*

(1) *If the social marginal rates of substitution are positive at a Pareto efficient allocation x^*,*[9] *then there is no destruction of w_1 in achieving Pareto efficient allocation x^*.*
(2) *If the social marginal rates of substitution are negative for any allocation x satisfying $x_{11} + x_{21} = w_1$, $x_{12} + x_{22} = w_2$, and the marginal equality condition* (21)*, then there is destruction of w_1 in achieving any Pareto efficient allocation x^*. That is, $x^*_{11} + x^*_{21} < w_1$.*

Thus, from the above proposition, we know that a sufficient condition for *all* Pareto efficient allocations to dispose of good 1 is

$$\text{SMRS}_1 = \text{SMRS}_2, x_{11} + x_{21} = w_1, x_{12} + x_{22} = w_2 \Rightarrow \frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}} < \frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}. \tag{24}$$

A sufficient condition for *all* Pareto efficient allocations to use up all resources is

$$\text{SMRS}_1 = \text{SMRS}_2, x_{11} + x_{21} \leq w_1, x_{12} + x_{22} = w_2 \Rightarrow \frac{\partial u_1}{\partial x_{11}} \frac{\partial u_2}{\partial x_{21}} > \frac{\partial u_1}{\partial x_{21}} \frac{\partial u_2}{\partial x_{11}}. \tag{25}$$

[9] As we discussed above, this is true if the consumption externality is positive, there is no externality, or there is only a one-sided externality.

Remark 5 Let us revisit our numerical example in Sect. 2.2 using the above sufficient conditions. By the marginal equality condition (21), we have

$$\left(\sqrt{\frac{x_{12}}{x_{11}}}+1\right)^2=\left(\sqrt{\frac{x_{22}}{x_{21}}}+1\right)^2$$

and thus

$$\frac{x_{12}}{x_{11}}=\frac{x_{22}}{x_{21}}. \tag{26}$$

Let $x_{11}+x_{21}\equiv x_1$. Substituting $x_{11}+x_{21}=x_1$ and $x_{12}+x_{22}=w_2$ into (26), we have

$$\frac{x_{12}}{x_{11}}=\frac{w_2}{x_1}. \tag{27}$$

Then, by (26) and (27), we have

$$\frac{\partial u_1}{\partial x_{11}}\frac{\partial u_2}{\partial x_{21}}=\frac{1}{4}\sqrt{\frac{x_{12}}{x_{11}}}\sqrt{\frac{x_{22}}{x_{21}}}=\frac{x_{12}}{4x_{11}}=\frac{w_2}{4x_1}$$

and

$$\frac{\partial u_1}{\partial x_{21}}\frac{\partial u_2}{\partial x_{11}}=1.$$

Thus, $\bar{x}_1=w_2/4$ is the critical point that makes $\frac{\partial u_1}{\partial x_{11}}\frac{\partial u_2}{\partial x_{21}}-\frac{\partial u_1}{\partial x_{21}}\frac{\partial u_2}{\partial x_{11}}=0$, or equivalently $\text{SMRS}_1=\text{SMRS}_2=0$. Hence, if $w_1>\frac{w_2}{4}$, then $\frac{\partial u_1}{\partial x_{11}}\frac{\partial u_2}{\partial x_{21}}-\frac{\partial u_1}{\partial x_{21}}\frac{\partial u_2}{\partial x_{11}}<0$, and thus, by (24), there is destruction in reaching any Pareto efficient allocation. If $w_1<\frac{w_2}{4}$, then $\frac{\partial u_1}{\partial x_{11}}\frac{\partial u_2}{\partial x_{21}}-\frac{\partial u_1}{\partial x_{21}}\frac{\partial u_2}{\partial x_{11}}>0$, and, by (25), no Pareto optimal allocation requires destruction. Finally, when $w_1=\frac{w_2}{4}$, any allocation that satisfies the marginal equality condition (21) and the balanced conditions $x_{11}+x_{21}=w_1$ and $x_{12}+x_{22}=w_2$ also satisfies (23), is a Pareto efficient allocation without destruction.

Roughly speaking, when the endowment of good 1, w_1, is relatively large, the sufficiency condition for destruction (24), is easily satisfied. Note that since $\frac{\partial u_1}{\partial x_{11}}$ and $\frac{\partial u_2}{\partial x_{21}}$ represent marginal benefits, they are usually diminishing in consumption of good 1. Since $\frac{\partial u_1}{\partial x_{21}}$ and $\frac{\partial u_2}{\partial x_{11}}$ are in the form of a marginal cost, their absolute values would be typically increasing in the consumption of good 1. Hence, when total endowment w_1 is small, the social marginal benefit would exceed the social marginal cost so that there is no destruction. As the total endowment of w_1 increases, the social marginal cost will ultimately outweigh the social marginal benefit, which results in the destruction of the endowment of w_1.

Alternatively, we can get the same result by using social marginal rates of substitution. When utility functions are strictly quasi-concave, marginal rates of substitution are diminishing. Therefore, in the presence of negative consumption externalities, social marginal rates of substitution may become negative when the consumption of good 1 becomes sufficiently large. When this occurs, it is better to destroy some units of good 1. The destruction of good 1 will in turn decrease the consumption of good 1 and consequently increase the social marginal rates of substitution. Eventually the SMRS's will become nonnegative.

3 Destruction involving production externalities

In this section we consider the destruction of resources to achieve Pareto efficient allocations for production economies with both consumption and production externalities. We start with production economies with two goods, two consumers and two firms, and move on to those with three goods (one input good and two consumption goods), two consumers, and two firms.

3.1 Destruction for $2 \times 2 \times 2$ production economies

There are two goods, two consumers, and two firms. Each firm produces only one good by using another good. We assume firm j produces good j. There are various externalities in this economy. As in the previous section, good 1 consumption of one consumer would affect the utility level of the other consumer; the production of good j would have externality on the production of good l; the consumption of good 1 would affect production of good 2; and the production of good 2 would influence the happiness of both consumers. A classic example is one in which both production processes produce pollution, which decreases the air quality.

