Economic Theory **24**, 561–581 (2004)
DOI: 10.1007/s00199-004-0503-7

Economic Theory

Economic implications of better information in a dynamic framework*

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Received: September 8, 2003; revised version: March 3, 2004

Summary. We consider an OLG model with accumulation in human capital and analyze the economic implications of information about individual skills. Agents in each period differ by the random innate ability assigned to each individual. When young, all agents are screened for their abilities and this screening process (signal) constitutes a public information which is used in choosing the level of private investment in education. We demonstrate that in the presence of risk sharing markets better information may be harmful for all in equilibrium, and find conditions under which better information either enhances growth or reduces growth.

Keywords and Phrases: Information system, Human capital accumulation, Risk sharing markets.

JEL Classification Numbers: D80, J24.

1 Introduction

The role of human capital in enhancing economic growth has been analyzed extensively in the literature of the last two decades. Following the seminal contributions of Becker emphasizing the link between education and productivity (see, for example, Becker, 1964), the role of human capital became central in endogenous growth models (see, for example, Razin, 1973; Lucas, 1988; Azariadis and Drazen, 1990). The assumptions regarding the process of human capital formation became

* We are pleased to acknowledge the useful comments and suggestions of R. Benabou, Z. Eckstein, E. Helpman, B. Ravikumar and D. Tsiddon. Also, special thanks are given to two anonymous referees. This research was supported by a Grant from G.I.F., the German-Israeli Foundation for Scientific Research and Development.

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significant in the evolution of these dynamic models. The production function of human capital is clearly a complex one since it is affected by many factors including the home and the social environment, provision of education, motivation etc. (see, for example, Jovanovic and Nyarko, 1995; Laitner, 1997; Orazem and Tesfatsion, 1997). This aspect of the human capital formation process is a central point in our work. We integrate two strands in the literature: endogenous growth with human capital accumulation and the role of information. In our framework, information affects the process of human capital formation via its effect on investments in education and, hence, economic growth.

Since the seminal contributions by Blackwell (1951, 1953) on the positive welfare implications of 'more information' for an individual decision maker, this topic has attracted substantial attention among economists. The decision maker observes a signal, correlated to the state of nature, and updates his/her probability distribution before taking an action. However, Blackwell's result holds in an economic environment where the signals are private information. If information is public signals affect the opportunity sets of decision makers and more information may result in a lower welfare. The negative value of public information in equilibrium has been established for certain types of exchange economies by Hirshleifer (1971,1975), Green (1981), Orosel (1996), Schlee (2001) and others. When the model includes production the welfare implications of better information are more gratifying. Eckwert and Zilcha (2001) demonstrate that whether signals reveal information about 'uninsurable risks' or 'insurable risks' is important in determining the value of information.

The aforementioned papers dealing with the value of information consider static models and therefore ignore the implications of information for growth. Economic growth affects the welfare of future generations, hence the welfare analysis should be extended. Our paper takes into account the impact of information on the process of human capital formation and thereby on the evolution of the economy over time. It is widely recognized that investment in human capital is subject to considerable risk. In addition, the possibilities for diversification are quite limited because human capital cannot be traded on markets and cannot be separated from individuals (see Levhari and Weiss, 1974). In some cases, insurance contracts which are contingent on the human capital of an individual may be tradable, thereby allowing the agents to share part of their idiosyncratic risks. Our study compares the welfare effects of information under two different scenarios. The first scenario is characterized by the absence of any risk sharing arrangements; and under the second scenario agents are able to obtain partial insurance for the risky returns of investments in human capital.

The framework we use for our analysis is an overlapping generations economy with production (see Diamond, 1965) and no population growth. Individuals in the same generation differ in their (random) innate abilities. We assume that the human capital of an individual depends upon his/her innate ability as well as the 'environment', represented by the average human capital level of the older generation (the generation of the teachers and parents).[1] When ability is still unknown each individual decides how much ‘effort’ to invest in his/her education and training. The return to this investment, in term of wages during the working period, is random since it depends on the realization of the ability. However, prior to making the decision about investment in education (i.e., effort in our case), each agent observes a signal which reveals in a Bayesian manner some information about his/her personal ability. Thus, the ‘screening’ process, and hence the accumulation of human capital, depends on the informativeness of the signals. In our model better information means better screening with respect to individual ability when education and training are being formed. In the extreme case where signals are uninformative private investment in education is uncorrelated to ability.

Better information affects economic welfare in two ways. First, as signals become more reliable the agents are exposed to less uncertainty when they make their decisions. This reduction in uncertainty has an impact on welfare which is called the *direct* effect. Second, better information creates an externality through its impact on the accumulation of human capital: future generations benefit from a higher accumulation rate because they inherit part of their human capital from the previous generation. This mechanism is called the *indirect* welfare effect. We show that if no risk sharing is available then (a) the direct effect on welfare is always positive; (b) the indirect effect, i.e., the effect via growth, is positive in economies with moderately risk averse agents, and negative in highly risk averse economies. We also demonstrate that the operation of a risk sharing market can potentially interfere with the informational structure that the economy displays and, hence, with the ability of the screening process to enhance welfare. More precisely, if part of the human capital risk can be insured, both the direct and the indirect welfare effects are negative in economies with highly risk averse agents, and positive in economies with moderately risk averse agents. Thus, if the consumers are highly risk averse, under certain conditions better screening during the youth period is harmful and will reduce growth and welfare of all generations. Our dynamic model demonstrates that, in equilibrium, the value of information depends heavily on the risk sharing arrangements that exist in the market. In particular, it matters whether the information relates to risks that can be insured or to risks that are uninsurable.

2 The model

Consider an overlapping generations economy with a single commodity and a continuum of individuals in each generation (but no population growth). The commodity can be either consumed or used as an input (physical capital) in a production process. Individuals live for three periods: ‘youth’ where they obtain education (while still supported by parents), ‘middle-age’ where they work and consume, and ‘retire-

[1] Endogenous growth models in which human capital operates as the engine of growth have been widely used in the literature to analyze various economic issues related to economic policy (see, e.g., Lucas, 1988; Azariadis and Drazen, 1990; Eckstein and Zilcha, 1994; Galor and Tsiddon, 1997; Orazem and Tesfatsion 1997). We use a similar framework for our study of the dynamic effects of better information.

ment' where they only consume. We denote generation t by $G_t, t = 0, 1, \cdots .G_t$ consists of all individuals born at date $t - 1$.

One of the main features of our economy is the heterogeneity of individuals with regard to their human capital generated by innate ability. Nature assigns abilities to agents deterministically but, when young, no agent knows nature's choice. The distribution of abilities across agents is the same in each generation. Let $\nu(A)$ denote the (time-invariant) density of agents with ability A and, for convenience, normalize the measure of agents in each generation to 1:

$$\int_{\mathbb{R}_+} \nu(A)\,\mathrm{d}A = 1.$$

Agents learn their abilities only at the beginning of their middle-age period and, hence, they act under uncertainty in their first period of life. Observe, however, that there is no risk in the aggregate since the distribution ν is fixed. This approach follows the modelling technique in Feldman and Gilles (1985, Proposition 2), which produces individual uncertainty but aggregate certainty.

