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Characterization and incentive compatibility of Walrasian expectations equilibrium in infinite dimensional commodity spaces*

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Summary. We consider a differential information economy with infinitely many commodities and analyze the veto power of the grand coalition with respect the ability of blocking non-Walrasian expectations equilibrium allocations. We provide two different Walrasian expectations equilibrium equivalence results. First by perturbing the initial endowments in a precise direction we show that an allocation is a Walrasian expectations equilibrium if and only if it is not "privately dominated" by the grand coalition. The second characterization deals with the fuzzy veto in the sense of Aubin but within a differential information setting. This second equivalence result provides a different characterization for the Walrasian expectations equilibrium and shows that the grand coalition privately blocks in the sense of Aubin any non Walrasian expectations equilibrium allocation with endowment participation rate arbitrarily close to the total initial endowment participation for every individual. Finally, we show that any no free disposal Walrasian expectations equilibria is coalitional Bayesian incentive compatible. Since the deterministic Arrow-Debreu-McKenzie model is a special case of the differential information economy model, one derives new characterizations of the Walrasian equilibria in economies with infinitely many commodities.

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1 Introduction

A differential (asymmetric) information economy consists of a set of agents, each of whom is characterized by a random utility function, a random initial endowment, a private information set and a prior. Such an economy is a generalization of the classical Walrasian deterministic economy as formulated rigorously by Arrow-Debreu and McKenzie. A natural extension of the competitive (Walrasian) equilibrium concept, which is appropriate for an asymmetric information economy is the *Walrasian expectations equilibrium* or *Radner equilibrium* introduced by Radner (1968).

The Radner equilibrium, like the Walrasian equilibrium, is a non cooperative solution concept capturing the idea that if each agent maximizes her ex ante utility function subject to her budget constraint by taking into account her own private information, then, this individualistic behavior will lead to a feasible redistribution of the initial endowments for each state of nature, (i.e., the total demand will balance the total initial endowment for each state of nature). It is important to notice that since agents make decisions before the state of nature is realized, (i.e., agents maximize ex-ante expected utility), prices do not reveal any private information ex ante. However, the equilibrium price reflects the private information as it has been obtained by maximizing expected utility subject to the budget constraint and also considering the private information of each agent.

Thus, the Radner equilibrium takes into account the private information of each agent, i.e., a change in the private information changes the Radner equilibrium. This is in sharp contrast with the traditional rational expectation equilibrium (REE) which, as it is well known, by now is not "sensitive" (does not take into account) to the private information of an agent (see Allen and Yannelis, 2001, and the reference there for a discussion of those issues).

The aim of this paper is to characterize the Radner equilibrium by means of cooperative solutions and also to analyze the incentive compatibility of the Radner equilibrium within an infinite dimensional commodity space setting.

Dealing with cooperative solution concepts with differential information, the basic problem which arises is, how agents within a coalition share their private information. Yannelis (1991) introduced the *private core* concept which is based on individual measurability requirements (i.e., when a coalition blocks an allocation each member in the coalition uses only her own private information - thus, we refer to this blocking as "private blocking"). The Radner equilibrium allocations have the property that they are not privately blocked by any coalition of agents and, therefore, the private core contains as a strictly subset the set of Radner equilibrium allocations. Throughout this paper, we consider that the way in which a coalition shares the information is the one leading to the private core solution, that is, every member in a coalition takes into account only her own private information. It turns out, that allowing individuals to make redistributions of their initial endowments based on their own private information results in allocations that are always Bayesian incentive compatible and also take into account the informational advantage of an individual (see Koutsougeras and Yannelis, 1993).

If one enlarges the number of coalitions, the possibilities of blocking an allocation increases and, then, the set of allocations which are not privately blocked is reduced. Addressing a finite set of agents and complete information economies, Debreu and Scarf (1963) enlarge the set of coalitions by replicating the original economy. By identifying the core allocations of each replicated economy with allocations in the initial economy, Debreu-Scarf showed that the set of non blocked allocations in every replicated economy converges to the set of Walrasian equilibria. A second development, was proposed by Aubin (1979) who also addresses a finite set of agents and complete information economies, stated an essentially similar approach although formally different than the one by Debreu and Scarf. By considering that, when forming a coalition, the agents in the economy can participate with any proportion of their endowments, the number of coalitions that may block an allocation is infinitely enlarge. This veto mechanism is referred in the literature to the confusing term fuzzy veto. Aubin (1979) showed that the allocations belonging to the core solution derived from this veto mechanism, called in this paper *Aubin core*, coincides with the Walrasian equilibrium allocations. Debreu-Scarf core convergence result and Aubin's result can be extended to differential information economies (see Meo, 2002).

The approaches of Debreu-Scarf and Aubin, enlarge the possibilities of blocking in order to obtain that the allocations which are not blocked by any coalition are precisely the Walrasian (or competitive) equilibrium allocations. In a companion paper Hervés-Beloso et al. (2003) showed that in a differential information economy with a finite dimensional commodity space, the veto power of just one coalition (the grand coalition) characterizes the Radner equilibrium. This result, not only corresponds to an extension of the Debreu-Scarf (1963) deterministic result to a differential information economy, but also has a different flavor. In particular, Debreu-Scarf in order to characterize the Walrasian equilibrium, replicate the economy i.e., enlarge the number of coalitions that agents can form. Hervés-Beloso et al. (2003) provide a characterization of the Radner equilibrium by considering the veto power (blocking power) of the grand coalition, by enlarging the possible redistribution of the initial endowments.

The main purpose of this paper is threefold: First, we provide a characterization of the Radner equilibrium for a differential information economy with an infinite dimensional commodity space, generalizing the Hervés-Beloso et al. (2003) finite dimensional commodity space characterization of the Radner equilibrium. It should be noted, that such a characterization is in general false for the deterministic Walrasian equilibrium in infinite dimensional spaces, as it was shown in Tourky and Yannelis (2001) and Podczeck (2003). In particular, the key observation is that unless the infinite dimensional commodity space is separable, one is bound not to obtain a characterization of the Walrasian equilibrium by means of the core and a fortiori of the Radner equilibrium. To overcome this difficulty, our commodity space in this paper is chosen to be the bounded sequence space ℓ^∞, endowed with

Mackey topology, which is separable. Moreover, the random utility function of each individual is assumed to be Mackey continuous. This is the standard set up for which Bewley (1972, 1973) has proved the existence of Walrasian equilibrium and its equivalence to the core. Recall that in this set up prices are in ℓ_1, and therefore one has a well defined price valuation of commodities. Indeed, for such a set up we show that an allocation x is a Radner equilibrium allocation of and only if x is not privately blocked by the grand coalition in any of the economies obtained by perturbing the original initial endowments in the direction of x (Theorem 4.1). The proof of this first equivalence theorem relies on an extension of the core-Walras equivalence showed by Bewley (1973) to differential information economies (Theorem 3.2) and on an extension of Vind's (1972) result to economies with infinitely many commodities and differential information (Theorem 3.3).

