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Approval voting reconsidered*

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Summary. The merit of approval voting has been widely discussed in the past 25 years. The distinct property of this rule is the extent of flexibility it allows; any voter can approve as many alternatives/candidates as he wishes. Nevertheless, this advantage is the very reason for two drawbacks of approval voting: its extreme vulnerability to majority decisiveness (Theorem 1) and its extreme vulnerability to erosion of the majority principle (Theorem 2). On the one hand, under some feasible voting strategies any majority of more than $1/2$ of the voters can guarantee the selection of its most favorable candidate, regardless of the preferences of the other voters. On the other hand, under alternative voting strategies even the largest majority cannot impose its common most preferred candidate. A simultaneous resolution of the two problems is possible by restricted approval voting (RAV), a voting rule that allows partial voter flexibility by restricting the minimal and maximal number of candidates that can be approved. Our main result (Theorem 3) clarifies how the foregone flexibility in voters' sovereignty mitigates the above mentioned drawbacks under sincere and insincere coordinated voting. Our findings suggest a new possible justification of a particular voting rule which is based on the significance assigned to three considerations: the advantages of voters' flexibility, immunity to majority decisiveness and immunity to erosion of the majority principle. Such justification can provide a possible explanation to the prevalent use of some special cases of RAV, notably, of the plurality rule and of approval voting.

Keywords and Phrases: Approval voting, Majority decisiveness, Erosion of the majority principle.

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1 Introduction

Voting rules allow voters variable degrees of preference revelation. In some, voters are required to vote only for their most preferred candidate (e.g. the plurality rule). In others, voters need to rank some or all of the candidates. However, most of the rules do not allow the voters the flexibility to vote for as many candidates as they wish. The only rule that allows such flexibility is "approval voting" where any voter can approve as many candidates as he wants. This rule has been axiomatized and extensively studied by Brams and Fishburn (1978, 1983). Nevertheless, its practical success has been mixed if judged on the basis of the extent of its adoption (Brams and Fishburn, 2003).

The two major distinct features of approval voting are its simple dichotomous nature and the unlimited flexibility it offers the voters. However, the second advantage viz., the maximal preservation of the voters' sovereignty, is the very reason for two of its drawbacks:

(i) Its vulnerability to the decisiveness of any majority, that is, even a simple majority can guarantee the selection of its most favorable candidate regardless of the other voters' preferences. In such a case, the majority members vote "as if" they use the plurality rule and it can be readily verified that any majority coalition that consists of more than 50% of the voters can guarantee the selection of its most favorable candidate, independent of the preferences of the other voters.[1]
(ii) Approval voting does not respect (and in an extreme way) the majority principle, because a candidate who is the most favorable one for even the largest possible majority coalition does not necessarily emerge as the voting outcome. This feature requires sincere voting, which is a plausible assumption in large voting bodies.

These two problems are two sides of the same coin. On the one hand, extreme proponents of the majority principle would not find any "evil" in majority decisiveness. On the contrary, they would consider it a plausible property, namely one that a voting mechanism should satisfy. While these proponents may feel comfortable with the existence of the first "drawback", they would consider the second drawback unacceptable. On the other hand, extreme opponents of even the largest possible majority decisiveness, namely those who insist on the right of even the smallest minority to effectively express its preferences would find the first drawback unacceptable. In general, election designers may adopt a non-extreme approach that reflects simultaneously some degree of aversion to majority decisiveness and some degree of aversion to erosion of the majority principle. Such more balanced non-extreme approach is frustrated by both drawbacks of approval voting. In any event the question is how can one or both drawbacks be eliminated or at least alleviated.

