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Economic Theory

Ambiguity aversion and the absence of indexed debt*

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Summary. Following the seminal works of Schmeidler (1989), Gilboa and Schmeidler (1989), roughly put, an agent's subjective beliefs are said to be *ambiguous* if the beliefs may not be represented by a unique probability distribution, in the standard Bayesian fashion, but instead by a set of probabilities. An *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it. In spite of wide and long-standing support among economists for indexation of loan contracts there has been relatively little use of indexation, except in situations of extremely high inflation. The object of this paper is to provide a (theoretical) explanation for this puzzling phenomenon based on the hypothesis that economic agents are *ambiguity averse*. The paper considers a general equilibrium model based on Magill and Quinzii (1997) with ambiguity averse agents, where both nominal and indexed bond contracts are available for trade and all relevant prices are determined endogenously. We obtain conditions which prompt an *endogenous* cessation of trade in indexed bonds: i.e., conditions under which there is no trade in indexed bonds in *any* equilibrium; only nominal bonds are traded.

Keywords and Phrases: Debt indexation, Ambiguity, Ambiguity aversion.

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1 Introduction

In spite of wide and long-standing support among economists for indexation of loan contracts there has been relatively little use of indexation, except in situations of extremely high inflation. Indeed, except in cases where inflationary circumstances forced them to do so, few governments, and fewer private borrowers, have issued indexed bonds. People seem to have a preference for specifying their obligations and opportunities in nominal units. As Shiller (1997) remarks:

> That the public should generally want to denominate contracts in currency units – despite all the evidence that it is not wise to do so and despite the obvious examples from nominal contracts of redistributions caused by unexpected inflation – should be regarded as one of the great economic puzzles of all time.

The object of this paper is to provide a (theoretical) explanation for this puzzling phenomenon based on the hypothesis that economic agents are *ambiguity averse*. The analysis throws up testable hypotheses and insights on policy that are distinctive, compared to what a more standard analysis based on the assumption that decision makers are (subjective) expected utility maximizers would suggest.

Suppose an agent's subjective knowledge about the likelihood of contingent events is consistent with more than one probability distribution. And further that, what the agent knows does not inform him of a (second order) probability distribution over the set of 'possible' (first order) probabilities. Roughly put, we say then that the agent's beliefs about contingent events are characterized by *ambiguity*. If ambiguous, the agent's beliefs are captured not by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Thus not only is the outcome of an act uncertain but *also* the expected payoff of the action, since the payoff may be measured with respect to more than one probability. An *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it: the decision rule is to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. This notion of ambiguity aversion, an intuition about behavior under subjective uncertainly famously noted in Ellsberg (1961) and earlier by Knight (1921), inspires the formal model of Choquet expected utility (CEU) preferences introduced in Schmeidler (1989). The present paper considers a competitive general equilibrium model of goods, bonds and money markets populated by agents with CEU preferences[1], where both nominal and indexed bonds are available for trade

[1] Recent literature has debated the merits of the CEU framework as a model of ambiguity aversion. For instance, Epstein (1999) contends that CEU preferences associated with *convex capacities* (see Sect. 2, below) do not always conform with a "natural" notion of ambiguity averse behavior. On the other hand, Ghirardato and Marinacci (2002) argue that ambiguity aversion is demonstrated by convex capacities.

Further on it will be seen that a property of portfolio inertia typical to the behavior of CEU agents is what crucially underpins the whole argument in the paper. Mukerji and Tallon (2003a) shows that this inertia property may be derived from the primitive notion of ambiguity presented in Epstein and Zhang (2001) without relying on a parametric preference model such as the CEU.

and prices of all goods and bonds are determined endogenously. We obtain conditions which prompt an *endogenous* cessation of trade in indexed bonds among private agents: i.e., conditions under which there is no trade in indexed bonds in *any* equilibrium and only nominal bonds are traded. It is worth clarifying, at this point, that while we "explain" the veritable absence of indexed debt by showing that no trade in indexed bonds is the unique equilibrium outcome under certain conditions, the analysis does not imply that this is an efficient outcome.

An important point of inspiration for the analysis was to note that indexing does not eliminate all (price) risk — rather it substitutes one risk for another – an observation, we believe, originally due to Magill and Quinzii (1997). An indexed bond, whose payoffs by definition are denoted in units of a reference bundle of goods and services, will be secure against the aggregate price level risk arising from changes in the money supply – *the monetary risk*, but unavoidably picks up the *real risks* arising from fluctuations in the relative prices of the goods in the reference bundle. A nominal contract on the other hand implies susceptibility to monetary risk but less so to real risk. The basic intuition is straightforward. Being paid in terms of an index essentially amounts to being paid units of the reference bundle of goods. Typically, the reference bundle contains items that are not part of a given individual's consumption basket. Hence, effectively the individual is left to exchange goods in the reference bundle not in his consumption basket with goods he actually consumes. Thus, a change in the price of goods not in his basket will affect the "worth" of his remuneration in terms of the goods he does consume. Since the presence of both types of risk is typical, standard portfolio analysis will advise that the optimal portfolio should contain *both* nominal and indexed contracts (or to put it somewhat differently, partial indexation). Given this one would expect, and a result in this paper confirms, trade in indexed bonds will always be observed in a market consisting of SEU (subjective expected utility) agents so long as there were *some* inflation, however small. Under ambiguity aversion the market outcome, though, may be dramatically different.

More specifically suppose, with respect to *any* two agents wishing to trade in indexed bonds, the following is true:

1. the indexation bundle contains at least one good which is not consumed by either of the agents;
2. the agents' beliefs about the change in the price of good(s) not consumed by either of them, relative to the average price level, is ambiguous;
3. agents are ambiguity averse.

The main result of the paper shows that, if agents believe general inflation will not exceed a given bound and if ambiguity of beliefs about the relative price movements is sufficiently high, then agents will have zero holdings of the indexed bond in *any* equilibrium.

Finally, we relate our paper to some of the literature that has applied CEU preferences to financial asset choice and financial market equilibrium. Although a fuller discussion of the precise relationship between this contribution and existing work is best deferred till the model has been spelled out, some preliminary remarks might help the reader to grasp the intuition behind our result. CEU preferences were identified by Dow and Werlang (1992) as a potential source of inertia in portfolio holding: an agent having a *riskless* endowment will not want to hold an uncertain asset on a (degenerate) price interval only if he perceives the asset return as ambiguous and if he is ambiguity averse. As noted in Mukerji and Tallon (2001) this inertia result does not translate to no trade in an equilibrium model unless some extra ingredients are added. Following a result in Epstein and Wang (1994) on sufficient conditions for an equilibrium to be supported by multiple asset prices, we precisely identified in that paper what these ingredients were: part of the asset return has to be idiosyncratic (i.e., uncorrelated with the agents' endowments), and that part has to be ambiguous. Then, we showed, it is possible to establish that assets that are of this form may not be traded at any equilibrium. The present paper may be seen to be following that line of research by specifying what form these idiosyncrasies might take in a more concrete setting: here, they are precisely the "noise" introduced in the return of an indexed asset *via* the presence of "irrelevant" goods (irrelevant in the sense that the agents trading the asset neither consume nor are endowed with them) in the indexation bundle. The way this paper "operationalizes" the idea is by restricting the financial market trade to agents who consume a bundle of goods that does not include some of the goods that are used in the indexation bundle. The "irrelevant" goods are consumed by some other "prop" agents who do not participate in financial markets; their role in the model is to endogenize the (stochastic) prices of these goods. Their role is crucial in the sense that without them we would lose the source of the noise and therefore the no trade result.

