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Axiomatic reference-dependence in behavior toward others and toward risk*

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Summary. This paper considers the applicability of the standard separability axiom for both risk and other-regarding preferences, and advances arguments why separability might fail. An alternative axiom, which is immune to these arguments, leads to a preference representation that is additively separable in a reference variable and the differences between the other variables and the reference variable. For other-regarding preferences the reference variable is the decision-maker's own payoff, and the resulting representation coincides with the Fehr-Schmidt model. For risk preferences the reference variable is initial wealth, and the resulting representation is a generalization of prospect theory.

Keywords and Phrases: Other-regarding preferences, Risk, Separability, Axiomatic foundation, Prospect theory.

JEL Classification Numbers: D81, D64.

1 Introduction

This paper presents a new preference axiom called self-referent separability. When combined with the usual axioms of completeness, transitivity, and continuity, it guarantees the existence of a preference representation that is additively separable in a reference variable and the difference between the other variables and the reference variable. In other words, the self-referent separability axiom generates reference-dependent preferences, and such preferences arise in the literature.

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Most prominently, prospect theory (Kahneman and Tversky, 1979) is a reference-dependent representation of preferences toward risk, and so self-referent separability can be used as part of a system of axioms for prospect theory. In a different branch of the literature, Fehr and Schmidt (1999) propose a reference-dependent representation for other-regarding preferences, and self-referent separability is the key axiom for generating their functional form.

The paper begins by stating the axiom and the main representation theorem. Since the applicability of the axiom depends on the choice setting, its rationale is left for two later sections. It is first applied to other-regarding (or interdependent or social) preferences, which arise from the voluminous literature on ultimatum, dictator, and trust games.[1] The upshot of this literature is that players in these games care not just about their own payoffs, but also about the payoffs of their opponents/partners in the game. Thus far, most of the attention on other-regarding preferences has been on constructing new experiments to identify their existence and characteristics and on generating highly-parameterized models to fit the data from the experiments.[2] From a purely decision-theoretic perspective, though, the possibility of preferences being other-regarding raises some interesting issues. In particular, do the standard preference axioms that are used in so many other areas of decision theory make sense in an other-regarding setting, or must they be replaced by something else? If they do need to be replaced, what should they be replaced with?[3]

The standard separability axiom states that if two bundles are identical on some dimensions but differ on others then preferences depend only on those dimensions that differ between the two bundles, and its primary appeal is that it has been used fruitfully in a variety of settings.[4] It does not, however, allow preferences to depend on the ordering or rank of the different dimensions. It seems reasonable that in an interpersonal setting a decision-maker cares about the rank of his own payoff relative to the payoffs of others affected by his decision, and Kahneman and Tversky (1979) established that when faced with risk a decision-maker cares about whether his new wealth level is above or below his previous wealth level. Self-referent separability allows position to matter.[5]

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[1] For overviews of the experiments, see Sobel (2004) and Camerer (2003). For recent experimental work see Bolton and Ockenfels (2005).

[2] Recent examples of models include Fehr and Schmidt (1999), Bolton and Ockenfels (2000), and Charness and Rabin (2002).

[3] To date there have been few studies providing axiomatic bases for other-regarding preferences, with exceptions including Segal and Sobel (1999), Ok and Kockesen (2000), Karni and Safra (2001), Sandbu (2003), and Neilson and Stowe (2004).

[4] Debreu (1959) shows how the separability axiom can lead to additively separable utility representations in consumer theory and to expected utility representations for behavior toward risk. Koopmans (1972) shows how it can lead to exponential discounting for preferences over time.

[5] Neilson and Stowe (2004) and Sandbu (2003) examine other axiomatic treatments of other-regarding preferences when rank matters, and the long literature on rank-dependent expected utility (Quiggin, 1982, 1993) explores preferences toward risk when the rank of the outcomes matters. In those studies the entire ranking vector matters, whereas here all that matters is whether each variable individually is above or below some reference value.