Human capital of individual $i \in G_t$ depends on ability $\tilde{A}^i$ (perceived as random and, therefore, marked by a ˜), effort $e^i \in \mathbb{R}_+$ invested in education by this individual when young, and the 'environment', represented here by the average human capital of agents in the previous generation (who are currently active economically). Thus we write,

$$\tilde{h}^i = \tilde{A}^i g(H_{t-1}, e^i) \tag{1}$$

with

$$g : \mathbb{R}_+^2 \to \mathbb{R}_+,\ g(H, e) = \hat{g}(H)e^{\alpha}, \tag{2}$$

where $\hat{g} : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, and $\alpha \in (0, 1)$. Agent i belongs to generation t, and H_{t-1} is the average human capital of G_{t-1}.

Before agent i chooses optimal effort in the youth period nature assigns to him a deterministic signal $y^i \in Y \subset \mathbb{R}$. The signals assigned to agents with ability A are distributed according to the density $\nu_A(y)$. The distribution of signals received by agents in the same generation has the density

$$\mu(y) = \int_{\mathbb{R}_+} \nu_A(y)\nu(A)\,\mathrm{d}A. \tag{3}$$

Denoting by $\nu_y(\cdot)$ the density of the conditional distribution of A given the signal y, average ability of all agents who have received the signal y is

$$\bar{A}(\nu_y) := \int_{\mathbb{R}_+} A\nu_y(A)\,\mathrm{d}A. \tag{4}$$

We assume that signals are public information while the effort employed by the individual is private information. This assumption will be relaxed later on.

Individuals derive negative utility from 'effort' while they are young. Denote their consumption in the working period by c_1, and in retirement period by c_2. For each agent, the lifetime utility function is given by

$$U(e, c_1, c_2) = v(e) + u_1(c_1) + u_2(c_2), \tag{5}$$

where the period utility functions belong to the family of CRRA:

$$u_1(c_1) = \frac{c_1^{1-\gamma_u}}{1-\gamma_u}; \quad u_2(c_2) = \beta\frac{c_2^{1-\gamma_u}}{1-\gamma_u}; \quad v(e) = -\frac{e^{\gamma_v+1}}{\gamma_v+1}. \tag{6}$$

γ_u and γ_v are strictly positive constants. γ_v parametrizes the curvature of the utility function in the youth period, v; and γ_u parametrizes the curvature of the utility functions in the middle age period and retirement period, u_i, $i = 1, 2$. In models with additively separable intertemporal preferences, the curvatures of the utility functions have elements of both risk aversion and intertemporal substitution: high relative risk aversion goes hand in hand with low intertemporal substitution in consumption (see Kihlstrom and Mirman, 1981; Hall, 1988; Epstein and Zin, 1989; Kocherlakota, 1990). However, since in our model the utility of an agent in his youth period is non-random, γ_v represents a measure for the reciprocal of the elasticity of intertemporal substitution (rather than a measure for relative risk aversion). By contrast, γ_u needs to be interpreted as a measure for the agent's relative risk aversion in his middle age period and retirement period. This is because in these periods price induced intertemporal substitution effects play no role as prices are fixed by the international rate of interest.[2]

In each period, production in our economy, is carried out by competitive firms who use two production factors: physical capital K and human capital H. The process is described by an aggregate production function $F(K, H)$, which exhibits constant returns to scale. If individual i supplies l^i units of labor in his 'working period', his supply of human capital equals $l^i h^i$. We assume inelastic labor supply, i.e., that l^i is a constant and it is equal to 1 for all i.

Assumption 1 *$F(K, H)$ is concave, homogeneous of degree 1, and satisfies $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.*

We assume throughout this paper full international capital mobility, while human capital is assumed to be immobile. Thus the interest rate $\bar{r}_t$ is exogenously given at each date t. This implies that marginal productivity of aggregate physical capital K_t must be equal to $1 + \bar{r}_t$ (assuming full depreciation of capital in each period). On the other hand, given the aggregate stock of human capital at date t, H_t, the stock K_t must adjust such that

$$1 + \bar{r}_t = F_K(K_t, H_t) \quad t = 1, 2, 3, \cdots \tag{7}$$

holds. But this implies, by Assumption 1, that $\frac{K_t}{H_t}$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate w_t (price of one unit of human capital),

[2] Below we shall assume that physical capital is internationally mobile and, hence, both the interest rate and the wage rate are fixed at each date. Note that it is this special feature of the model which allows us to interpret γ_u as a pure risk aversion measure.

given in equilibrium by the marginal product of aggregate human capital, is also determined once $\bar{r}_t$ is given. Thus we may write

$$w_t = F_L\Big(\frac{K_t}{H_t}, 1\Big) =: \zeta(\bar{r}_t) \quad t = 1, 2, 3, \cdots . \tag{8}$$

Now let us consider the optimization problem that each $i \in G_t$ faces, given $\bar{r}_t, w_t$, and H_{t-1}. At date $t-1$, when 'young', this individual chooses the optimal level of effort employed in obtaining education. This decision is made under random ability $\tilde{A}$, but after the signal y^i has been observed.[3] The decision about saving, s^i, to be used for consumption when 'old' is taken in the second period, after the realization of $\tilde{A}$, and hence when the human capital h^i is known. Thus s^i will depend on h^i via the wage earnings $w_t h^i$.

For given levels of h^i, w_t and $\bar{r}_t$, the optimal saving decision of individual $i \in G_t$ is determined by

$$\max_{s^i} \; u_1(c_1^i) + u_2(c_2^i), \qquad \text{s.t. } c_1^i = w_t h^i - s^i, \;\; c_2^i = (1+\bar{r}_t)s^i \tag{9}$$

and satisfies the necessary and sufficient first order condition

$$-u_1'(w_t h^i - s^i) + (1+\bar{r}_t)u_2'((1+\bar{r}_t)s^i) = 0 \tag{10}$$

for all h^i. From equation (10) we find optimal saving as a function of each realized h^i, i.e., $s^i = s_t(h^i)$. The optimal level of effort invested in education, e^i, is determined by

$$\max_{e^i} \; E[v(e^i)+u_1(\tilde{c}_1^i)+u_2(\tilde{c}_2^i)|y^i], \qquad \text{s.t. } \tilde{c}_1^i = w_t\tilde{h}^i - \tilde{s}^i, \;\; \tilde{c}_2^i = (1+\bar{r}_t)\tilde{s}^i, \tag{11}$$

where $\tilde{h}^i$ is given by equation (1) and $\tilde{s}^i$ satisfies equation (10). Due to the Envelope theorem and the strict concavity of the utility functions, problem (9) has a unique solution determined by the first order condition

$$v'(e^i) + w_t g_2(H_{t-1}, e^i)E[\tilde{A}u_1'(w_t\tilde{h}^i - \tilde{s}^i)|y^i] = 0. \tag{12}$$

Since u_1' is a decreasing function we also conclude from (10) that $s_t(h^i)$ and $w_t h^i - s_t(h^i)$ are both increasing in h^i. This implies, in particular, that the LHS in (12) is strictly decreasing in e^i. Similarly, from equation (12) we obtain the optimal level of effort as a function of the conditional distribution ν_{yi}, i.e., $e^i = e_t(\nu_{yi})$. Note that any two agents in generation t who receive the same individual signal will choose the same effort level.