The rest of the paper is organized as follows. The following section provides a brief introduction to the formal model of Choquet expected utility. Section 3 works through a leading example with the aim of conveying the essential intuition of the argument in a partial equilibrium setting. Section 4 contains the general equilibrium model and the main result. Section 5 concludes the paper with a discussion of the related literature and of the empirical significance of the findings.[2]

2 Choquet expected utility

Let $\Omega = \{\omega_i\}_{i=1}^N$ be a finite state space, and assume that the decision maker (DM) chooses among acts with state contingent payoffs, $z : \Omega \to \mathbb{R}$. In the CEU model an ambiguity averse DM's subjective belief is represented by a *convex non-additive probability* function (or a *convex capacity*), ν such that, (i) $\nu(\emptyset) = 0$, (ii) $\nu(\Omega) = 1$ and, (iii) $\nu(X \cup Y) \geq \nu(X) + \nu(Y) - \nu(X \cap Y)$, for all $X, Y \subset \Omega$. Define the *core* of ν, (notation: $\Delta(\Omega)$ is the set of all additive probability measures on Ω):

$$\mathcal{C}(\nu) = \{\pi \in \Delta(\Omega) \mid \pi(X) \geq \nu(X), \text{ for all } X \subseteq \Omega.\}$$

Hence, $\nu(X) = \min_{\pi \in \mathcal{C}(\nu)} \pi(X)$. Hence, convex capacity may be interpreted as representing a convex set of (additive) probabilities. The *ambiguity*[3] of the belief

[2] A longer version of the paper, with all the proofs is available at http://eurequa.univ-paris1.fr/membres/tallon/indexation1.pdf

[3] Fishburn (1993) provides an axiomatic framework for this definition of ambiguity and Mukerji (1997) demonstrates its equivalence to a more primitive and epistemic notion of ambiguity (expressed in term's of the DM's knowledge of the state space).

about an event X is measured by the expression $\mathcal{A}(X;\nu) \equiv 1-\nu(X)-\nu(X^c) = \max_{\pi\in\mathcal{C}(\nu)}\pi(X)-\min_{\pi\in\mathcal{C}(\nu)}\pi(X)$.

Like in SEU, a *utility function* $u:\mathbb{R}_+\rightarrow\mathbb{R}$, $u'(\cdot)\geq 0$, describes DM's attitude to risk and wealth. Gilboa and Schmeidler (1989) showed, that given a convex non-additive probability ν, *the Choquet expected utility*[4] of an act is simply the minimum of all possible 'standard' expected utility values obtained by measuring the contingent utilities possible from the act with respect to each of the additive probabilities in the core of ν:

$$\mathbb{CE}_\nu u(z) = \min_{\pi\in\mathcal{C}(\nu)}\left\{\sum_{\omega\in\Omega} u(z(\omega))\,\pi(\omega)\right\} \equiv \int_\Omega u(z(\omega))\,d\nu$$

The fact that the same additive probability in $\mathcal{C}(\nu)$ will not in general 'minimize' the expectation for two different acts, explains why the Choquet expectations operator is not additive, i.e., given any acts z, w : $\mathbb{CE}_\nu(z)+\mathbb{CE}_\nu(w)\leq\mathbb{CE}_\nu(z+w)$. The operator is additive, however, if the two acts z and w are *comonotonic*, i.e., if $(z(\omega_i)-z(\omega_j))(w(\omega_i)-w(\omega_j))\geq 0$.

Next, we state the notion of independence of convex non-additive probabilities, proposed by Gilboa and Schmeidler (1989), used in this paper. Essentially, the idea is as follows. Start with the set of probabilities in the core of each capacity, select a probability from each such set and multiply to obtain the corresponding product probability. Repeat for all possible selections, thereby obtaining a set of product probabilities: the lower envelope of the set of product probabilities is the product capacity.

Definition 1 *Let ν and μ be two convex non-additive probabilities, defined on Ω_ν and Ω_μ respectively. The independent product of ν and μ is*

$$\forall A\subseteq\Omega_\nu\times\Omega_\mu,\;\;(\nu\otimes\mu)(A)\equiv\min\left\{(\pi_\nu\times\pi_\mu)(A):\pi_\nu\in\mathcal{C}(\nu),\pi_\mu\in\mathcal{C}(\mu)\right\}$$

It is well-known that it is possible to define more than one notion of independence for non-additive beliefs (Ghirardato (1997)). However, the formal analysis in the present paper, given the primitives of our model, does not hinge on this particular choice of the notion of independence. The capacity we use in our model is a product of two two-point capacities (i.e., each capacity is defined on a state-space consisting of two states). A two-point capacity is a convex capacity and (trivially) a belief function. As is explicit in Theorems 2 and 3 in Ghirardato (1997), if marginals satisfy the structural properties the marginals we use do, then uniqueness of product capacity obtains.

3 The single decision maker's problem: the intuition in a simplified set up

In this section we consider the problem of a decision maker who wants to transfer an amount S from Period 0 to Period 1. Goods prices in Period 1 are uncertain

[4] The Choquet expectation operator may be directly defined with respect to a non-additive probability, see Schmeidler (1989). Also, for an intuitive introduction to the CEU model see Section 2 in Mukerji (1998).

and, for the purposes of this section, taken to be exogenously determined. We will examine the DM's choice between only two kinds of assets, nominal bonds and indexed bonds, whose prices are known and exogenously given. While the model in this section is simpler than the one in the next section, it is instructive in that it will reveal to us how the trade-offs involved, given ambiguity aversion, are such that the DM will strictly prefer to maintain a zero holding of the indexed bond over a *non-degenerate* interval of indexed bond prices. This is a key intuition to understanding why no trade in indexed bonds might emerge as an equilibrium outcome.

3.1 A simple portfolio problem

We assume that there are just two goods in the economy: x and y. The agent consumes only good x and is endowed in Period 1 with a (non-random) endowment of that good, $\bar{x}$. The agent does not consume good y nor is he endowed with that good. However, the indexed bond pays off a unit of good x and a unit of good y. The nominal bond pays in units of money.

The money supply in the economy in Period 1 can be either high (M) or low (m). When the money supply is low, suppose that prices can be equal to either (p_x, p_y^L) or (p_x, p_y^H), with $p_y^H > p_y^L$, *i.e.*, we assume that the price of good y can be affected by factors that do not affect the price of good x. When the money supply is high, we assume that prices can be either equal to $(\lambda p_x, \lambda p_y^L)$ or $(\lambda p_x, \lambda p_y^H)$ where $\lambda = M/m > 1$. This is reminiscent of the quantity theory of money.

The following four states exhaustively describe the price uncertainty faced by the individual:

State	Prices	Return from an indexed bond
1	(p_x, p_y^H)	$p_x + p_y^H$
2	$(\lambda p_x, \lambda p_y^H)$	$\lambda \times (p_x + p_y^H)$
3	(p_x, p_y^L)	$p_x + p_y^L$
4	$(\lambda p_x, \lambda p_y^L)$	$\lambda \times (p_x + p_y^L)$

In this section we leave the decision problem concerning the Period 0 consumption unspecified and simply assume that the agent wants to save a given amount $S > 0$. Let x^s denote the agent's consumption in state s, b^i the agent's indexed bond holding, q^i its price, b^n the agent's nominal bond holding, and q^n its price. The budget constraints are given by:

$$x^1 = \bar{x} + \left(1 + \frac{p_y^H}{p_x}\right) b^i + \frac{b^n}{p_x} = \bar{x} + \left(1 + \frac{p_y^H}{p_x} - \frac{q^i}{q^n p_x}\right) b^i + \frac{S}{q^n p_x}$$

$$x^2 = \bar{x} + \left(1 + \frac{p_y^H}{p_x}\right) b^i + \frac{b^n}{\lambda p_x} = \bar{x} + \left(1 + \frac{p_y^H}{p_x} - \frac{q^i}{q^n \lambda p_x}\right) b^i + \frac{S}{q^n \lambda p_x}$$

$$x^3 = \bar{x} + \left(1 + \frac{p_y^L}{p_x}\right) b^i + \frac{b^n}{p_x} = \bar{x} + \left(1 + \frac{p_y^L}{p_x} - \frac{q^i}{q^n p_x}\right) b^i + \frac{S}{q^n p_x}$$

$$x^4 = \bar{x} + \left(1 + \frac{p_y^L}{p_x}\right) b^i + \frac{b^n}{\lambda p_x} = \bar{x} + \left(1 + \frac{p_y^L}{p_x} - \frac{q^i}{q^n \lambda p_x}\right) b^i + \frac{S}{q^n \lambda p_x}$$