The new axiom does place one restriction on preferences that may or may not be deemed restrictive, depending on the setting. For risk preferences, self-referent separability implies constant absolute risk aversion. Since it does not allow for the standard expected utility formulation with asset integration (i.e. in which the carrier of value in the utility function is the final wealth level), however, constant absolute risk aversion places no restrictions on the functional forms of the underlying utility functions. For other-regarding preferences, self-referent separability implies constant absolute reallocation preferences, which is a generalization of the idea that adding \$100 to everyone's payoff should have no effect on the decision-maker's willingness to take \$20 away from one opponent and give it to another.

Section 2 introduces the self-referent separability axiom and presents the main representation theorem. Section 3 discusses the applicability and applications of the axiom to other-regarding preferences, and Section 4 does the same for risk preferences. Section 4 also shows that in the setting of risk preferences, self-referent separability implies constant absolute risk aversion, and Section 5 discusses constant absolute reallocation preferences. The paper concludes in Section 6.

2 The axiom and the representation theorem

Let $N = \{0, ..., n\}$, and let $x = (x_0, ..., x_n)$ denote a vector of real numbers with $x_i \in X_i$ for each $i \in N$. Define $X = \prod_{i=0}^{n} X_i$. The vector x is referred to as an *allocation*, and the components are referred to as *payoffs*.

Let S be a subset of N, and let $\sim S$ be its complement. Let $(x_S, y_{\sim S})$ denote the allocation $z \in X$ such that $z_i = x_i$ when $i \in S$ and $z_i = y_i$ when $i \in\sim S$. For the special case when S contains only a single element, so that $S = \{i\}$, use the notation $(x_i, y_{\sim i})$ to denote the allocation $(y_0, ..., y_{i-1}, x_i, y_{i+1}, ..., y_n)$. For any scalar k, let $(k + x_S, y_{\sim S})$ denote the allocation z where $z_i = k + x_i$ when $i \in S$ and $z_i = y_i$ when $i \in\sim S$. Accordingly, $k + x$ denotes the allocation $(k + x_0, ..., k + x_n)$.

Let $\succeq$ be a complete, transitive, and continuous preference ordering defined over X. The assumptions of completeness, transitivity, and continuity are standard in the literature, and will be assumed throughout so that attention can be restricted to the axioms that are new in this paper.

It is also assumed throughout that each component is *essential*, that is, for every $i \in N$ there exist $x_i, x_i' \in X_i$ and an allocation $y \in X$ such that $(x_i, y_{\sim i})(x_i', y_{\sim i})$. In words, payoff i is essential if the decision-maker is not always indifferent between two allocations that differ only in their i-th components.

The point of departure is the standard separability axiom, which is given below.

Separability. For any nonempty $S \subset N$ and any x, x',$y \in X$, $(x_S, y_{\sim S}) \succeq (x_S', y_{\sim S})$ implies $(x_S, y_{\sim S}') \succeq (x_S', y_{\sim S}')$ for any $y' \in X$.

Debreu (1959) proves that if $n \geq 2$ (so that the total number of components is at least three), the preference ordering satisfies separability if and only if there exist functions $u_0, ..., u_n$ such that preferences can be represented by a function U

of the form

$$U(x) = \sum_{i=0}^{n} u_i(x_i). \tag{1}$$

Furthermore, the utility functions $u_0, ..., u_n$ are unique up to a joint increasing affine transformation; that is, if $v_0, ..., v_n$ also represent preferences, then $v_i = au_i + b$ for some scalar $a > 0$ and some scalar b. In short, the separability axiom implies that preferences have an additive representation.

The primary purpose of this paper is to introduce an alternative axiom that is similar to the standard separability axiom but treats component 0 differently.[6]

Self-Referent Separability [SRS]. For any nonempty $S \subset N$ and any $x, y, z \in X$,

(i) if $0 \in\sim S$, then $(z_0 + x_S, z_{\sim S}) \succeq (z_0 + y_S, z_{\sim S})$ implies $(w_0 + x_S, w_{\sim S}) \succeq (w_0 + y_S, w_{\sim S}))$ for any $w \in X$; and
(ii) if $0 \in S$, then $(x_S, x_0 + z_{\sim S}) \succeq (y_S, y_0 + z_{\sim S})$ implies $(x_S, x_0 + w_{\sim S}) \succeq (y_S, y_0 + w_{\sim S})$ for any $w \in X$.

