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Economic Theory

Stock options and managerial optimal contracts*

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Summary. In this paper we are concerned with the performance of stock option contracts in the provision of managerial incentives. In our simple framework, we restrict the space of contracts available to the principal to those conformed by a fixed payment and a call option on the firm's stock. As compared to the fixed payment and the option grant, we find that the strike price plays an intermediate role in the provision of insurance and incentives. We also develop some methods for the calibration of a standard principal-agent model based upon observed CEO earnings schedules and the volatility of the firm's value in the stock market. These methods are useful to address some important issues such as the performance of stock option contracts, the degree of risk aversion compatible with current earnings profiles and the sensitivity of compensation to changes in firm's characteristics.

Keywords and Phrases: Stock option contract, Optimal contract, CEO earnings schedule, Stock price.

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1 Introduction

Most principal-agent models yield rather complex payment schedules, and it seems quite challenging to test the predictions of these models.[1] In practice there is a widespread use of simple compensation schemes such as linear or piecewise linear

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[1] Different components of compensation (e.g., base salary, bonus payments, restricted stock, stock options of various maturities, probability of termination, severance payments) taken together may define

(stock option) contracts. As is well understood (e.g., Stiglitz, 1991) these simple contracts can only be generated under some specialized assumptions that seem unsuitable for the construction of economic theories. Hence, a question that follows naturally is if simpler contracts can be optimal or nearly optimal. We address this question in a standard principal agent framework in which we evaluate the cost of restricting compensation to a family of contracts composed of a fixed salary and a call option on the company's stock. Our analysis relies on numerical techniques, since the optimal contract cannot be calculated analytically. We also develop some methods for the calibration of a principal agent model. These methods should be of independent interest.

In our simple setting, the principal represents the shareholders of a firm, and the agent plays the role of the CEO. The random outcome, realized after the agent's action choice, is interpreted as the firm's stock price. One major task is to understand the incentives provided by each component of the stock option compensation scheme. Clearly, the fixed payment discourages high effort. The strike price and the option grant[2] perform better as instruments to induce high effort, although the option grant turns out to be the most powerful instrument. Thus, a marginal increment in cost devoted to lower the strike price has a greater impact on the provision of incentives but it yields less utility than a marginal increment devoted to increase the fixed component, and it has a lesser impact on the provision of incentives but it yields more utility than a marginal increment invested in the option grant.

At a later stage in our analysis we address the issue of optimality of stock option contracts. Our main purpose is to compare the cost of the stock option contract with the cost of the optimal contract. First, some environments where the welfare loss approaches zero are discussed. As one may anticipate, if the agent is risk neutral, there exists a continuum of stock option contracts which are all first-best optimal. Moreover, in some rather limited environments the stock option contract may be the outcome of a general maximization program even if the agent is risk averse. Second, we develop a methodology for the calibration of a general principal-agent model, and we carry out several quantitative experiments to evaluate the performance of the stock option contract. The most critical issue in the calibration of a principal-agent model is the specification of the manager's effort technology, that is, the quantification of the effects on the firm's stock value of the effort exerted by the manager (cf. Haubrich, 1994; Haubrich and Popova, 1998; Margiotta and Miller, 2000). We assess these effects using a new approach based upon observed CEO earnings schedules and the volatility of the firm's stock value. This calibration exercise should be of interest for further quantitative experiments in models of agency.

In an influential paper, Jensen and Murphy (1990) claimed that observed contracts are fairly insensitive to changes in shareholders' wealth. Hall and Liebman (1998) report a higher pay-performance sensitivity in more recent years. We analyze this empirical issue using our quantitative framework. We find that observed CEO earnings schedules can be generated by plausible risk aversion coefficients,

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a very complex contract (cf. Kole, 1997; and Murphy, 1999) but isolating the mechanisms generating such contract seems an ambitious task.

[2] The option grant is the number of shares specified by the option.

which may range between 0 and 7. These numerical experiments confirm that observed pay schedules can be generated by principal-agent models. We then analyze the performance of the stock option contract. In all numerical exercises, our stock option contract fares well in cost with the optimal contract, even though both payoff profiles may take on different shapes, and may be quite distant from each other. The small extra cost associated with restricting the choice space to three-parameter contracts may account for the widespread use of stock option contracts.

Finally, we perform some other comparative numerical exercises of certain general interest. Thus, from some controlled numerical exercises it appears that fixing arbitrarily the strike price within a reasonable range does not result in a substantial additional cost of the contract. As discussed above, in the optimal provision of insurance and incentives the strike price p plays an intermediate role between the option grant q and the fixed wage T. Hence, deviations from the optimal p may be partially offset by a suitable reshuffling of both q and T. This seems key to explain why in these contracts most options are granted at the money (i.e., at the grant-date price), since these options receive a better fiscal treatment and the cost of departing from the optimal p may be minor. Also, we find that the optimal wage schedule is steeper if the gains to the agent of exerting low effort are higher and if low effort is less detrimental to the firm. These results will be useful to understand how the stock option contract varies with shifts in the mean and variance of the firm's returns, and with changes in the rank and productivity of the agent.

The rest of the present article is organized as follows. In the next section we describe our environment along with the basic assumptions about the primitives. Section 3 studies qualitative properties of the stock option contract. Section 4 presents several numerical experiments related to the calibration of the model and the aforementioned comparative analysis exercises. In Section 5 we conclude and discuss some ideas for future research. All proofs are gathered in a separate appendix.

2 The framework of analysis

The present section begins with a standard principal-agent model with fully-contingent payment schemes. This model will serve as a useful benchmark to evaluate some dimensions of the stock option contract in the provision of insurance and incentives and in a further quantitative study of its welfare properties.

2.1 A standard principal-agent model

We portray a stylized model of agency (e.g., see Mas-Collel et al., 1995, Ch. 14; Hart and Holmström, 1987) for the managing of a corporation or a production activity. The observable outcome y is assumed to be the market price of the firm's stock, and often called the value of the firm. The principal, representing the shareholders, offers a contract specifying a compensation scheme, $w(\cdot)$, and a recommended action, a. If the agent accepts the contract and chooses action a, the value of the firm is realized according to the cumulative distribution $F(\cdot; a)$. Then, the payoff of the principal is given by $y - w(y)$, and the payoff of the agent is $v(w(y)) - a$, where $v(\cdot) - a$ is the agent's utility function representing preferences over monetary payments and effort.

The details concerning the fundamentals of the contracting problem in this simple static framework are specified by the following assumptions:

Assumption 2.1 *The agent selects an action from a finite set* $A = \{a_1, \ldots, a_N\}$*, where* $a_i \in \mathbb{R}$ *for all* $i = 1, \ldots, N$ *and* $a_1 > \cdots > a_N$*.*

Assumption 2.2 *The value of the firm,* y*, is a continuous random variable, which takes on values in the interval* $Y = [\underline{y}, \overline{y}] \subseteq \mathbb{R}_+$*. For each fixed* $a \in A$*, function* $F(\cdot; a)$ *is the probability distribution of* y *with continuous density* $f(\cdot; a)$ *such that* $f(y; a) > 0$ *for all* $y \in Y$*.*

Assumption 2.3 *Strong monotone likelihood ratio property (SMLRP): For all pairs of actions* a_i *and* a_j*, if* $a_i < a_j$ *then the ratio* $f(y; a_i)/f(y; a_j)$ *is non-increasing in* y*.*

Assumption 2.4 *Concavity of distribution function condition (CDFC): For any triple of actions* a^**,* a'*, and* a''*, if* $a^* = \lambda a' + (1 - \lambda)a''$*, for* $\lambda \in (0, 1)$*, then it must hold that* $F(y; a^*) < \lambda F(y; a') + (1 - \lambda)F(y;\ a'')$ *for all* $y \in int(Y)$*.*

Assumption 2.5 *The agent's utility of consumption,* c*, and effort,* a*, is given by* $v(c) - a$*, where* $v : \mathbb{R} \to \mathbb{R}$ *is bounded, strictly increasing, strictly concave, and continuously differentiable.*

These assumptions are commonly used in the literature of principal-agent models (e.g., see Grossman and Hart, 1983), and ensure that the optimal contract is monotone in the observable outcome. Regarding Assumption 2.1, our analysis is easily extended to a countable number of actions, and it seems plausible to consider a continuum convex domain of actions. Note that in Assumption 2.2 the observable outcome y is a continuous variable. This condition will be convenient for the computation and calibration of the model. Assumptions 2.3 and 2.4 are crucial to guarantee the monotonicity of the optimal contract, and to build the analysis on the comparison of two carefully chosen actions. Finally, the separable form of the utility function in Assumption 2.5 will simplify the analysis considerably but it is not needed for most of our results.