Let x denote the consumers' consumption of goods, y the outputs of firms, v the input used by firms, and w the endowment vector. Again, when double subscripts are used, the first subscript is used to index individuals (consumers or firms) and the second one is used to index goods. For example, x_{ij} means the amount of good j consumed by consumer i and v_{jl} means the amount of good l used by firm j when producing good j.

Let the utility functions $u_i\,(x_{11}, x_{21}, x_{i2}, y_2)$ be defined on R^4_{++}. Assume the utility functions are continuously differentiable, strictly quasi-concave, and differentiably increasing in their own consumption, i.e., $\frac{\partial u_i(x_{11},x_{21},x_{i2},y_2)}{\partial x_{il}} > 0$ for $l = 1, 2$. Since we mainly study the destruction issue in the presence of negative consumption externalities, we also assume $\frac{\partial u_i(x_{11},x_{21},x_{i2},y_2)}{\partial x_{j1}} \leq 0$ for $j \neq i$. Production functions $y_1 = y_1\,(v_{12}, v_{21})$ and $y_2 = y_2(v_{12}, v_{21}, x_1)$ are defined on R^2_{++} and R^3_{++}, which are continuously differentiable and strictly concave, where $x_1 = x_{11} + x_{21}$. Thus, we only require a negative consumption externality, while the production externality and cross externality between production and consumption may be negative or positive, because we believe the consumption externality is essential for destructing resources.

The endowments for goods are $w_1 \geq 0$ and $w_2 \geq 0$. Assume that at least one of them is strictly positive, depending on the preferences and technologies, in order to make this economy meaningful.

In this economy, an allocation $(x, y, v) \equiv (x_{11}, x_{12}, x_{21}, x_{22}, y_1, y_2, v_{12}, v_{21})$ is *feasible* if $(x, y, v) \in \mathbb{R}^8_{++}$, $y_1 = y_1\,(v_{12}, v_{21})$, $y_2 = y_2(v_{12}, v_{21}, x_1)$, $x_{11} + x_{21} + v_{21} \leq y_1 + w_1$, and $x_{12} + x_{22} + v_{12} \leq y_2 + w_2$. An allocation (x, y, v) is balanced if $x_{11} + x_{21} + v_{21} = y_1 + w_1$ and $x_{12} + x_{22} + v_{12} = y_2 + w_2$. An allocation (x, y, v) is *Pareto efficient* if it is feasible and there does not exist another feasible allocation (x', y', v') such that $u_i\left(x'_{11}, x'_{21}, x'_{i2}, y'_2\right) \geq u_i\,(x_{11}, x_{21}, x_{i2}, y_2)$ for all $i = 1, 2$ and $u_i\left(x'_{11}, x'_{21}, x'_{i2}, y'_2\right) > u_i\,(x_{11}, x_{21}, x_{i2}, y_2)$ for some i.

By an argument similar to that in the previous section, we could show that the first-order conditions characterizing Pareto optimality boil down to the equality of the *social marginal rates of substitution* and the *social marginal rates of transformation* (SMRT henceforth) for all consumers and producers. Moreover, in order for these conditions to guarantee Pareto efficient allocations, we need that the social marginal rates of substitution and the social marginal rates of transformation are not only equal but also nonnegative; i.e.,

$$\lambda_1/\lambda_2 = \text{SMRS}_1 = \text{SMRS}_2 = \text{SMRT}_1 = \text{SMRT}_2 \geq 0,$$

where λ_1 and λ_2 are the Kuhn–Tucker multipliers associated with the two resource constraints,

$$\begin{aligned}
\text{SMRS}_1 &= \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} + \frac{(\partial u_2/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_2/\partial x_{22}} \\
&\quad + \frac{\partial y_2}{\partial x_1} + \frac{(\partial u_1/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_1/\partial x_{12}}, \\
\text{SMRS}_2 &= \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}} + \frac{(\partial u_2/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_2/\partial x_{22}} \\
&\quad + \frac{\partial y_2}{\partial x_1} + \frac{(\partial u_1/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_1/\partial x_{12}}, \\
\text{SMRT}_1 &= \frac{1 - \partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}} - \frac{\partial u_2/\partial y_2}{\partial u_2/\partial x_{22}} \frac{\partial y_2/\partial v_{12}}{(\partial y_1/\partial v_{12})} - \frac{\partial u_1/\partial y_2}{\partial u_1/\partial x_{12}} \frac{\partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}},
\end{aligned}$$

and

$$\text{SMRT}_2 = \frac{\partial y_2/\partial v_{21}}{1 - \partial y_1/\partial v_{21}} + \frac{\partial u_2/\partial y_2}{\partial u_2/\partial x_{22}} \frac{\partial y_2/\partial v_{21}}{1 - \partial y_1/\partial v_{21}} + \frac{\partial u_1/\partial y_2}{\partial u_1/\partial x_{12}} \frac{\partial y_2/\partial v_{21}}{1 - \partial y_1/\partial v_{21}}.$$

The derivation for the SMRS's and SMRT's are given in the Appendix. Further there is no destruction for good 1 when the social marginal rates are positive, or equivalently, $\lambda_1 > 0$. Formally, we have the following proposition:

Proposition 3 *For* $2 \times 2 \times 2$ *production economies, suppose that utility functions* $u_i(x_{11}, x_{21}, x_{i2}, y_2)$ *are continuously differentiable, strictly quasi-concave,* $\frac{\partial u_i(x_{11},x_{21},x_{i2},y_2)}{\partial x_{il}} > 0$ *for* $l = 1, 2$*, and* $\frac{\partial u_i(x_{11},x_{21},x_{i2},y_2)}{\partial x_{j1}} \leq 0$ *for* $j \neq i$*. Suppose production functions are continuously differentiable and strictly concave. Then we have the following statements.*

(1) *If the social marginal rates of substitution and social marginal rate of transformation are positive at a Pareto efficient allocation* (x^*, y^*, v^*) *, then there is no destruction of resources in achieving the Pareto efficient allocation* (x^*, y^*, v^*).
(2) *If the social marginal rates of substitution and social marginal rate of transformation are negative for any allocation* $(x_1, x_2, y_1, y_2, v_1, v_2)$ *satisfying* $x_{11} + x_{21} + v_{21} = w_1 + y_1$, $x_{12} + x_{22} + v_{12} = w_2 + y_2$*, and the marginal equality*

conditions, then there is destruction of resources in achieving the Pareto efficient allocation (x^*, y^*, v^*).