The budget constraints reveal how each of the two kinds of bonds provide a hedge against a particular type of risk while simultaneously making the agent vulnerable to another type of risk. The agent does not consume y, hence given that the indexed bond pays a unit each of x and y, on maturity (of the indexed bond) the agent is effectively left to exchange units of y obtained for units of x. Therefore, even though payoff from an indexed bond is immune to monetary shocks (it is independent of λ) it changes with changes in the price of y, relative to the price of x. On the other hand, while the payoff (to the agent) of a nominal bond is not affected by shocks to the relative price of y, it is affected by monetary shocks (i.e., the value of λ). Hence, if $b^i = 0$, $x^1 = x^3$ and $x^2 = x^4$, while, if $b^n = 0$, then $x^1 = x^2$ and $x^3 = x^4$. Notice also that if $b^i > 0$, then $x^1 > x^3$ and $x^2 > x^4$, while, if $b^i < 0$, then $x^1 < x^3$ and $x^2 < x^4$; i.e., the agent's ranking of the states (1,3 and 2,4) according to consumption *reverses* when switching from a long to a short position on the indexed bond.

We next explore the consequences of ambiguity of beliefs about relative price movements on the agent's decision whether or not to hold indexed bonds. We assume that the agent is risk averse, with a utility index $u : \mathbb{R}_+ \to \mathbb{R}$, which is increasing, strictly concave and differentiable. Suppose the agent has precise probabilistic beliefs concerning the money supply[5], that is, the agent can assess the probability of the event $\{1, 3\}$ to be, say, μ and that of event $\{2, 4\}$ to be $1 - \mu$. On the other hand, conditional on a monetary state, we assume that the agent has only vague beliefs on whether the price of good y is high or low, which is represented by the fact that subjective beliefs are described by capacities $\nu^H \equiv \nu(\{1, 2\})$ and $\nu^L \equiv \nu(\{3, 4\})$, with $\nu^L + \nu^H < 1$. The overall beliefs of the agent are the independent product of μ and ν. The preferences of the agent are then represented by a utility functional, denoted $V(x^1, x^2, x^3, x^4)$, obtained by taking the Choquet integral of $u(x^s)$ with respect to the independent product belief $\mu \otimes \nu$.

If $b^i > 0$, then $V(x^1, x^2, x^3, x^4)$ is given by:

$$\mu\left(\nu^H u(x^1) + (1 - \nu^H)u(x^3)\right) + (1 - \mu)\left(\nu^H u(x^2) + (1 - \nu^H)u(x^4)\right)$$

If $b^i < 0$, then $V(x^1, x^2, x^3, x^4)$ is given by:

$$\mu\left((1 - \nu^L)u(x^1) + \nu^L u(x^3)\right) + (1 - \mu)\left((1 - \nu^L)u(x^2) + \nu^L u(x^4)\right)$$

Note that if $\nu^H + \nu^L = 1$ then the two expressions above coincide.

3.2 A price interval supporting zero holding of the indexed bond

We now establish, following Dow and Werlang (1992) that there is a non-degenerate interval of relative bond prices, $\frac{q^i}{q^n}$, at which the agent optimally wants to hold a zero position in the indexed bond. Below, we present an informal, intuitive argument.[6]

[5] The actual equilibrium model in the next section allows beliefs about the money supply to be ambiguous too.

[6] A more formal argument appears in the working paper version of this work.

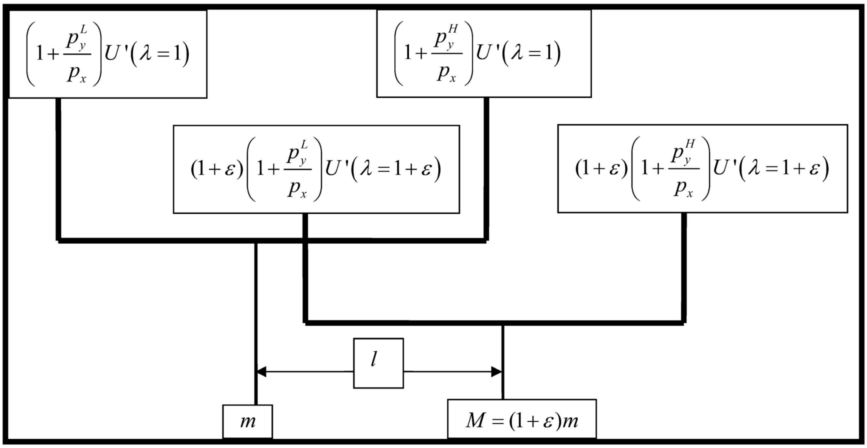

Figure 1. Contingent payoffs from an indexed bond

Suppose the agent holds only nominal bonds and is considering buying/selling an arbitrarily small unit of indexed bonds. The agent's present utility level in each of the four states may then be represented generically by $U(\lambda) \equiv u(\bar{x} + \frac{S}{q^n \lambda p_x})$, where $\lambda = 1$ in states 1 and 3 and $\lambda = 1 + \varepsilon$, $\varepsilon > 0$, in states 2 and 4. Since $u''(x) < 0$, the marginal utility in each state, $U'(\lambda) \equiv u'(\bar{x} + \frac{S}{q^n \lambda p_x})$, is increasing in λ, i.e., a higher inflation will affect the saver adversely. Now consider the vector of gross increase (decrease) in welfare at each state if the agent were to buy (sell) an infinitesimal unit of an indexed bond:

$$\left(\left(1+\frac{p_y^H}{p_x}\right) U'(1), \left(1+\frac{p_y^H}{p_x}\right) U'(1+\varepsilon), \left(1+\frac{p_y^L}{p_x}\right) U'(1), \left(1+\frac{p_y^L}{p_x}\right) U'(1+\varepsilon)\right)$$

Figure 1, depicts these "payoffs" for an ε "small enough".

Since $U'(\lambda)$ is increasing in λ and $\varepsilon > 0$, $(1+\frac{p_y^L}{p_x})U'(1) < (1+\frac{p_y^L}{p_x})U'(1+\varepsilon)$ and $(1+\frac{p_y^H}{p_x})U'(1) < (1+\frac{p_y^H}{p_x})U'(1+\varepsilon)$. Since u is continuous and $p_y^H > p_y^L$, for ε small enough $(1+\frac{p_y^L}{p_x})U'(1+\varepsilon) < (1+\frac{p_y^H}{p_x})U'(1)$. Now, to simplify matters dramatically, suppose $\nu^H = \nu^L = 0$. Hence, if the agent were to go long (resp. short) in the indexed bond, the payoffs when the monetary shock is low are $(1+\frac{p_y^L}{p_x})U'(1)$ (resp. $(1+\frac{p_y^H}{p_x})U'(1)$) and the payoffs when the monetary shock is high are $(1+\frac{p_y^L}{p_x})U'(1+\varepsilon)$ (resp. $(1+\frac{p_y^H}{p_x})U'(1+\varepsilon)$). Hence, the most the agent would want to bid for an unit of the indexed bond is $(1+\frac{p_y^L}{p_x})U'(1+\varepsilon)$. On the other hand the minimum the agent would ask for going short on an indexed bond is $(1+\frac{p_y^H}{p_x})U'(1)$. Hence, there must be a non-degenerate interval of prices at which the agent strictly prefers to maintain a zero holding of the indexed bond. The effect of increasing ε would be to increase the distance l. This implies that, for ε large enough, the portfolio inertia interval will collapse. Notice, it is not necessary that $\mathcal{A} = 1 - \nu^H - \nu^L = 1$ for the portfolio inertia interval to emerge: it will exist for $\mathcal{A} < 1$, as long $\mathcal{A}$ is high enough. Since the ranking of states (according to consumption) reverses when switching between a long position and a short position, the "relevant" probability also switches between evaluating a long and a short position. It is this feature which causes a "kink" in the utility functional at the zero holding position and leads to the portfolio inertia interval.[7]