In this paper we suggest a rule that partially resolves both the problem of erosion of the majority principle and the problem of majority decisiveness. The voting rule that resolves the two problems imposes lower and upper bounds on

[1] This would not be the case under other scoring rules (e.g., the Borda rule), as shown in Baharad and Nitzan (2002).

the number of candidates that voters can approve. The proposed rule can maintain some flexibility for the voters. It is based on the two basic features of approval voting, namely simplicity (dichotomous voting) and flexibility as well as on the basic feature of scoring rules, that is scores rigidity as a means of ameliorating majority decisiveness and erosion of the implementation of the majority principle. We refer to this rule as "Restricted Approval Voting", henceforth RAV. Approval voting and all "vote for m candidates" rules, such as the plurality rule where $m = 1$ and the inverse plurality rule where $m = k - 1$, k being the number of candidates, are special cases of RAV. The vote for m candidates rules are dichotomous scoring rules that can be referred to as non-flexible approval voting.

In the next section we present our general framework and introduce the definitions of scoring rules and approval voting. Section 3 contains two results that formally state the two drawbacks of approval voting. The main result is presented in Section 4. It clarifies how the parameters of RAV, the lower and upper bounds on the number of permissible approvals alleviate the two drawbacks of approval voting. Section 5 contains a brief conclusion.

2 Scoring rules and approval voting

Let A be a finite set of k alternatives/candidates, $k \geq 3$, and let $N = \{1, \ldots, n\}$ be a finite set of voters. Suppose that the preference relation L_i of voter i, $i \in N$, is a complete and transitive relation over A. In addition to L_i voter i is also characterized by t_i^* – the number of candidates who are unacceptable from his point of view. The set of preference profiles is denoted $\mathcal{L}^n$. A social choice rule V is a mapping from $\mathcal{L}^n$ to the set of non-empty subsets of A. This rule specifies the collective choice for any preference profile. A widely discussed family of voting rules is the family of scoring rules.

Let $\{S_1, S_2, \ldots, S_k\}$ be a monotone sequence of real numbers, $S_1 \leq S_2 \leq \ldots \leq S_k$ and $S_1 < S_k$. Each of the n voters ranks the candidates assigning S_1 points to the one ranked last, S_2 points to the one ranked next to the last, and so on. Under a *scoring rule* a candidate with a maximal total score is elected. If the sequence $\{S_1, S_2, \ldots, S_k\}$ is strictly monotone, that is, $S_1 < S_2 < \ldots < S_k$, the scoring rule is called a *strict scoring rule*.[2] The *plurality rule* is the most common scoring rule. It is defined by the sequence $S^p = \{S_1, S_2, \ldots, S_k\} = \{0, \ldots, 0, 1\}$. That is, the candidate who is ranked first by the largest number of voters is elected. Another well known special case of a scoring rule is *the Borda rule* which is defined by the scores $S^B = \{S_1, S_2, \ldots, S_{k-1}, S_k\} = \{0, 1, \ldots, k-2, k-1\}$. A scoring rule that hitherto has not attracted much attention is the *inverse plurality rule*. This rule is defined by $S^{ip} = \{S_1, S_2, \ldots, S_k\} = \{0, 1, \ldots, 1\}$.

There are two features that might enhance the appeal of scoring rules:

(i) Dichotomous voting that does not require voters to fully rank the candidates. The only requirement is to approve or disapprove every candidate. Such voting schemes obviously reduce the complexity of the voting process.

[2] For recent analysis of scoring rules and, in particular, of the plurality and Borda rules, see Brams and Fishburn (2002), Nurmi (2002) and Saari (2001).

(ii) Flexible voting that increases voters' sovereignty. Under flexible rules voters can approve as many candidates as they wish.

The combination of these two features with the basic characteristic of scoring rules yields the class of flexible dichotomous scoring rules. Approval voting is the most well known such rule.

Approval voting (AV) is a flexible scoring rule defined by $S^i = \{S^i_1, S^i_2, \ldots, S^i_k\}$, such that: $S^i_j = \begin{cases} 0 & if \quad j = 1, ..., t_i \\ 1 & if \quad j = t_i + 1, ..., k \end{cases}$.

That is, voter i assigns 0 points to t_i candidates and 1 point to the remaining $(k - t_i)$ candidates (the approved candidates). Note that the flexibility of approval voting is represented by each t_i being individually specific.