To have some explicit results, let us consider a special case in which there is no cross externality between consumption and production; that is, the cross derivatives $\frac{\partial y_2}{\partial x_1} = 0$ and $\frac{\partial u_i}{\partial y_2} = 0$ for all $i = 1, 2$. Then the SMRT's for production economies with the above simplified consumption and production externalities become

$$\frac{\lambda_1}{\lambda_2} = \frac{1 - \partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}} = \frac{\partial y_2/\partial v_{21}}{1 - \partial y_1/\partial v_{21}}. \tag{28}$$

Now when $\frac{\partial y_1}{\partial v_{21}} < 1$, $\frac{\partial y_2}{\partial v_{12}} < 1$, $\frac{\partial y_2}{\partial v_{21}} > 0$, and $\frac{\partial y_1}{\partial v_{12}} > 0$, the social marginal rates of transformation are positive, and thus $\lambda_1 > 0$ because $\lambda_2 > 0$. Therefore, the marginal equality conditions (28) and the two resource balanced conditions fully characterize the set of Pareto efficient allocations. In this case, there is no destruction of resources in achieving Pareto efficient allocations. Thus, nonpositive production externalities together with positive marginal product of inputs eliminate the problem of destruction of resources in the presence of negative consumption externalities. Formally, we have the following corollary:

Corollary 1 *For* $2 \times 2 \times 2$ *production economies, suppose that* (1) *utility functions* $u_i\,(x_{11}, x_{21}, x_{i2})$ *are continuously differentiable, strictly quasi-concave, and differentiably increasing in their own consumption;* (2) *production functions* $y_1 = y_1\,(v_{12}, v_{21})$ *and* $y_2 = y_2(v_{12}, v_{21})$ *are continuously differentiable and strictly concave;* (3) *production externalities are nonpositive and marginal product of inputs are positive. Then, there is no destruction in achieving efficient allocations.*

The above corollary could be understood intuitively. Good 2 is always of social value to the economy, and the production of good 2 needs good 1 as input. If the production of good 2 has a negative effect on the production of good 1, this would reduce the destruction pressure on good 1. Thus, negative production externalities offset the problem of destruction of resources in the presence of negative consumption externalities. On the other hand, positive production externalities would aggravate the destruction pressure, because when the economy is producing the socially valuable good, the positive externality in the production process results in an extra amount of undesired good 1. The following example verifies this intuition. This situation is of particular interest when studying the phenomenon of economic growth without happiness, because as argued by the endogenous growth models (Romer 1986; Barro 1990; Turnovsky 1996), the drastic postwar economic growth in developed countries is partly due to the presence of positive production externalities.

Example 1 Suppose the utility functions are the same as in the numerical example in Sect. 2

$$u_i(x_{11}, x_{21}, x_{i2}) = \sqrt{x_{i1}x_{i2}} - x_{j1}, \quad j \neq i,$$

and the endowments are $(w_1, w_2) = (1, 10)$. By Proposition 1 in Sect. 2, we know that, when there is no production, there is no destruction since $w_1 = 1 < w_2/4 = 2.5$.

However, when a production with positive externalities is allowed, one may have to destroy some resources. To see this, let production functions be given by

$$y_1 = 2\sqrt{v_{12}} + 2v_{21}$$
$$y_2 = 2\sqrt{v_{21}} - v_{21} + 2\sqrt{v_{12}}.$$

Since $\frac{\partial y_2/\partial v_{21}}{1-\partial y_1/\partial v_{21}} = 1 - \frac{1}{\sqrt{v_{21}}} < 0$ and $\frac{1-\partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}} = \sqrt{v_{12}} - 1 < 0$ for all $0 < v_{12} < 1$ and $0 < v_{21} < 1$, the social marginal rates of transformation are negative when inputs are less than one. Thus, to have Pareto efficient allocations, we must require $v_{12} \geq 1$ and $v_{21} \geq 1$.

Since consumers' supply for good 2, $w_2+y_2-v_{12} = 10+2\sqrt{v_{21}}-v_{21}+2\sqrt{v_{12}}-v_{12}$ is maximized at $v_{21} = 1$ and $v_{12} = 1$, which is equal to 12, the critical point for destruction $(w_2 + y_2 - v_{12})/4$ is also maximized at $v_{21} = 1$ and $v_{12} = 1$, which is equal to 3. Also, because the resource constraint for good 2 is always binding at Pareto efficient allocations, $v_{12} > 1$ and $v_{21} > 1$ cannot be Pareto efficient inputs. We now show $(v_{12}, v_{21}) = (1, 1)$ is the only input vector that results in Pareto efficient allocations. Indeed, when $(v_{12}, v_{21}) = (1, 1)$, $y_1 = 4$ and $y_2 = 3$, and the feasible conditions become $x_{11} + x_{21} \leq 4$ and $x_{12} + x_{22} \leq 12$. Since the critical level for the destruction of good 1 is $12/4 = 3 < 4$, by applying Proposition 1 again, we need to destroy one unit of good 1 in order to achieve Pareto efficient allocations. Thus the set of Pareto efficient allocations is given by

$$\begin{aligned}\mathrm{PO} = \{(x, v, y) \in R^8_{++} : {}& x_{12} = 4x_{11},\ x_{22} = 4x_{21},\ x_{12} + x_{22} = 12,\ x_{11} + x_{21} = 3,\\ & v_{21} = 1,\ v_{12} = 1,\ y_1 = 4,\ y_2 = 3.\}.\end{aligned}$$

3.2 Destruction for 3 × 2 × 2 production economies

From Corollary 1, it seems that introducing production may solve the destruction problem as long as production externalities are negative. However, this may not be true in general. In this subsection, we show that when production and consumption both have negative externalities, the destruction of resources is still necessary in order to achieve Pareto efficiency in some cases.