The expected utility of the agent under compensation scheme $w(\cdot)$ and action a is defined as

$$V(w; a) = \int_Y [v(w(y)) - a] f(y; a) dy.$$

The agent will accept the contract offer and take action a if two conditions are satisfied. The first condition is the *individual rationality constraint*,

$$V(w; a) \geq \overline{v}$$

for given $\overline{v} \in \mathbb{R}$, which represents the CEO's reservation utility; and the second condition is the *incentive compatibility constraint*,

$$V(w; a) \geq V(w; a') \text{ for all } a' \in A.$$

The principal's expected payoff is defined as

$$U(w; a) = \int_Y [y - w(y)] f(y; a) dy.$$

Let a_H be the action that the principal wants to implement. Then, the optimal payment scheme is a function $w^*(\cdot)$ that solves[3] the optimization program

$$\max_w \quad U(w; a_H) \tag{P1}$$

$$s.\ t. \quad V(w; a_H) \geq \overline{v} \tag{IR}$$

$$V(w; a_H) \geq V(w; a') \text{ for all } a' \in A. \tag{IC}$$

From existing theoretical work, it is known that a multiple action framework may become intractable in many respects unless ranking conditions are imposed on the set of actions. The significance of Assumptions 2.3–2.4 is enhanced by the following result:

Proposition 2.1 *Assume that there exists a unique optimal action $a_H = a_j > a_N$ for (P1). Then, the optimal contract $w^*(\cdot)$ is unique and nondecreasing in y. Moreover, constraint (IC) only binds for action a_{j-1}.*

Let $a_L = a_{j-1}$ be the next most costly action after a_H. An interior solution must satisfy the following first-order condition:

$$\frac{1}{v'(w^*(y))} = \gamma + \phi \left[1 - \frac{f(y; a_L)}{f(y; a_H)} \right] \tag{2.1}$$

where $\gamma > 0$ is the multiplier on constraint (IR), and $\phi > 0$ is the multiplier on constraint (IC). By the strict concavity of v, from (2.1) the compensation scheme can be expressed as a function of the multipliers and the likelihood ratio:

$$w^*(y) = (v')^{-1} \left(\left[\gamma + \phi \left[1 - \frac{f(y; a_L)}{f(y; a_H)} \right] \right]^{-1} \right). \tag{2.2}$$

2.2 *The stock option contract*

We now restrict the principal to use compensation schemes consisting of a fixed payment and a call option on the firm's stock. A stock option contract $\sigma = (p, q, T)$ specifies the strike price p, the option grant q, and the fixed payment T. We assume that $p \in Y$, $q \geq 0$, and that the fixed component T of the payment schedule is bounded below by some number $\underline{T} \in \mathbb{R}$.[4] Hence, the agent's payoff is $v(T + q(y - p)^+)$ where $(y - p)^+ = max(0, y - p)$.

[3] See Page (1987) for a rigorous discussion on the existence of an optimal contract in principal-agent models with continuous random variables.

[4] The possibility of a negative fixed component will allow us to approximate certain situations discussed below in which the CEO has a positive initial wealth, and for some regions the payment schedule may in fact take on negative values following a fall in price of the CEO's stock holdings. In what follows we suppose that the optimal choice involves $T > \underline{T}$. Of course, certain interesting contractual issues arise when the constraint $T \geq \underline{T}$ is binding.

Let $V(\sigma; a)$ be the expected utility of the agent under compensation scheme σ and action a, and let $U(\sigma; a)$ be the expected utility of the principal. Let a_S be the action that the principal wants to implement over the space of all feasible stock option contracts. An optimal stock option contract $\widetilde{\sigma}$ solves the optimization program

$$\max_{\sigma} \quad U(\sigma; a_S) \tag{P2}$$

$$s.\ t. \quad V(\sigma; a_S) \geq \overline{v} \tag{IR'}$$

$$V(\sigma; a_S) \geq V(\sigma; a') \text{ for all } a' \in A \tag{IC'}$$

$$p \in Y, \quad q \geq 0, \text{ and } T \geq \underline{T}.$$

Remark 2.1 As is typical in the principal-agent literature, in the sequel we assume that a feasible σ exists. Then we can show (cf., Aseff and Santos, 2002) that the maximization actually involves a continuous function $U(\sigma; a_S)$ on a compact set. Therefore, $(P2)$ always has a solution. Moreover, if $a_S > a_N$, then $q > 0$ and $p < \overline{y}$; for if not the contract becomes constant and (IC') is violated.

The monotonicity embedded in the stock option contract implies that only one incentive compatibility constraint is binding.

Proposition 2.2 *Assume that there exists a unique optimal action $a_S = a_i > a_N$ for (P2). Then, at an optimal contract $\widetilde{\sigma}$ constraint (IC') binds for action a_{i-1} only.*

One should realize that the optimal action a_H for $(P1)$ may not be equal to the optimal action a_S for $(P2)$. The next result, however, provides an upper bound for the welfare loss of the stock option contract. Let L be the extra cost to the principal of restricting the decision space to stock option contracts over the set of all actions A; more precisely, from $(P1)-(P2)$ above let $L = U(w^*; a_H) - U(\widetilde{\sigma}; a_S)$. Also, let $\widehat{L}$ denote the corresponding extra cost when the optimization is confined to the action set $\widehat{A} = \{a_H, a_L\}$; by virtue of Proposition 2.1, $\widehat{L} = U(w^*; a_H) - U(\widehat{\sigma}; a_H)$, where $\widehat{\sigma}$ solves the version of $(P2)$ in which only actions a_H and a_L are considered.

Proposition 2.3 *Under all the above conditions, $\widehat{L} \geq L$.*

Hence, it follows from Proposition 2.3 that the welfare loss for the principal from solving $(P2)$ under actions a_H and a_L is an upper bound of the true loss of restricting the decision space to stock option contracts over all possible actions. In what follows the analysis will be confined to actions a_H and a_L.

3 Characterization of the optimal stock option contract

If the agent is risk neutral, it is well known that the first-best solution (i.e., a solution in which the principal extracts the maximum feasible surplus) is to sell the firm to the agent. With stock options, such contract can always be realized; i.e., set $p = \underline{y}$, $q = 1$ and solve for T from the individual rationality constraint.

Furthermore, under risk neutrality there is usually a continuum of first-best stock option contracts (see Aseff and Santos, 2002), since the agent values only the overall expected revenue of the contract, independently of the relative size of the fixed and contingent components. The main goal of this section is to explore the role played by each variable defining the stock option contract in the provision of insurance and incentives for a risk averse agent.

Let us first recast $(P2)$ under actions a_H and a_L in the framework of the Kuhn-Tucker conditions. Let λ be the multiplier on constraint (IR'), μ the multiplier on constraint (IC'), and φ the multiplier on constraint $p \geq \underline{y}$. Note that by Remark 2.1 constraints $p \leq \overline{y}$ and $q \geq 0$ need not be considered. For $T > \underline{T}$, we now write the Lagrangean

$$\mathcal{L}(\sigma, \lambda, \mu, \varphi) = U(\sigma, a_H) + \lambda \left[V(\sigma; a_H) - \overline{v}\right] + \mu \left[V(\sigma; a_H) - V(\sigma; a_L)\right] + \varphi(p - \underline{y}).$$

Differentiating with respect to p, q, and T, after some simple rearrangements we obtain the following necessary conditions for a solution[5]

$$\frac{U_p(\sigma; a_H)}{V_p(\sigma; a_H)} + \lambda + \mu \left[1 - \frac{V_p(\sigma; a_L)}{V_p(\sigma; a_H)}\right] + \varphi \frac{1}{V_p(\sigma; a_H)} = 0 \tag{3.1}$$

$$\frac{U_q(\sigma; a_H)}{V_q(\sigma; a_H)} + \lambda + \mu \left[1 - \frac{V_q(\sigma; a_L)}{V_q(\sigma; a_H)}\right] = 0 \tag{3.2}$$

$$\frac{U_T(\sigma; a_H)}{V_T(\sigma; a_H)} + \lambda + \mu \left[1 - \frac{V_T(\sigma; a_L)}{V_T(\sigma; a_H)}\right] = 0. \tag{3.3}$$

These conditions should be familiar from related models of risk sharing and agency (cf. Borch, 1962; Holmström, 1979), and reflect the usual trade-offs in the provision of insurance and incentives.