Remark 1 Is there a range of prices at which there is a zero holding of the nominal bond? Indeed, in the set up we have described so far, if we were to have, in addition, ambiguous beliefs about the inflation (i.e., $\mu(\{1,3\}) + \mu(\{2,4\}) < 1$), then for a high enough level of this ambiguity, by an argument analogous to the one given above, there will be an interval of relative bond prices, $\frac{q^i}{q^n}$, at which the agent will only have a zero holding of the nominal bond. However, this is not very compelling as the result is not robust in an important way. It does not hold any more if the agent were to have some second period income, preset in nominal terms, that is not derived from bond holdings.

Suppose that the agent receives a state contingent income stream, $\{\overline{m}^s\}_{s=1}^4$, preset in nominal terms, e.g., house rent, social security benefits, payments which have a nominal component. In this richer model, the second period budget constraint in state 1 is as follows (the constraints in the three other states are obtained similarly):

$$x^1 = \bar{x} + \frac{\left(p_x + p_y^H\right)}{q^i p_x} S + \left(1 - \left(p_x + p_y^H\right) \frac{q^n}{q^i}\right) \frac{b^n}{p_x} + \frac{\overline{m}^1}{p_x}$$

Now, it follows immediately, except in the very specific case wherein $(\overline{m}^2, \overline{m}^4) = \lambda(\overline{m}^1, \overline{m}^3)$, having a zero holding of nominal bonds does not allow the agent to be rid of the inflation risk. Indeed, $b^n = 0$ implies $x^1 = x^2$ and $x^3 = x^4$ if and only if $(\overline{m}^2, \overline{m}^4) = \lambda(\overline{m}^1, \overline{m}^3)$. This, of course, is the case only if all income is fully indexed or there is no preset nominal income ($\overline{m}^s = 0, \forall s$). Hence, whenever, $(\overline{m}^2, \overline{m}^4) \neq \lambda(\overline{m}^1, \overline{m}^3)$ and $b^n = 0$ consumption is strictly ordered across states 1 and 2 and across states 3 and 4. This strict ordering is preserved in the ϵ-neighborhood $b^n \in (-\epsilon, \epsilon)$. Hence, there is no switch in the probability the DM "applies" when evaluating going short and going long on the nominal bond. Thus the usual expected utility logic applies and there is no non-degenerate interval of (bond) prices at which the agent holds zero nominal bond.

Remark 2 We have assumed constant x endowment. This is essentially for expositional ease. If one were to introduce uncertain endowment of good x, a similar reasoning would hold: given a value of the endowment, there would still be two other (orthogonal) sources of uncertainty, namely, the price of good x and the value of the indexed bundle (the price of good y). While it is true that Dow and Werlang's result assumed that the agent's endowment was riskless, a crucial contribution of Epstein and Wang (1994) was to show that the result could be generalized to the case of a non-stochastic endowment so long as there were a part of the asset payoff that

[7] An analogous argument shows that one would also obtain a portfolio inertia interval for a lender (i.e., $S < 0$).

was orthogonal to the endowment. If one were to introduce say two values for the level of endowment (say, $\bar{x}, \underline{x}$), the actual number of states would be eight, and for sufficiently high ambiguity aversion, the agent would not be willing, for an interval of prices, to use the indexed bond to hedge the risk linked to his x-endowment.

4 No-trade in indexed bonds: a general equilibrium framework

4.1 The institutional setup

The previous section shows that for both types of agents, those who save and those who borrow, there exists a range of relative bond prices, corresponding to each agent, at which the agent maintains a zero holding in indexed bonds. However, this does not immediately translate into a conclusion about conditions under which no-trade in indexed bonds is the *unique equilibrium* outcome. To be able to get to that conclusion several questions remain to be answered. What would ensure that the bid-ask price intervals of the various agents "overlap"? Why should equilibrium bond price fall within the zone of "overlap"? Further, since we know that for a portfolio inertia interval to emerge the goods prices have to vary across states in particular ways, a related question is are such state-contingent price variations consistent with competitive equilibrium in money, goods and bond markets? To deal with such issues we turn next to a two-period monetary general equilibrium model, general in the sense that *all* prices are obtained endogenously by (simultaneous) market clearing in bond, goods and money markets. Since the overall aim is to lay out the logic of no trade as transparently as possible, we have chosen the simplest model we could: an exchange economy without any production, wherein relative-price movements are derived by perturbing endowments.

There are two groups of agents in the model. The first group (whose agents are indexed by $h = 1, \dots, H$) are those who trade on financial markets, while the second group (whose agents are indexed by $k = 1, \dots, K$) has no access to any financial markets and therefore all the agents in this group consume all the revenue from their endowment spot by spot. There are three goods in this economy, x, y, and z. Agents h consume only goods x and z while agents k consume only goods y and z. We also assume that agents h have real endowments only in goods x and z, while agents k have real endowments only in goods y and z. In addition, agents h may have nominal endowments. Nominal endowments are any precontracted transfers, positive or negative, between agents that are set in nominal terms.

To see the rationale of "type-casting" agents as above recall, from what was noted in the introductory section, we want to ensure in the model that with respect to *any* two agents wishing to trade in indexed bonds, it is true that the indexation bundle contains at least one good which is not consumed by either of the agents. This condition, of course, would not be satisfied if an h-type agent were to trade bonds with a k-type agent. We have each type of agent consuming two goods rather than one, unlike in the model in the previous section, so that there may be market exchange among agents, thereby obtaining well-defined prices at equilibrium (reflecting the common utility gradients). Informally put, the focus of the "show" will be the intertemporal exchange between the h-type agents, with the role of k-type agents being essentially that of a necessary "prop", enabling the determination of the relative price of the good not figuring in the consumption baskets of agents trading bonds.

There are two periods in the model; uncertainty essentially comes into play in the final period. The endowment of h-type agents is uncontingent, given by $((\bar{x}_h^0, \bar{z}_h^0), (\bar{x}_h, \bar{z}_h, \overline{m}_h))$, where $(\bar{x}_h^0, \bar{z}_h^0)$ is the endowment in the initial period, Period 0, and $(\bar{x}_h, \bar{z}_h, \overline{m}_h)$ is the endowment in the final period, Period 1. $\overline{m}_h$ denotes the nominal endowment, so that $\overline{m}_h \lesseqqgtr 0$ and since transfers should balance across households, we have

$$\sum_{h=1}^{H} \overline{m}_h = 0.$$

Note though, the endowments vary across households; this heterogeneity is the reason why h-type agents trade intertemporal transfers. The endowment of k-type agents are given by $(\bar{y}_k^0, \bar{z}_k^0)$ in the initial period[8]. Their final period endowment in good z is uncontingent and equal to $\bar{z}_k$. We assume, though, their endowments in good y *is* contingent: in state t, $\bar{y}_k^t$, are such that $\sum_{k=1}^{K} \bar{y}_k^t = y^L$ for, say, $t = 1, \ldots, \tau$ and $\sum_{k=1}^{K} \bar{y}_k^t = y^H$ for $t = \tau + 1, \ldots, T$. Thus, in terms of total endowments, there are two "aggregate" states: one where the total endowment of good y is low (y^L) and another, where the total endowment of y is high (y^H). As will be seen, it is this variation in aggregate endowment which completely determines the variation in the relative price of y.