Consider production economies with three goods (two consumption goods and one input), two consumers, and two firms. Each consumer consumes both consumption goods. Firm j produces consumption good j. There are no initial consumption goods available. However, firms can produce both consumption goods from common raw materials, or natural resources, which are denoted by r. The initial resource endowment is $w_r > 0$. Assume the consumption of good 1 by one consumer negatively affects consumption of good 1 by the other consumer, and the production of one good imposes negative externalities on the production of the other good. We specify the preferences and technologies as follows:

$$u_i = u_i\left(x_{11}, x_{21}, x_{i2}\right),$$
$$y_j = y_j\left(r_1, r_2\right),$$

where $\frac{\partial u_i(x_{11},x_{21},x_{i2})}{\partial x_{il}} > 0$, $\frac{\partial u_i(x_{11},x_{21},x_{i2})}{\partial x_{j1}} \leq 0$, $\frac{\partial y_j}{\partial r_j} > 0$, and $\frac{\partial y_j}{\partial r_l} \leq 0$ for $i \neq j,\ j \neq l$.

In this economy, an allocation $(x, y, r) \equiv (x_{11}, x_{12}, x_{21}, x_{22}, y_1, y_2, r_1, r_2)$ is *feasible* if $(x, y, r) \in \mathbb{R}^8_{++}$, $y_1 = y_1(r_1, r_2)$, $y_2 = y_2(r_1, r_2)$, $x_{11} + x_{21} \leq y_1$, $x_{12} + x_{22} \leq y_2$ and $r_1 + r_2 \leq w_r$. An allocation (x, y, r) is *balanced* if the feasibility conditions hold with equality. Pareto efficiency can be similarly defined.

In this model, we want to characterize when there is destruction for consumption goods or the raw material resource. This economy is very similar to a pure exchange economy discussed in Sect. 2. The raw material r could not be produced by any technology. We expect to see a similar condition as (23) for characterizing the destruction of raw material. Not surprisingly, there is no destruction in consumption good 2. In addition, it turns out that there is no destruction in consumption good 1 either. To see this, by manipulating the first-order conditions of the program that determine the Pareto efficient allocations, we have the following marginal equality condition:

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}} = \frac{\partial y_2/\partial r_2 - \partial y_2/\partial r_1}{\partial y_1/\partial r_1 - \partial y_1/\partial r_2}, \tag{29}$$

in which the last equality guarantees $\lambda_1 > 0$ by the assumptions on technology. Formally, we have the following proposition, which is proved in the appendix:

Proposition 4 *For the production economies specified in this subsection, let* (x^*, y^*, r^*) *be a Pareto efficient allocation. Then we have the following results*

(1) *There is no destruction of consumption goods.*
(2) *Suppose* $\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2} > \frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}$ *at* (x^*, y^*, r^*). *Then, there is also no destruction of* w_r *in achieving the Pareto efficient allocation* (x^*, y^*, r^*).
(3) *Suppose* $\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2} < \frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}$ *for any allocation* (x, y, r) *that satisfies* $r_1 + r_2 = w_r$, $x_{11} + x_{21} = y_1(r_1, r_2)$, $x_{12} + x_{22} = y_2(r_1, r_2)$, *and the marginal equality conditions* (29). *Then there is destruction of* w_r *in achieving a Pareto efficient allocation* (x^*, y^*, r^*).

In Proposition 4, whether there is destruction of raw materials depends on the magnitudes of $\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2}$ and $\frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}$, two terms similar to the joint marginal benefit and joint marginal cost in (23), respectively. The interpretation is also the same.

Remark 6 If the production process imposes externalities on the satisfaction of consumers, such as $u_i = u_i(x_{11}, x_{21}, x_{i2}, y_1)$ with $\frac{\partial u_i}{\partial y_1} > 0$, or $u_i = u_i(x_{11}, x_{21}, x_{i2}, y_2)$ with $\frac{\partial u_i}{\partial y_2} < 0$, we areable to recover the possibility of destruction in consumption of good 1.

4 Conclusion

In this paper, we considered the problem of obtaining Pareto efficient allocations in the presence of various externalities. We provided specific conditions for determining whether there should be destruction in achieving Pareto efficiency. Roughly speaking, a sufficiently large endowment of a good with negative consumption externalities requires destruction. This occurs because in Pareto efficient allocations, social marginal rates of substitution may diminish from positive to negative when consumption of the good with negative externalities increases.

These conclusions are somewhat surprising. In contrast to conventional wisdom, we showed that even with preferences which are monotonically increasing in own consumption, negative consumption externalities—and some types of production externalities—demand disposal of a certain amount of resources in order to achieve Pareto efficieny. Furthermore, even if information is complete and there are no transaction costs, there is no way to allocate resources efficiently without throwing away some portion of goods. Thus, all the existing alternative solutions cannot solve the market failure without destroying resources in the presence of externalities considered in this paper. These solutions must be appropriately modified.

As an application, our results also provide a possibility of building a formal foundation for the aspiration theory developed in psychology literature, which has been used to study the economics of happiness. The aspiration theory says that an individual's utility is determined by the difference between her own income and the average income (aspiration level) of the society. When the society becomes wealthier, the aspiration level also increases over time, yielding no additional utility overall. Thus, the aspiration theory is essentially based on the assumption that there are negative consumption externalities. As a result, our results explain Easterlin's happiness–income paradox: average happiness levels do not increase as countries grow wealthier.

To conclude the paper, we mention some possible future research on negative consumption externalities and destruction of resources in achieving Pareto efficiency. This paper only establishes a benchmark or criterion for achieving social optimality for economies with negative consumption externalities and has ignored what decentralized mechanism could be used to achieve efficient allocations. We have also neglected the incentive issue of implementing Pareto efficient allocations with destruction. It is still a challenge to answer the question of how to design an incentive mechanism that implements Pareto efficient allocations when utility functions and productions are unknown to the designer. However, some techniques developed in Tian (2003, 2004) may be useful in developing such a mechanism.

A Appendix

A.1 Proof of Remark 1

A boundary Pareto efficient allocation refers to a Pareto efficient point $x^* = \left(x^*_{11}, x^*_{12}, x^*_{21}, x^*_{22}\right)$ with at least one of the four coordinates equal to zero. It suffices to find boundary Pareto efficient allocations to prove Remark 1, and we do this following a number of claims.