Proposition 3.1 *The derivatives of U and V must satisfy the following properties*

$$\left|\frac{V_T(\sigma; a_H)}{U_T(\sigma; a_H)}\right| \geq \left|\frac{V_p(\sigma; a_H)}{U_p(\sigma; a_H)}\right| > \left|\frac{V_q(\sigma; a_H)}{U_q(\sigma; a_H)}\right|. \tag{3.4}$$

Suppose that the principal wants to increase marginally the expected cost of the contract. Consider first an infinitesimal increase in T. The surplus of the principal decreases by $U_T(\sigma; a_H)$. Thus, $\left|\frac{V_T(\sigma; a_H)}{U_T(\sigma; a_H)}\right|$ denotes the change in the agent's marginal utility after an infinitesimal increase in T. Alternatively, the principal could devote this marginal increase in cost to lower the strike price. In this case, the effect on the agent's marginal utility is $\left|\frac{V_p(\sigma; a_H)}{U_p(\sigma; a_H)}\right|$. In the same way $\left|\frac{V_q(\sigma; a_H)}{U_q(\sigma; a_H)}\right|$ represents the marginal utility of an infinitesimal increase in the cost of the contract invested in q. The inequalities indicate that for a marginal rise in compensation the agent would prefer an increase in T to a reduction in p or an increase in q. The

[5] Subscripts denote partial derivatives of functions U and V with respect to $i \in \{p, q, T\}$.

more interesting feature is that the agent prefers a lower p to a larger q. Essentially, if p goes down, then the agent's compensation goes up uniformly over the set $E = \{y : y \geq p\}$; but if q goes up, then the gains are slanted towards the high end of the distribution. Therefore, for the same marginal increase in compensation a risk averse agent will prefer a reduction in p.

There are two other aspects of the characterization of the stock option contract that we would like to address. The first one relates to the values of λ, μ, and φ. The second aspect relates to the incentives provided by the different components of the compensation scheme.

Proposition 3.2 *At an optimum,* (i) $\lambda > 0$*, which implies that constraint* (IR') *will be binding;* (ii) $\mu > 0$*, which implies that constraint* (IC') *will be binding; and* (iii) $\varphi = 0$.

Parts $(i) - (ii)$ of this proposition seem a minimal requirement for the specification of contracts with good optimality properties and facilitate the characterization of the optimal stock option contract from the first-order conditions. Concerning (iii), note that $\varphi = 0$ regardless of whether the constraint $p \geq \underline{y}$ binds. Hence, relaxing the constraint by a very small amount (i.e., a small change in the distribution of the company's returns that lowers marginally the bound $\underline{y}$) will not yield extra profits to the principal, since at point $p = \underline{y}$ an infinitesimal decrease in p can be fully offset by a corresponding reduction in T [see (3.1) and (3.3)].

Finally, the provision of incentives is reflected in expressions (3.1)–(3.3) in all the ratios of the form $[1 - V_i(\sigma; a_L)/V_i(\sigma; a_H)]$ for $i \in \{p, q, T\}$. With regard to the fixed payment T, in the proof of Proposition 3.2 we show that $V_T(\sigma; a_H) < V_T(\sigma; a_L)$, and thus $[1 - V_T(\sigma; a_L)/V_T(\sigma; a_H)] < 0$. Therefore, an increment in T discourages high effort. Indeed, a larger T increases the payoff of the agent in all states, and as a consequence of risk aversion and stochastic dominance, it yields a larger marginal value under action a_L. Concerning the strike price p, as already pointed out the effects of a decrease in p near $\underline{y}$ can be nullified by a corresponding increase in T. Hence, the continuity of V_p implies that $[1 - V_p(\sigma; a_L)/V_p(\sigma; a_H)] < 0$ for p sufficiently close to $\underline{y}$. In the opposite situation, for p near $\overline{y}$ a marginal unit spent in lowering p confines all extra wage gains at the high-end of the distribution. Therefore, $[1 - V_p(\sigma; a_L)/V_p(\sigma; a_H)] > 0$ for p sufficiently close to $\overline{y}$, since $f(y; a_H) > f(y; a_L)$ by Assumption 2.3. Similarly, one may expect that changes in the option grant may have indeterminate effects in the provision of incentives. And, indeed, we have not been able to rule out the possibility that in cases of extreme risk aversion, for $p = \underline{y}$ an increase in the option grant may lead to the implementation of action a_L.[6] We nevertheless show that if $[1 - V_p(\sigma; a_L)/V_p(\sigma; a_H)] \geq 0$ then $[1 - V_q(\sigma; a_L)/V_q(\sigma; a_H)] > 0$ (Proposition 3.3 in the Appendix). Also, $[1 - V_q(\sigma; a_L)/V_q(\sigma; a_H)] \geq 0$ if the coefficient of relative risk aversion is always less than or equal to one (Proposition 3.4 in the Appendix). These results corroborate the role of the option grant as the most powerful instrument in the provision of incentives.

[6] Observe that the expression, $V_q(\sigma; a_H) - V_q(\sigma; a_L) = \int_E v'(q(y-p)+T)(y-p)[f(y; a_H) - f(y; a_L)]dy$, is highly non-linear.

4 A quantitative analysis

Will the stock option contract be optimal or nearly optimal? Under some rather restrictive conditions, problem $(P1)$ may generate a piecewise linear contract. For instance, assume that the utility function $v(c) = \log(c)$ and the term $1 - \frac{f(y;a_L)}{f(y;a_H)}$ has the form $\max\{-k_0, (y - k_1) - k_0\}$ for some constant $k_0 > 0$ and some point $k_1 \in Y$. Then (2.1) implies that the solution $w^*(\cdot)$ takes the form of a stock option contract. Analogous conditions can be imposed on the likelihood ratio for the more general class of CES utility functions.[7] Since these conditions on the likelihood ratio are rather fragile, we assess the optimality of stock option contracts numerically. We find that observed CEO payment profiles can be generated under plausible coefficients of risk aversion; moreover, the additional costs of implementing the stock option contract are always negligible. We also consider some other comparative exercises on the structure of the stock option contract for changes in the rank and productivity of the manager and the distribution of the firm's value.

4.1 Calibration of a standard principal-agent model

Some previous work on the calibration of models of agency (e.g., Haubrich, 1994; Haubrich and Popova, 1998; Robe, 1999) postulate functional forms for the manager's contribution to the company. A major objection to these studies is that there is a lack of evidence about the form of $f(\cdot; a_H)$ and $f(\cdot; a_L)$; moreover, the shape of the optimal contract is very sensitive to the functional forms of these densities and the assigned parameter values (e.g., Haubrich and Popova, 1998). Our calibration imposes no functional restrictions on these technologies. In the absence of reliable data to estimate these densities, we introduce independent estimates for all the other primitives of the model. Then, from a postulated optimal wage schedule, we infer the likelihood ratio $\frac{f(y;a_L)}{f(y;a_H)}$ from optimality conditions. Hence, the likelihood ratio is obtained indirectly from available data on all other specifications of the model. Margiotta and Miller (2000) propose a related estimation procedure but their generalized method of moments estimator is defined under a restrictive utility function and a parametric form for the likelihood ratio. Also, their data set is quite limited.

(A) *Calibration of density* $f(\cdot; a_H)$: As in our previous analysis we presuppose that the shareholders offer an incentive scheme to implement action a_H. If our principal-agent model is intended to predict observed earnings schedules, it seems then pertinent to impose the identifying restriction that the probability distribution $F(\cdot; a_H)$ mimics the observed distribution of returns of a certain representative firm. Hence, density $f(\cdot; a_H)$ should be estimated from a proposed distribution of stock price returns. But even when averaged out over decades, equity returns have

[7] In insurance models Holmström (1979) shows that there are scenarios in which pure deductible contracts (i.e., option-like contracts) are optimal. For a similar result to hold in our context, a mass point at $\overline{y}$ is needed. Also, the agent's action should only affect the probability of departing from $\overline{y}$, and not the magnitude by which y departs from $\overline{y}$. Such an environment is clearly unattractive in a managerial compensation framework.

fluctuated wildly (cf., Mehra and Prescott, 1985, Table 1), and so any proposed distribution of returns can just be considered as a rough approximation of the volatility of a firm's stock. Numerous studies have gathered observed distributions of returns for various samples of firms and time periods (e.g., Fama. 1976, Ch. 1; Hull, 1993, Ch. 11). Here we follow Hall and Liebman (1998, Table V) who report the nine decile cutoffs of observed annual returns corresponding to the pool of firms in the S&P 500 for the period 1970 to 1994. Then, using some simple bootstraping procedures we construct density $f(\cdot; a_H)$ defined over a selected domain $[0.55, 1.70]$ of possible annual stock price returns. This density function is depicted as the solid line in Figure 1. The mean of this distribution is equal to 1.0811 and the standard deviation is equal to 0.2750. These values fall roughly within the accepted region of various empirical studies on annual stock price returns.