Using (3) and (4) the aggregate stock of human capital at date t can be expressed as

$$H_t = E_y[\bar{h}_t(\nu_y)] = \int\limits_Y \bar{h}_t(\nu_y)\mu(y)\mathrm{d}y, \tag{13}$$

[3] Strictly speaking, at this date the agent *perceives* his ability to be randomly distributed according to $\nu_y(\cdot)$.

where

$$\bar{h}_t(\nu_y) := \bar{A}(\nu_y)g(H_{t-1}, e_t(\nu_y)) \tag{14}$$

is the average human capital of agents in G_t who have received the signal y.[4]

Definition 1 *Given the international interest rates $(\bar{r}_t)$ and the initial stock of human capital H_0, a competitive equilibrium consists of a sequence $\{(e^i, s^i)_{i\in G_t}\}_{t=1}^{\infty}$, and a sequence of wages $(w_t)_{t=1}^{\infty}$, such that:*

(i) *At each date t, given $\bar{r}_t$, H_{t-1}, and w_t, the optimum for each $i \in G_t$ in problems (11) and (9) is given by (e^i, s^i).*
(ii) *The aggregate stocks of human capital, $H_t, t = 1, 2, \cdots$, satisfy (13).*
(iii) *Wage rates $w_t, t = 1, 2, \cdots$, are determined by (8).*

2.1 Information systems

Since a young individual is ignorant about what ability nature has assigned to him, he perceives his ability as a random variable with prior distribution ν. The distributions of signals and of abilities across agents in the same generation are correlated. Therefore, each agent uses his signal, y, to update the prior distribution, ν, of his ability. The updated distribution has density

$$\nu_y(A) = \nu(A|y) = \nu_A(y)\nu(A)/\mu(y). \tag{15}$$

An information system, which will be represented by ν_A throughout the paper, specifies for each level of ability $A \in \mathbb{R}_+$ a conditional density function over the set of signals. The positive real number $\nu_A(y)$ is the conditional density of all agents with ability A to whom nature has assigned the signal y.

Hence, the positive real number $\nu_A(y)$ defines the perceived conditional probability (density) that if ability is A, then the signal y will be sent. We assume that the densities $\{\nu_A(\cdot), A \in \mathbb{R}_+\}$ have the strict monotone likelihood ratio property (MLRP): $y' > y$ implies that for any given (nondegenerate) prior distribution for A, the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-order stochastic dominance.[5] As a consequence, $\int_{\mathbb{R}_+} \varphi(A)\nu(A|y')\mathrm{d}A > \int_{\mathbb{R}_+} \varphi(A)\nu(A|y)\mathrm{d}A$ holds for any strictly increasing function φ.

Following Blackwell (1953) a criterion can be defined that compares different information systems by their informational contents.[6] Suppose $\bar{\nu}_A$ and $\hat{\nu}_A$ are two information systems with associated density functions $\bar{\nu}_y$, $\hat{\nu}_y$, $\bar{\mu}$, $\hat{\mu}$. The informativeness of an information system can be defined as follows:

[4] The fact that human capital constitutes the only state variable is a limiting feature of our model. In particular, due to short planning horizons of agents, individual beliefs do not constitute state variables. Bertocchi and Spagat (1998) and Datta, Mirman and Schlee (2002) have recently studied dynamic models which include beliefs as one component of the state vector.

[5] For details see Milgrom (1981).

[6] The Blackwell-criterion is quite demanding. In particular, it does not allow a comparison of *any* two information structures and, therefore, induces an incomplete ordering on the set of information systems. For a generalization of this concept see Athey and Levine (1998), and Persico (2000).

Definition 2 *(informativeness:) Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems. $\bar{\nu}_A$ is said to be more informative than $\hat{\nu}_A$(expressed by $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$), if there exists an integrable function $\lambda : Y^2 \to \mathbb{R}_+$ such that*

$$\int_Y \lambda(y', y)\mathrm{d}y' = 1 \tag{16}$$

holds for all y, and

$$\hat{\nu}_A(y') = \int_Y \bar{\nu}_A(y)\lambda(y', y)\mathrm{d}y \tag{17}$$

holds for all $A \in \mathbb{R}_+$.

The concept of informativeness is based on a simple intuitive idea: consider a stochastic mechanism, compatible with equation (16), that transforms a signal y into another signal y' according to the probability density $\lambda(y', y)$. If the y'-values are generated in this way, the information system $\hat{\nu}_A$ can be interpreted as being obtained from the information system $\bar{\nu}_A$ by adding some random noise. The following criterion turns out to be a useful tool for the analysis of our model:

Lemma 1 *Information system $\bar{\nu}_A$ is more informative than information system $\hat{\nu}_A$, if and only if*

$$\int_Y F(\bar{\nu}_y)\bar{\mu}(y)\mathrm{d}y \geq \int_Y F(\hat{\nu}_y)\hat{\mu}(y)\mathrm{d}y$$

holds for every convex function F on the set of density functions over $\mathbb{R}_+$.

A proof of Lemma 1 can be found in Kihlstrom (1984). Note that $\bar{\nu}_y$ and $\hat{\nu}_y$ are the posterior beliefs under the two information systems. Thus, Lemma 1 implies that a more informative structure (weakly) raises the expectation of any convex function of posterior beliefs. For concave functions, F, the inequality is reversed, and for linear functions it holds with equality.

3 Information in the absence of risk sharing

Let us analyze first the effect of better information on the welfare of the first generation G_1 and on the welfare of future generations. Recall that all agents in the same generation are identical *ex-ante*, i.e., before individual signals are received. Therefore, the welfare of generation G_t can be defined in a natural way as the ex-ante expected utility of each member in G_t. Note that this welfare concept differs from the commonly used (conditional) Pareto criterion. According to our concept a welfare improvement of an equilibrium allocation implies that ex-ante expected utilities of (agents in) all generations increase.

Consider the optimization in (9) and (11) under some given information system ν_A, and denote by $e_t(\nu_{yi})$ and $s_t(h^i)$ the decision rules for agents in generation t.

The value function, V_t, of generation t associates to any realization of an individual signal, $y^i, i \in G_t$, the level of i's expected utility,[7]

$$V_t(\nu_{yi}) = v\big(e_t(\nu_{yi})\big) + E_{\tilde{A}^i}\Big[u_1\big(w_t\tilde{h}^i - s_t(\tilde{h}^i)\big) + u_2\big((1+\bar{r}_t)s_t(\tilde{h}^i)\big)|y^i\Big], \tag{18}$$

where $\tilde{h}^i = \tilde{A}^i g(H_{t-1}, e_t(\nu_{yi}))$. Economic welfare, W_t, of an individual in generation t is defined as the ex ante expected utility at the outset of his lifetime:

$$\begin{aligned} W_t(\nu_A) &= E_y[V_t(\nu_y)] \\ &= E_y\left\{v(e_t(\nu_y)) + E_{\tilde{A}}\left[u_1(w_t\tilde{h}_t - s_t(\tilde{h}_t)) + u_2((1+\bar{r}_t)s_t(\tilde{h}_t))|y\right]\right\}, \end{aligned} \tag{19}$$

where $\tilde{h}_t = \tilde{A}g(H_{t-1}, e_t(\nu_y))$. Observe that W_t does not depend on the particular agent i chosen from G_t, i.e., all individuals within the same generation attain the same level of welfare.