There is also (outside) money in the model, whose supply in the Period 0 is fixed at M^0 but may take on two values in the Period 1, m or M, where $M \equiv \lambda m$, $\lambda > 1$. The role of money is simply to facilitate exchange. Hence, at each spot, we assume the standard fiction that agents sell to a central authority *all* their endowments against currency issued by the central authority and then buy back from that authority the goods they want to consume (see Magill and Quinzii (1992)). The money obtained from the central authority by agent h (respectively, k) from the sale of endowments in state s is denoted m_h^s (respectively, m_k^s).

Uncertainty in the model is exhaustively represented by the state space

$$\mathcal{S} \equiv \{0\} \cup \{\{1, ..., T\} \times \{m, M\}\},$$

where, $\{0\}$ refers to Period 0, $\{1, ..., T\}$ indexes contingencies in Period 1 obtaining due to variation in real endowments of agents, $\{m, M\}$ indexes the variation in money supply. Let $s \in \mathcal{S}$ be an index for states, $s = 0, 1, \ldots, S$. We denote the prices of goods x, y, and z as p_x^s, p_y^s, and p_z^s, respectively, in state s.

There are two financial assets in the model, traded in Period 0. The first is a nominal bond, b^n, that pays off one unit of money in all states and with its price denoted q^n. The second is an indexed bond, b^i, that pays off a bundle of goods at each state in Period 1. We take this bundle to be state-independent and comprising

[8] Note, h-type agents do not have nominal endowments in the initial period. k-type agents do not have nominal endowments at all. This is just to save on notation; introducing such endowments would not make the slightest difference to any of our results.

of a unit of each good traded in the economy. Hence, the monetary return to holding a unit of this indexed bond is $p_x^s + p_y^s + p_z^s$ in state $s = 1, \ldots, S$. Its price is denoted q^i.

For the moment, denote an agent h's preferences by a functional $V_h((x_h^0, z_h^0), \ldots, (x_h^S, z_h^S))$, on which we'll impose assumptions detailed later on. His maximization problem is hence :

$$Max_{x_h, z_h, b_h^i, b_h^n} V_h\left((x_h^0, z_h^0), \ldots, (x_h^S, z_h^S)\right)$$
$$s.t. \begin{cases} p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 = m_h^0 \\ p_x^0 x_h^0 + p_z^0 z_h^0 = m_h^0 - q^i b_h^i - q^n b_h^n \\ p_x^s \bar{x}_h^s + p_z^s \bar{z}_h + \bar{m}_h = m_h^s \\ p_x^s x_h^s + p_z^s z_h^s = m_h^s + b_h^n + (p_x^s + p_y^s + p_z^s) b_h^i \ , s = 1, \ldots, S \end{cases}$$

Agents k, who have no access to financial markets, have to solve $S+1$ separate maximization programs. We assume that their preferences at each spot take the simple form of a Cobb-Douglas function: $(y_k^s)^\alpha (z_k^s)^{1-\alpha}$ for $\alpha \in (0,1)$. Hence, their maximization problem for $s \in \mathcal{S}$, is:

$$Max_{y_k^s, z_k^s} \ (y_k^s)^\alpha (z_k^s)^{1-\alpha}$$
$$s.t. \quad \begin{cases} p_y^s \bar{y}_k^s + p_z^s \bar{z}_k = m_k^s \\ p_y^s y_k^s + p_z^s z_k^s = m_k^s \end{cases}$$

An equilibrium of this model is therefore an allocation $(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{m}, \mathbf{b}^i, \mathbf{b}^n)$ and prices $(p_x, p_y, p_z, q^i, q^n)$ such that, given these prices agents solve their maximization problems and markets clear.

Observe that money as we introduced it is simply a veil and we can rewrite the budget constraints as follows, for agent h:

$$\begin{cases} p_x^0 x_h^0 + p_z^0 z_h^0 = p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 - q^i b_h^i - q^n b_h^n \\ p_x^s x_h^s + p_z^s z_h^s = \overline{m}_h + p_x^s \bar{x}_h^s + p_z^s \bar{z}_h + b_h^n + (p_x^s + p_y^s + p_z^s) b_h^i \quad s = 1, \ldots, S \end{cases}$$

and, for agent k in state s ($s = 0, 1, \ldots, S$):

$$p_y^s y_k^s + p_z^s z_k^s = p_y^s \bar{y}_k^s + p_z^s \bar{z}_k$$

One can also use the particular structure of the model to simplify the market clearing condition for good z. Indeed, adding the budget constraints in state s of agents h, one gets, at equilibrium, $\sum_{h=1}^H z_h^s = \sum_{h=1}^H \bar{z}_h$ (under the assumption that $p_x^s > 0$, which is met since preferences are assumed strictly monotonic). Similarly, for agents k, one obtains, from adding their budget constraints in state s and using equilibrium condition on the market for good y (plus the fact that, at an equilibrium, $p_z^s > 0$), that $\sum_{k=1}^K z_k^s = \sum_{k=1}^K \bar{z}_k$. Thus, the market clearing conditions on the market for good z can be split in two equalities as follows :

$$\sum_{h=1}^H z_h^s = \sum_{h=1}^H \bar{z}_h \ \text{ and } \ \sum_{k=1}^K z_k^s = \sum_{k=1}^K \bar{z}_k \quad s = 0, \ldots, S$$

Hence, the market for good z can be "divided in two", agents h exchanging among themselves, and similarly for agents k. The intuition for this is fairly obvious once one lifts the "veil of money" and considers the nature of "real" exchange in the model. The point is, given that the two types of agents share only one good between their respective consumption baskets, there cannot be any "real" exchange between these groups on spot markets.

Finally, notice that the market clearing condition on the money market can be written as

$$p_x^s \sum_{h=1}^{H} \bar{x}_h + p_y^s \sum_{k=1}^{K} \bar{y}_k^s + p_z^s \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right) = M^s \quad s = 0, \ldots, S$$

while the market clearing condition on the bond markets are $\sum_{h=1}^{H} b_h^i = \sum_{h=1}^{H} b_h^n = 0$.

4.2 Equilibrium prices in goods markets

We can further reduce the model by noticing that only aggregate states "matter". Indeed, note that there are two sources of (aggregate) uncertainty in this model: one is linked to the money supply, the second stems from the randomness in the (aggregate) endowment in good y of agents k. As we will be only interested in the equilibrium allocation of the h agents (and in particular whether they hold indexed bonds or not), the only way this last source of uncertainty is relevant to h agents is through the effect it has on prices. Now, observe that we can solve for the equilibrium relative price of y with respect to z, spot by spot. Indeed, agents k demand functions are easily computed and are equal to:

$$y_k^s(p_y^s, p_z^s) = \alpha \frac{p_y^s \bar{y}_k^s + p_z^s \bar{z}_k}{p_y^s} \text{ and } z_k^s(p_y^s, p_z^s) = (1-\alpha) \frac{p_y^s \bar{y}_k^s + p_z^s \bar{z}_k}{p_z^s}$$

Hence, at equilibrium,

$$\frac{p_y^s}{p_z^s} = \frac{\alpha}{1-\alpha} \frac{\sum_{k=1}^{K} \bar{z}_k}{\sum_{k=1}^{K} \bar{y}_k^s}$$

and therefore, the ratio of the prices p_y^s and p_z^s depends only on the *aggregate* (among k agents) endowments of good y and z, and thus, can take on only two values, whether aggregate endowment in y is high (y^H) or low (y^L). Note that the price levels do depend on the money supply. To sum up, we need, for our purposes, concentrate only on four states $\omega \in \Omega \equiv \{1, 2, 3, 4\}$ defined in as follows [9] : ω_1 has low money supply (m) and low aggregate y-endowment (y^L), ω_2 has (M,y^L), ω_3 has (m,y^H), and ω_4 has (M,y^H).