(B) $The\ optimal\ contract$: For the calibration of our model we identify the optimal contract with a payments profile constructed by Hall and Liebman (1998). Under the observed distribution of returns from corporate proxy statements and 10-K forms on CEO's holdings of stock and stock options these authors compute compensation schedules for a median manager for the years 1980 and 1994. We should remark that these earnings profiles are not observed compensation schemes, since they have been simulated from information of median salary, bonus and CEO's company portfolios as applied to the hypothesized distribution of returns. In 1994 dollars, this representative CEO would get an expected annual compensation of about $968,269 in 1980, and $3,695,540 in 1994. Hall and Liebman compute the gains from stocks and stock options with the Black and Scholes formula. However, these gains are not liquid and thus may be less valuable to a risk averse manager. To correct for this bias all deferred compensation is scaled down by 0.85.[8] Also, in our analysis below we assume that preferences are defined over earnings and initial wealth. Then, following related back-of-the-envelope calculations by these authors we consider that in real terms the initial wealth for the median CEO is $10,000,000 in 1980 and $20,000,000 in 1994. The solid lines in Figure 2 display the wage-over-wealth schedules for both 1980 and 1994. Observe that relative earnings are close to zero for low realizations of returns, and they represent a substantial fraction of estimated wealth for high realizations. As discussed by these authors the 1994 schedule is steeper, and this is reflected in more extreme values at both ends.

(C) $Calibration\ of\ the\ utility\ function$: The following procedures are used to specify the parameter values corresponding to the agent's utility of consumption and disutility of effort. First, the value a_H is determined from an optimization condition in which the marginal disutility of effort must be equal to its marginal benefit. More specifically, for the above utility function $v(c) - a$ let $a = l\alpha$ where

[8] This adjusting factor is derived from several calculations based on vesting periods of stock option contracts, and standard values for the interest rate and coefficients of risk aversion. This linear approximation is exact for a CES utility function in which the initial wealth and the fixed payment are equal to zero. In our static model, this approximation is not exact since we aggregate all the components of the contract.

l is quantity of time worked and α is the marginal disutility. Then,

$$\int_Y v'(lw(y))w(y)f(y;a_H)dy = \alpha. \tag{4.1}$$

Assuming that the solution is attained at $l = 1$, we get α from (4.1); furthermore, $a_H = \alpha$ and $\overline{v} = \int_Y v(w(y))f(y;a_H)dy - a_H$. Second, the value of the less desirable action a_L is taken from Margiotta and Miller (2000, Table 7). They estimate a three-action model and obtain that $a_H/a_L = 1.172$. Finally, for the utility of consumption we postulate the following functional form

$$v(c) = \begin{cases} \log(c+m) & \text{if } \rho = 1, \\ \dfrac{(c+m)^{1-\rho}-1}{1-\rho} & \text{otherwise,} \end{cases}$$

where m denotes the initial wealth of the agent, and without loss of generality will be normalized to unity. Hence, c will be interpreted as the ratio of earnings over initial wealth. We let the risk aversion coefficient $\rho = 1$, which is within the range of feasible estimates considered in the finance literature (e.g., Mehra and Prescot, 1985).

(D) $\textit{Derivation of density}$ $f(\cdot;a_L)$: This density is not observable but may be recovered from $(IR)-(IC)$ and the first-order condition (2.1) using our estimates of $f(\cdot;a_H)$, the relative compensation schedule $w(\cdot)$, and the utility function $v(c) - a$. Thus,

$$f(y;a_L) = \left[1 - \frac{[v'(w(y))]^{-1} - \gamma}{\phi}\right] f(y;a_H). \tag{4.2}$$

Therefore, for any (γ, ϕ) we obtain $f(\cdot;a_L)$ from (4.2). Since $f(\cdot;a_L)$ is a probability density function, by condition (IC) it follows that

$$\int_Y \left[1 - \frac{[v'(w(y))]^{-1} - \gamma}{\phi}\right] f(y;a_H)dy = 1 \tag{4.3}$$

$$\int_Y v(w(y)) \left[1 - \frac{[v'(w(y))]^{-1} - \gamma}{\phi}\right] f(y;a_H)dy = a_L + \overline{v}. \tag{4.4}$$

We can now obtain the pair (γ^*, ϕ^*) that solves the system of equations (4.3)-(4.4) from the following expressions,

$$\gamma^* = \int_Y [v'(w(y))]^{-1} f(y;a_H)dy \tag{4.5}$$

$$\phi^* = \frac{\int_Y v(w(y)) \left[[v'(w(y))]^{-1} - \gamma^*\right] f(y;a_H)dy}{a_H - a_L}. \tag{4.6}$$

(E) $\textit{A consistency test}$: This procedure does not guarantee that $f(y;a_L) > 0$ for all $y \in Y$. Since $v(\cdot)$ is a concave function, expression (4.5) implies that $[v'(w(y))]^{-1} - \gamma^* > 0$ for large values of y; hence, by (4.2) we may have that

$f(y; a_L) < 0$. If for a given compensation schedule $w(\cdot)$, function $f(y; a_L)$ takes on negative values over a certain region of the domain, then we may conclude that under the postulated functional forms for the observed density $f(\cdot; a_H)$ and the utility function $v(\cdot) - a$, contract $w(\cdot)$ cannot be generated by problem $(P1)$. This is a rather weak consistency test but we are unwilling to impose further restrictions on $f(\cdot; a_L)$ as such density is not observable.

A key parameter in this calibration exercise is the agent's degree of risk aversion, ρ. In fact, following the above steps we can search for the feasible region of degrees of risk aversion ρ which may generate the given earnings profile, i.e., the range of values of $\rho > 0$ under which $f(y; a_L) \geq 0$ for all $y \in [0.55, 1.70]$. We find that the compensation schedule[9] of 1994 is compatible with values of ρ in the interval $(0, 7.34]$ and the less steep compensation schedule of 1980 is compatible with values of ρ in the interval $(0, 15.2]$. This simple test confirms that observed CEO compensation schedules can be explained by models of agency.

Before closing this part, let us offer a more detailed comparison of these methods to those of Haubrich and Popova (1998) whose analysis seems closest to ours. Technically, Haubrich and Popova propose an estimation procedure in which parameter values are determined as solutions to the minimization of the squared sum of errors between model-predicted data and data from a sample of firms, and matching data's first and second moments. Our parameters are calibrated from a median CEO's payment schedule and further independent evidence. Also, Haubrich and Popova map states and probabilities to company payoffs in a way that seems quite close to ours. These are, however, some minor differences. The main point of departure is in the calibration of the manager's effort technology. We back out this technology in a non-parametric way from the first-order conditions of the decision problem subject to the above consistency condition. Given the lack of independent evidence, this procedure seems very reasonable. In contrast, Haubrich and Popova search over various arbitrary functional forms until they consider that a solution is adequate.[10]

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[9] To evaluate variations in the risk aversion coefficient, a more satisfactory approach would be to consider a utility function of the following form,

$$\frac{[v(c) - a]^{1-\rho} - 1}{1 - \rho}$$

All results reported below did not experience substantial variations under the following alternative functional form,

$$\frac{[\log(c+1) - a]^{1-\rho} - 1}{1 - \rho}.$$

[10] The limitations of this analysis are actually acknowledged by the authors. Thus, one can read (see Haubrich and Popova, 1998, p. 564)

> Of central concern here is the proper specification of the $\pi_i(a)$ function: Measuring CEO's contribution to the firm is the most problematic aspect of calibrating the principal-agent problem. The available evidence on how CEOs affect firm value is rare... Finding an intuitively appealing specification proved difficult. Even after restricting the search to probability structures that satisfy the "spanning condition",... many structures had only degenerate feasible solutions or required implausible CEO productivity.