Claim 1 *For a boundary Pareto optimal allocation x^*, if $x^*_{i1} = 0$ or $x^*_{i2} = 0$, then $x^*_{i1} = x^*_{i2} = 0$ and $x^*_{j1} > 0$, $x^*_{j2} = w_2$ for any $i \neq j$.*

Proof Suppose $x^*_{i1} = 0$. We first show $x^*_{j1} > 0$. Suppose not, then $x^*_{j1} = 0$. By recalling the form of utility functions, $u^*_i = u^*_j = 0$. Then the allocation $x'_{i1} = x'_{j1} =$

$\varepsilon, x'_{i2} = x'_{j2} = 4\varepsilon$ will be Pareto superior, since $u'_i = u'_j = \varepsilon > 0$ for any arbitrarily small positive number ε.

Second, show $x^*_{i2} = 0$. Suppose $x^*_{i2} > 0$. Then, $x^*_{j2} < w_2$ by the resource constraint. So, $u^*_i = -x^*_{j1}$, and $u^*_j = \sqrt{x^*_{j1}x^*_{j2}} < \sqrt{x^*_{j1}w_2}$ by $x^*_{j1} > 0$. Hence, we find another superior allocation by assigning all of good 2 to consumer j.

Finally, show $x^*_{j2} = w_2$. If $x^*_{j2} < w_2$, we can find the allocation $x'_{i1} = x'_{i2} = 0$, $x'_{j1} = x^*_{j1}, x'_{j2} = w_2$ superior to x^*.

As for the case $x^*_{i2} = 0$, a similar argument applies. □

Claim 1 helps us to shrink the potential boundary Pareto optimal points to a rather small set. We need only to check which of such points ($x^*_{i1} = x^*_{i2} = 0, 0 < x^*_{j1} \leq w_1$, $x^*_{j2} = w_2$) are Pareto optimal.

Claim 2 *An allocation x^* such that $x^*_{i1} = x^*_{i2} = 0$, $0 < x^*_{j1} \leq w_1$, $x^*_{j2} = w_2$ is Pareto optimal if and only if it solves the following problem:*

$$(Q1)\begin{cases} \max\limits_{x\in R^4_+} \sqrt{x_{j1}x_{j2}} - x_{i1} \\ s.t.\ x_{11} + x_{21} \leq w_1, \\ x_{12} + x_{22} \leq w_2, \\ \sqrt{x_{i1}x_{i2}} - x_{j1} \geq -x^*_{j1}. \end{cases}$$

Proof The "only if" part follows directly from Varian (1992, p. 330).

As for the "if" part, by Varian (1992, p. 330), we need only to show that if x^* solves (Q1), then it also solves

$$(Q2)\begin{cases} \max\limits_{x\in R^4_+} \sqrt{x_{i1}x_{i2}} - x_{j1} \\ s.t.\ x_{11} + x_{21} \leq w_1, \\ x_{12} + x_{22} \leq w_2, \\ \sqrt{x_{j1}x_{j2}} - x_{i1} \geq \sqrt{x^*_{j1}w_2}. \end{cases}$$

Suppose not. Then there is another allocation x' such that $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} \geq \sqrt{x^*_{j1}w_2}$ and $\sqrt{x'_{i1}x'_{i2}} - x'_{j1} > \sqrt{x^*_{i1}x^*_{i2}} - x^*_{j1} = -x^*_{j1}$. We know x'_{j1} should be positive since $x^*_{j1} > 0$. Thus, if $x'_{i2} > 0$, then we can subtract x'_{i2} a little bit and increase x'_{j2} the same amount, resulting in $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} > \sqrt{x^*_{j1}w_2}$ and $\sqrt{x'_{i1}x'_{i2}} - x'_{j1} > \sqrt{x^*_{i1}x^*_{i2}} - x^*_{j1}$. So, it is contradicted by the fact x^* solves (Q1). If $x'_{i2} = 0$, then $x'_{j1} < x^*_{j1}$. So, $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} \leq \sqrt{x'_{j1}x'_{j2}} \leq \sqrt{x'_{j1}w_2} < \sqrt{x^*_{j1}w_2}$, a contradiction. □

Thus, Claim 2 allows us only consider problem (Q1) in order to check boundary Pareto optimal points.

Claim 3 *An allocation x^* satisfying $x^*_{i1} = x^*_{i2} = 0$, $0 < x^*_{j1} \leq w_1$, $x^*_{j2} = w_2$ is a boundary Pareto optimal allocation if and only if for any interior Pareto optimal points $\tilde{x}$, at least one of the following inequalities holds:*

$$\sqrt{\tilde{x}_{j1}\tilde{x}_{j2}} - \tilde{x}_{i1} \leq \sqrt{x^*_{j1}w_2},$$
$$\sqrt{\tilde{x}_{i1}\tilde{x}_{i2}} - \tilde{x}_{j1} < -x^*_{j1}.$$

Before we give a formal proof of Claim 3, this claim itself needs further interpretation. By Claim 2, after taking the contrapositive, Claim 3 says that the allocation x^* satisfying $x^*_{i1} = x^*_{i2} = 0$, $0 < x^*_{j1} \leq w_1$, $x^*_{j2} = w_2$ does not solve problem (Q1) if and only if there is some *interior Pareto optimal allocation* $\tilde{x}$, *such that*

$$\sqrt{\tilde{x}_{j1}\tilde{x}_{j2}} - \tilde{x}_{i1} > \sqrt{x^*_{j1}w_2} \text{ and } \sqrt{\tilde{x}_{i1}\tilde{x}_{i2}} - \tilde{x}_{j1} \geq -x^*_{j1}. \tag{30}$$

Since any interior Pareto optimal allocation satisfies the resource constraint conditions, then (30) is equivalent to saying that x^* does not solve problem (Q1) if and only if there is some *interior Pareto optimal allocation* $\tilde{x}$ does better than x^* in (Q1).

The "if" part in the above sentence is obvious. We prove the "only if" part by the following claim:

Claim 4 *If an allocation x^* satisfying $x^*_{i1} = x^*_{i2} = 0$, $0 < x^*_{j1} \leq w_1$, $x^*_{j2} = w_2$ does not solve problem (Q1), then there is some interior Pareto optimal allocation $\tilde{x}$ better than x^* in (Q1).*

Proof First, we show that any boundary allocation x' can never do better than x^* in (Q1); that is, if such an allocation satisfies the constraint, then it fails to attain a higher objective function value. We finish the discussion case by case.