[9] We denote these aggregate states by ω to distinguish them from the underlying states s which encompass some heterogeneity among k-type endowments. We'll continue to use the notation $\omega = 0$ to denote the first period.

We now describe agent's h preferences, following partly a specification due to Magill and Quinzii (1997). At state 0, h-type agents' utility function is written as $u(x_h^0, z_h^0)$, where $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, concave, differentiable and homogeneous of degree 1. In state $\omega = 1, 2, 3, 4$, the spot utility function of agent h is given by $f_h(u(x_h^\omega, z_h^\omega))$, where $f_h : \mathbb{R} \to \mathbb{R}$ is strictly increasing, strictly concave and differentiable, and u is as defined above. The linear homogeneity of u will facilitate the tractability of the equilibrium (contingent) price function (essentially, the assumption ensures that goods prices, in each spot market, are independent of the distribution of wealth among agents) while the concavity of f_h is a simple way of endowing agents with risk aversion as well as a desire to smooth consumption across periods.

Type-h agents are endowed with (common) beliefs about the money supply process and the process generating the (aggregate) y-endowments. The capacity $\mu = (\mu^m, \mu^M)$ denotes their (marginal) belief about the money supply: μ^m is the (possibly non-additive) probability the money supply is m and μ^M is the probability that it is M; $\mu^m + \mu^M \leq 1$. Their (marginal) beliefs on the process generating the y-endowments (a good they do not consume and are not endowed with) are represented[10] by $\nu = (\nu^L, \nu^H)$, with $\nu^L + \nu^H \leq 1$. The overall beliefs on Ω is given by the independent product $\mu \otimes \nu$. Preferences of agent h are thus represented by the functional V_h, where,

$$V_h\left((x_h^0, z_h^0), \ldots, (x_h^4, z_h^4)\right) \equiv u(x_h^0, z_h^0) + \mathbb{CE}_{\mu\otimes\nu} f_h\left(u(x_h^\omega, z_h^\omega)\right).$$

Under our assumptions (essentially, the linear homogeneity of u) at an equilibrium, the maximization problem of each h agent can be decomposed to separate the financial decision about the allocation of resources across states from the decision about the consumption mix at each state. Turning first to the latter, given a stream $(R_h^0, R_h^1, \ldots, R_h^4)$ of income with $R_h^0 = p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 - q^i b_h^i - q^n b_h^n$ and $R_h^\omega = p_x^\omega \bar{x}_h^\omega + p_z^\omega \bar{z}_h + (p_x^\omega + p_y^\omega + p_z^\omega) b_h^i + b_h^n + \bar{m}_h$, $\omega = 1, \ldots, 4$, the agent solves the problem :

$$\begin{aligned} &Max_{x_h^\omega, z_h^\omega} \; u(x_h^\omega, z_h^\omega) \\ &\text{s.t.} \qquad p_x^\omega x_h^\omega + p_z^\omega z_h^\omega = R_h^\omega \end{aligned}$$

At the optimal choice, one gets for all $\omega = 0, 1, ..., 4$:

$$\frac{\nabla_1 u(x_h^\omega, z_h^\omega)}{\nabla_2 u(x_h^\omega, z_h^\omega)} = \frac{p_x^\omega}{p_z^\omega}$$

where $\nabla_i u(x_h^\omega, z_h^\omega)$ is the derivative of u with respect to its i^{th} component. By homogeneity of degree one, the gradients are collinear among agents only if their consumption vectors are collinear as well. Recall, at an equilibrium agents h only trade among themselves and do not trade with the k agents. Hence, each agent h's consumption in state ω is a fraction α_h^ω of total endowment of h-agents with

$$\alpha_h^0 = \frac{p_x^0 \bar{x}_h + p_z^0 \bar{z}_h - q^i b_h^i - q^n b_h^n}{p_x^0 \sum_{h=1}^H \bar{x}_h^0 + p_z^0 \sum_{h=1}^H \bar{z}_h^0}$$

[10] Note that, ν^H refers now to the state with high y-endowment, and therefore low p_y, while it referred to the high p_y in the previous section.

$$\alpha_h^\omega = \frac{p_x^\omega \bar{x}_h + p_z^\omega \bar{z}_h + (p_x^\omega + p_y^\omega + p_z^\omega) b_h^i + b_h^n + \bar{m}_h}{p_x^\omega \sum_{h=1}^H \bar{x}_h + p_z^\omega \sum_{h=1}^H \bar{z}_h}, \quad \omega = 1, ..., 4$$

Hence, at an equilibrium,

$$(x_h^\omega, z_h^\omega) = \alpha_h^\omega \left(\sum_{h=1}^H \bar{x}_h, \sum_{h=1}^H \bar{z}_h \right)$$

Therefore, agent h's utility, at an equilibrium, can be rewritten in state $\omega = 1, ..., 4$, $u(x_h^\omega, z_h^\omega) = \alpha_h^\omega u$, where u is simply the utility at the endowment point, i.e., $u \equiv u(\sum_{h=1}^H \bar{x}_h, \sum_{h=1}^H \bar{z}_h)$. Observe that the same holds in the first period, i.e., $u(x_h^0, z_h^0) = \alpha_h^0 u^0$, with $u^0 \equiv u(\sum_{h=1}^H \bar{x}_h^0, \sum_{h=1}^H \bar{z}_h^0)$.

Finally, since relative prices of good x and z in state ω are equal to the gradient of an agent h's utility function, it is easy to see that

$$\frac{p_x^1}{p_z^1} = \frac{p_x^2}{p_z^2} = \frac{p_x^3}{p_z^3} = \frac{p_x^4}{p_z^4} \equiv \zeta$$

given that endowments of goods x and z are constant across states. In fact, it turns out that, the absolute price of goods x and z do not depend on the amount of good y available in the economy. In other words, and since there is no uncertainty on the total endowments of goods x and z, their price depends only on the money supply. This is the content of Proposition 1, below. A direct corollary is that the price of y, conditional on the monetary state, is completely determined by the aggregate endowment in y. Proposition 2, which essentially shows that monetary equilibrium requires the price vector in state 2 (respectively, 4) is simply a λ-multiple of prices in state 1 (respectively, 3), completes the required characterization of equilibrium prices.

Proposition 1 *At an equilibrium,* $p_x^1 = p_x^3, p_x^2 = p_x^4,\ p_z^1 = p_z^3$, *and* $p_z^2 = p_z^4$.

From this proposition, it is easy to show that, at an equilibrium, $\frac{p_y^1}{p_y^3} = \frac{y^H}{y^L} = \frac{p_y^2}{p_y^4}$ (this follows from the fact that $\frac{p_y^1}{p_z^1} = \frac{p_y^3}{p_z^3} \frac{y^H}{y^L}$ and $p_z^1 = p_z^3$).

Proposition 2 *At an equilibrium,* $(p_x^1, p_y^1, p_z^1) = \frac{1}{\lambda}(p_x^2, p_y^2, p_z^2)$ *and* $(p_x^3, p_y^3, p_z^3) = \frac{1}{\lambda}(p_x^4, p_y^4, p_z^4)$.

The following table, then, summarizes the equilibrium prices, and the corresponding return from an unit of an indexed bond at each state $\omega,\ \omega \in \Omega$. The reader will recall the table is identical (but for price of the good z) to the one presented in Section 3.