Moreover, in page 566 (see op. cit.) one also reads: "This admittedly falls short of the the ideal calibration, which would optimize over functional forms as well as parameters, but the experiment was enough to establish some level of robustness..."

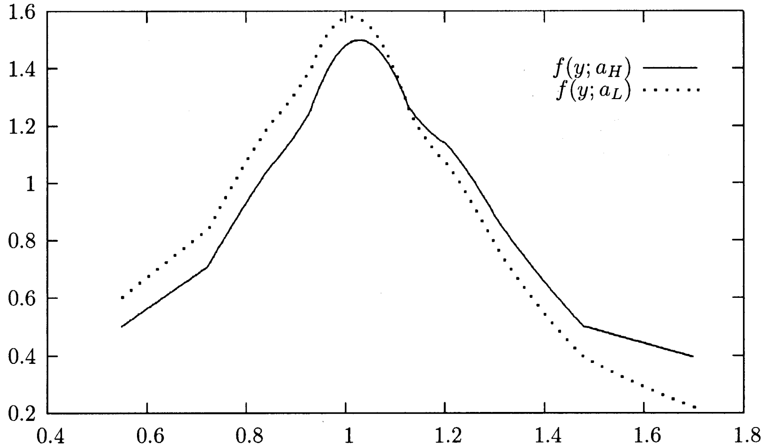


Figure 1. Densities $f(y; a_H)$ and $f(y; a_L)$

4.2 The performance of stock option contracts: a baseline parameterization

For the above density $f(\cdot; a_H)$, and the relative compensation schedule of year 1994, we now focus on our baseline parameterization in which $\rho = 1$, and a_H and a_L are obtained from the above calibration methods. Using this parameterization, we examine the implied costs of agency and the distribution of returns under action a_L. Then we consider several numerical exercises to assess the performance of stock option contracts.

We first compute the optimal flat wage that the CEO would receive in the absence of agency problems. That is, the constant payment $\overline{w}$ under the same a_H and $\overline{v}$ such that

$$\int_Y v(\overline{w}) f(y; a_H) dy - a_H = \overline{v}. \tag{4.7}$$

We find that $\overline{w} = 0.155705$. Since the expected value of $w(\cdot)$ under $f(\cdot; a_H)$ is equal to 0.164909, our model predicts that agency problems increase expected compensation by approximately 5.9%. Note that this figure is obtained from an evaluation of the manager's earnings profile, and ignores supervision and coordination activities and other measures that may mitigate the costs of moral hazard. Haubrich (1994), Haubrich and Popova (1998), Margiotta and Miller (2000) and Robe (1999) present alternative analyses of the importance of moral hazard, but as already pointed out their calibration exercises are substantially different.

Next, from (4.2)–(4.6) we compute the implied density function $f(\cdot; a_L)$. This density is also portrayed in Figure 1. The mean of this distribution is equal to 1.04047 and the standard deviation is equal to 0.263295. As compared to $f(\cdot; a_H)$, the mean for the new distribution drops by approximately 4%. Considering that in our sample the value of a median company in the S&P 500 is of about 2 billion dollars, we have that the expected loss for the shareholders of implementing action a_L would be around 80 million dollars. Furthermore, under the above parameter values we can

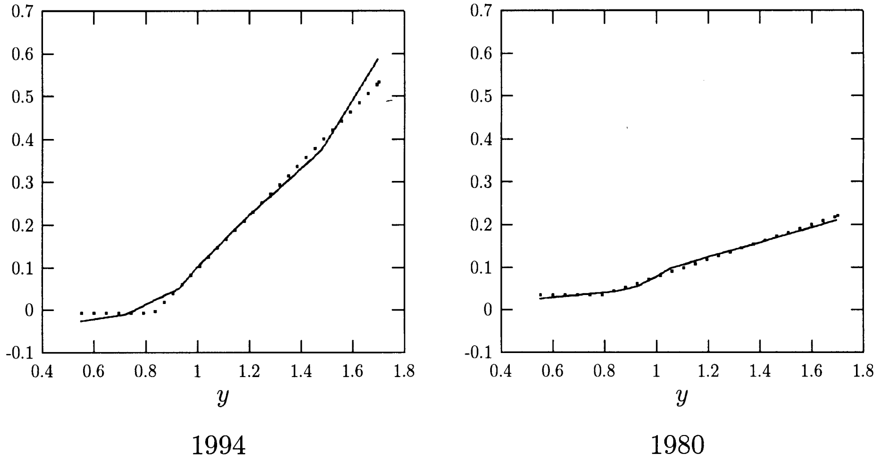


Figure 2. Observed earnings profiles and stock option contracts

easily compute from (4.7) that the extra gain for the shareholders of implementing action a_L would amount to a 23.9% drop on the expected wage, which is about \$788,872. Hence, our simple model is able to rationalize a commonly disputed paradox in agency theory that a relatively small change in the manager's expected pay may result in a large gain to the shareholders. This gain to the shareholders is over 100 times greater than the cost.

By construction, for our baseline parameterization and above densities $f(\cdot; a_H)$ and $f(\cdot; a_L)$, the solution to $(P1)$ is the payment schedule of 1994 normalized by the initial wealth. We now evaluate the performance of the stock option contract by computing the solution to $(P2)$ under the same conditions. This is the stock option contract depicted in the left panel of Figure 2. The first row of Table 1 reports values for p, q and T and the costs of the stock option contract and the optimal contract. Restricting the space of contracts to a fixed salary and a call option does not increase the cost of compensation significantly. The upper bound $\widehat{L} = C(\sigma; a_H) - C(w; a_H)$ from Proposition 2.3 is of order of 0.0478%.

From the first row of Table 1, we can observe that the fixed component of the contract, T is actually negative. In our static setting in which all components of compensation are aggregated, a negative T means that pronounced falls in the value of the stock may lead to actual losses in the CEO's total compensation. Also, note that $q = 0.618804$. Given that our estimate for an initial wealth of 20 million dollars has been normalized to unity, a stock value of 2 billion dollars for a representative company in S&P 500 would imply that an increase of \$1,000 in value to the shareholders would result in an extra gain of \$6.19 for the CEO. This latter estimate is of course fairly close to the ones in Hall and Liebman (1998), since our stock option contract is a fairly good piecewise linear approximation of their simulated payment schedule (cf. Fig. 2).

In our baseline parameterization, the value of the strike price, $p = 0.827586$, is substantially below the current value of the firm, which has been normalized to unity. In our view, there are at least two plausible explanations as to why in practice