Second, we provide another characterization of Radner equilibria (Theorem 4.2) which deals with the Aubin veto mechanism within a differential information framework. Following this veto mechanism each agent in a coalition uses her own private information and can participate with a determined weight in the coalition. If we consider (as in the original definition by Aubin) the possibility of null weights or contributions, the grand coalition contains implicitly any other coalition. In this case, consider the veto power of the grand coalition as equivalent to the veto power of all coalitions. This is the reason why, in this paper, we modify Aubin's definition by requiring any participation (representing the contribution of an agent in a coalition) to be strictly positive. Even with non-null participation, the intuition underlying Aubin's result suggests that the grand coalition is able to block any non equilibrium allocation with arbitrarily small participation of some of the agents. However, this equivalence result provides a second characterization for the Radner equilibria and shows that the grand coalition privately blocks any non Radner equilibrium allocation with participation as close to the total participation as one wants for every individual.

Since the deterministic Arrow-Debreu-McKenzie model is a special case of the differential information economy model, Theorems 4.1 and 4.2 yield to new characterizations of the Walrasian equilibria in economies with infinitely many commodities.

Thirdly, we analyze the Bayesian incentive compatibility of the Radner equilibria. As it was shown in Glycopantis et al. (2002), the finite dimensional Radner equilibrium need not be Bayesian incentive compatible because of the free disposal requirement that Radner (1968) imposes. By redefining the Radner equilibrium to exclude free disposal, we show that any no free disposal Radner equilibrium allocation is Coalitional Bayesian Incentive Compatible (CBIC). Note that the private core is always CBIC (see Koutsougeras and Yannelis (1993)) and in finite dimensional commodity spaces the CBIC of the no free disposal Radner equilibrium follows from Koutsougeras and Yannelis. However, in this paper not only we use a stronger definition of CBIC, and allow for infinitely many commodities but also prove directly that the no free disposal Radner equilibrium is CBIC.

The paper is organized as follows. Section 2 states the model of a differential information economies with infinitely many commodities, contains the main concepts and a discussion of the assumptions. Section 3 focuses on the interpretation of the economy stated as a continuum differential information economy with a finite number of types of agents. Moreover, in this section, a private core-Walras expectations equilibrium equivalence and an extension of Vind's (1972) result are given for a differential information economy with infinitely many commodities. Section 4 contains two different characterizations of Radner equilibrium by using the private blocking power of the grand coalition. Finally, Section 5 shows the Bayesian incentive compatibility property of the free disposal Radner equilibria.

2 The model

Consider a differential information economy $\mathcal{E}$ with n consumers. Let $(\Omega, \mathcal{F})$ be a measurable space, where Ω denotes the states of nature of the world and the algebra $\mathcal{F}$ denotes the set of all events. Hence, $(\Omega, \mathcal{F})$ describes the exogenous *uncertainty*. The set of states of nature, Ω, is finite and there are infinitely many commodities in each state. $N = \{1, \ldots, n\}$ is the *set of* n *traders* or agents and ℓ^∞ will denote the *commodity space* which is the set of all bounded sequences.

The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty over the states of nature and agents make contracts (agreements) that may be contingent on the realized state of nature at period $\tau = 1$ (that is, ex ante contract arrangement).

In this paper, we consider that for every state of nature $\omega \in \Omega$ and for every agent $i \in N$, the consumption set is ℓ^∞_+ which is the positive cone of the set of all bounded sequences ℓ^∞. Note that infinitely many commodities arise whenever one allows an infinite variation in any of the characteristics describing commodities. This characteristics could be physical properties, locations or the time of delivery. In fact, an infinite variation in time could arise if an infinite time horizon is allowed by considering the case of infinitely many time periods in each state of nature. Hence, it is economically natural to restrict commodity bundles to ℓ^∞_+, in each state, since we can assume that only bounded bundles would ever appear in an economy. For instance, if we restrict our economy to earth, then the availability of primary resources puts an upper bound on the quantity of any single commodity that can be produced. If an infinite number of physical commodities appear in the economy, then the units of these commodities can be chosen in such a way that only bounded bundles are possible.

Thus, a *differential information exchange economy* $\mathcal{E}$ with a finite number of agents and infinitely many commodities in every state of nature is defined by

$$\mathcal{E} = \{((\Omega, \mathcal{F}), \ell^\infty_+, \mathcal{F}_i, U_i, e_i, q) : i = 1, \ldots, n\}, \quad \text{where:}$$

1. ℓ^∞_+ is the *consumption set* for every state of nature ω and for every agent $i = 1, \ldots, n$.
2. $\mathcal{F}_i$ is a partition of Ω, denoting the *private information* of agent i;
3. $U_i : \Omega \times \ell^\infty_+ \to \mathbb{R}$ is the *random utility function* of agent i;
4. $e_i : \Omega \to \ell^\infty_+$ is the *random initial endowment* of agent i, assumed to be constant on elements of $\mathcal{F}_i$.

5. q is a probability function on Ω giving the (common) *prior* of every agent. It is assumed that q is positive on all elements of Ω.

   For any $x : \Omega \to \ell_+^\infty$, the *ex ante expected utility* of agent i is given by

$$V_i(x) = \sum_{\omega \in \Omega} U_i(\omega, x(\omega)) q(\omega).$$

An *allocation* is a function $x = (x_1, \ldots, x_n)$ which associates to every agent i a random consumption bundle $x_i \in (\ell_+^\infty)^\Omega$.

We will refer to a function with domain Ω, constant on elements of $\mathcal{F}_i$, as $\mathcal{F}_i$*-measurable*, although, strictly speaking, measurability is with respect to the σ-algebra generated by the partition. We can think of such a function as delivering information to trader i, who can not discriminate between the states of nature belonging to any element of $\mathcal{F}_i$.

Let $\mathcal{X}_i$ denote the *set of all* $\mathcal{F}_i$*-measurable selections from the random consumption set of agent* i, that is:

$$\mathcal{X}_i = \left\{ x_i : \Omega \to \ell_+^\infty, \text{ such that } x_i \text{ is } \mathcal{F}_i\text{-measurable} \right\}.$$

Let $\mathcal{X} = \prod_{i=1}^n \mathcal{X}_i$. Any allocation x in $\mathcal{X}$ is called an *informationally feasible* allocation. An allocation x is said to be *physically feasible* if $\sum_{i=1}^n x_i \le \sum_{i=1}^n e_i$. An allocation x is *feasible* if it is both informationally and physically feasible.

A coalition $S \subset N$ *privately blocks* an allocation $x \in \mathcal{X}$ if there exists $(y_i)_{i \in S} \in \prod_{i \in S} \mathcal{X}_i$ such that $\sum_{i \in S} y_i \le \sum_{i \in S} e_i$ and $V_i(y_i) > V_i(x_i)$ for every $i \in S$.

The *private core* of the differential information exchange economy $\mathcal{E}$ is the set of all feasible allocations which are not privately blocked by any coalition (see Yannelis (1991)).