The self-referent separability axiom leads to a different preference representation.

Theorem 1. *Suppose $n \geq 2$. The preference ordering $\succeq$ satisfies SRS if and only if there exist functions $u_0, ..., u_n$ such that preferences can be represented by a function U of the form*

$$U(x) = u_0(x_0) + \sum_{i=1}^{n} u_i(x_i - x_0). \tag{2}$$

The utility functions $u_0, ..., u_n$ are unique up to a joint increasing affine transformation; that is, if $v_0, ..., v_n$ also represent preferences, then $v_i = au_i + b$ for some scalar $a > 0$ and some scalar b.

Proof. The "if" part of the proof is straightforward. For the "only if" part, define the function $f: X \to X$ by $f(x) = (x_0, x_1 - x_0, ..., x_n - x_0)$. Let $\succeq^*$ be a derived preference ordering defined by $f(x) \succeq^* f(y)$ if and only if $x \succeq y$. Since $\succeq$ is complete, transitive, and continuous, so is $\succeq^*$, and therefore there exists a continuous preference function U^* representing $\succeq^*$. Furthermore, since each component is essential for $\succeq$, each component is also essential for $\succeq^*$.

Let S be a subset of N with $0 \in S$. Without loss of generality, and for notational purposes only, let $S = \{0, ..., m\}$, so that $\sim S = \{m+1, ..., n\}$. Then $f((z_S, z_0 + x_{\sim S},)) = (z_0, z_1 - z_0, ..., z_m - z_0, x_{m+1}, ..., x_n)$. By condition (ii) of SRS, $(z_0, z_1 - z_0, ..., z_m - z_0, x_{m+1}, ..., x_n) \succeq^* (z_0, z_1 - z_0, ..., z_m - z_0, y_{m+1}, ..., y_n)$ implies that $(w_0, w_1 - w_0, ..., w_m - w_0, x_{m+1}, ..., x_n) \succeq^* (w_0, w_1 - w_0, ..., w_m - w_0, y_{m+1}, ..., y_n)$ for any w.

Now let S be a subset of N with $0 \in\sim S$. For notational purposes, let $S = \{m + 1, ..., n\}$ so that $\sim S = \{0, ..., m\}$. Then $f((x_S, x_0 +$

[6] For other-regarding preferences, the component x_0 is the decision-maker's own payoff while the other components are his opponents' payoffs, and for decisions toward risk x_0 is initial wealth.

$z_{\sim S})) = (x_0, z_1, ..., z_m, x_{m+1} - x_0, ..., x_n - x_0)$. By condition (i) of SRS, $(x_0, z_1, ..., z_m, x_{m+1} - x_0, ..., x_n - x_0) \succeq^* (y_0, z_1, ..., z_m, y_{m+1} - y_0, ..., y_n - y_0)$ implies that $(x_0, w_1, ..., w_m, x_{m+1} - x_0, ..., x_n - x_0) \succeq^* (y_0, w_1, ..., w_m, y_{m+1} - y_0, ..., y_n - y_0)$ for any w.

Combining the two cases yields that for any subset S of N, $(x_S, z_{\sim S}) \succeq^* (y_S, z_{\sim S})$ implies that $(x_S, w_{\sim S}) \succeq^* (y_S, w_{\sim S})$ for any w. By Debreu (1959, Theorem 3), U^* has an additively separable representation

$$U^*(t) = \sum_{i=0}^{n} u_i(t_i)$$

with the u_i's unique up to a joint increasing affine transformation. Let $t = f(x)$ and let

$$U(x) = U^*(f(x)) = u_0(x_0) + \sum_{i=1}^{n} u_i(x_i - x_0).$$

Then U represents $\succeq$. $\square$

Theorem 1 states that under SRS, preferences over the allocation of payoffs can be represented by a function that is additively separable in the level of payoff 0 and the difference between payoff i and payoff 0. Payoff 0 can therefore be thought of as a *reference payoff*, and the representation states that the decision-maker cares about the level of the reference payoff and differences from the reference payoff.