An AV strategy is called *sincere* if it is consistent with the true preference relations of the voters (the L_i 's and the t^*_i's). That is, the $k(t_i) = k - t_i$ actually approved candidates are the top $(k - t_i)$ candidates according to L_i, such that $t_i \geq t^*_i$.

Let us use an example to clarify the distinction between these two components of sincerity:

Example 1. Let $\{a, b, c, d, e\}$ be a set of 5 candidates. The individual's true preferences are $a \succ b \succ c \succ d \succ e$, and he truly approves candidates a and b ($t^*_i = 3$). If his voting strategy is to approve candidates a, b, c then only one requirement of sincerity is satisfied: the approved 3 candidates are the top ones according to his true preferences. However, the second requirement of sincerity ($t_i \geq t^*_i$) is not satisfied: he votes for 3 candidates whereas sincere voting requires approval of no more than 2 candidates.

3 The problems

As mentioned above, we focus on two drawbacks of approval voting: its vulnerability to majority decisiveness and to erosion of the majority principle. In this section we clarify these two problems.

3.1 Majority decisiveness

When examining the problem of majority decisiveness, we consider the occurrence of the problem under sincere voting or under coordinated strategic voting. It has been shown in Baharad and Nitzan (2002) that in the context of scoring rules, under sincere voting a majority of size α can guarantee the selection of its most favorable candidate when

(1) $$\alpha > \frac{S_k - S_1}{2S_k - S_{k-1} - S_1}$$

and that under coordinated strategic voting a majority of size α can guarantee the selection of its most favorable candidate when

(2) $$\alpha > \frac{S_k - S_1}{2S_k - S_1 - \overline{S}},$$

where α is assumed to be a fraction with a denominator n and $\overline{S}$ denotes the average of the scores $S_1, \ldots, S_{k-1}$ (that is, the average score of all the candidates but the top ranked one). Based on equations (1) and (2), the first drawback of approval voting, namely its vulnerability to majority decisiveness can be easily established:

Theorem 1. Under sincere or coordinated strategic voting, approval voting is vulnerable to simple majority decisiveness.

Proof. The proof is straightforward because approval of the best single candidate, which is an admissible sincere voting strategy under approval voting, implies that both inequalities (1) and (2) are satisfied for $\alpha = 0.5$ and $S_k = 1$, $S_{k-1} = S_1 = \overline{S} = 0$ (the majority size and the scores that correspond to the plurality rule). □

Under approval voting, the majority can guarantee the selection of its most favorable candidate by adopting a strategy of voting that is feasible under the plurality rule. In such a case the members of even the smallest simple majority always guarantee the selection of their most preferred candidate. In other words, approval voting is vulnerable to α-majority decisiveness, where $\alpha = 0.5$.

Example 2. Suppose there are 4 candidates $\{a, b, c, d\}$ and 5 voters whose preferences are as follows:

For 3 voters: $a \succ b \succ c \succ d$ and $t_i^* = 2$, that is, their two most preferred candidates are acceptable.

For 2 voters: $b \succ c \succ d \succ a$ and $t_i^* = 3$, that is, only their most preferred candidate is acceptable.

Under sincere AV strategy, the 3 voters apply one of the following scores {0,0,1,1} or {0,0,0,1}. If they assign one point to candidates a and b and the 2 remaining voters apply the scores {0,0,0,1}, assigning one point to candidate b, it can be readily verified that under AV and sincere voting strategies candidate b is selected. However, the majority members prefer candidate a, which might induce them to vote (still sincerely, since $t_i \geq t_i^*$ is satisfied) just for candidate a. In other words, the majority members can apply the coordinated sincere AV strategy where the applied scores are {0,0,0,1}and only candidate a receives one point . In such a case, independent of the minority votes, candidate a is chosen.