Next we shall define a Walrasian equilibrium notion in the sense of Radner (see Radner (1968, 1982)). For this, we need the following notations and definitions. Let ℓ_1 denote the space of absolutely summable sequences and let ℓ_1^+ denote the positive cone of ℓ_1. For any $a = (a_j)_{j=1}^\infty \in \ell_+^\infty$, $b = (b_j)_{j=1}^\infty \in \ell_1$, let $a \cdot b = \sum_{j=1}^\infty a_j b_j$. A *price system* is a non-zero function $p : \Omega \to \ell_1^+$. For a price system p, the *budget set* of agent i is given by

$$B_i(p) = \left\{ x_i \in \mathcal{X}_i, \text{ such that } \sum_{\omega \in \Omega} p(\omega) \cdot x_i(\omega) \le \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega) \right\}.$$

Notice that traders must balance the budget ex-ante.

Definition 2.1 *A pair (p, x), where p is a price system and $x = (x_1, \ldots, x_n) \in \mathcal{X}$ is an allocation, is a Walrasian expectations equilibrium (or a Radner equilibrium) if*

(i) *for all i the consumption function x_i maximizes V_i on $B_i(p)$,*
(ii) $\sum_{i=1}^n x_i \le \sum_{i=1}^n e_i$ *(free disposal), and*
(iii) $\sum_{\omega \in \Omega} p(\omega) \cdot \sum_{i=1}^n x_i(\omega) = \sum_{\omega \in \Omega} p(\omega) \cdot \sum_{i=1}^n e_i(\omega)$.

Throughout this paper we will refer explicitly to the following assumptions on preferences and endowments:

(A.1) *Continuity.* For every consumer i, her utility function $U_i(\omega, \cdot) : \ell_+^\infty \to \mathbb{R}$ is Mackey continuous for every state ω.
(A.2) *Monotonicity.* For every consumer i, her utility function $U_i(\omega, \cdot) : \ell_+^\infty \to \mathbb{R}$ is monotone for every state ω, that is, for every individual i, if $x, y \in \ell_+^\infty$ and $y \gg 0$, then $U_i(\omega, x + y) > U_i(x)$.
(A.3) *Convexity.* For every consumer i, her utility function $U_i(\omega, \cdot) : \ell_+^\infty \to \mathbb{R}$ is concave for every state ω.
(A.4) *Interiority of initial endowments.* For every i and w, $e_i(w)$ belongs to the interior of ℓ_+^∞, i.e., there exists $a > 0$ such that $e_{ij}(w) > a$ for all $j \geq 1$ and for every $i = 1, \ldots, n$.

The hypothesis (A.3) and (A.4) requiring monotonicity and convexity of preferences will be used in the proof of our main results where we will refer explicitly to them. Note also, that assumption (A.3) is a weak monotonicity condition; given a consumption bundle in some state of nature, if the amount of every coordinate increases then the utility increases. This assumption is required, for instance, in Section 3 in which a result by Vind (1972) is extended to differential information economies with infinitely many commodities. Vind's (1972) result was stated for economies with complete information and a finite number of commodities, under a stronger monotonicity assumption: If the amount of only one commodity increases, then the utility increases. In our setting, the stronger requirement (A.5) on initial endowments allows us to use a weaker monotonicity condition. This assumption (A.5) requiring that initial endowments are strictly positive was also used by Araujo (1985) addressing complete information economies with ℓ^∞ as commodity space.

The topological dual of ℓ^∞ depends, of course, on the topology considered on ℓ^∞. It is well known that the Mackey topology is the strongest of the locally convex topologies on ℓ^∞ having ℓ_1 as dual space. The stronger the topology is chosen, the larger the set of preference relations continuous with respect to it. Assumption (A.2) is stronger than the norm continuity of the utility functions, but, as Araujo (1985) remarked, if we relax this assumption, allowing in this way for a larger class of preferences, the equilibrium might fail to exist. On the other hand, Bewley (1972) proved, within a complete information scenario, an existence of equilibrium theorem for economies with ℓ^∞ as commodity space. Besides the usual assumptions for the existence of equilibrium, Bewley assumes preferences to be Mackey continuous. The Mackey topology is sufficiently strong to admit interesting preference relations. In words of Araujo (1985) "we can say that continuity with respect the Mackey topology is the best assumption of this kind."

The Mackey continuity of preferences can be interpreted in terms of impatience of consumers. To see this, given a consumption bundle $z \in \ell_+^\infty$ let $z^{(m)}$ denote the m-tail of z, i.e., the bundle defined as $z_j^{(m)} = 0$, $1 \leq j \leq m$, and $z_j^{(m)} = z_j$ for $j > m$. It is known that if a consumer has a preference that is Mackey continuous then she exhibits impatience behavior in the sense that if x is preferred to y, then x is also preferred to $y + z^{(m)}$ for any sufficiently large m; that is, Mackey upper semicontinuity (usc) of preferences implies upper myopia. On the other hand, Mackey lower semicontinuity (lsc) of preferences implies lower myopia; that is, if x is preferred to y and $x - z^{(n)}$ belongs to the consumption set, then $x - z^{(n)}$ is also preferred to y for any sufficiently large n.

In particular, let y be a consumption bundle and, for a small $\varepsilon > 0$, let $x = y + \bar{\varepsilon}$ (where $\bar{\varepsilon}_n = \varepsilon$ for every n). Then, x is strictly preferred to y. Let $A_n = \{n+1, n+2, \dots\}$. Then, $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Hence, χ_{A_n} converges to zero when n goes to ∞; where $\chi_{A_n}(\omega) \subset \ell_+^\infty$ is the function which is one on A_n and zero elsewhere. From the Mackey continuity of preferences it follows that x is also preferred to $y + \chi_{A_n}$ for any sufficiently large n. In other words, a little bit more in the near future would be preferred to a large constant amount more in every period after some date in the distant future.

For this kind of myopic preferences and addressing continuum economies with infinitely many commodities and complete information, Hervés-Beloso et al. (2000) showed that in order to get the core it is enough to consider, for any ε, the veto power of coalitions with measure less than ε. On the other hand, Bewley (1973) showed that Aumann's (1964) theorem on the equality of the core and the set of equilibria in atomless markets can be made to apply to complete information economies whose commodity space is ℓ^∞, under monotonicity, convexity and Mackey continuity of preferences. Hence, loosely speaking, existence and core equivalence of equilibria as well as blocking efficacy of small coalitions, for complete information economies with ℓ^∞ as commodity space, tend to be hold only in situations where the consumers "discount" the future in the sense that gains in the distant future are negligible.

In the next section we will deal with the continuum economies introduced by Aumann (1964, 1966). Our aim is to use this continuum approach in order to obtain the main results for the economy described in our model.

3 A continuum approach

In this section, we interpret differential information economies with n agents and infinitely many commodities as continuum or atomless economies with differential information and infinitely many commodities in which only a finite number of different characteristics can be distinguished (see García-Cutrín and Hervés-Beloso, 1993, for the deterministic case).