Whether or not the SRS axiom, and by implication the preference representation in (2), is a useful description of behavior depends on the choice setting. The next section argues that the axiom is suitable for describing other-regarding preferences, and Section 4 contends that it is also a suitable description of preferences toward risk.

3 Other-regarding preferences and the Fehr-Schmidt model

Other-regarding preferences are used to capture the notion that players in games sometimes care about the payoffs that the other players receive. Let x_0 be the decision-maker's own payoff, and let x_i be the payoff to player i for $i = 1, ..., n$, where the other players are referred to as opponents. The standard separability axiom's interpretation in this setting is straightforward. It states that if there is a subset of players whose payoffs are not affected by the choice, then the choice is independent of what those payoffs actually are. Put another way, the payoffs of unaffected players do not matter to the decision. There are reasons, though, that the levels of the unaffected payoffs might matter in an other-regarding framework.

First suppose that the decision-maker has two opponents and prefers $(60, 60, 60)$ to $(60, 80, 40)$, so that he prefers the allocation that has the more-equal payoffs for his opponents. One interpretation of this preference is that he is willing to take 20 from opponent 1 and give it to opponent 2 in order to make their payoffs more equal. Separability implies that $(x_0, 60, 60) \succeq (x_0, 80, 40)$ for all x_0, which has some additional implications besides the interpretation just given. In particular, if

$x_0 = 50$, separability implies that he is willing to take 20 from opponent 1 and give it to opponent 2 *and* guarantee himself the lowest payoff of the three in order to make his opponents' payoffs more equal. In other words, separability makes the decision-maker's choices independent of his rank in the allocation. Self-referent separability allows rank to matter.

To see how, rewrite the original decision as a preference for $(60, 60+0, 60+0)$ over $(60, 60+20, 60-20)$. Condition (i) of the SRS axiom implies that $(x_0, x_0+0, x_0+0) \succeq (x_0, x_0+20, x_0-20)$. This can be interpreted as the decision-maker preferring to take 20 away from opponent 1 and giving it to opponent 2 as long as doing so does not change his own position in the rank ordering of the payoffs.

Now consider the preference $(60, 60, 60) \succeq (70, 40, 70)$, which can be interpreted as a willingness on the part of the decision-maker to take 10 away from both himself and opponent 2 in order to increase opponent 1's payoff by 20. Condition (ii) of SRS states that $(60, 60, 60+k) \succeq (70, 40, 70+k)$ for all k, which means that the decision-maker is willing to make that transaction as long as it does not change his own position relative to opponent 2's.

In general, the SRS axiom ensures that changes in the payoffs that are unaffected by the choice do not alter the ranking of the players. When $0 \in \sim S$, condition (i) holds and the payoffs for players in S are then manipulated so that they stay in the same positions relative to the decision-maker across choice pairs. When $0 \in S$, condition (ii) holds and the payoffs of players outside S also change so as to keep their positions relative to the decision-maker constant, so that in each choice pair each player in $\sim S$ has the same payoff relative to the decision-maker in both lotteries.