3.2 Erosion of the majority principle

The proponents of the majority principle may not consider majority decisiveness as a problem. However, the dual problem, namely the erosion of the majority principle might be considered as a disturbing drawback of a voting rule because in their view a compelling demand is that a voting rule should select the simple-majority first-best consensus candidate, when one exists. In other words, a minimal requirement for a voting rule is that it satisfies the simple-majority decisiveness property. In this paper this requirement is referred to as “the majority principle”. Approval voting is vulnerable to erosion of the majority principle, in addition to being vulnerable to majority decisiveness. The vulnerability to this second dual problem is also caused by the extreme flexibility allowed by approval voting.

Theorem 2. Under sincere voting, approval voting is vulnerable to erosion of the majority principle by the smallest possible minority, that is by a single voter.

Proof. The proof is straightforward because whenever the sincere voting strategy under approval voting is approval of at least two candidates, that is, whenever the applied scores are such that $S_k = S_{k-1} = 1$ and $S_1 = 0$, inequality (1) cannot be satisfied. This means that approval voting is vulnerable to erosion of the majority principle even by a single voter. □

Example 3. Suppose that a majority of $n - 1$ members has the following preferences: $a \succ b \succ c \succ d$. Suppose that all the majority members sincerely approve one or two candidates ($t_i^* = 2$). Suppose that they apply the scores $\{$0,0,1,1$\}$. Now suppose that the single minority member has the following preferences: $b \succ c \succ d \succ a$ and that he chooses to approve only his most preferred candidate b, that is, he applies the scores (0,0, ... ,1). In such a case, candidate b (that is not the majority most favorable candidate) is selected.

4 Mitigating the two problems

A simultaneous partial resolution of the two problems presented in the preceding section is possible by maintaining the dichotomous nature of approval voting while reducing the extreme flexibility it allows. Specifically, under *restricted approval voting* (RAV) the number of candidates that can be assigned 1 point is bounded from below and from above. That is, let l and u be integers such that $1 \leq l \leq u \leq k - 1$ and voters can assign 1 point to no less than l candidates and to no more than u candidates. Some special cases of RAV are:

(i) When 1< $l = u = m < k$, the applied rule is the "vote for m candidates" rule which is a particular form of non-flexible approval voting. In this rule voters are required to approve (that is, to assign 1 point) a pre-fixed number of m candidates.
(ii) When $l = u = 1$, the applied rule is the plurality rule.
(iii) When $l = u = k - 1$, the applied rule is referred to as the "inverse plurality" rule.
(iv) When $l = 1$ and $u = k - 1$, the rule is the celebrated approval voting.

An RAV voting strategy is called sincere if it is admissible under RAV and consistent with the true preference relations of the voters (the L_i's and the t_i^*'s).

The potential advantage of RAV in simultaneously alleviating the two drawbacks of approval voting under sincere and coordinated voting is clarified by our main result:

Theorem 3.

(i) Under sincere voting, if $l > 1$ RAV is not vulnerable to any α-majority decisiveness.
(ii) Under coordinated strategic voting, RAV is vulnerable to α-majority decisiveness if $\alpha > \frac{k-1}{2k-l-1}$.
(iii) Under sincere voting, the vulnerability of RAV characterized by $l > 1$ to erosion of the majority principle cannot be eliminated. However, the proportion of preference profiles under which the erosion problem exists can be decreased by reducing u.
(iv) Under coordinated strategic voting, RAV characterized by $l > 1$ is vulnerable to erosion of the majority principle by an $\frac{k-l}{2k-l-1}$ minority.

Proof.