Given the economy $\mathcal{E} = \{((\Omega, \mathcal{F}), \ell_+^\infty, \mathcal{F}_i, U_i, e_i, q) : i = 1, \dots, n\}$ with a finite number of agents, we define a continuum economy $\mathcal{E}_c$, where the ith agent is the representative of infinitely many identical agents, as follows. The set of agents is represented by the real interval $[0, 1]$, with the Lebesgue measure μ. We write $I = [0, 1] = \bigcup_{i=1}^{n} I_i$, where $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $I_n = \left[\frac{n-1}{n}, 1\right]$. Each consumer $t \in I_i$ is characterized by her *private information* which is described by $\mathcal{F}_t = \mathcal{F}_i$, her *consumption set* ℓ_+^∞, for every $\omega \in \Omega$; her *random initial endowment* $e(t, \cdot) = e_i \in (\ell_+^\infty)^\Omega$ and her *expected utility function* $V_t = V_i$. We will refer to I_i as the set of agents of type i in the atomless economy $\mathcal{E}_c$. Then, the *continuum economy* $\mathcal{E}_c$ with a finite number of types is given by $\mathcal{E}_c = \left\{(\Omega, \mathcal{F}), \ell_+^\infty, I = \bigcup_{i=1}^{n} I_i,\ \mathcal{F}_i,\ e_i, V_i, q) :\ i = 1, \dots, n\right\}$.

An *allocation* in the continuum economy $\mathcal{E}_c$ is a Bochner integrable function $f : I \to (\ell_+^\infty)^\Omega$ or, alternatively, $f : I \times \Omega \to \ell_+^\infty$, where $f(t, \omega) \in \ell_+^\infty$ is the consumption bundle for agent t associated to the state of nature ω (see Diestel and Uhl, 1977, for the definition of Bochner integral as an extension of the Lebesgue integral to Banach spaces).

An allocation f is *feasible* in the economy $\mathcal{E}_c$ if: *(i)* for almost all $t \in I$ the function $f(t,\cdot)$ is $\mathcal{F}_t$-measurable, and *(ii)* $\int_I f(t,\omega)d\mu(t) \leq \int_I e(t,\omega)d\mu(t)$ for all $\omega \in \Omega$.

A *coalition* S is a measurable subset $S \subset I$, with $\mu(S) > 0$. An allocation f is *privately blocked* by a coalition S in the economy $\mathcal{E}_c$ if there exists $g : S \times \Omega \to \ell_+^\infty$ such that $g(t,\cdot)$ is $\mathcal{F}_t$-measurable for every $t \in S$, $\int_S f(t,\cdot)d\mu(t) \leq \int_S e(t,\cdot)d\mu(t)$ and $V_t(g(t,\cdot)) > V_t(f(t,\cdot))$ for every $t \in S$. The set of all feasible allocations that are not privately blocked by any coalition of agents is the *private core* of the economy $\mathcal{E}_c$.

A *Walrasian expectations equilibrium* in the sense of Radner (or a *Radner equilibrium*) in the associated continuum economy $\mathcal{E}_c$ is a pair (f,p) where f is a feasible allocation and $p \neq 0$ is a price system such that, for every consumer $t \in I$, the consumption bundle $f(t,\cdot)$ maximizes the expected utility function V_t on the budget set $B_t(p) = \{y \in (\ell_+^\infty)^\Omega$ such that $\sum_{\omega\in\Omega} p(\omega)\cdot y(\omega) \leq \sum_{\omega\in\Omega} p(\omega)\cdot e(t,\omega)\}$.

An allocation f in $\mathcal{E}_c$ can be interpreted as an allocation $x = (x_1,\dots,x_n)$ in $\mathcal{E}$, where $x_i = \frac{1}{\mu(I_i)}\int_{I_i} f(t,\cdot)d\mu(t)$. Reciprocally, an allocation x in $\mathcal{E}$ can be interpreted as an allocation f in $\mathcal{E}_c$, where f is the step function given by $f(t,\cdot) = x_i$, if $t \in I_i$.

Next result shows that the continuum and the discrete approach can be considered equivalent with respect to Radner equilibria.

Theorem 3.1 *Under assumptions (A.2) and (A.4) the following statements hold:*

If (x,p) is a Radner equilibrium for the economy $\mathcal{E}$, then (f,p) is a Radner equilibrium for the continuum economy $\mathcal{E}_c$, where $f(t,\cdot) = x_i$ if $t \in I_i$.

Reciprocally, if (f,p) is a Radner equilibrium for the atomless economy $\mathcal{E}_c$, then (x,p) is a Radner equilibrium for $\mathcal{E}$, where $x_i = \frac{1}{\mu(I_i)}\int_{I_i} f(t,\cdot)d\mu(t)$.

Proof. Let $((x_1,\dots,x_n),p) \in (\ell_+^\infty)^{\Omega\times n} \times \ell_1$ be a Radner equilibrium for $\mathcal{E}$. Then, $\int_I f(t,\omega)d\mu(t) = \sum_{i=1}^n \mu(I_i)x_i(\omega) \leq \sum_{i=1}^n \mu(I_i)e_i(\omega) = \int_I e(t,\omega)d\mu(t)$ for every state $\omega \in \Omega$; and the consumption function $f(t,\cdot)$ maximizes V_t on $B_t(p) = B_i(p)$ for all $t \in I_i$. Therefore, (f,p) is a Radner equilibrium for the continuum economy $\mathcal{E}_c$.

Conversely, let (f,p) be a Radner equilibrium for $\mathcal{E}_c$. Then, $x = (x_1,\dots,x_n)$, with $x_i = \frac{1}{\mu(I_i)}\int_{I_i} f(t,\cdot)d\mu(t)$, is a feasible allocation in the economy $\mathcal{E}$. Since $\sum_{\omega\in\Omega} p(\omega)\cdot x_i(\omega) = \sum_{\omega\in\Omega} \frac{1}{\mu(I_i)}\int_{I_i} p(\omega)\cdot f(t,\omega)d\mu(t) \leq \sum_{\omega\in\Omega} p(\omega)\cdot e_i(\omega)$, we can deduce that $x_i \in B_i(p)$ for every agent i. Let $z \in (\ell_+^\infty)^\Omega$ be a random consumption bundle such that $V_i(z) > V_i(x_i)$. Then, since V_i is a concave and continuous function, there exists $S \subset I_i$, with $\mu(S) > 0$, such that $V_i(z) > V_i(f(t,\cdot))$ for every $t \in S$; and thus $\sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) > \sum_{\omega\in\Omega} p(\omega)\cdot e_i(\omega)$. Otherwise, observe that if $V_i(z) \leq V_i(f(t,\cdot))$ for almost all $t \in I_i$ then $V_i(z) \leq V_i(x_i)$ (see Lemma in García-Cutrín and Hervés-Beloso, 1993, p. 582). □

3.1 Equal treatment private core equivalence

Considering complete information economies, different papers point out the core-Walras equivalence in continuum economies. Aumann (1964) showed the equivalence between the core and the Walrasian equilibria for atomless economies with a finite dimensional commodity space. Bewley (1973) proved a core-Walras equivalence for economies in which the commodity space is the space of essentially bounded, real-valued, measurable functions on a measure space. Rustichini and Yannelis (1991, 1992) generalized Aumann's result for economies in which the commodity space is an ordered separable Banach space.