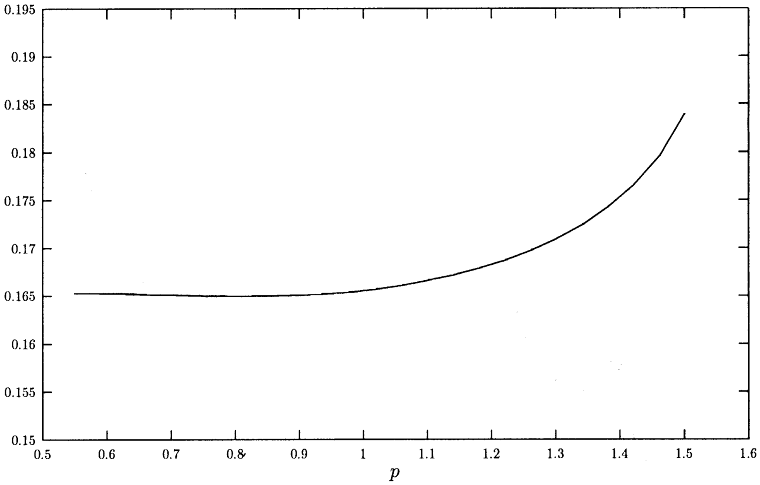


Figure 3. Cost of compensation as a function of p

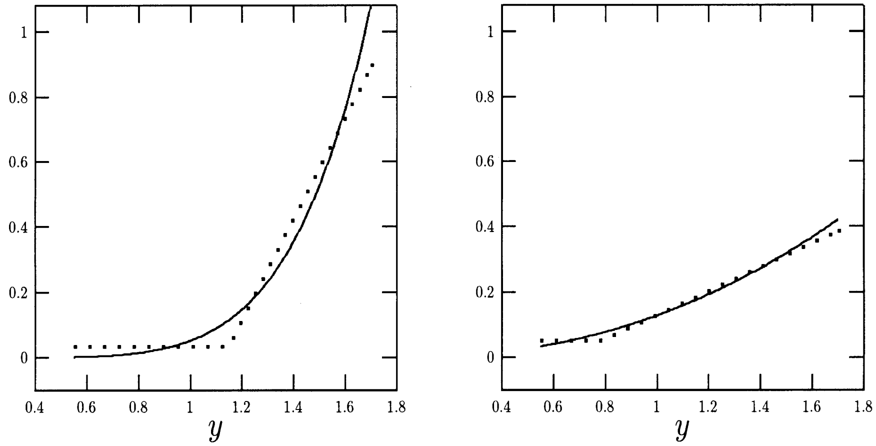
options are usually granted at the money. First, for each p let $C(p, q(p), T(p); a_H)$ be the cost of a stock option contract that satisfies $(IC') - (IR')$. This function is depicted in Figure 3. As one can see, the function contains a rather flat basin around the point of minimal cost, which seems to confirm a point stressed in this paper that in the provision of insurance and incentives p plays an intermediate role between q and T. Hence, controlled changes in p will result in relatively small incremental costs, since the variation in p may be partially offset by countervailing changes in q and T. Second, in a real world situation, a deviation from the optimal strike price may be supported by other compensation mechanisms. For instance, a package of stock options granted at the money may be complemented with a bonus policy so that in effect the strike price is below the actual price of the share.

The same computational experiment has been replicated under the compensation scheme of 1980 (see Fig. 2) with similar results concerning the extra cost of restricting the optimization to a simple stock option contract. As pointed out in Hall and Liebman (1998), the 1980 contract is less sensitive to changes in the stock value. With respect to our previous exercise the most noticeable change is the value of q that drops to 0.204442.

4.3 Constant elasticity contracts

The shape of the contract in Hall and Liebman (1998) is relatively simple and hence the good performance of the stock option contract should not be so surprising. Then, it seems useful to test the robustness of the good performance of stock options against the general class of constant elasticity contracts, $w_\theta(y) = By^\theta$. Parameter θ represents the elasticity of the compensation scheme with respect to the value of the firm. Parameter B is used to set the level of compensation.

To determine the value of θ we use information on the fraction of compensation that corresponds to fixed wages (cf. Murphy, 1999). Let κ denote the fraction of



$\kappa = 0.01$ $\kappa = 0.20$

Figure 4. Constant elasticity and stock option contracts

compensation attributed to fixed wages. Then,

$$\kappa \int_Y y^\theta f(y; a_H) dy = \underline{y}^\theta$$

Thus, parameter θ is univocally determined by κ. In fact, θ moves inversely with κ; a convex contract $w_\theta(y)$ is obtained for low values of κ, and a concave contract $w_\theta(y)$ entails a high enough value for κ. Regarding B, it is normalized so that $E[By^\theta; a_H] = E[w(y); a_H)]$. Therefore, θ is defined from κ, and B is determined by the level of expected pay.

Our numerical experiments are as follows. We vary parameter θ and leave unchanged density $f(\cdot; a_H)$ and $\rho = 1$. Then, we recover values for $\overline{v}$, a_H, a_L, and density $f(\cdot, a_L)$ using the computational procedure (4.1)-(4.6).[11] We employ this procedure for the cases in which 1%, 5%, and 20% of compensation is fixed; i.e., for $\kappa = 0.01$, $\kappa = 0.05$, and $\kappa = 0.20$. Even in the extreme case in which $\kappa = 0.01$ the extra cost of implementing action a_H under the stock option contract is of the order of 0.45%. This is a relatively small deviation in cost, although as illustrated in the left panel of Figure 4 this latter contract may be quite distant from the postulated optimal non-linear contract.

4.4 Comparative statics

We now explore numerically how the stock option contract varies with changes in the model's primitives. These findings may hold analytically under further specific

[11] Hence, with respect to the preceding analysis, the interpolated contract $w(y)$ from Hall and Liebman is now replaced by contract w_θ.

Table 1. Changes in the reservation utility

p	q	T	$\overline{v}$	$C(\sigma; a_H)$	$C(w(\cdot); a_H)$
0.827586	0.618804	−0.006723	0.009983	0.164979	0.164909
0.827586	0.615724	−0.011669	0.004991	0.159174	
0.827586	0.625009	0.003243	0.019966	0.176663	

Table 2. Shifts in the distribution

λ	p	q	T	$C(\sigma; a_H)$
0.3	0.827586	0.897756	−0.074251	0.174846
0.5	0.827586	1.281470	−0.161775	0.193789
0.7	0.827586	2.224275	−0.352230	0.264932

Table 3. Changes in the effort ratios

a_H/a_L	p	q	T	$C(\sigma; a_H)$
1.002	0.827586	0.008051	0.153474	0.155707
1.080	0.827586	0.306487	0.072993	0.158033
1.172	0.827586	0.618804	−0.006723	0.164974
1.260	0.827586	0.882907	−0.070740	0.174237

assumptions, but our main interest is to offer some simple illustrations on the following empirical issues: the widespread use of stock options for non CEO workers and the more prevalent use of stock options in riskier companies.

(I) *Changes in the reservation utility* $\overline{v}$: Table 1 displays three examples in which the structure of compensation varies with the level of $\overline{v}$. All other parameters are fixed. We can see that an increase in $\overline{v}$ results in higher q and T with no noticeable change in p. As already discussed controlled variations in p do not have a first-order effect on utility, and so p may be assumed constant for present purposes.

(II) *Changes in density* $f(\cdot; a_L)$: Table 2 reports a similar experiment for variations in $f(\cdot; a_L)$. We let this density vary over the parameterized family of functions $f_\lambda(y; a_L) = \lambda f(y; a_H) + (1-\lambda) f(y; a_L)$. Note that lower λ-values relax constraint (IC), and action a_L becomes more detrimental to the firm. Then, the shareholders can afford to offer a flatter, less costly wage profile, since action a_L is also more harmful to the agent. Another way to see this result is that for lower λ-values it is easier for the shareholders to infer the action taken by the agent.

(III) *Changes in the value of shirking*: Monitoring and supervision are more prevalent for non-executives. An executive usually enjoys more autonomy in the use of the company resources; hence, shirking may be substantially more attractive for higher-level employees. In their quantitative study, Margiotta and Miller (2000) obtain a point estimate $a_H/a_L = 1.172$ for a CEO worker, and $a_H/a_L = 1.002$ for a non CEO worker. As we let a_L move closer to a_H we are relaxing constraint (IC) and so the payment schedule should be flatter. This is illustrated in Table 3.

We can now combine (I)–(III) to evaluate whether stock options should be granted to non-executives. Several authors (e.g., Core and Guay, 2001; Hall and Murphy, 2003; Oyer and Schaefer, 2003) have argued that stock options are an expensive way to provide incentives to these employees, and have suggested some other reasons such as retention for the widespread use of stock option contracts. Their arguments, however, ignore some of the above considerations.

As shown above, in our calibrated model the differential gain to the shareholders of taking action a_H, instead of a_L, is over 100 times greater than the extra cost to induce the manager to take such action. In view of this productivity gap between the two actions, it may be worth granting options to less influential workers. Indeed, from computations along the lines of Table 2 we can tell that stock options will be granted for values of λ up to 0.98, which means that these options should be awarded for a wide range of levels of workers' efficiency. The problem is that as noted in (II) for higher λ-values the compensation schedule becomes steeper, and hence the model would predict that less productive workers should receive a higher fraction of their pay in stock options. This is counterfactual (e.g., see Oyer and Schaefer, 2003), since stock options have less weight in the compensation of non-executives. But apart from changes in the reservation utility $\overline{v}$, an executive may enjoy privileges and fringe benefits from shirking that are not available to a middle manager or a lower level employee. As discussed in (III), Margiotta and Miller (2000) obtain that $a_H/a_L = 1.172$ for a CEO worker and $a_H/a_L = 1.002$ for a non CEO worker. Hence, the shareholders should be less concerned with the provision of incentives to non-executives and should offer less variable compensation. For illustrative purposes consider an employee for which the productivity gap between actions a_H and a_L is only 10 percent of our original calibration so that for $f_\lambda(\cdot; a_L)$ defined above we let $\lambda = 0.9$; also, assume that for such an employee $a_H/a_L = 1.01$, which is one order of magnitude higher than the estimate obtained by Margiotta and Miller. Letting constant all other calibrated values, we now get that $p = 0.827586$, $q = 0.412382$ and $T = 0.0444628$. Hence, as compared to our original baseline calibration, the present contract involves a lower option grant. These computations highlight the crucial role played in this analysis by the agent's differential utility of actions a_H and a_L, and the differential gains to the shareholders from densities $f(\cdot; a_H)$ and $f(\cdot; a_L)$.