If the SRS axiom holds in this setting, then by Theorem 1 the preference ordering can be represented by an additively separable function in which the carriers of value are the decision-maker's own payoff, x_0, and the differences between the other players' payoffs and the decision-maker's payoff, $x_i - x_0$:

$$U(x) = u_0(x_0) + \sum_{i=1}^{n} u_i(x_i - x_0).$$

This functional form has been used by other researchers. Most prominently in the economics literature, Fehr and Schmidt (1999) propose the following preference function for analyzing behavior in experiments:

$$U(x) = x_0 - \frac{\alpha}{n}\sum_{i=1}^{n} \max\{x_i - x_0, 0\} - \frac{\beta}{n}\sum_{i=1}^{n} \max\{x_0 - x_i, 0\} \tag{3}$$

where $0 \le \beta \le \alpha$. The basic intuition is that the individual gets utility from his own monetary payoff but loses utility whenever his payoff is different from his opponents' payoffs. The second term measures his disutility from receiving less than his opponents, and the third term measures his disutility from receiving more. The inequalities $0 \le \beta \le \alpha$ capture the properties that the individual dislikes both receiving less and receiving more than his opponents (inequality aversion), and that receiving less is worse than receiving more. Clearly the preferences in Eq. (3) are a special case of the preferences in Eq. (2), and Theorem 1 provides an axiomatization of the Fehr-Schmidt functional form.

A similar functional form appears earlier in the psychology literature, with the first appearance (to my knowledge) being a bivariate form in Conrath and Deci (1969). In their experimental study of social utility, Loewenstein et al. (1989) estimate several functional forms, including

$$U(x) = a + b_1 x_0 + b_2 x_0^2 + \sum_{i=1}^{n} \left[b_3 x_i + b_4 x_i^2 \right] \tag{4}$$

and

$$\begin{aligned} U(x) = a + b_1 x_0 + \sum_{i:x_i < x_0} \left[b_2 (x_i - x_0) + b_3 (x_i - x_0)^2 \right] \\ + \sum_{i:x_i \geq x_0} \left[b_4 (x_i - x_0) + b_5 (x_i - x_0)^2 \right]. \end{aligned} \tag{5}$$

The representation in Eq. (4) is additively separable while the representation in (5) is self-referent separable. Their preferred functional form is the one given in Eq. (5). Consequently, their paper provides (weak) evidence that self-referent separability outperforms standard separability, at least when the preference representation is restricted to being piecewise quadratic.

4 Risk preferences and reference-dependence

A lottery (or prospect) consists of two components, labeled 0 and 1. Component 0 corresponds to initial wealth, which is assumed to be nonstochastic, and component 1 corresponds to the final wealth position, which can be random. Preferences are defined over vectors of the form $(w_0, \tilde{w})$, where w_0 is initial wealth and $\tilde{w}$ is the random final wealth variable. We constrain both w_0 and $\tilde{w}$ to be positive but no greater than some finite $M > 0$. Let W be the space of random final wealth variables whose support is in $(0, M]$. When we write $\tilde{w} = w_0 + \tilde{x}$ we constrain $-w_0 < \tilde{x} < M - w_0$.

In this context self-referent separability takes the following form:

(i) $(w_0, w_0 + \tilde{x}) \succeq (w_0, w_0 + \tilde{y})$ implies $(w_0', w_0' + \tilde{x}) \succeq (w_0', w_0' + \tilde{y})$ for all w_0', and
(ii) $(w_0, w_0 + \tilde{x}) \succeq (w_0', w_0' + \tilde{x})$ implies $(w_0, w_0 + \tilde{y}) \succeq (w_0', w_0' + \tilde{y})$ for all $\tilde{y}$.

The first condition states that if the individual prefers the change in wealth $\tilde{x}$ to the change $\tilde{y}$ when initial wealth is w_0, he prefers that change for any initial wealth level. The second condition states that if he prefers having initial wealth w_0 to having initial wealth w_0' when the change in wealth is $\tilde{x}$, he prefers having initial wealth w_0 to having initial wealth w_0' no matter what the change in wealth is. There is little evidence about how experimental subjects respond to changes in initial wealth, but the second condition is consistent with a preference for increasing initial wealth.