Addressing economies with differential information, Einy et al. (2001) showed the equivalence between the Walrasian expectations equilibria (in the sense of Radner) and the private core for continuum economies with a finite number of commodities.

Next we state a result which shows that the set of Walrasian expectations equilibrium allocations with the equal treatment property coincides with the private core for the differential information continuum economy $\mathcal{E}_c$ (associated to the economy $\mathcal{E}$ with a finite number of agents) with infinitely many commodities.

Theorem 3.2 *Consider the differential information economy $\mathcal{E}$ under assumptions (A.1)–(A.4). Let f be a feasible allocation in the associated continuum economy $\mathcal{E}_c$ with $f(t,\cdot) = f_i$ for every $t \in I_i$. Then, f is a Walrasian expectations allocation if and only if f belongs to the private core of $\mathcal{E}_c$.*

Proof. Let (f, p) be a Walrasian expectations equilibrium in $\mathcal{E}_c$. Assume that f does not belong to the private core of $\mathcal{E}_c$. Then, there exists a coalition $S \subset I$ which privately blocks f via y. By concavity of the expected utility functions, we can consider that y is an equal treatment allocation, i.e., $y(t,\cdot) = y_i \in \mathcal{X}_i$ for every $t \in S_i = S \bigcap I_i$, $\sum_{i=1}^{n} \mu(S_i) y_i \leq \sum_{i=1}^{n} \mu(S_i) e_i$ and $V_i(y_i) > V_i(f_i)$ for every i with $\mu(S_i) > 0$. This implies that $\sum_{\omega \in \Omega} p(\omega) \cdot y_i(\omega) > \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega)$, for every i with $\mu(S_i) > 0$; which is a contradiction to the physical feasibility of y for the coalition S.

Now, let f be an equal treatment allocation belonging to the private core of the continuum economy $\mathcal{E}_c$. Let $\mathcal{H}$ be the set of finite dimensional subspaces of ℓ^∞ containing $f_i(\omega)$ and $e_i(\omega)$, for every $\omega \in \Omega$ and every $i = 1, \ldots, n$. For any $H \in \mathcal{H}$ let $\mathcal{E}_c^H$ denote the continuum economy obtained from $\mathcal{E}_c$ by restriction of the consumption sets to $\ell_+^\infty \bigcap H$. That is,

$$\mathcal{E}_c^H = \left\{ (\Omega, \mathcal{F}), \ell_+^\infty \bigcap H, I = \bigcup_{i=1}^{n} I_i,\ \mathcal{F}_i,\ e_i, V_i^H, q,\ i = 1, \ldots, n \right\},$$

where V_i^H is the expected utility function of the agents of type i restricted to the finite dimensional positive cone $(\ell_+^\infty \bigcap H)^\Omega$. Obviously, since f belongs to the private core of $\mathcal{E}_c$, the allocation f belongs also to the private core of $\mathcal{E}_c^H$ for any $H \in \mathcal{H}$. On the other hand, the economy $\mathcal{E}_c^H$ satisfies the assumptions which guarantee that the set of Walrasian expectations equilibrium allocations coincides with the private core (see Einy et al., 2001). For each $H \in \mathcal{H}$, let p^H, with $\|p^H\| = 1$, be the price system such that (p^H, f) is a competitive equilibrium for the economy $\mathcal{E}_c^H$. By the Hahn-Banach theorem (see Aliprantis and Burkinshaw, 1985), p^H can be extended to the whole ℓ^∞. In this way, we obtain a bounded subset $\{p^H, H \in \mathcal{H}\}$ of ℓ_1. By the Alaoglu theorem (see Aliprantis and Burkinshaw, 1985) the set of prices $\{p^H, H \in \mathcal{H}\}$ is relatively compact in the weak* topology denoted by $\sigma(ba, \ell^\infty)$. Then there exists a $\sigma(ba, \ell^\infty)$ convergent subnet of $\{p^H, H \in \mathcal{H}\}$. Let p be the point to which it converges. Let us show that (p, f) is a Walrasian expectations equilibrium for the economy $\mathcal{E}_c$. Since the positive cone of ba is $\sigma(ba, \ell^\infty)$ closed, $p \geq 0$; and since $\|p^H\| = 1$ for every $H \in \mathcal{H}$, we deduce that $p \neq 0$. Assume that (p, f) is not a Walrasian expectations equilibrium for $\mathcal{E}_c$. Then, for some i, there exists $g \in \mathcal{X}_i$ such that $\sum_{\omega \in \Omega} p(\omega) \cdot g(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega)$ and $V_i(g) > V_i(f)$. Actually, by assumption (A.4) and continuity of preferences, we can take g such that $\sum_{\omega \in \Omega} p(\omega) \cdot g(\omega) < \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega)$. Then, there exists a subspace $\hat{H}$, such that $g \in \hat{H}$ and g belongs to the budget set of agent i for the price p^H for every H containing $\hat{H}$, which is a contradiction to the fact that (p^H, f) is a Walrasian expectations equilibrium for the economy $\mathcal{E}_c^H$. Finally, as in the proof of Theorem 2 in Bewley (1972), monotonicity and Mackey continuity of preferences allow us to conclude that $p \in \ell_1$. □

3.2 Infinite dimensional extension of Vind's theorem

In the case of considering $\mathbb{R}^\ell$ as commodity space and a complete information framework, Schmeidler (1972) and Grodal (1972) enforced the Aumann's (1964) core equivalence theorem. Vind (1972) completed the previous results by Schmeidler and by Grodal and showed that, for atomless economies, it is enough to consider the veto power of coalitions of any measure, in order to obtain the core; in particular, the blocking power of arbitrarily big coalitions is enough to get the core. Next we state an extension of this result to differential information continuum economies with infinitely many commodities and a finite number of types of agents. For this, we need some notation. Given $x = (x_h)_{h=1}^\infty \in \ell_+^\infty$ and $n \in \mathbb{N}$, we denote by x^n the element of ℓ^∞ defined by $x_h^n = x_h$ if $1 \leq h \leq n$ and $x_h^n = 0$ if $h > n$. Given a set $J \subset I = [0, 1]$, we denote by J_i the set of agents of type i belonging to J, that is, $J_i = J \bigcap I_i$.