(i) Let $S_k = S_{k-1} = 1$, and let a majority of $n-1$ voters share the same preference regarding its two most favorable candidates a and b, such that $a \succ b$. Since $l > 1$, this majority assigns 1 point to a and b. Suppose that the two most preferred candidates for the single voter minority are b and c, such that $b \succ c$. Thus, this voter assigns 1 point to b and c. In such a case, candidate b who is not the majority most preferred candidate, is selected under the RAV. A similar proof applies for any α-majority where $\frac{1}{2} < \alpha < \frac{n-1}{n}$.
(ii) By (2), the minimal degree of decisiveness is positively related to $\overline{S}$ which, in turn, is positively related to the lower bound on the number of approvals l. Hence, under coordinated strategic voting, the minimal degree of decisiveness is positively related to l. This means that the most effective coordinated voting strategy for the majority is to approve exactly l candidates. In such a case, by substituting into (2) the corresponding scores $S_k = 1, S_1 = 0$ and $\overline{S} = \frac{l-1}{k-1}$ we obtain that that RAV is vulnerable to α-majority decisiveness if $\alpha > \frac{k-1}{2k-l-1}$.
(iii) The proof of the first claim is directly obtained from (i) and the definitions of majority decisiveness and erosion to the majority principle.
Since, by assumption $l > 1$, $S_k = S_{k-1} = 1$. The number of feasible voting strategies that are consistent with this constraint (the number of feasible voting strategies that satisfy $S_j = 1$ for $j = 1, 2$) clearly increases with the upper limit on the number of approvals u. In other words, the proportion of preference profiles under which the erosion problem exists can be decreased by reducing u.
(iv) The existence of α-majority decisiveness implies erosion of the majority principle by an $(1-\alpha)$-minority. By (ii), we get that $(1-\alpha)$ is equal to $\frac{k-l}{2k-l-1}$.
□

Theorem 3 clarifies the significance of imposing upper and lower bounds on the number of approvals that can be made by the voters. In particular, it sheds new light on the implications of the most distinguished feature of approval voting, namely the unrestricted flexibility of the voters in determining the approved candidates. Under sincere voting, an effective lower bound l, $l > 1$, that eliminates any voting profile that is admissible under the plurality rule, is sufficient to eliminate vulnerability to any α-majority decisiveness. Under coordinated strategic voting, this lower bound ameliorates (but does not eliminate) such vulnerability.

The upper bound on the number of approvals is intended to handle the problem of erosion of the majority principle. Under sincere voting, if $l > 1$, the upper bound cannot eliminate this problem, however it can reduce its severity, in the sense that given the number of candidates and the number of voters, it reduces the number of preference profiles that give rise to such erosion. Under coordinated strategic voting, a decrease in the upper bound u ameliorates (but does not eliminate) the erosion of the majority principle.

The plausibility of RAV hinges both on the relative weight assigned to the two drawbacks and on whether the voters are expected to vote sincerely or insincerely. By Theorem 3 we obtain the following implications.

When majority decisiveness and erosion of the majority principle are not considered as problems or drawbacks whereas voter flexibility is considered as a virtue[3], approval voting is clearly a very desirable voting rule.

If majority decisiveness is disregarded, while immunity to erosion of the majority principle is considered as a crucial property, the natural RAV is the plurality rule where $l = u = 1$. By (i), this is true under sincere voting and by (ii), this conclusion is valid under coordinated strategic voting.

If majority decisiveness as well as erosion of the majority principle are considered as drawbacks and, in addition voters are expected to vote sincerely, then by (i) and (iii), the desirable RAV is the plurality rule when the weight assigned to the problem of erosion of the majority principle is sufficiently high. However, when the weight assigned to the problem of majority decisiveness is sufficiently high, the desirable rule is an intermediate RAV, non-flexible approval voting where $l = u = 2$.

If majority decisiveness as well as erosion of the majority principle are considered as drawbacks and, in addition voters are expected to coordinate votes, by (ii) and (iv), the desirable RAV is non-flexible approval voting where $1 < l = u < k - 1$. The exact values of the fixed number of approvals hinge on the relative weights assigned to the two drawbacks.[4]

Finally, when both problems are considered as drawbacks yet it is not clear whether voters resort to sincere or coordinated strategic voting, as argued above, if sincere voting is the expected voters' behavior, then by (i) and (iii) the desirable RAV is defined by $l = 1$ or $l = 2$. However, if the expected behavior of the voters is insincere coordinated strategic voting, then the desirable RAV is non-flexible approval voting characterized by $1 < l = u < k - 1$. But since there is uncertainty

[3] For a discussion of three specific desirable implications of this flexibility (individual non-manipulability, appropriate representation of the electorate preferences by the election outcome and moderate positioning by the candidates), see Weber (1995).