State ω	Prices	Return from an indexed bond
1	(p_x, p_y^H, p_z)	$p_x + p_y^H + p_z$
2	$(\lambda p_x, \lambda p_y^H, \lambda p_z)$	$\lambda \times (p_x + p_y^H + p_z)$
3	(p_x, p_y^L, p_z)	$p_x + p_y^L + p_z$
4	$(\lambda p_x, \lambda p_y^L, \lambda p_z)$	$\lambda \times (p_x + p_y^L + p_z)$

4.3 The nature of equilibrium in bond markets

We now turn to the intertemporal maximization problem of agent h and derive the principal formal conclusions of our analysis. The first result, which is in the nature of a benchmark, shows that, for generic endowments, if beliefs are not ambiguous, there will always be trade in indexed bonds. The second and main result shows that, at equilibrium, if ambiguity of belief about the $y-$prices ($\mathcal{A}(\nu) \equiv 1 - \nu^L - \nu^H$) is large enough and inflation risk (λ) is not too high, the indexed bond is not traded and only the nominal bond is traded. As we show, this result holds irrespective of the degree of ambiguity about the money supply. In what follows, we first explain an intuition of the equilibrium reasoning underlying the results and then state the theorems.

We begin by considering the nature of the equilibrium in the indexed bond market at two values of λ, $\lambda = 1$ and $\lambda = 1 + \varepsilon$, where ε is a positive number arbitrarily close to 0. Consider, first, the case wherein $\lambda = 1$. Without any inflation risk at all, clearly, all borrowing and lending will be done through nominal bonds, at equilibrium. Take two h-type agents, h' and h who save and borrow, respectively, in the initial period. Their utility in the final period, with slight abuse of the notation, may be written as $f_{h'}(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_{h'}}{\lambda q^n}))$ and $f_h(u(\bar{x}_h, \bar{z}_h; \frac{S_h}{\lambda q^n}))$, where $S_{h'} > 0$ and $S_h < 0$ denote the amount saved and the amount borrowed by h' and h respectively. Define the "marginal utilities", $U'_h(\lambda) \equiv f'_h(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_h}{\lambda q^n}))$ and $U'_{h'}(\lambda) \equiv f'_{h'}(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_{h'}}{\lambda q^n}))$. Notice, $U'_{h'}(\lambda) \uparrow$ in λ while $U'_h(\lambda) \downarrow$ in λ, since $S_{h'} > 0$ and $S_h < 0$; intuitively, inflation affects the welfare of savers and borrowers differently. Furthermore, it must be necessarily true at an equilibrium that $U'_h(\lambda = 1) = U'_{h'}(\lambda = 1)$. This is so since if there is no inflation risk, h-agents are effectively trading in a complete market when trading only in nominal bond, and thus the equilibrium is Pareto optimal.

First suppose, ceteris paribus, there were no ambiguity, i.e., $1-\nu^L-\nu^H = 0$ and $1 - \mu^m - \mu^M = 0$, so that the DM's behavior were that of an SEU agent. At $\lambda = 1$, the "utility return" from an (infinitesimal) unit of an indexed bond at equilibrium must be $U'_h(\lambda = 1) \times \mathbb{E}(p_x + p_y^\omega) = U'_{h'}(\lambda = 1) \times \mathbb{E}\left(p_x + p_y^\omega\right) \equiv q^i(\lambda = 1; h, h')$. Putting it differently,
$q^i(\lambda = 1; h, h')$ is the price at which the agents (h and h') are indifferent between not trading and trading an infinitesimal amount of indexed bonds. Similarly, for an arbitrary λ, define $q^i(\lambda; h)$ (respectively, $q^i(\lambda; h')$) as the minimum (maximum) price h (h') is willing to accept (pay) to trade in the indexed bond. Next, consider a perturbation of λ to $\lambda = 1 + \varepsilon$. Recalling the effect of a change in λ on $U'_h(\lambda)$ and $U'_{h'}(\lambda)$, it is straightforward to see that

$$\begin{aligned} q^i\left(\lambda = 1 + \varepsilon; h'\right) &\equiv U'_{h'}\left(\lambda = 1 + \varepsilon\right) \times \mathbb{E}\left(p_x + p_y^\omega\right) \\ &> q^i\left(\lambda = 1; h, h'\right) \\ &> U'_h\left(\lambda = 1\right) \times \mathbb{E}\left(p_x + p_y^\omega\right) \geq q^i\left(\lambda = 1 + \varepsilon; h\right). \end{aligned}$$

Intuitively, since the saver is affected adversely by inflation, relative to the debtor, the indexed bond is more valuable to the saver in the presence of inflation, and also, more valuable than it is to the debtor. Hence, inevitably, with inflation risk creeping up there will be gains from trading in indexed bonds and indexed bonds will be traded at equilibrium under SEU. This is depicted in Figure 2, below, in the left-hand-side panel and formally stated in our first theorem[11].

Theorem 1 *Suppose,* $\mu^m + \mu^M = 1$ *and* $\nu^L + \nu^H = 1$. *Then, for generic first period aggregate endowments, there is trade in the indexed bond whenever* $\lambda \neq 1$.

Next suppose, agents have CEU preferences and beliefs about the y-prices are ambiguous, i.e., $1 - \nu^L - \nu^H > 0$. As before, we first consider the equilibrium at $\lambda = 1$. As would be evident from our discussion in Section 3, there would exist a portfolio inertia interval: there will be a bid price corresponding to (perceived) marginal gain from moving (infinitesimally) into a indexed bond, and an ask price, that is strictly lower, corresponding to the (perceived) marginal gain from going short on the indexed bond. The bid-ask interval will correspond to an interval of expected marginal utilities, where the lower end of the interval is evaluated by applying the probability measure that minimizes the expectation for an agent going long and the upper end is evaluated by applying the probability measure that minimizes the expectation for an agent going short.

Consider now the equilibrium given the perturbation $\lambda = 1 + \varepsilon$. It is straightforward to see that for h', the saver, the *entire* interval moves *up*, whereas for h, the debtor, the *entire* interval moves *down*. The extent of movement is greater, the

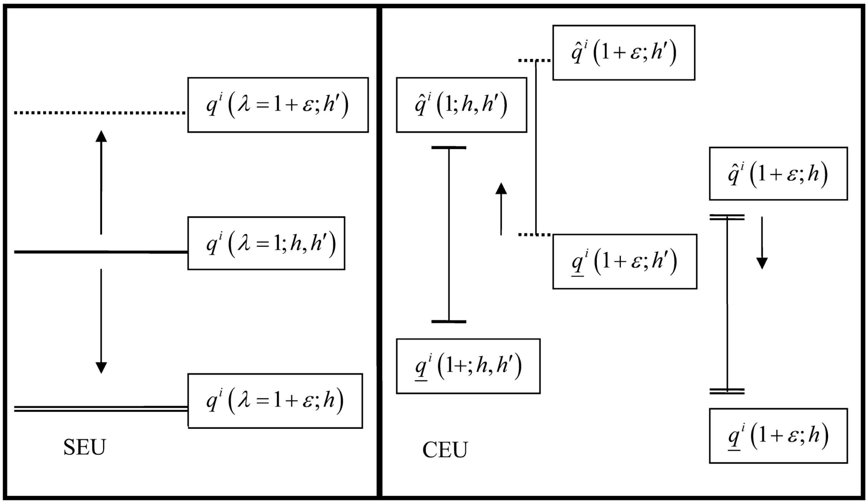

Figure 2. Equilibrium in the indexed bond market

[11] Both theorems refer to properties that hold "generically". The term is applied in a way that is now standard in economic theory. Notice, endowments are points in $\mathbb{R}^6$ and the μ-beliefs are simply points on the 2-dimensional simplex. We say that a property is satisfied for generic endowments (respectively, μ's) if, for every endowment (respectively, μ) vector there is an open dense neighborhood of endowment (respectively, μ) vectors that generate economies that satisfy this property. Thus, if a property is satisfied for all generic endowments (respectively, μ's), small perturbations of the endowment (respectively, μ) in any economy can generate a new economy that satisfies this property robustly, even if the original economy does not.

greater increase in λ. Hence, for $\lambda > 1$, but small enough, the intervals overlap and the bid price of the saver remains strictly lower than the ask price required by the lender:$\underline{q}^i(1+\varepsilon; h')\hat{q}^i(1+\varepsilon; h)$. This is represented in Figure 2 in the right-hand side panel. It is also evident in the figure that if the increase in λ were large enough then the intervals move apart enough not to overlap. Hence, we have, for λ small enough, there is no trade in indexed bonds at equilibrium. This is formally stated in the theorem below.