We say that the value of information is positive for G_t, if $W_t(\bar{\nu}_A) \geq W_t(\hat{\nu}_A)$, whenever $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$.

Proposition 1 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$. Given any initial conditions, all members of G_1 are better-off (or at least nobody is worse-off) under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

Remark. This is a general result which does not require the parametrizations in (2) and (6).

Thus, for all agents in G_1 information has positive value, i.e., these agents will benefit from a more informative system. *Future* generations $G_t,\ t > 1$, differ from G_1 only by their inherited stock of human capital, H_{t-1}. The welfare of future generations therefore depends on two, possibly conflicting, factors. The first factor represents the mechanism characterized in Proposition 1. This factor which, in the absence of risk sharing, has a positive impact on the welfare of all generations will be called the *direct* welfare effect. The second factor is the aggregate stock of human capital, H_{t-1}, which affects human capital, and hence welfare, of agents in G_t. This factor will be called the *indirect* welfare effect. Future generations unambiguously benefit from a better information system only if these two factors work in the same direction, i.e., if under a more informative system the aggregate stock of human capital is higher at all dates $t > 1$.

Using the functional forms of $u_j, j = 1, 2$, in (6), it follows from equation (10) that, given $\bar{r}_t$ and w_t, the saving s^i is proportional to the human capital level h^i. In other words, for each t and for each $i \in G_t$ we have:

$$s^i = m_t h^i, \quad 0 < m_t < w_t, \quad t = 1, 2, \cdots \tag{20}$$

Define $\rho = \alpha/[\gamma_v + \alpha(\gamma_u - 1) + 1]$. Obviously, $\rho \geq 1$ implies that γ_u and γ_v are both less than 1, while $\gamma_v > 1$ and $\gamma_u > 1$ both imply that $\rho < 1$. In the remainder of this section we will distinguish between two constellations which turn out to be critical for our analysis:

[7] Note that any two agents in generation t who receive the same signal face the same posterior distribution of ability. Therefore, all agents in generation t have the same value function.

(a) $\rho \geq 1$: the economy is moderately risk-averse and exhibits strong intertemporal substitution;
(b) $\gamma_u > 1$: the economy is highly risk-averse.

Since ρ depends both on γ_u and γ_v, under constellation (a) relative risk aversion and intertemporal substitution are jointly restricted. In particular, risk aversion alone is not an effective device to separate the above constellations. Since $\rho \geq 1$ implies $\gamma_u < 1$ and $1/\gamma_v > 1$, we shall refer to constellation (a) as the moderate risk aversion/strong substitution case and to constellation (b) as the high risk aversion case.

In moderately risk-averse economies with strong intertemporal substitution better information has positive effects on growth and welfare:[8]

Proposition 2 (moderate risk aversion/strong substitution) *Assume $\rho \geq 1$, let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$, and denote by $H_t(\bar{\nu}_A)$ and $H_t(\hat{\nu}_A)$, $t = 0, 1, \cdots$, the corresponding stocks of human capital in equilibrium.*

(i) *Better information enhances growth, i.e., $H_t(\bar{\nu}_A) > H_t(\hat{\nu}_A)$ for all $t \geq 1$.*
(ii) *In any competitive equilibrium information has positive value in the sense that all generations are (weakly) better-off under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

For highly risk-averse economies our next proposition implies an inverse link between information and growth:

Proposition 3 (high risk aversion) *Assume $\gamma_u > 1$. Better information reduces growth: $H_t(\bar{\nu}_A) < H_t(\hat{\nu}_A)$, for all $t \geq 1$, whenever $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$.*

Better information changes the return on investment in education, which in turn affects effort decisions by means of intertemporal substitution. Through its impact on effort the information also affects the volatility of random human capital, thereby causing risk effects. This is why the parameter ρ depends on both γ_v and γ_u implying that in Proposition 2 intertemporal substitution effects and risk effects show up in combination. If we rule out intertemporal substitution effects by setting $1/\gamma_v = 0$, the risk effects also disappear. As a consequence, aggregate human capital (equation (31) in the Appendix) becomes a constant and, hence, information does not affect growth. Similarly, for $\gamma_u = 1$, human capital $\bar{h}_t(\nu_y)$ (equation (29) in the Appendix) is linear in the posterior belief ν_y and, hence, Lemma 1 implies that better information does not affect growth. With $1/\gamma_v > 0$ and $\gamma_u \neq 1$, however, intertemporal substitution and risk aversion jointly determine the growth effects of better information.

In equilibrium there are two mechanisms through which the precision of information signals affects economic growth. We illustrate these mechanisms separately for the respective constellations specified in Proposition 2 and Proposition 3. Consider the case $\rho \geq 1$ (moderate risk aversion/strong substitution). Firstly, under a

[8] In our framework, where the stock of physical capital is fixed by the assumption of international capital mobility, the notion 'growth' refers solely to aggregate human capital accumulation. This implies that the accumulation process by which the economy grows, namely investment in education, is not governed by market mechanisms.

more informative system private investment in education will be better in line with the distribution of talent across agents: when signals are more reliable, it is less likely that an agent with low ability receives a signal which suggests high talent, and which induces him to invest heavily in education; or that an agent with high ability receives a signal which suggests low talent, thereby inducing him to invest too little. This allocative effect has a positive impact on growth.

Secondly, from (30) in the Appendix we conclude that the conditional expectation $\tilde{\tilde{A}}^{1-\gamma_u}(\nu_y) = E\left[\tilde{A}^{1-\gamma_u}|y\right]$ aggregates all relevant information conveyed by the signal y. The higher is $\tilde{\tilde{A}}^{1-\gamma_u}(\nu_y)$, the more favorable is the signal y. When signals become more informative agents with good signals invest more in education, and agents with bad signals invest less. However, $\rho \geq 1$ implies that $\bar{h}_t$ in (29) is a convex function of $\tilde{\tilde{A}}^{1-\gamma_u}(\nu_y)$. Thus, the additional effort of agents with good signals adds more to the stock of human capital than is detracted from it through the reduced effort of agents with bad signals. The strength of this positive effect on aggregate human capital is inversely related to risk aversion and positively related to intertemporal substitution because ρ is decreasing in γ_u and γ_v.

The overall impact of better information on growth combines these two effects: on the one hand the allocation of investment in education becomes more efficient; and on the other hand the distribution of individual effort levels becomes more dispersed.[9] If the economy is moderately risk-averse, these two effects work in the same direction and stimulate economic growth.