Case 1 $x'_{i1} = 0$. By constraint $\sqrt{x'_{i1}x'_{i2}} - x'_{j1} = -x'_{j1} \geq -x^*_{j1}$, we know $x'_{j1} \leq x^*_{j1}$. Then, $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} = \sqrt{x'_{j1}x'_{j2}} \leq \sqrt{x^*_{j1}x'_{j2}} \leq \sqrt{x^*_{j1}w_2}$.

Case 2 $x'_{i2} = 0$. Similarly, $x'_{j1} \leq x^*_{j1}$. Then, $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} \leq \sqrt{x'_{j1}x'_{j2}} \leq \sqrt{x^*_{j1}x'_{j2}} \leq \sqrt{x^*_{j1}w_2}$.

Case 3 $x'_{j1} = 0$. $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} = -x'_{i1} \leq 0 < \sqrt{x^*_{j1}w_2}$.

Case 4 $x'_{j2} = 0$. $\sqrt{x'_{j1}x'_{j2}} - x'_{i1} = -x'_{i1} \leq 0 < \sqrt{x^*_{j1}w_2}$.

So, if the allocation x^* does not solve problem (Q1), then there must be some interior allocation $\tilde{x}'$ (which may not be Pareto optimal) that does better than it. If $\tilde{x}'$ itself is Pareto efficient, then we are done. If not, there should be some Pareto efficient allocation $\tilde{x}$ superior to $\tilde{x}'$. By the definition of Pareto superiority and the fact any allocation satisfies the resource constraints, $\tilde{x}$ will do better than x^* in problem (Q1). Note that by the argument of the first part, $\tilde{x}$ never be a boundary point. The proof is completed. □

Claim 5 *The set of boundary Pareto efficient allocations is given by*

(1) *when* $w_1 > w_2/4$

$$\begin{aligned}\text{PO}_B = & \{x_{11} = x_{12} = 0, x_{22} = w_2, w_2/4 \le x_{21} \le w_1\} \\ & \cup \{x_{21} = x_{22} = 0, x_{12} = w_2, w_2/4 \le x_{11} \le w_1\}.\end{aligned}$$

(2) *when* $w_1 \le w_2/4$

$$\begin{aligned}\text{PO}_B = & \{x_{11} = x_{12} = 0, x_{22} = w_2, x_{21} = w_1\} \\ & \cup \{x_{21} = x_{22} = 0, x_{12} = w_2, x_{11} = w_1\}.\end{aligned}$$

Proof By Claim 3, *an allocation* x^* satisfying $x^*_{i1} = x^*_{i2} = 0$, $0 < x^*_{j1} \le w_1$, $x^*_{j2} = w_2$ *is a boundary Pareto optimal allocation if and only if for* any *interior Pareto optimal allocation* $\tilde{x}$, *at least one of the following inequalities holds*

$$\begin{aligned}\sqrt{\tilde{x}_{j1}\tilde{x}_{j2}} - \tilde{x}_{i1} \le \sqrt{x^*_{j1} w_2}, \\ \sqrt{\tilde{x}_{i1}\tilde{x}_{i2}} - \tilde{x}_{j1} < -x^*_{j1}.\end{aligned}$$

Our task is to pin down the range of x^*_{j1}. According to Proposition 1, we discuss two cases.

Case 1 $w_1 > w_2/4$

By Proposition 1, any *interior Pareto Optimal points* $\tilde{x}$ satisfies $\tilde{x}_{j1} = \frac{1}{4}\tilde{x}_{j2}$, $\tilde{x}_{i2} = w_2 - \tilde{x}_{j2}$, $\tilde{x}_{i1} = \frac{1}{4}\left(w_2 - \tilde{x}_{j2}\right)$. Thus, we need a positive x^*_{j1} which has the property that for all $0 < \tilde{x}_{j2} < w_2$, at least one of the following inequalities holds

$$\frac{3}{4}\tilde{x}_{j2} - \frac{w_2}{4} \le \sqrt{x^*_{j1} w_2} \tag{31}$$

$$\frac{w_2}{2} - \frac{3}{4}\tilde{x}_{j2} < -x^*_{j1} \tag{32}$$

For $\tilde{x}_{j2} \in (0, \frac{w_2}{3}]$, inequality (31) holds. So, no restriction is imposed by $\tilde{x}_{j2}$ in this range. Thus, we only consider the range $\tilde{x}_{j2} \in (\frac{w_2}{3}, w_2)$. We require that for any $\tilde{x}_{j2} \in (\frac{w_2}{3}, w_2)$, either $x^*_{j1} \ge \frac{9}{16 w_2}\left(\tilde{x}_{j2} - \frac{w_2}{3}\right)^2$ or $0 < x^*_{j1} < \frac{3}{4}\tilde{x}_{j2} - \frac{w_2}{2}$. So, x^*_{j1} must falls into the interval $\left[\frac{w_2}{4}, w_1\right]$.

Case 2 $w_1 \le w_2/4$

By Proposition 1, $\tilde{x}_{j1} = \frac{w_1}{w_2}\tilde{x}_{j2}$, $\tilde{x}_{i2} = w_2 - \tilde{x}_{j2}$, $\tilde{x}_{i1} = \frac{w_1}{w_2}\left(w_2 - \tilde{x}_{j2}\right)$. Thus, (31) and (32) become

$$\begin{aligned}\left(\sqrt{\frac{w_1}{w_2}} + \frac{w_1}{w_2}\right)\tilde{x}_{j2} - w_1 \le \sqrt{x^*_{j1} w_2}, \\ \sqrt{w_1 w_2} - \left(\sqrt{\frac{w_1}{w_2}} + \frac{w_1}{w_2}\right)\tilde{x}_{j2} < -x^*_{j1}.\end{aligned}$$