In the agency model of Holmströn and Milgrom (1987) the slope of the optimal contract varies inversely with the variance of the firm's returns. But many "new economy" firms are highly volatile and rely heavily on stock options for compensating their employees. As seen from (2.2), in our principal-agent model the shape of the optimal contract depends on the likelihood ratio $f(y; a_H)/f(y; a_L)$ rather than on $f(\cdot; a_H)$ directly. As the stock option contract can be viewed as an approximation of the optimal contract, then the structure of the stock option contract will also be expected to depend on the likelihood ratio $f(y; a_H)/f(y; a_L)$. To illustrate these complex effects of uncertainty on the fundamentals, several numerical exercises are reported in Table 4. For simplicity, in all these experiments we let $p = 1$. In the first three rows we increase progressively the mean of density $f(\cdot; a_H)$. As already known from (II), after these changes the value of the option grant q goes down. In the second three rows we increase at the same time the variance generated

Table 4. Shifts in the distributions

$f(y; aH)$		$f(y; a_L)$		Contract		
Mean	Std. Dev.	Mean	Std. Dev.	p	q	T
1.08	0.274	1.04	0.261	1	0.814548	0.0307247
1.09	0.274	1.04	0.261	1	0.659639	0.0490021
1.10	0.274	1.04	0.261	1	0.552484	0.0619524
1.08	0.274	1.04	0.261	1	0.814548	0.0307247
1.09	0.311	1.04	0.261	1	0.455818	0.0738587
1.10	0.324	1.04	0.261	1	0.363543	0.0854285
1.08	0.274	1.04	0.261	1	0.814548	0.0307247
1.09	0.311	1.04	0.305	1	0.742235	0.0263439
1.10	0.324	1.04	0.315	1	0.582124	0.0458039
1.08	0.274	1.04	0.261	1	0.814548	0.0307247
1.09	0.311	1.05	0.305	1	0.900826	0.0012279
1.10	0.324	1.06	0.315	1	0.880244	−0.00530156

by $f(\cdot; a_H)$. As compared to the first set of experiments, and since the difference in the means generated by $f(\cdot; a_H)$ and $f(\cdot; a_L)$ is quite large, an increase in the volatility of the distribution of $f(\cdot; a_H)$ allows the shareholders to infer more easily the action taken by the agent, and hence q goes further down. In the third set of rows the additional change is in the variance generated by $f(\cdot; a_L)$. This higher volatility makes harder to infer the action taken by the agent, and for each corresponding row the option grant is always greater than in the other two sets of experiments. Finally, in the last three rows we shift the mean of $f(\cdot; a_L)$. Then, from (II) this shift increases the slope of the contract. In summary, an increase in the volatility of the firm's returns may have rather complex effects on the structure of compensation as it may affect the likelihood ratio $f(y; a_H)/f(y; a_L)$ in various ways. An increase in volatility results in a steeper compensation profile if it becomes harder for the shareholders to infer the action taken by the agent.

5 Concluding remarks

This paper is concerned with the workings and performance of stock option contracts in the provision of managerial incentives. In our simple stylized model a marginal dollar spent in lowering the strike price has a greater impact on the provision of incentives but it yields less utility to the manager than a dollar spent in increasing the fixed component, and it has a lesser impact on the provision of incentives but it yields more utility than a dollar spent in increasing the option grant. This intermediate role of the strike price may explain why most stock options are granted at the money, since these options have a favorable tax and accounting treatment (cf. Hall and Murphy, 2003) and the costs of deviating from the optimal strike price seem to be minimal. These costs are accommodated by an appropriate reshuffling of the option grant and the fixed component of the contract.

In our principal-agent model, a contract cannot be dismissed simply because the sensitivity of pay-performance seems low. There are other components in the configuration of the optimal contract such as the opportunity cost of the manager relative to the size of the corporation, the disutility of effort, the degree of risk aversion, and the manager's technology linking productivity to effort. In order to assess numerically all these effects, we develop some methods for the calibration of the model. Our numerical experiments attest that if the agent takes the low action then the cost to the shareholders is over 100 times greater than the implied extra savings in compensation. Therefore, a relatively small additional amount in compensation may be enough to incentivize the agent to exert high effort. This aspect seems crucial to understand why observed payment schemes can be generated by plausible parameter values. Thus, we consider a representative compensation contract from Hall and Liebman (1998), and we get that such a wage profile can be generated by coefficients of relative risk aversion that range between 0 and 7. Moreover, as compared to the optimal non-linear contract, we should note the remarkable performance of the simple stock option contract. In all our numerical exercises the additional cost of implementing this latter contract is always below 0.5%. Of course, such small costs coupled with the benefits of implementing this simple contract may account for the popularity of stock options.

We would like to emphasize that the main intuition behind the remarkable performance of the stock option contract seems to go beyond our calibration exercises. It is just the simple insight that it is possible to implement action a_H over action a_L via a piecewise linear contract and at a very small extra cost. To simply separate these two actions, it seems even preposterous to resort to an infinite-dimensional contract. Furthermore, our results demonstrate that this underlying reasoning prevails in a setting with a finite or countable number of actions provided that certain standard regularity conditions hold.[12]

These results may lend further insights to ongoing discussions on the nature of executive compensation. As an illustration, we considered the structure of compensation for rank-and-file workers and for riskier firms. The shape of the stock option contract will depend upon the effects that the low action has on the loss of revenues to the firm and on the extra utility to the employee. These two margins have been overlooked by previous studies, and play a crucial role in the present analysis. Also, it is not clear that a riskier company should offer higher incentives, since the slope of the optimal contract depends on the variability of the likelihood ratio $f(y; a_L)/f(y; a_H)$, rather than on the observed volatility of the firm's returns defined by $f(\cdot; a_H)$. Therefore, the structure of compensation across the spectrum of employees in a company will be influenced by monitoring costs and hence by the differential gains of exerting low effort for each category of workers. Top executives and consultants are usually harder to monitor, and hence their pay should be more sensitive to performance. Moreover, to explain the shape of the stock option contract what counts is not the volatility of the company but rather how the actions of each worker may interact with this volatility.

[12] There are other environments in which payment schedules may be linear or nearly linear; e.g., see Laffont and Tirole (1986), Holmström and Milgrom (1987), McAfee and McMillan (1987) and Diamond (1998).

At the heart of our contribution, there is the idea that high-powered incentives induce desired levels of effort, and relieve agency contracts from the problem of moral hazard. This is not to deny the existence of other considerations that may shape executive compensation. For instance, a body of the literature has focused on career concerns (cf., Holmström, 1999; Holmström and Ricart-i-Costa, 1986; Gibbons and Murphy, 1992; Prendergast and Stole, 1996; Caruana and Celentani, 2002), and on multi-task efficiency (cf., Holmström and Milgrom, 1991).

Let us now conclude with some extensions for further research. In the above models the agent is not allowed to borrow or to invest in other assets. In practical situations, however, the CEO may be willing to hedge against the firm's risk stemming from the contingent compensation scheme. Therefore, it is of interest to analyze the investment decision problem of the agent, and the optimal restrictions on portfolio holdings that should be imposed by the principal in order to implement the most desirable action. Also, further research is needed to understand why the strike price is not indexed to other variables such as the evolution of stock prices in the sector.

Dynamic considerations may play an important role in current compensation schemes. Most executive stock options are not tradable, and cannot be exercised over certain time periods. Moreover, CEOs usually face rather undiversified portfolios over time, and most option grants and pay revisions are based upon past performance. These issues cannot be addressed properly in our static framework. Some recent contributions (e.g., Wang, 1997; Clementi and Cooley, 2001; Cadenillas et al., 2002) aim at the computation of solutions in infinite-horizon principal-agent models. This seems to be a research area in which the construction and analysis of structural models should further our understanding on executive compensation, and for which the results of the present paper should be useful.