Consider now the case of high risk aversion where $\gamma_u \geq 1$ (which implies $\rho < 1$). Regarding the second effect discussed above observe that, again, the dispersion of individual effort levels increases with better information. However, now the resulting effect on the stock of human capital is negative, because $\rho < 1$ implies that $\bar{h}_t$ in (29) is concave as a function of the aggregated information $\tilde{\tilde{A}}^{1-\gamma_u}(\nu_y)$. The first effect which is due to a more efficient allocation of effort also works in the opposite direction as before: according to (30) agents who have received a good signal (and will probably be highly talented) invest less in education than agents with bad signals (and low talent). By responding to low expected talent with higher investment in education agents attempt to achieve a satisfactory level of human capital in their second period of life. When the signals become more reliable, agents who have received bad signals will step up their effort and invest more in education. By contrast, agents who have received good signals will cut back on their investment in education. While this kind of behavior is efficient from the decision makers' point of view it is, of course, detrimental to economic growth. Thus, again, the two effects work in the same direction. However, in a highly risk averse economy they depress economic growth.

4 Information with risk sharing

We now study the case where part of the perceived uncertainty of an agent's ability is insurable. Let individual ability be composed of two factors, $A = A_1 \cdot A_2$, with

[9] Note that all agents choose the same effort level if the signals are uninformative.

$(A_1, A_2) \in \mathcal{A} := \mathbb{R}^2_+$. We assume that the distributions of A_1 and of A_2 across agents in H_t are stochastically independent. Before agents make decisions about effort they can insure the perceived risk which is associated with the A_1- component of their (unknown) ability. Since there is no aggregate risk in the economy the insurance market for the A_1-component of ability will be unbiased, i.e., the agents can share part of the perceived risk on fair terms. In Section 3 the signals affected only uninsurable risks. In this section we assume that the signals contain only information about the insurable risk factor A_1.[10]

In order to introduce the risk sharing market we need to assume that the A_1-component of individual ability is verifiable by the insurers. The future income of each individual, perceived as random at young age, will then have an insurable component as well as an uninsurable component.[11] Denote by $\bar{A}_1(\nu_y)$ the expected value of $\tilde{A}_1$ if the signal y has been observed,

$$\bar{A}_1(\nu_y) := \int_{\mathcal{A}} A_1 \nu_y(A)\, \mathrm{d}A. \tag{21}$$

Since the insurance market is unbiased, all agents find it optimal to completely eliminate the (perceived) A_1- risk from income in their second period of life. Thus the optimal saving and effort decisions of individual $i \in G_t$ satisfy the following first order conditions:

$$(1+\bar{r}_t)u_2'\left((1+\bar{r}_t)s^i\right) - u_1'\left(w_t\bar{A}_1(\nu_{y^i})A_2 g(H_{t-1}, e^i) - s^i\right) = 0 \qquad (A_2 \in \mathbb{R}_+) \tag{22}$$

$$v'(e^i) + w_t g_2(H_{t-1}, e^i) E\left[\tilde{\bar{A}}(\nu_{y^i})u_1'\left(w_t\tilde{\bar{A}}(\nu_{y^i})g(H_{t-1}, e^i) - s^i\right) |y^i\right] = 0 \qquad (y^i \in Y), \tag{23}$$

where

$$\tilde{\bar{A}}(\nu_{y^i}) := \bar{A}_1(\nu_{y^i}) \cdot \tilde{A}_2. \tag{24}$$

Does better information enhance economic welfare when a risk sharing market for the $\tilde{A}_1$-risk is available? On this issue recent papers have produced ambiguous results. Schlee (2001) showed that under certain conditions in exchange economies with efficient risk sharing arrangements better information will *always* be harmful. Eckwert und Zilcha (2001) demonstrate for a class of production economies that the welfare effects of information critically depend on the degree of risk aversion of the consumers. However, neither of these papers takes into account the externality created by private investment into the human capital stock.

Let us first consider the *direct* welfare effect which was shown to be positive in the absence of risk sharing (cf. Proposition 1). Denote by $e(\cdot)$ and $s(\cdot)$ the optimal effort and saving decision of an agent in generation 1 (omitting the indices i and t). Recall that the signal affects only the insurable risk $\tilde{A}_1$. Thus, according to (23),

[10] The framework in Section 3 is formally equivalent to the one used here with the signals containing only information about the uninsurable (perceived) risk factor A_2.

[11] Again, we shall mark with a ˜ those variables which are perceived as random by the agent.

$e(\cdot)$ depends on the posterior belief ν_y only via $\bar{A}_1(\nu_y)$. Similarly, in view of (22), $s(\cdot)$ depends on ν_y only via $\tilde{\bar{h}} := \tilde{\bar{A}}(\nu_y)g(H_0, e(\cdot))$, where $\tilde{\bar{A}}(\nu_y)$ has been defined in (24). Thus we may write the value function as

$$V(\bar{A}_1(\nu_y)) = v(e(\cdot)) + E\left[u_1\left(w_1\tilde{\bar{h}}(\cdot) - s(\tilde{\bar{h}}(\cdot))\right)\right] + E\left[u_2\left((1+\bar{r})s(\tilde{\bar{h}}(\cdot))\right)\right], \tag{25}$$

where $e(\cdot)$ and $\tilde{\bar{h}}(\cdot)$ are functions of $\bar{A}_1(\nu_y)$.

Proposition 4 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$, and assume that all agents have access to the insurance market. Given any initial conditions, all members of G_1 are worse-off (or at least nobody is better-off) under $\bar{\nu}_A$ than under $\hat{\nu}_A$, if $\gamma_u \geq 1/2$ holds.*

Remark. The validity of Proposition 4 is not restricted to the parametric family of economies studied here: for arbitrary twice differentiable strictly concave period utility functions denote by $R_j(c_j) := -u_j''(c_j)c_j/u_j'(c_j)$, $j = 1, 2$, the relative measures of risk aversion in the agent's working period and retirement period. Then the claim in Proposition 4 holds true if (i) $R_1(c_1) \geq 1/2$, for all $c_1 \geq 0$ and (ii) $R_2(c_2) \geq R_1(c_1)$, for all $c_1, c_2 \geq 0$ are satisfied.

Proposition 1 and Proposition 4 suggest that the *direct* welfare effect, i.e., the impact of better information on the welfare of G_1, is less favorable (or even harmful) when agents are able to hedge against the risk on which information is revealed. This result can be interpreted in terms of two opposing mechanisms which affect economic welfare. The first mechanism was pointed out by Blackwell (1953): when agents receive more reliable information they are able to improve the quality of their effort and saving decisions. And better individual decisions result in higher welfare.

The second mechanism captures the so-called Hirshleifer-effect (Hirshleifer, 1971, 1975). The Hirshleifer-effect rests on a deterioration of the risk allocation due to better information: more reliable information signals typically restrict the risk sharing opportunities in an economy, which leads to lower welfare. In our model the risk sharing market opens after the signals have been observed. Thus, on this market the agents can only insure that part of the $\tilde{A}_1$-risk which has not yet been resolved through the signals. Accordingly, with more informative signals the insurable part of the $\tilde{A}_1$-risk will be smaller and, hence, economic welfare will be lower. The welfare loss caused by the uninsured risks is small, if the economy is only slightly risk- averse, but may assume significant proportions in highly risk-averse economies.