Since λ refers to inflationary risk, part (a) of the theorem states that if this risk is sufficiently small then, ambiguity about relative price movement prevents trade in indexed assets. Finally, recalling Remark 1, if there is at least one agent with a positive nominal endowment (and hence, at least one other agent with a negative nominal endowment) it follows that there is at least a pair of agents who do not optimally choose a zero-holding of the nominal bond over a non-degenerate interval of relative bond prices. Hence, for these agents the situation is no different from the SEU case described earlier. "Typically" such agents would want to trade in nominal bonds, as stated in part (b).

Theorem 2 *Suppose,* $\mu^m + \mu^M \leq 1$ *and* $\nu^L + \nu^H < 1$. *Then, there exists a bound* $\delta, \delta > 1$, *such that, if* $\lambda < \delta$, *there exists* $\gamma, 0 < \gamma < 1$, *such that if* $\mathcal{A}(\nu) > \gamma$, *then at an equilibrium,*

(a) the indexed bond is not traded, i.e., $b^i_h = 0$ *for all* h,
(b) for generic μ*-beliefs, there is trade in the nominal bond as long as there exists an* h *such that* $\bar{m}_h > 0$.

Remark 3 Notice, both parts of the theorem hold regardless of the level of ambiguity w.r.t. the μ-beliefs. Indeed, the formal proof is a lot shorter (and simpler) if one were to assume that μ-beliefs were unambiguous. We, however, do not impose this restriction in the analysis so that one may obtain a more informed idea of the nature of robustness of the result. Of course, nominal endowments would no longer play a role in the argument if μ-beliefs were assumed to be unambiguous.

Remark 4 The logic underlying the result in Theorem 2(b) is actually instructive, indirectly, as to why ambiguity about the price movements of goods not in the (h-agents') consumption basket was the crucial factor in obtaining no-trade in indexed bonds. Putting it differently, if we allowed the absolute prices of say, x, to vary in response to supply shocks and assumed agents had ambiguous beliefs about such price movements, that would not obtain the no-trade in indexed bonds (without ambiguity about price of y). The reasoning here is analogous to the one showing that the presence of nominal endowments precludes no-trade in nominal bonds. Since x is present in the endowment and/or affects utility directly (of h-agents), maintaining a zero position on the indexed bond would not get rid of the risk due to the variability of the price of x. This is why we need the "prop" agents in the model: they are the ones who are the source of volatility of the price of good y, which is the crucial factor underlying the result. If we dropped these prop agents we would be taking away the y-good and therefore, the risk in an indexed asset orthogonal to the asset traders' endowment and consumption. If the asset did not contain this idiosyncratic risk, there would be trade in the two bonds, much like in a SEU economy (for further clarification see the discussion related to Figure 1 on page 887 and Example 2 in Mukerji-Tallon 2001).

5 Concluding discussion

As has already been noted, Dow and Werlang (1992) showed that a zero position may be held on a price interval if the agent's endowments were riskless. Obviously, an economy where all agents' endowments were unvarying across *all* states the question of asset trading and risk sharing is an uninteresting question. Epstein and Wang (1994) significantly generalized the Dow and Werlang (1992) result to find that price intervals supporting the zero position occurred (in equilibrium) if there were *some* states across which asset payoffs differ while endowments remain identical; in other words, asset payoffs have component of idiosyncratic risk. However, the focus of Epstein and Wang (1994) was the issue of asset pricing. In their model endowments are Pareto optimal, and consequently, the issue of whether ambiguity aversion cause assets not to be traded is not examined. Mukerji and Tallon (2001), building on the results in the two papers cited, finds conditions for an economy wherein the agents' price intervals overlap in such a manner such that *every* equilibrium of the economy involves no trade in an asset, and more importantly, conditions under which ambiguity aversion *demonstrably* "worsens" risk sharing and incompleteness of markets. One of the conditions, the presence of idiosyncratic risk, identified in Mukerji and Tallon (2001), is essentially the same as in the result of Epstein and Wang (1994) explained above. As has been suggested, it is possible to see that, for h-type agents, payoffs of indexed bonds contain an element of idiosyncratic risk derived from the risk inherent in the relative price of y.

The paper closest to ours, within the Savage paradigm, which seeks to explain the lack of indexed debt is Magill and Quinzii (1997). That paper compares the welfare improvements obtained from introducing within an incomplete markets setting, *in turn*, a nominal bond and an indexed bond. The welfare improvements derive from, essentially, the increase in the span of available assets (or, in other words, the "lessening" of incompleteness) that comes about due to the introduction of each type of bond. The more relevant result is that the welfare gain from introducing the indexed bond may be less (respectively, more) than that from introducing a nominal bond if the inflation risk was "small" (respectively, large) compared to the relative price risk. In contrast to the analysis in this paper, Magill and Quinzii (1997) does not actually obtain a equilibrium with no-trade in indexed bonds; indeed, as we confirm in Theorem 1, Savage rational agents will necessarily trade in indexed bonds as long as there is some inflation. Also, Magill and Quinzii (1997) do not allow *both* indexed and nominal bonds to be available for trade simultaneously; one or the other is available.

The present paper also complements the finding in Mukerji and Tallon (2004) which shows that ambiguity aversion (with CEU preferences) may help to explain why we see so little wage indexation. Among other things, the framework in that paper is one of bilateral contracting, not a general equilibrium market environment like it is here. Hence, the result there does not follow from the result here even. However, as we understand it, the same intuition explains both results, a point that is significant in so far as it shows that the intuition is robust across seemingly different trading environments. Finally, the paper adds to the growing literature on the economics applications of the idea of ambiguity aversion (see Mukerji and Tallon (2003b) for a survey).

Recall the intuition underlying the main result. Taking a long or a short position on the indexed bond implies betting on or against the (ambiguous) event wherein the (relative) price of good y will be high. To decide whether to bet on or against a particular event one has to reach a fine judgement about the relative likelihood of the event compared to its complement. Hence, the attraction of the zero holding position to the ambiguity averse agent. Moving from the zero position, in either direction, requires a compensating "ambiguity premium". Hence, the portfolio inertia intervals for the indexed bond. At low levels of inflation the bid-ask intervals of the borrower and the saver overlap and agents only trade in the nominal bond. As inflation rises, the saver is affected adversely while the borrower is made better off. As a consequence, the saver's bid price goes up and the borrower's ask price decreases. Hence if inflation were high enough, agents do trade indexed bonds. We also argued that, so long as agents held (non-zero) nominal endowments, this reasoning does not apply quite symmetrically to trade in nominal bonds.

Thus, according to the theory presented in this paper, it is the comparative lack of information about relative, as opposed to average, price movements, the comparative preponderance of nominal, as opposed to indexed, endowments that explains why trade in indexed bonds is observed only in exceptional circumstances but trade in nominal bonds is so widespread. Much of what we know about trade in indexed bonds is consistent with the theory. The theory is consistent with the fact that typically indexed bonds are traded almost exclusively under extreme inflationary circumstances. Also, while trade in indexed bonds is negligible in most non-inflationary economies, it is more than negligible (though still quite small) in the few such economies where, in addition, there are some instances of indexed endowments, statutory wage indexation, as is the case, for example, in the U.K. and in Israel (statutory wage indexation in a limited number of sectors of the economy). One may also argue that an analogous reasoning explains why in countries, like Turkey, where use of dollar is widespread in spot market transactions and so is the use of dollar-indexed debt.

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