By the same argument, we can find $x^*_{j1} = w_1$ in this case. □

A.2 Proof of Proposition 3

Under the conditions imposed on utility functions and production functions in Sect. 3.1, an allocation $\left(x_{11}^*, x_{12}^*, x_{21}^*, x_{22}^*, v_{12}^*, v_{21}^*\right)$ [10] is Pareto efficient if and only if it solves the following program:

$$\max_{(x,v)\in R_{++}^6} u_2\left(x_{11}, x_{21}, x_{22}, y_2\right)$$
$$s.t.\ \ x_{11}+x_{21}+v_{21} \le y_1 + w_1,$$
$$x_{12}+x_{22}+v_{12} \le y_2 + w_2,$$
$$u_1\left(x_{11}, x_{21}, x_{12}, y_2\right) \ge u_1\left(x_{11}^*, x_{21}^*, x_{12}^*, y_2^*\right).$$

The Lagrangian function of the above program is

$$\begin{aligned} L = {} & u_2\left(x_{11}, x_{21}, x_{22}, y_2\right) + \lambda_1\left(y_1 + w_1 - x_{11} - x_{21} - v_{21}\right) \\ & + \lambda_2\left(y_2 + w_2 - x_{12} - x_{22} - v_{12}\right) \\ & + \mu\left(u_1\left(x_{11}, x_{21}, x_{12}, y_2\right) - u_1\left(x_{11}^*, x_{21}^*, x_{12}^*, y_2^*\right)\right) \end{aligned}$$

and keeping in mind the fact that $y_1 = y_1\left(v_{12}, v_{21}\right)$, $y_2 = y_2\left(v_{12}, v_{21}, x_{11}+x_{21}\right)$, we have the following first-order conditions:

$$x_{11}: \frac{\partial u_2}{\partial x_{11}} + \frac{\partial u_2}{\partial y_2}\frac{\partial y_2}{\partial x_1} - \lambda_1 + \lambda_2\frac{\partial y_2}{\partial x_1} + \mu\left(\frac{\partial u_1}{\partial x_{11}} + \frac{\partial u_1}{\partial y_2}\frac{\partial y_2}{\partial x_1}\right) = 0 \quad (33)$$

$$x_{12}: -\lambda_2 + \mu\frac{\partial u_1}{\partial x_{12}} = 0 \quad (34)$$

$$x_{21}: \frac{\partial u_2}{\partial x_{21}} + \frac{\partial u_2}{\partial y_2}\frac{\partial y_2}{\partial x_1} - \lambda_1 + \lambda_2\frac{\partial y_2}{\partial x_1} + \mu\left(\frac{\partial u_1}{\partial x_{21}} + \frac{\partial u_1}{\partial y_2}\frac{\partial y_2}{\partial x_1}\right) = 0 \quad (35)$$

$$x_{22}: \frac{\partial u_2}{\partial x_{22}} - \lambda_2 = 0 \quad (36)$$

$$v_{12}: \frac{\partial u_2}{\partial y_2}\frac{\partial y_2}{\partial v_{12}} + \lambda_1\frac{\partial y_1}{\partial v_{12}} + \lambda_2\left(\frac{\partial y_2}{\partial v_{12}} - 1\right) + \mu\frac{\partial u_1}{\partial y_2}\frac{\partial y_2}{\partial v_{12}} = 0 \quad (37)$$

$$v_{21}: \frac{\partial u_2}{\partial y_2}\frac{\partial y_2}{\partial v_{21}} + \lambda_1\left(\frac{\partial y_1}{\partial v_{21}} - 1\right) + \lambda_2\frac{\partial y_2}{\partial v_{21}} + \mu\frac{\partial u_1}{\partial y_2}\frac{\partial y_2}{\partial v_{21}} = 0 \quad (38)$$

$$\lambda_1: \lambda_1 \ge 0, x_{11}+x_{21}+v_{21} \le y_1 + w_1, \lambda_1\left(y_1 + w_1 - x_{11} - x_{21} - v_{21}\right) = 0 \quad (39)$$

$$\lambda_2: \lambda_2 \ge 0, x_{12}+x_{22}+v_{12} \le y_2 + w_2, \lambda_2\left(y_2 + w_2 - x_{12} - x_{22} - v_{12}\right) = 0 \quad (40)$$

$$\mu: \mu \ge 0, u_1 \ge u_1^*, \mu\left(u_1 - u_1^*\right) = 0.$$

By (34) and (36),we have $\lambda_2 = \frac{\partial u_2}{\partial x_{22}} > 0$ and $\mu = \frac{\partial u_2/\partial x_{22}}{\partial u_1/\partial x_{12}}$.

[10] The output y_i^* can be found by production function accordingly. So, for the sake of simplicity in calculation, we drop the output variables in allocation.

Then, by (33) and (34)

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} + \frac{(\partial u_2/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_2/\partial x_{22}} + \frac{\partial y_2}{\partial x_1} + \frac{(\partial u_1/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_1/\partial x_{12}} \equiv SMRS_1 \tag{41}$$

and by (35) and (36)

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}} + \frac{(\partial u_2/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_2/\partial x_{22}} + \frac{\partial y_2}{\partial x_1} + \frac{(\partial u_1/\partial y_2)\,(\partial y_2/\partial x_1)}{\partial u_1/\partial x_{12}} \equiv SMRS_2. \tag{42}$$

Equations (41) and (42) are the social marginal rates of substitution for both consumers, corrected by the various external effects.

Also, by (37) and (38), we have

$$\frac{\lambda_1}{\lambda_2} = \frac{1-\partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}} - \frac{\partial u_2/\partial y_2}{\partial u_2/\partial x_{22}}\frac{\partial y_2/\partial v_{12}}{(\partial y_1/\partial v_{12})} - \frac{\partial u_1/\partial y_2}{\partial u_1/\partial x_{12}}\frac{\partial y_2/\partial v_{12}}{\partial y_1/\partial v_{12}}, \tag{43}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial y_2/\partial v_{21}}{1-\partial y_1/\partial v_{21}} + \frac{\partial u_2/\partial y_2}{\partial u_2/\partial x_{22}}\frac{\partial y_2/\partial v_{21}}{1-\partial y_1/\partial v_{21}} + \frac{\partial u_1/\partial y_2}{\partial u_1/\partial x_{12}}\frac{\partial y_2/\partial v_{21}}{1-\partial y_1/\partial v_{21}}, \tag{44}$$

which are the social marginal rates of transformation for firm 1 and firm 2, respectively.