Theorem 3.3 *Consider the differential information economy $\mathcal{E}$ under assumptions (A.1)–(A.4). Let f be a step function defined by $f(t, \cdot) = f_i$ if $t \in I_i$. Suppose that f is a feasible allocation in the associated atomless economy $\mathcal{E}_c$ and does not belong to the private core of $\mathcal{E}_c$. Then, for any ε, with $0 < \varepsilon < 1$, there exists a coalition S, with $\mu(S) = \varepsilon$, privately blocking the allocation f.*

Proof. Let f an equal treatment allocation which does not belong to the private core of the economy $\mathcal{E}_c$. Then, there exist a coalition of agents $A \subset I$ and an allocation $\widetilde{g} : A \to (\ell_+^\infty)^\Omega$ such that $\widetilde{g}(t, \cdot) \in \mathcal{X}_i$ for every $t \in A_i$ (i.e, $\widetilde{g}$ is informationally feasible for the coalition A), $\int_A \widetilde{g}(t, \cdot) d\mu(t) \leq \int_A e(t, \cdot) d\mu(t)$ (i.e, $\widetilde{g}$ is physically feasible for the coalition A) and $V_t(\widetilde{g}(t, \cdot)) > V_t(f(t, \cdot))$ for every $t \in A$.

For each state $\omega \in \Omega$, let $g_i(\omega) = \frac{1}{\mu(A_i)} \int_{A_i} \widetilde{g}(t,\omega)d\mu(t)$. Consider the allocation g given by $g(t,\cdot) = g_i$ if $t \in A_i = A \bigcap I_i$. Note that $g_i \in \mathcal{X}_i$ and $\int_A g(t,\cdot)d\mu(t) \leq \int_A e(t,\cdot)d\mu(t)$. Furthermore, by the convexity property of preferences, $V_i(g(t,\cdot)) > U_i(f_i)$ for every $t \in A_i$. On the other hand, observe that, by the Mackey continuity of preferences and assumption (A.4), we can take g such that $\int_A (e(t,w) - g(t,w))\, d\mu(t) \leq z(\omega) = z \gg 0$, for every $\omega \in \Omega$. Therefore, the coalition A privately blocks the allocation f via the allocation g which is constant on types and $\sum_{i=1}^n \mu(A_i)g_i \leq \sum_{i=1}^n \mu(A_i)e_i - z$, where z is a non null constant sequence.

Since g_i^n converges to g_i for the Mackey topology , Mackey continuity of preferences implies that there exists n_0 such that for every $n \geq n_0$ $V_i(g_i^n) > V_i(f_i)$ for every i with $\mu(A_i) > 0$. Hence, coalition A privately blocks f via g^n for every $n \geq n_0$. In particular, we have the following inequality between n-dimensional Lebesgue integrals, $\int_A g^n(t,\cdot)d\mu(t) \leq \int_A e^n(t,\cdot)d\mu(t)$, where $g^n(t,\omega) = g_i^n(\omega)$ for every $t \in A_i$.

Let the atomless measure $\eta(H) = \left(\mu(H), \int_H e^n(t,\cdot)d\mu(t), \int_H g^n(t,\cdot))d\mu(t)\right)$, restricted to A. Applying Lyapunov theorem to η, we obtain that for any α, with $0 < \alpha < 1$, there exists a coalition $\overline{A} \subset A$, with $\mu(\overline{A}) = \alpha\mu(A)$, that privately blocks f via g^n. This proves the result for $\varepsilon \leq \mu(A)$.

Then, if $\mu(A) = 1$ the proof is complete. Otherwise, we have that $\mu(I \setminus A) > 0$. In this case, given $\varepsilon > 0$, consider the allocation $g_\varepsilon : A \times \Omega \to \ell_+^\infty$ defined by

$$g_\varepsilon(t,\omega) = \varepsilon g(t,\omega) + (1-\varepsilon)f(t,\omega).$$

By convexity of preferences (assumption (A.3)), $V_t(g_\varepsilon(t,\cdot)) > V_t(f(t,\cdot))$ for every $t \in A$. Moreover, by continuity of preferences, there exists n_1 such that $V_t(g_\varepsilon^n(t,\cdot)) > V_t(f(t,\cdot))$ for every $t \in A$ and for every $n \geq n_1$. Consider also the consumption bundle given by

$$h_i(\omega) = f_i(\omega) + \frac{\varepsilon\mu(A)}{\mu(I \setminus A)}\, z, \quad \text{for each } \omega \in \Omega.$$

By monotonicity of preferences (assumption (A.2)), we have that $V_i(h_i) > V_i(f_i)$. Again by Mackey continuity of preferences, there exists n_2 such that for every $n \geq n_2$ one has that $V_i(h_i^n) > V_i(f_i)$ for every $i = 1, \ldots, n$.

Consider now $n > \max\{n_1, n_2, n_3\}$ and the vector measure ν restricted to $I \setminus A$ and defined by

$$\nu(C) = \left(\mu(C), \int_C e^n(t,\cdot)d\mu(t), \int_C f^n(t,\cdot)d\mu(t)\right) \in \mathbb{R}^{2nk+1} \text{ for each } C \subset I \setminus A,$$

where k is the number of states of nature, that is, the cardinal of Ω.

Applying Lyapunov's convexity theorem to the atomless measure ν, we obtain that, given $\varepsilon > 0$, there exists $B \subset I \setminus A$ such that

(i) $\mu(B) = (1-\varepsilon)\mu(I \setminus A)$ and
(ii) $\int_B (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t) = (1-\varepsilon) \int_{I \setminus A} (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t)$.

Consider the coalition $S = A \bigcup B$. Note that $\mu(S) = \mu(A) + (1-\varepsilon)\mu(I \setminus A)$. It remains to show that the coalition S blocks the allocation f. For this, let $y : S \times \Omega \to \ell_+^\infty$ be the allocation given by:

$$y(t,\cdot) = \begin{cases} g_\varepsilon^n(t,\cdot) = \varepsilon g_i^n + (1-\varepsilon) f_i^n \text{ if } t \in A_i = A \bigcap I_i \\ \\ y_i = f_i^n + \frac{\varepsilon\mu(A)}{\mu(B)}\, z^n \qquad \text{if } t \in B_i = B \bigcap I_i. \end{cases}$$

Observe that $h_i^n = f_i^n + \frac{\varepsilon\mu(A)}{\mu(I \setminus A)}\, z^n \leq y_i = f_i^n + \frac{\varepsilon\mu(A)}{\mu(B)}\, z^n$ for every i. Thus, by construction, the members in the coalition S prefer the allocation y to the allocation f, that is, $V_i(y(t,\cdot)) > V_i(f_i)$ for every $t \in S_i,\ i = 1, \ldots, n$. Since, g_i and f_i belong to $\mathcal{X}_i$ and z is a constant sequence, we have that $y(t,\cdot) \in \mathcal{X}_t$ for every $t \in S$. In order to conclude that S privately blocks f via y it remains to show that y is physically feasible for the coalition S. Actually, we have the following inequalities:

$$\begin{aligned}
&\int_S (e(t,\cdot) - y(t,\cdot))\, d\mu(t) \geq \int_S (e^n(t,\cdot) - y(t,\cdot))\, d\mu(t) \\
&\geq \int_A (e^n(t,\cdot) - g_\varepsilon^n(t,\cdot))\, d\mu(t) + \int_B (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t) - \varepsilon\mu(A)\, z^n(\cdot) \\
&\geq (1-\varepsilon) \int_A (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t) + (1-\varepsilon) \int_{I \setminus A} (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t) \\
&= (1-\varepsilon) \int_I (e^n(t,\cdot) - f^n(t,\cdot))\, d\mu(t) \geq 0.
\end{aligned}$$