Appendix

The proof of Proposition 2.1 parallels that of Grossman and Hart (1983, Prop. 8), and it is offered here since it is relevant for further purposes. By an approximation argument, these results could in fact be extended to a setting with a countable, infinite number of actions.

Proof of Proposition 2.1. As is well known, program $(P1)$ can be formulated in the space of utility levels using the change of variable $u(y) = v(w(y))$ for all $y \in Y$, and defining $s(u(y)) = v^{-1}(u(y))$. Then, $(P1)$ involves the maximization of a strictly concave function over a convex set conformed by a set of linear constraints. Hence, the optimal solution is unique whenever only one action $a_H = a_j$ is the optimal one.

Consider now a version of $(P1)$ in which the action space is limited to those actions a' such that $a' \leq a_H$. By Assumption 2.3, it is easy to show from the first-order conditions of the Lagrangean that for the restricted maximization problem the optimal contract $w^*(y)$ is monotone, given that constraint (IC) must be binding for at least one action a'. We next show that there is not any other action $a'' > a_H$ for which the agent obtains a higher utility under contract $w^*(y)$. By way of contradiction, assume that for $a'' > a_H = a^* > a'$ we have

$$\int_Y v(w^*(y))f(y;a'')dy - a'' \geq \int_Y v(w^*(y))f(y;a^*)dy - a^* \qquad (A.1)$$
$$= \int_Y v(w^*(y))f(y;a')dy - a'.$$

Now, observe that there is $\lambda \in (0,1)$ such that

$$a^* = \lambda a' + (1-\lambda)a''. \qquad (A.2)$$

Also, as $w^*(y)$ is increasingly monotone, the CDFC condition of Assumption 2.4 implies that

$$\int_Y v(w^*(y))f(y;a^*)dy > \lambda \int_Y v(w^*(y))f(y;a')dy + (1-\lambda)$$
$$\times \int_Y v(w^*(y))f(y;a'')dy. \qquad (A.3)$$

Then, combining $(A.2)$ and $(A.3)$ we get the desired contradiction with $(A.1)$. As a matter of fact, this same argument can be used to show that if $a_H = a_j$ is the optimal action, then constraint (IC) can only bind for action $a_L = a_{j-1}$. For if there were an additional action $a' < a_L < a_H$ for which constraint (IC) is binding, it follows from Assumption 2.4 that the agent would strictly prefer to take the intermediate action a_{j-1}. □

Proof of Proposition 2.2. The proof of this proposition follows from the previous arguments, since the stock option contract generates a nondecreasing payment schedule.

Proof of Proposition 2.3. This result follows immediately from the definitions of $\widehat{L}$ and L.

Proof of Proposition 3.1. Let $s(y;a_H) = \frac{f(y;a_H)}{1-F(p;a_H)}$. Then, the first inequality in (3.4) may be rewritten as

$$\int_E v'(q(y-p)+T)s(y;a_H)dy \leq \int_Y v'(q(y-p)^+ + T)f(y;a_H)dy.$$

Notice that $s(\cdot;a_H)$ is a density in $E \subseteq Y$ with $s(y;a_H) \geq f(y;a_H)$ for all $y \in E$. Hence, by concavity of $v(\cdot)$, the expected marginal utility over Y must be at least as large as the expected marginal utility over E, which is the desired result.

Now, let $\delta(y;a_H) = \frac{(y-p)f(y;a_H)}{\int_E (y-p)f(y;a_H)dy}$. Notice that both $s(\cdot;a_H)$ and $\delta(\cdot;a_H)$ are densities in E. Moreover, one readily shows that the distribution resulting from $\delta(\cdot;a_H)$ stochastically dominates the distribution resulting from $s(\cdot;a_H)$. Therefore, by the concavity of $v(\cdot)$, we get

$$\int_E v'(q(y-p)+T)\delta(y;a_H)dy < \int_E v'(q(y-p)+T)s(y;a_H)dy.$$

This latter expression validates the second inequality in (3.4). □

Proof of Proposition 3.2. (i) From equation (3.3) we obtain

$$\lambda = \frac{1 - \mu[V_T(\sigma; a_H) - V_T(\sigma; a_L)]}{V_T(\sigma; a_H)}. \tag{A.4}$$

Note that by Assumption 2.3 the distribution induced by action a_H stochastically dominates the distribution induced by action a_L. Then, the concavity of $v(\cdot)$ implies that $V_T(\sigma, a_H) - V_T(\sigma, a_L) = \int_Y v'(q(y-p)^+ + T)[f(y; a_H) - f(y, a_L)]dy < 0$. Therefore, the desired result follows now from $(A.4)$.

(ii) Assume that $\mu = 0$. Then, combining expressions (3.2) and (3.3) we obtain

$$\frac{V_q(\sigma; a_H)}{U_q(\sigma; a_H)} = \frac{V_T(\sigma; a_H)}{U_T(\sigma; a_H)},$$

which contradicts Proposition 3.1.

(iii) Suppose that $\varphi > 0$. Then, by definition of φ it must be the case that $p = \underline{y}$. But if $p = \underline{y}$, it is easy to show that

$$\frac{U_p(\sigma; a_H)}{V_p(\sigma; a_H)} = \frac{U_T(\sigma; a_H)}{V_T(\sigma; a_H)}.$$

Therefore, combining conditions (3.1) and (3.3) we obtain $\varphi/V_p(\sigma; a_H) = 0$, which contradicts the hypothesis $\varphi > 0$, since $V_p(\sigma; a_H) < 0$. □

Proposition 3.3 *If* $[1 - V_p(\sigma; a_L)/V_p(\sigma; a_H)] \geq 0$, *then* $[1 - V_q(\sigma; a_L)/V_q(\sigma; a_H)] > 0$.

Proof. Assumption 2.3 implies that there exists $\widehat{y}$ such that $f(y; a_L) \geq f(y; a_H)$ for $y \in [\underline{y}, \widehat{y}]$, and $f(y; a_L) \leq f(y; a_H)$ for $y \in [\widehat{y}, \overline{y}]$. Suppose that $p < \widehat{y}$ since otherwise the result follows trivially. Let $E_1 = [p, \widehat{y}]$ and $E_2 = [\widehat{y}, \overline{y}]$. Then, by the asserted condition in this proposition,[13]

$$\int_{E_1} v'[f(y; a_L) - f(y; a_H)]dy \leq \int_{E_2} v'[f(y; a_H) - f(y; a_L)]dy. \tag{A.5}$$

It follows that

$$\begin{aligned}\int_{E_1} (y - p)v'[f(y; a_L) - f(y; a_H)]dy &< \int_{E_1} (\widehat{y} - p)v'[f(y; a_L) - f(y; a_H)]dy \\ &\leq \int_{E_2} (\widehat{y} - p)v'[f(y; a_H) - f(y; a_L)]dy \\ &< \int_{E_2} (y - p)v'[f(y; a_H) - f(y; a_L)]dy.\end{aligned}$$

Here, the first and third inequalities come from the definitions of $\widehat{y}$, E_1, and E_2. The second inequality comes from expression $(A.5)$, since this expression has been multiplied by a constant term. Thus, $[1 - V_q(\sigma; a_L)/V_q(\sigma; a_H)] > 0$. □

[13] For convenience, v' denotes $v'(q(y - p) + T)$.

Proposition 3.4 *Let $T \geq 0$. Assume that $|c\, v''(c)/v'(c)| \leq 1$ for all $c \geq 0$. Then,* $[1 - V_q(\sigma; a_L)/V_q(\sigma; a_H)] \geq 0$.

Proof. Observe that

$$V_q(\sigma; a_H) - V_q(\sigma; a_L) = \int_E [F(y; a_L) - F(y; a_H)](q(y-p)v'' + v')dy.$$

Therefore, we will have that $V_q(\sigma; a_H) - V_q(\sigma; a_L) \geq 0$ whenever the term $[q(y-p)v'' + v'] \geq 0$. By the asserted condition on the degree of risk aversion, we have $1 \geq |[q(y-p) + T]v''/v'|$. Moreover, for $T \geq 0$ it follows that $|[q(y-p) + T]v''/v'| \geq |q(y-p)v''/v'|$. Consequently,

$$[q(y-p)v'' + v'] = [q(y-p)v''/v' + 1]\, v' \geq 0.$$

The result is thus established. □

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