Let $\tilde{\varepsilon}$ be a symmetric, mean-zero random variable with support in $[-50, 50]$ and suppose that the decision-maker's preferences prescribe $(1000, 1100) \succeq (1000, 1100+\tilde{\varepsilon})$. This pattern is consistent with risk aversion, since the final wealth variable in the second vector is a mean-preserving spread of the final wealth variable in the first vector. Separability would imply that $(1200, 1100) \succeq (1200, 1100+\tilde{\varepsilon})$. However, such a pattern is violated by the reflection effect of Kahneman and Tversky (1979). In the first choice pair the decision-maker starts with 1000 and decides between a sure gain of 100 and a risky gain of $100 + \tilde{\varepsilon}$ and, assuming the typical pattern of risk aversion over gains, prefers the sure gain of 100. In the second choice pair he starts with 1200 and decides between a sure loss of 100 and a risky loss of $100 + \tilde{\varepsilon}$ and, assuming the typical pattern of risk seeking over losses, prefers the risky loss. The reflection effect violates the standard separability axiom in this setting.

Self-referent separability allows for the reflection effect. Under part (i) of SRS, $(1000, 1100) \succeq (1000, 1100+\tilde{\varepsilon})$ implies that $(x_0, x_0+100) \succeq (x_0, x_0+100+\tilde{\varepsilon})$ for all x_0. No matter what the decision-maker's reference wealth level is, he prefers a sure gain of 100 to a risky gain of $100+\tilde{\varepsilon}$. More generally, if the decision-maker prefers a nonstochastic change in wealth to a random change with the same mean at one level of reference wealth, he prefers to avoid that random change at every level of reference wealth.

Since Theorem 1 applies only to allocations with three or more components, we need a new theorem to govern the case of two components. It requires an additional assumption which is a counterpart of what has been called both the Thomsen condition and the hexagon condition (e.g. Wakker, 1989):

Self-Referent Thomsen Condition [SRTC]. For all w_0, w_0', and $w_0'' \in \mathbb{R}_+$ and all random variables $\tilde{x}$, $\tilde{y}$, and $\tilde{z}$ with support in the interval $(-\min\{w_0, w_0', w_0''\}, \infty)$, if $(w_0', w_0'+\tilde{x}) \sim (w_0, w_0+\tilde{y})$, $(w_0', w_0'+\tilde{y}) \sim (w_0, w_0+\tilde{z})$, and $(w_0'', w_0''+\tilde{x}) \sim (w_0', w_0'+\tilde{y})$, then $(w_0'', w_0''+\tilde{y}) \sim (w_0', w_0'+\tilde{z})$.

Theorem 2. *The preference ordering $\succeq$ satisfies SRS and SRTC if and only if there exist functions u_0 and U such that preferences can be represented by a function V of the form*

$$V(w_0, \tilde{w}) = u_0(w_0) + U(\tilde{w} - w_0). \tag{6}$$

The utility functions u_0 and U are unique up to a joint increasing affine transformation; that is, if u_0^* and U^* also represent preferences, then $u_0^* = au_0 + b$ and $U^* = aU + b$ for some scalar $a > 0$ and some scalar b.

Proof. The "if" part of the proof is straightforward. For the "only if" part, define the function $f : \mathbb{R}_+ \times X \to \mathbb{R}_+ \times X$ by $f(w_0, \tilde{w}) = (w_0, \tilde{w} - w_0)$. Let $\succeq^*$ be a derived preference ordering defined by $f(x) \succeq^* f(y)$ if and only if $x \succeq y$. Since $\succeq$ is complete, transitive, and continuous, so is $\succeq^*$, and therefore there exists a continuous preference function V^* representing $\succeq^*$. Furthermore, since each component is essential for $\succeq$, each component is also essential for $\succeq^*$.