[4] One such intermediate restricted approval voting is the "Borda-equivalent" scoring rule. When k is even, this scoring rule requires that every voter approves exactly half of the candidates, i.e., the rule is defined by $S_j = 0$ for $j \leq k/2$ and $S_j = 1$ for $j > k/2$. Although the "Borda-equivalent" scoring rule differs from the celebrated Borda method of counts which is defined by $S^B = \{S_1, S_2, \dots, S_{k-1}, S_k\} = \{0, 1, \dots, k-2, k-1\}$, it can be readily verified by (2) that the two rules are characterized by the same minimal α-majority decisiveness which is equal to $\frac{2k-2}{3k-2}$.

regarding the expected behavior of the voters, risk aversion may result in a plausible insurance strategy, namely selecting RAV that guarantees both that the decisiveness problem and the erosion problem are not too severe. This RAV is characterized by l and u that satisfy $1 < l < u < k - 1$.

5 Conclusion

Approval voting is based on three principles: simplicity in the sense of restricting the voting to be dichotomous; flexibility in the sense of allowing the voters to determine the number of approvals; and majoritarianism in the sense of basing the selection criterion on the largest number of accumulated scores/approvals. We began by exposing two drawbacks of approval voting that were not noticed in the voting literature, namely its vulnerability to extreme decisiveness of the majority and extreme erosion of the majority principle. In light of these drawbacks the mixed success of approval voting that has been recently described by Brams and Fishburn (2003) is somewhat more understandable. Our main argument is that restricted approval voting (RAV) mitigates the two drawbacks. A typical RAV is a flexible dichotomous scoring rule so it fully respects the simplicity and majoritarianism inherent in approval voting. It only partly respects the flexibility allowed by approval voting because it imposes lower and upper bounds (l and u) on the number of candidates voters can approve. Put differently, it only partly respects the principle of rigid scores that essentially serves as a means of ameliorating the decisiveness of the majority. The former parameter l is the means of limiting the severity of the problem of extreme majority decisiveness both under sincere and strategic coordinated voting. Clearly, this instrument is more effective under sincere voting than under coordinated voting. The latter parameter u is the means of alleviating the problem of extreme erosion of the majority principle.

Under sincere voting, if $l > 1$, u cannot eliminate the problem of extreme erosion of the majority principle, but it can limit its severity as measured by the proportion of preference profiles that give rise to such erosion. Under strategic coordinated voting, if $l > 1$, l becomes the means of eliminating the erosion problem in its extreme form. The severity of the problem is positively related to the size of l because an increase in l implies that a smaller minority can veto the majority will. Under such circumstances the parameter u is ineffective both in terms of coping with the majority decisiveness problem and in terms of coping with the problem of erosion of the majority principle.

When majority decisiveness and erosion of the majority principle are not considered as drawbacks whereas voter flexibility is considered as a virtue, approval voting is a very appealing voting rule. However, the use of alternative voting rules becomes sensible if majority decisiveness or erosion of the majority principle are considered as drawbacks. If majority decisiveness is disregarded, while immunity to erosion of the majority principle is considered as a crucial property, the natural RAV is the plurality rule where $l = u = 1$. If majority decisiveness as well as erosion of the majority principle are considered as drawbacks and, in addition, voters are expected to vote sincerely, the desirable RAV is either the plurality rule or an intermediate rule, non-flexible approval voting where $l = u = 2$. If majority decisiveness as well as erosion of the majority principle are considered as drawbacks and, in addition voters are expected to coordinate votes, the desirable RAV is non-flexible approval voting where $1 < l = u < k - 1$. The exact values of the fixed number of approvals hinge on the relative weights assigned to the two drawbacks. Finally, when both problems are considered as drawbacks, yet it is not clear whether voters resort to sincere or coordinated strategic voting, the desirable RAV is one characterized by $1 < l < u < k - 1$.

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