In economies where no risk sharing arrangements are operative, the Hirshleifer-effect is nil and, hence, better information increases welfare (Proposition 1). If, by contrast, the $\tilde{A}_1$-risk can be insured, then the *direct* impact of better information on economic welfare depends on a subtle interaction between the positive Blackwell-effect and the negative Hirshleifer-effect: in weakly risk-averse economies welfare will rise; and in strongly risk-averse economies, where the Hirshleifer-effect dominates the Blackwell-effect, welfare will decline. According to Proposition 4, the critical value of relative risk aversion, beyond which the Hirshleifer-effect outweighs the Blackwell-effect, is less than 1/2.

We now turn to the indirect welfare effect which is due to economic growth.

Proposition 5 (moderate risk aversion) *Assume that an insurance market for the $\tilde{A}_1$-risk exists and let $\gamma_u < 1$. Better information enhances growth for all $t \geq 1$. When $\gamma_u = 1$, information has no effect on growth.*

Proposition 5 contains a similar message as Proposition 2(i): loosely speaking, better information enhances growth in moderately risk averse economies. In the absence of a risk sharing market for the $\tilde{A}_1$-risk, moderate risk aversion was a necessary condition for higher information-induced growth, too. Therefore, the conditions which guarantee positive growth effects are somewhat less restrictive if a risk sharing market exists.

In strongly risk-averse economies better information has negative effects on growth and welfare:

Proposition 6 (high risk aversion) *Assume that an insurance market for the $\tilde{A}_1$-risk exists and let $\gamma_u > 1$, i.e., the economy is strongly risk-averse.*

(i) *Better information reduces growth for all $t \geq 1$.*
(ii) *Information has negative value in the sense that all generations are (weakly) worse-off under a more informative system.*

It is interesting to note, that the results in Propositions 5 and 6 depend only on relative risk aversion, γ_u, although there is interaction among the risk effects and the intertemporal substitution effects that are caused by better information. Yet, with regard to economic growth these effects always work in the same direction if a risk sharing market is available. This explains why the growth effects of better information can be characterized solely in terms of relative risk aversion.

Better information may stimulate growth and, at the same time, reduce welfare of the agents in G_1. Combining Propositions 4 and 5, this happens if $1/2 \leq \gamma_u \leq 1$. If γ_u exceeds 1, better information depresses growth according to Proposition 6(i). In this case the direct and the indirect welfare effects are both negative and, hence, all generations are worse-off under a more informative system. Future generations are hit harder than the current generation: they suffer not only from the negative direct welfare effect but also from the indirect welfare effect induced by lower growth.

Better information unambiguously raises economic welfare (in the Pareto sense) only if the direct and the indirect effects are both positive. Under both market structures considered in sections 3 and 4 this requirement will be violated unless relative risk aversion, γ_u, is sufficiently small.[12] In the more realistic case considered in Proposition 6 where the $\tilde{A}_1$-risk is insurable and γ_u exceeds 1, information has negative value in the sense that the economy is better off under a less informative system. This result may explain why in some countries (like Israel) the use of aptitude tests as a screening device for entrance to high education has recently been subjected to a critical reevaluation.

[12] Observe from eq. (33) that, in the presence of an insurance market, $\gamma_u = 0$ implies the convexity of V. Thus, by Lemma 1, the direct welfare effect is positive.

5 Concluding remarks

There is extensive literature which examines the role of information in the presence of risk sharing markets. Most of these studies are conducted within a static theoretical set up. We argue in this paper that static models are of limited value for an analysis of the welfare implications of better information: these models do not explain economic growth which, however, contributes to the welfare of future generations. Our paper therefore proposes such analysis in a dynamic framework in which the role of information in enhancing growth and economic welfare can be studied.

Better information creates a *direct* and an *indirect* welfare effect. The direct welfare effect arises because, under a more informative system, agents are able to anticipate the uncertain future economic environment in a more reliable way. Better information also has implications for economic growth via more efficient investment in human capital (indirect effect) thereby affecting the welfare of future generations. We have shown that the direct and the indirect welfare effects can be in conflict with each other. This happens in strongly risk-averse economies if no risk sharing takes place (direct effect positive, indirect effect negative); and, if risk sharing is possible, it also happens in moderately risk-averse economies where γ_u exceeds 1/2 but is less than 1 (direct effect negative, indirect effect positive).

In either case, i.e., with and without risk sharing, both effects are positive, if γ_u and γ_v are sufficiently small. Thus, in 'slightly risk averse economies with strong intertemporal substitution' better information enhances welfare. By contrast, if a risk sharing market exists and γ_u exceeds 1, both effects are negative which means that all generations are worse-off under a more informative system. As a rule of thumb we may paraphrase these findings as follows: The impact of better information on welfare is less favorable (or even harmful), when risk sharing arrangements are more effective and/or when risk aversion is 'high'.

The analysis in this paper is greatly simplified by the 'small open economy assumption' which fixes interest rates and wages at each date. This specification raises the question whether the results are robust to the inclusion of physical capital accumulation and, hence, of equilibrium price effects. While the effects of information on welfare and growth studied here will still be present and play an important role in a more general framework with endogenous capital accumulation, we do not expect our results to carry over without significant modifications. The Blackwell-effect, for example, which in our model always enhances welfare may become ambiguous if the agents' opportunity sets depend upon information via the induced price variations. We plan to address this issue in a separate work.

Appendix

In this Appendix we prove Propositions 1–5.

Proof of Proposition 1. Denote by $e(\nu_y)$ and $s(h)$ the optimal decision of an agent in G_1 (omitting the indices i and t), and define $U\big(\tilde{h}, s(\tilde{h})\big) := u_1\big(w\tilde{h} - s(\tilde{h})\big) +$

$u_2\big((1+\bar{r})s(\tilde{h})\big)$. With this notation we may state the value function as

$$V(\nu_y) = v\big(e(\nu_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, s(\tilde{h})\big)\nu_y(A)\mathrm{d}A.$$

We show that the value function is convex in the posterior belief ν_y. Assume $\nu_y = \alpha\bar{\nu}_y + (1-\alpha)\hat{\nu}_y, \alpha \in [0,1]$, and denote by $(e(\bar{\nu}_y), \bar{s}(h))$ and $(e(\hat{\nu}_y), \hat{s}(h))$ the optimal decisions under the posterior beliefs $\bar{\nu}_y$ and $\hat{\nu}_y$. We obtain

$$\begin{aligned} V(\nu_y) &= v(e(\nu_y)) + \int\limits_{\mathbb{R}_+} U(\tilde{h}, s(\tilde{h}))[\alpha\bar{\nu}_y(A) + (1-\alpha)\hat{\nu}_y(A)]\mathrm{d}A \\ &= \alpha\left[v(e(\nu_y)) + \int\limits_{\mathbb{R}_+} U(\tilde{h}, s(\tilde{h}))\bar{\nu}_y(A)\mathrm{d}A\right] \\ &\quad +(1-\alpha)\left[v(e(\nu_y)) + \int\limits_{\mathbb{R}_+} U(\tilde{h}, s(\tilde{h}))\hat{\nu}_y(A)\mathrm{d}A\right] \\ &\leq \alpha\left[v(e(\bar{\nu}_y)) + \int\limits_{\mathbb{R}_+} U(\tilde{h}, \bar{s}(\tilde{h}))\bar{\nu}_y(A)\mathrm{d}A\right] \\ &\quad +(1-\alpha)\left[v(e(\hat{\nu}_y)) + \int\limits_{\mathbb{R}_+} U(\tilde{h}, \hat{s}(\tilde{h}))\hat{\nu}_y(A)\mathrm{d}A\right] \\ &= \alpha V(\bar{\nu}_y) + (1-\alpha)V(\hat{\nu}_y). \end{aligned}$$

The inequality holds because $(e(\bar{\nu}_y), \bar{s}(h))$ and $(e(\hat{\nu}_y), \hat{s}(h))$ maximize expected utility, if the posterior belief is given by $\bar{\nu}_y$ and $\hat{\nu}_y$, respectively.