Now note that $f(w_0, w_0 + \tilde{x}) = (w_0, \tilde{x})$. By condition (i) of SRS, $(w_0, \tilde{x}) \succeq^* (w_0, \tilde{y})$ implies that $(w_0', \tilde{x}) \succeq^* (w_0', \tilde{y})$ for all w_0'. By condition (ii) of SRS,

$(w_0, \tilde{x}) \succeq^* (w_0', \tilde{x})$ implies that $(w_0, \tilde{y}) \succeq^* (w_0', \tilde{y})$ for all $\tilde{y}$. Together these imply that $\succeq^*$ satisfies the separability axiom. Applying the same technique to SRTC, if $(w_0', \tilde{x}) \sim^* (w_0, \tilde{y})$, $(w_0', \tilde{y}) \sim^* (w_0, \tilde{z})$, and $(w_0'', \tilde{x}) \sim^* (w_0', \tilde{y})$, then $(w_0'', \tilde{y}) \sim^* (w_0', \tilde{z})$. Consequently, $\succeq^*$ also satisfies the Thomsen condition. By Wakker (1989) V^* has an additively separable representation

$$V^*(w_0, \tilde{x}) = u_0(w_0) + U(\tilde{x})$$

with u_0 and U unique up to a joint increasing affine transformation. Finally,

$$V(w_0, \tilde{w}) = V^*(f(w_0, \tilde{w})) = u_0(w_0) + U(\tilde{w} - w_0).$$

Then V represents $\succeq$. □

If SRS holds, by Theorem 2 preferences can be represented by a function of the form (6). This is the functional form posited by Markowitz (1952) in his seminal paper, where in his terminology w_0 corresponds to "customary wealth." If one assumes that U has a Choquet expected utility representation (e.g. Schmeidler, 1989; Wakker, 1989; Diecidue and Wakker, 2001), then the result is very similar to cumulative prospect theory (Tversky and Kahneman, 1992):[7]

$$V(w_0, \tilde{w}) = u_0(w_0) + \int_0^M u(w - w_0)dg(F_{\tilde{w}}(w)), \tag{7}$$

where g is a strictly increasing function with $g(0) = 0$ and $g(1) = 1$. In fact, since it does not explicitly account for changes in reference wealth, cumulative prospect theory would be a special case in which $u_0(x_0) = 0$ for all x_0.

Because it can allow for the reflection effect and because it generates a functional form compatible with that of prospect theory, it seems that the SRS axiom is appropriate for applying to behavior toward risk. However, SRS places an additional restriction on preferences toward risk – it forces preferences to exhibit constant absolute risk aversion.

Proposition 1. *SRS implies constant absolute risk aversion.*

Proof. Following Pratt (1964), let $\tilde{z}$ be a random variable with mean μ and define $\pi(w_0, \tilde{z})$ to be the value of π that solves

$$(w_0, w_0 + \tilde{z}) = (w_0, w_0 - \pi). \tag{8}$$

Preferences exhibit constant absolute risk aversion if $\pi(w_0, \tilde{z})$ is constant with respect to w_0. But condition (i) of SRS states that if (8) holds then $(y_0, y_0 + \tilde{z}) \sim (y_0, y_0 - \pi)$ for any y_0. Consequently, $\pi(w_0, \tilde{z}) = \pi(y_0, \tilde{z})$ for all w_0 and y_0, and so preferences exhibit constant absolute risk aversion. □

[7] The literature already contains axiomatizations of cumulative prospect theory. See Luce and Fishburn (1991), Wakker and Tversky (1993), Groes et al. (1998), Chateauneuf and Wakker (1999), Chateauneuf et al. (2003), and Maccheroni (2004). None contains anything like the SRS axiom, and the primary focus is on obtaining the probability transformation function g.

SRS leads to a much more general form of constant absolute risk aversion than the standard expected utility model does.[8] In ordinary expected utility with asset integration, that is, when the argument of the only utility function is final wealth, constant absolute risk aversion holds only if the utility function takes on a specific functional form. According to Proposition 1, when SRS holds so that asset integration fails, constant absolute risk aversion holds for *any* functions u_0 and U in Eq. (6).

It remains to be seen whether or not constant absolute risk aversion is a desirable feature of risk preferences or not. Most classroom explanations of risk preferences contend that people exhibit declining absolute risk aversion, and many empirical papers use constant relative risk averse utility functions (in a standard expected utility framework with asset integration), and these utility functions imply decreasing absolute risk aversion. All of these assumptions and arguments are based on the premise of asset integration, though, and once asset integration fails the property of constant absolute risk aversion deserves further attention.