Therefore, the coalition S, with $\mu(S) = \mu(A) + (1-\varepsilon)\mu(I \setminus A)$, blocks the allocation f via the allocation y. Since ε is arbitrary, we have construct an arbitrarily large coalition privately blocking f. □

4 Equivalence results

In this section, we provide two different characterizations of the Walrasian expectations equilibria (Radner equilibrium). Both characterizations are obtained in terms of the private blocking power of the grand coalition. In order to obtain the Radner equilibrium equivalence theorems, the veto power of the coalition formed by all the agents is strengthened. In the first characterization, the blocking power of the grand coalition is made stronger by considering perturbations of the original initial endowments. The second characterization is obtained by considering that agents in a coalition can participate with a fraction of their resources, instead. Since the deterministic Arrow-Debreu-McKenzie model is a special case of the differential information economy model, one derives insights which yield to new characterizations of the Walrasian equilibria in economies with infinitely many conmodities.

4.1 Non-dominated allocations and equilibria

In this subsection, we obtain a first characterization of Walrasian expectations equilibrium in differential information economies with a finite number of traders and infinitely many commodities. This characterization is obtained by exploiting the veto power of only one coalition, i.e., the coalition formed by all the agents in the economy. Precisely, the main result stated in this subsection, Theorem 4.1, shows that an allocation is a Walrasian expectations allocation if and only if it is non dominated by the grand coalition in any economy which results from altering the initial endowments, as slightly as one wants, in a precise direction. Welfare theorems become particular cases of our main result.

Consider the differential information economy $\mathcal{E}=\{((\Omega,\mathcal{F}),\ell_+^\infty,\mathcal{F}_i,U_i,e_i,q) : i = 1,\dots,n\}$ defined in Section 2, with n consumers and infinitely many commodities.

In order to obtain our first equivalence result, we introduce some additional notation. Given an allocation $x = (x_1,\dots,x_n)$ in the economy $\mathcal{E}$ and a vector $a = (a_1,\dots,a_n)$, with $0 \le a_i \le 1$, let $\mathcal{E}(a,x)$ be a differential information economy which coincides with $\mathcal{E}$ except for the random initial endowment of each agent i that is given by the following convex combination of e_i and x_i.

$$e_i(a_i,x_i) = a_i e_i + (1-a_i)x_i,$$

i.e., given the state $\omega \in \Omega$, $e_i(a_i,x_i)(\omega) = a_i e_i(\omega) + (1-a_i)x_i(\omega) \in \ell_+^\infty$.

That is, $\mathcal{E}(a,x) \equiv \{((\Omega,\mathcal{F}),\ell_+^\infty,\mathcal{F}_i,U_i,e_i(a_i,x_i) = a_i e_i + (1-a_i)x_i,q) : i = 1,\dots,n\}$.

Definition 4.1 *An allocation $z \in \mathcal{X}$ is privately dominated (or privately blocked by the grand coalition) in the economy $\mathcal{E}(a,x)$ if there exists a feasible allocation y in $\mathcal{E}(a,x)$ such that $V_i(y_i) > V_i(z_i)$ for every $i = 1,\dots,n$.*

The meaning of the definition is clear. An allocation z is dominated in an economy if the total resources can be distributed in such a way that every agent is strictly better off with respect to z. That is, z is dominated if it is blocked by the grand coalition.

Observe that to be physically feasible and to be dominated are independent conditions for an allocation $z \in \mathcal{X}$. According to the definition above, a (privately) Pareto optimal allocation is a feasible and non-dominated allocation. That is, if z is feasible in an economy and it is not dominated then z is a Pareto optimum.

The next theorem states that a feasible allocation x in the economy $\mathcal{E}$ is a Radner equilibrium allocation if and only if it is not blocked by the grand coalition in any economy $\mathcal{E}(a,x)$ obtained by perturbing the initial endowments in the direction of x. In this way, we provide a characterization of Walrasian expectations equilibria by means of the veto power of the coalition formed by all the agents in a set of economies, which are defined from the initial economy by altering the original endowments following a precise direction.

Theorem 4.1 *Let x be a feasible allocation in the differential information economy $\mathcal{E}$ satisfying assumptions (A.1)–(A.4). Then x is a Walrasian expectations equilibrium allocation in $\mathcal{E}$ if and only if x is a non privately dominated allocation for every economy $\mathcal{E}(a,x)$.*

Proof. Let (p, x) be a Walrasian expectations equilibrium for the economy $\mathcal{E}$. Suppose that there exists $a = (a_1, \dots, a_n)$, such that x is privately dominated in the economy $\mathcal{E}(a, x)$. Then, there exists $y = (y_1, \dots, y_n) \in \mathcal{X}$ such that

(i) $\sum_{i=1}^{n} y_i \leq \sum_{i=1}^{n} e_i(a_i, x_i)$ and
(ii) $V_i(y_i) > V_i(x_i)$ for every agent $i \in \{1, \dots, n\}$.

Since x is a Walrasian expectations equilibrium allocation in the economy $\mathcal{E}$, we have that $p \cdot x_i = \sum_{w \in \Omega} p(\omega) \cdot x_i(\omega) \leq \sum_{w \in \Omega} p(\omega) \cdot e_i(\omega) = p \cdot e_i$, for every agent i; and from *(ii)* we deduce that $p \cdot y_i = \sum_{w \in \Omega} p(\omega) \cdot y_i(\omega) > \sum_{w \in \Omega} p(\omega) \cdot e_i(\omega)$, for every agent $i = 1, \dots, n$. Multiplying these inequalities by $(1 - a_i)$ and a_i, respectively, we obtain that $p \cdot (1 - a_i) y_i > p \cdot (1 - a_i) x_i$ and $p \cdot a_i y_i > p \cdot a_i e_i$. Thus, $p \cdot y_i > p \cdot a_i e_i + p \cdot (1 - a_i) x_i$, for every agent i. Therefore, $\sum_{i=1}^{n} p \cdot y_i > \sum_{i=1}^{n} p \cdot e_i(a_i, x_i)$, which is a contradiction to *(i)*, that is, a contradiction with the physical feasibility of y in the economy $\mathcal{E}(a, x)$.

Now, let $x \in \mathcal{X}$ be a non privately dominated allocation for every economy $\mathcal{E}(a, x)$. Let f be a step function on the real interval $I = [0, 1]$, defined by $f(t, \cdot) = x_i$ if $t \in I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$, if $i \neq n$, and $f(t, \cdot) = x_n$ if $t \in I_n = \left[\frac{n-1}{n}, 1\right]$.