We have shown that the value function is convex in the posterior beliefs. Now the claim in Proposition 1 follows from Lemma 1. □

The proof of Proposition 2 requires some preparatory work. Define

$$\phi(y, y') := \lambda(y', y)\bar{\mu}(y)/\hat{\mu}(y').$$

Note that for any $y' \in Y$, the function $\phi(\cdot, y')$ constitutes a probability density over Y, i.e., $\int_Y \phi(y, y')\mathrm{d}y = 1$.[13] For any integrable function $\vartheta : Y \to \mathbb{R}$, let $\Gamma\big(\vartheta(y); y'\big)$ be its expectation with respect to the probability density $\phi(\cdot, y')$, i.e.,

$$\Gamma\big(\vartheta(y); y'\big) := \int_Y \vartheta(y)\phi(y, y')\,\mathrm{d}y.$$

[13] The interpretation of $\phi : Y \times Y \to \mathbb{R}_+$ is the following: If the signal y has realized under the information system $\bar{\nu}_A$, then $\phi(y, y')$ is the probability (density) that y' would have been observed under the information system $\hat{\nu}_A$.

Direct computation yields

$$\hat{\nu}_{y'}(A) = \Gamma\big(\bar{\nu}_y(A); y'\big) \tag{26}$$

$$\bar{A}(\hat{\nu}_{y'}) = \Gamma\big(\bar{A}(\bar{\nu}_y); y'\big). \tag{27}$$

Proof of Proposition 2.

(i) Since the initial stock of human capital, H_0, is fixed, it suffices to show that for any given $H_{t-1},\ t \geq 1$, the aggregate stock of human capital at date t,

$$H_t = \int\limits_Y \bar{h}_t(\nu_y)\mu(y)\mathrm{d}y, \tag{28}$$

is higher under the more informative system. Using (19) in (12) we get

$$\bar{h}_t(\nu_y) = \bar{A}(\nu_y)\hat{g}(H_{t-1})\big(e_t(\nu_y)\big)^{\alpha}, \tag{29}$$

where $e_t(\nu_y)$ is given by[14]

$$e_t(\nu_y) = \delta_t \left(E[\tilde{A}^{1-\gamma_u}|y] \right)^{\frac{1}{\gamma_v + \alpha\gamma_u + 1 - \alpha}} \tag{30}$$

with

$$\delta_t := \left[\frac{\alpha w_t(\hat{g}(H_{t-1}))^{1-\gamma_u}}{(w_t - m_t)^{\gamma_u}} \right]^{\frac{1}{\gamma_v + \alpha\gamma_u + 1 - \alpha}}.$$

Combining (29) and (30) with (28) we arrive at

$$H_t(\nu_A) = \delta_t^{\alpha}\hat{g}(H_{t-1}) \int_Y \bar{A}(\nu_y) \left[\bar{A}^{1-\gamma_u}(\nu_y)\right]^{\rho} \mu(y)\mathrm{d}y. \tag{31}$$

By assumption, $\rho \geq 1$ holds which implies $\gamma_u < 1$. From this in combination with strict MLRP we conclude that $\bar{A}(\bar{\nu}_y)$ and $\big(\bar{A}^{1-\gamma_u}(\bar{\nu}_y)\big)^{\rho}$ are co-monotone in the signal y. The representation in (31) then implies the following assessment with regard to the information systems $\bar{\nu}_A$ and $\hat{\nu}_A$:

$$\begin{aligned} H_t(\bar{\nu}_A)/\delta_t^{\alpha}\hat{g}(H_{t-1}) &= \int_Y \bar{A}(\bar{\nu}_y)(\bar{A}^{1-\gamma_u}(\bar{\nu}_y))^{\rho}\bar{\mu}(y)\,\mathrm{d}y \\ &= \int_Y \Gamma(\bar{A}(\bar{\nu}_y)(\bar{A}^{1-\gamma_u}(\bar{\nu}_y))^{\rho}, y')\hat{\mu}(y')\,\mathrm{d}y' \\ &> \int_Y \Gamma(\bar{A}(\bar{\nu}_y), y')\Gamma((\bar{A}^{1-\gamma_u}(\bar{\nu}_y))^{\rho}, y')\hat{\mu}(y')\,\mathrm{d}y' \\ &\geq \int_Y \bar{A}(\hat{\nu}_{y'})(\bar{A}^{1-\gamma_u}(\hat{\nu}_{y'}))^{\rho}\hat{\mu}(y')\,\mathrm{d}y' \\ &= H_t(\hat{\nu}_A)/\delta_t^{\alpha}\hat{g}(H_{t-1}) \end{aligned} \tag{32}$$

In (32) the first inequality follows from the co-monotonicity property and the second inequality follows from $\rho \geq 1$.

[14] Equation (30) can be derived from (12) using (2), (6) and (20).

(ii) The future generations G_t, $t \geq 2$, differ from G_1 only by their inherited stocks of human capital. Since the factor prices $\bar{r}_t$ and w_t do not depend on H_t, all future generations will benefit from higher growth. Proposition 1 and Proposition 2(i) therefore imply the claim. □

Proof of Proposition 3. $\gamma_u > 1$ implies $\rho < 1$. In this case both inequalities in (32) are reversed and the second inequality becomes strict. Thus the same reasoning as in the proof of Proposition 2(i) yields the result claimed in the proposition. □

Proof of Proposition 4. We prove the more general claim in the Remark. Under the specification in (6), conditions (i) and (ii) in the Remark boil down to the restriction in Proposition 4.