5 Constant absolute reallocation preferences

Since the SRS axiom implies that risk preferences exhibit constant absolute risk aversion, it is worthwhile looking back at other-regarding preferences to see if they exhibit a similar property and, if so, to determine if the property is a reasonable one. We say that the n-dimensional vector $z_{\sim 0}$ is a *reallocation vector* if $\sum_{i=1}^{n} z_i = 0$. A reallocation vector, then, is simply a plan that describes how money will be taken from some players and given to others without involving the decision-maker. Now consider the allocation $(x_0, x_0 + y_{\sim 0}) = (x_0, x_0 + y_1, ..., x_0 + y_n)$. If $z_{\sim 0}$ is a reallocation vector and the decision maker prefers the allocation $(x_0, x_0 + y_{\sim 0} + z_{\sim 0}) = (x_0, x_0 + y_1 + z_1, ..., x_0 + y_n + z_n)$ to the original allocation, we can say that he prefers the reallocation given x_0 and $y_{\sim 0}$.

The decision-maker exhibits the property of *constant absolute reallocation preference* if a preference for the reallocation $z_{\sim 0}$ given x_0 and $y_{\sim 0}$ implies a preference for the reallocation $z_{\sim 0}$ given x_0' and $y_{\sim 0}$ for any x_0'. Such a property seems reasonable. If the decision-maker is willing to take 20 away from one opponent and give it to another opponent when his own payoff is x_0, it is hard to see why he would be unwilling to do that when his own payoff is x_0'. The reallocation does not involve him, and the way it is defined it has the same effect on his payoff ranking no matter what his own payoff is, so the value of his own payoff should not matter.

Just as SRS implies constant absolute risk aversion in a risk preference setting, it implies constant absolute reallocation preference in an other-regarding setting, as shown by the next proposition.

Proposition 2. *SRS implies constant absolute reallocation preference.*

Proof. Let $z_{\sim 0}$ be a reallocation vector, and suppose that $(x_0, x_0 + y_{\sim 0} + z_{\sim 0}) \succeq (x_0, x_0 + y_{\sim 0})$. Then by part (i) of the SRS axiom, $(x_0', x_0' + y_{\sim 0} + z_{\sim 0}) \succeq (x_0', x_0' + y_{\sim 0})$ for all x_0', which is constant absolute reallocation preference. □

[8] Nielsen (2005) considers constant absolute risk aversion in a standard expected utility framework but without the standard differentiability restrictions on the utility function.

6 Conclusion

This paper presents a new preference axiom, self-referent separability, as an alternative to the standard separability axiom. The axiom is based on the notion that when deciding between two multidimensional alternatives, the decision-maker identifies one of the dimensions as a reference outcome. He then cares about not only the values of all the outcomes, but also the positions of the other outcomes relative to the reference outcome. The resulting preference representation depends on the value of the reference outcome and the differences between the other outcomes and the reference outcome.

In the context of other-regarding preferences the new axiom implies that the decision-maker's preference ordering over payoff allocations has a representation that is a nonlinear generalization of the one proposed by Fehr and Schmidt (1999). In the context of risk preferences it implies a generalization of prospect theory preferences (Kahneman and Tversky, 1979). The axiom implies that risk preferences exhibit constant absolute risk aversion, and for the same reason it implies that other-regarding preferences exhibit constant absolute reallocation preference.

The self-referent separability axiom retains some of the logic behind the standard separability axiom, but it modifies the standard axiom in a way that preserves the ordering between the component outcomes. Because of this, the approach used here is, in a sense, dual to the rank-dependent expected utility approach which also pays attention to the ordering between the component outcomes. Whereas the rank-dependent expected utility approach leads to restrictions on the probability transformation function, the approach used here leads to restrictions on the arguments of the utility functions. The two approaches are not mutually exclusive, however, and when taken together they imply a generalization of cumulative prospect theory.

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