Thus, $\text{SMRS}_1 = \text{SMRS}_2 = \text{SMRT}_1 = \text{SMRT}_2 \geq 0$, $x_{12} + x_{22} + v_{12} = y_2 + w_2$, $SMRS_1 \cdot (y_1 + w_1 - x_{11} - x_{21} - v_{21}) = 0$ form a system which characterizes the Pareto efficient allocations.

A.3 Proof of Proposition 4

Define the Pareto optimal allocations by solving the following program:

$$\begin{aligned}
&\max_{(x,r)\in\mathbb{R}^6_{++}} u_2(x_{11}, x_{21}, x_{22})\\
&s.t.\ x_{11} + x_{21} \leq y_1\,(r_1, r_2)\\
&x_{12} + x_{22} \leq y_2\,(r_1, r_2)\\
&r_1 + r_2 \leq w_r\\
&u_1(x_{11}, x_{21}, x_{12}) \geq u_1\left(x^*_{11}, x^*_{21}, x^*_{12}\right)
\end{aligned}$$

The Lagrangian is

$$L = u_2(x_{11}, x_{21}, x_{22}) + \lambda_1\,[y_1\,(r_1, r_2) - x_{11} - x_{21}] + \lambda_2\,[y_2\,(r_1, r_2) - x_{12} - x_{22}] + \delta\,(w_r - r_1 - r_2) + \mu\left[u_1(x_{11}, x_{21}, x_{12}) - u_1\left(x^*_{11}, x^*_{21}, x^*_{12}\right)\right].$$

The first-order conditions are

$$x_{11} : \frac{\partial u_2}{\partial x_{11}} - \lambda_1 + \mu \frac{\partial u_1}{\partial x_{11}} = 0 \tag{45}$$

$$x_{12} : -\lambda_2 + \mu \frac{\partial u_1}{\partial x_{12}} = 0 \tag{46}$$

$$x_{21} : \frac{\partial u_2}{\partial x_{21}} - \lambda_1 + \mu \frac{\partial u_1}{\partial x_{21}} = 0 \tag{47}$$

$$x_{22} : \frac{\partial u_2}{\partial x_{22}} - \lambda_2 = 0 \tag{48}$$

$$r_1 : \lambda_1 \frac{\partial y_1}{\partial r_1} + \lambda_2 \frac{\partial y_2}{\partial r_1} - \delta = 0 \tag{49}$$

$$r_2 : \lambda_1 \frac{\partial y_1}{\partial r_2} + \lambda_2 \frac{\partial y_2}{\partial r_2} - \delta = 0 \tag{50}$$

$$\lambda_1 : x_{11} + x_{21} \leq y_1, \lambda_1 \geq 0, \lambda_1 (y_1 - x_{11} - x_{21}) = 0 \tag{51}$$

$$\lambda_2 : x_{12} + x_{22} \leq y_2, \lambda_2 \geq 0, \lambda_2 (y_2 - x_{12} - x_{22}) = 0 \tag{52}$$

$$\delta : r_1 + r_2 \leq w_r, \delta \geq 0, \delta (w_r - r_1 - r_2) = 0 \tag{53}$$

$$\mu : u_1 - u_1^* \geq 0, \mu \geq 0, \mu \left(u_1 - u_1^*\right) = 0 \tag{54}$$

The first four first-order conditions are the same as those in Sect. 2 so that we have

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}}. \tag{55}$$

By (49) and (50)

$$\frac{\lambda_1}{\lambda_2} = \frac{\partial y_2/\partial r_2 - \partial y_2/\partial r_1}{\partial y_1/\partial r_1 - \partial y_1/\partial r_2}. \tag{56}$$

Thus, equalizing (55) and (56) yields the marginal equality conditions

$$\frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}} = \frac{\partial y_2/\partial r_2 - \partial y_2/\partial r_1}{\partial y_1/\partial r_1 - \partial y_1/\partial r_2}, \tag{57}$$

which is positive. Thus, there are no destructions in consumptions goods 1 and 2 so that

$$x_{11} + x_{21} = y_1$$

and

$$x_{12} + x_{22} = y_2.$$

Solving (49) and (50) for δ, we have

$$\delta = \frac{\partial u_2}{\partial x_{22}} \frac{(\partial y_1/\partial r_1)(\partial y_2/\partial r_2) - (\partial y_1/\partial r_2)(\partial y_2/\partial r_1)}{\partial y_1/\partial r_1 - \partial y_1/\partial r_2}.$$

The sign of δ is indeterminate and depends on the magnitudes of $\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2}$ and $\frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}$. We will call these two terms the joint marginal benefit and joint marginal cost, respectively. Now, the marginal equality conditions (57) and the two balanced consumption conditions, along with (52) and (53), constitute the system which determines all Pareto efficient allocations,

$$
\text{(PPO)}\begin{cases}
\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2} \geq \frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}, \\
\frac{\partial u_1/\partial x_{11}}{\partial u_1/\partial x_{12}} + \frac{\partial u_2/\partial x_{11}}{\partial u_2/\partial x_{22}} = \frac{\partial u_2/\partial x_{21}}{\partial u_2/\partial x_{22}} + \frac{\partial u_1/\partial x_{21}}{\partial u_1/\partial x_{12}} = \frac{\partial y_2/\partial r_2 - \partial y_2/\partial r_1}{\partial y_1/\partial r_1 - \partial y_1/\partial r_2}, \\
x_{11} + x_{21} = y_1\left(r_1, r_2\right), \\
x_{12} + x_{22} = y_2\left(r_1, r_2\right), \\
r_1 + r_2 \leq w_r, \\
\left(\frac{\partial y_1}{\partial r_1}\frac{\partial y_2}{\partial r_2} - \frac{\partial y_1}{\partial r_2}\frac{\partial y_2}{\partial r_1}\right)\left(w_r - r_1 - r_2\right) = 0.
\end{cases}
$$

Then, by the same discussion as in Sect. 2, we have Proposition 4.

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