In view of Lemma 1 we have to show that under the conditions of the proposition the value function in (25) is concave in the posterior belief ν_y. Since $\bar{A}_1(\nu_y)$ is linear in ν_y, the value function will be concave in ν_y if it is concave in $\bar{A}_1$. Making use of the Envelope theorem, differentiation of (25) with respect to $\bar{A}_1$ yields

$$V'(\bar{A}_1) = E\Big[u_1'\Big(w_1\tilde{\tilde{A}}g\big(H_0, e(\bar{A}_1)\big) - s(\cdot)\Big)w_1\tilde{A}_2 g\big(H_0, e(\bar{A}_1)\big)\Big] \tag{33}$$

and (omitting the arguments of all functions)

$$V'' = E\Big[w_1\tilde{A}_2 g_2 u_1' e' + w_1\tilde{A}_2 g u_1''\big\{w_1\big(\tilde{A}_2 g + \tilde{\tilde{A}}g_2 e'\big) - s'\big\}\Big]. \tag{34}$$

e' and s' denote, respectively, the derivatives of $e(\cdot)$ and $s(\cdot)$ with respect to $\bar{A}_1$. Differentiate equation (23) with respect to $\bar{A}_1$ and multiply by e' to obtain

$$\begin{aligned} 0 = v''e'^2 &+ E\left[\left(\tilde{A}_2 w_1 g_2 e' + \tilde{\tilde{A}}w_1 g_{22}e'^2\right)u_1'\right] \\ &+ E\Big[\big\{w_1\big(\tilde{A}_2 g + \tilde{\tilde{A}}g_2 e'\big) - s'\big\}w_1 g_2\tilde{\tilde{A}}e'u_1''\Big]. \end{aligned} \tag{35}$$

Adding (34) and (35), and rearranging, yields

$$\begin{aligned} V'' &= e'^2 v'' + w_1 g_{22} e'^2 E\left[\tilde{\tilde{A}}u_1'\right] \\ &+ E\left[\left(w_1\tilde{A}_2 g + w_1\tilde{\tilde{A}}g_2 e'\right)^2 u_1'' - s'u_1''w_1\left(\tilde{A}_2 g + g_2\tilde{\tilde{A}}e'\right) + 2\tilde{A}_2 w_1 g_2 e' u_1'\right]. \end{aligned} \tag{36}$$

The two terms in the first line on the RHS of (36) are negative. We show that the sign of the term in the second line of (36) is negative as well. From (22) we get

$$s' = \hat{m}\big(\tilde{\tilde{A}}\big)\big(\tilde{A}_2 g + g_2 e'\tilde{\tilde{A}}\big), \tag{37}$$

where

$$\hat{m}\big(\tilde{\tilde{A}}\big) := \frac{u_1'' w_1}{(1+\bar{r}_1)^2 u_2'' + u_1''} \leq w_1.$$

Using (37) we rewrite the term in the second line of (36) as

$$E\Big[\big(w_1 - \hat{m}(\tilde{\bar{A}})\big)w_1\big(\tilde{A}_2 g + \tilde{\bar{A}} g_2 e'\big)^2 u_1'' + 2\tilde{A}_2 w_1 g_2 e' u_1'\Big]. \tag{38}$$

Noting that $\tilde{c}_1 = w_1 \tilde{\bar{A}} g - s$, the expression in (38) can be transformed into

$$E\left[\frac{u_1' w_1 g(\tilde{A}_2 + B)^2}{\tilde{\bar{A}}}\left\{\frac{2B\tilde{A}_2}{(\tilde{A}_2 + B)^2} - R_1(\tilde{c}_1) + \frac{u_1''}{u_1'}\big[s - \hat{m}\big(\tilde{\bar{A}}\big)\tilde{\bar{A}} g\big]\right\}\right], \tag{39}$$

where $B := \tilde{\bar{A}} g_2 e'/g$. Since $2B\tilde{A}_2/(\tilde{A}_2 + B)^2 = 2(B/\tilde{A}_2)/[1 + (B/\tilde{A}_2)]^2$ is bounded from above by $1/2$, $[2B\tilde{A}_2/(\tilde{A}_2 + B)^2] - R_1(\tilde{c}_1)$ is negative under the assumptions of the proposition. Thus the proof is complete if we can show that

$$s \geq \hat{m}\big(\tilde{\bar{A}}\big)\tilde{\bar{A}} g \tag{40}$$

is satisfied.

By assumption, $R_2(\tilde{c}_2) \geq R_1(\tilde{c}_1)$ holds for all $\tilde{c}_1, \tilde{c}_2$. From this we conclude, using (22),

$$\begin{aligned} R_2(\tilde{c}_2) \geq R_1(\tilde{c}_1) &\iff \frac{u_1'}{u_2'} u_2''(1 + \bar{r}_1) \leq u_1''\left[\frac{w_1 \tilde{\bar{A}} g}{s} - 1\right] \\ &\iff (1 + \bar{r}_1)^2 u_2'' \leq u_1''\left[\frac{w_1 \tilde{\bar{A}} g}{s} - 1\right]. \end{aligned} \tag{41}$$

If $\tilde{\bar{A}} g$ is locally increasing (decreasing) in $\bar{A}_1$, then (41) in combination with (22) implies that $w_1 \tilde{\bar{A}} g/s$ is locally increasing (decreasing) in $\bar{A}_1$. Hence we obtain

$$\begin{aligned} 0 \leq \frac{d}{d\bar{A}_1}\big(\tilde{\bar{A}} g\big)\frac{d}{d\bar{A}_1}\left(\frac{w_1 \tilde{\bar{A}} g}{s}\right) &= \big(\tilde{A}_2 g + \tilde{\bar{A}} g_2 e'\big)\frac{w_1}{s^2}\big[\big(\tilde{A}_2 g + \tilde{\bar{A}} g_2 e'\big)s - s'\tilde{\bar{A}} g\big] \\ &= w_1\left(\frac{\tilde{A}_2 g + \tilde{\bar{A}} g_2 e'}{s}\right)^2\big[s - \hat{m}\big(\tilde{\bar{A}}\big) g \tilde{\bar{A}}\big], \end{aligned} \tag{42}$$

where in the last equality we have made use of (37). Obviously, (42) implies the inequality in (40). □

Proof of Proposition 5. Proceeding along the same line as in the proof of Proposition 2(i) we obtain

$$\bar{h}_t(\nu_y) = \delta_t^{\alpha} \hat{g}(H_{t-1}) \bar{A}_2 \big(\bar{A}_1(\nu_y)\big)^{1+\rho(1-\gamma_u)}\Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^{\rho}, \tag{43}$$

where $\rho > 0$ has been defined in Section 3. $\bar{h}_t$ depends on the posterior belief ν_y only via $\bar{A}_1(\nu_y)$. Since $\bar{A}_1(\cdot)$ is linear in ν_y, $\bar{h}_t$ is convex (linear) in ν_y, if and only if $\hat{h} : \mathbb{R}_+ \to \mathbb{R}_+$,

$$\hat{h}(\bar{A}_1) := \bar{A}_2 \bar{A}_1^{1+\rho(1-\gamma_u)}\Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^{\rho}, \tag{44}$$

is a convex (linear) function. Obviously $\hat{h}$ is convex, if $\gamma_u \leq 1$, and linear, if $\gamma_u = 1$ holds. Lemma 1 therefore implies the claims in the proposition. □

Proof of Proposition 6. $\gamma_u > 1$ implies that $\hat{h}$ in (44) is a concave function. Thus, the same reasoning as in the proof of Proposition 5 yields the result in part (i).

(ii) By Proposition 4, the direct welfare effect is negative for $\gamma_u > 1$. In addition, we have just seen in part (i) of this proof that the (growth-induced) indirect welfare effect is also negative. Hence, all generations are worse-off under a more informative system. □

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