Assume that x is not an equilibrium allocation for the economy $\mathcal{E}$. Then, by Theorem 3.1, the step allocation f given by x is not an equilibrium allocation for the associated continuum economy $\mathcal{E}_c$ with n different types of agents. Applying Theorem 3.2, we have that f does not belong to the private core of the associated continuum economy. Furthermore, by Theorem 3.3, there exists a coalition $S \subset I = [0, 1]$, with $\mu(S) > 1 - \frac{1}{n}$, privately blocking the allocation f via an allocation $g : S \to (\ell_{+}^{\infty})^{\Omega}$, such that for each state of nature $\omega \in \Omega$, $g(t, \omega) = g_i(\omega)$ for every $t \in S_i = S \bigcap I_i$. That is, $\int_S g(t, \cdot) d\mu(t) = \sum_{i=1}^{n} \mu(S_i) g_i \leq \int_S e(t, \cdot) d\mu(t) = \sum_{i=1}^{n} \mu(S_i) e_i$ and $V_i(g_i) > V_i(x_i)$ for all $i = 1, \dots, n$. Let $a_i = n\mu(S_i)$. Notice that, since $\mu(S) > 1 - \frac{1}{n}$, we obtain that $a_i > 0$ for every i.

In the economy $\mathcal{E}$ with a finite number of agents, let us consider the allocation $(g_1, \dots, g_n)$. Let $z_i = a_i g_i + (1 - a_i) x_i$. By construction, $\sum_{i=1}^{n} z_i \leq \sum_{i=1}^{n} a_i e_i + (1 - a_i) x_i$ and $z_i \in \mathcal{X}_i$ for every i. By convexity of preferences, $V_i(z_i) > V_i(x_i)$, for every agent $i \in \{1, \dots, n\}$.

Therefore, the grand coalition privately blocks x via z in the economy $\mathcal{E}(a, x)$, which is a contradiction. □

It should be noted that we characterize the equilibrium allocations as those non-dominated allocations in the economies given by infinitesimal perturbations in a precise direction of the original random initial endowments. In fact, the parameters a_i in the statement of Theorem 4.1 can be chosen arbitrarily close to one for every agent i. Indeed, note that given δ, with $0 < \delta < 1$, it is enough to consider the privately blocking coalition S such that $\mu(S) > 1 - \frac{\delta}{n}$ in order to guarantee $a_i = n\mu(S_i) > 1 - \delta$ for every i.

Notice also that the first welfare theorem is an immediate consequence of Theorem 4.1. In fact, if x is a Radner equilibrium allocation in the economy $\mathcal{E}$, then x is a Pareto optimal allocation not only in the economy $\mathcal{E}$ but also in any economy $\mathcal{E}(a, x)$ where x is feasible.

Moreover, observe that if x is a privately Pareto optimal allocation in $\mathcal{E}$, then x is also a privately Pareto optimal allocation in the economy in which the initial endowment allocation is x, that is, in the economy $\mathcal{E}(0, x)$. Thus, by taking $x_i = e_i$, for all i, all the economies $\mathcal{E}(a, x)$ are equal to $\mathcal{E}(0, x)$ and x is not privately blocked by the grand coalition. Then, if $x \gg 0$, we can apply Theorem 4.1 to the economy $\mathcal{E}(0, x)$ and we obtain, exactly, the second welfare theorem.

Therefore, Theorem 4.1 not only provides a characterization of equilibria in terms of the blocking power of the grand coalition but also allows us to obtain both welfare theorems as particular cases.

4.2 Fuzzy core and equilibria

Aubin (1979), addressing complete information economies with a finite number of agents and commodities, introduced the pondered veto concept and showed that the core obtained by this veto mechanism coincides with the Walrasian equilibria (see also Florenzano, 1990, for more general economies). The veto system proposed by Aubin extends the notion of ordinary veto in the sense that it is allowed a participation of the agents with a fraction of their endowments when forming a coalition. This veto mechanism is referred in the literature to fuzzy veto. On the other hand, the term fuzzy is usually used when elements belong to a set with certain probability. Then, this term may lead the reader to situate within another different scenario. In fact, regarding the veto mechanism introduced by Aubin, the agents actually (and not probably) participate in a coalition with a fraction of their endowments . Thus, as it is known, this veto mechanism is equivalent to the classical (Debreu-Scarf) veto system applied to the sequence of replicated economies. Therefore, we will refer this veto system as Aubin veto or veto in the sense of Aubin.

Following Aubin (1979), we define the privately Aubin blocking for differential information economies as follows.

Definition 4.2 *An allocation x is privately blocked in the sense of Aubin by the coalition S via the allocation y if there exist $\alpha_s \in (0, 1]$, for each $s \in S$, such that $\sum_{s \in S} \alpha_s y_s \leq \sum_{s \in S} \alpha_s e_s$, and $V_s(y_s) > V_s(x_s)$, for every $s \in S$.*

The Aubin private core of the economy $\mathcal{E}$ is the set of all feasible allocations which cannot be privately blocked in the sense of Aubin.

This definition of Aubin private veto and the consequent Aubin private core solution extend the notion of veto mechanism due to Aubin (1979) to a differential information setting. However, as it was noticed in the introduction, it is important to remark that we require the coefficients α_i to be strictly positive for every agent forming the coalition. Otherwise, the grand coalition contains implicitly the set of all possible coalitions.

Definition 4.3 *A feasible allocation x is Aubin dominated (or* dominated in the sense of Aubin*) in the differential information economy $\mathcal{E}$ if x is privately blocked in the sense of Aubin, by the grand coalition.*

The next result shows that the set of Radner equilibrium allocations for the economy $\mathcal{E} = \{((\Omega, \mathcal{F}), \ell_+^\infty, \mathcal{F}_i, U_i, e_i, q) : i = 1, \ldots, n\}$, coincides with the set of allocations which are not Aubin dominated. Therefore, in order to obtain the Walrasian equilibria in the sense of Radner it suffices to consider the privately Aubin blocking power of just one coalition, namely, the grand coalition. Moreover, as we will show, from the proof we can deduce that the participation of every agent i can be taken as close to one as one wants.

Theorem 4.2 *Let $\mathcal{E}$ be an economy under assumptions (A.1)–(A.4). Then x is a Walrasian expectations equilibrium allocation in $\mathcal{E}$ if and only if x is not a dominated allocation in the sense of Aubin in the economy $\mathcal{E}$.*

Proof. Let x be dominated allocation in the sense of Aubin in $\mathcal{E}$. Then, the corresponding step function f given by x does not belong to the private core of the associated continuum economy $\mathcal{E}_c$. Hence, by Theorem 3.2, the step function f is not a Radner equilibrium in the continuum economy $\mathcal{E}_c$. Therefore, applying Theorem 3.1, x is not a Radner equilibrium allocation in $\mathcal{E}$.

Reciprocally, let x be a non Radner allocation in the economy $\mathcal{E}$. Then, by Theorem 3.1., the step function f defined by x is not a Radner allocation in the continuum economy $\mathcal{E}_c$. Hence, f does not belong to the private core of $\mathcal{E}_c$, that is, there exists a coalition S, with $\mu(S) > 0$ blocking f. By Theorem 3.3, the coalition S can be chosen such that $\mu(S_i) > 0$, for every $i = 1, \ldots, n$. Then, there exists an allocation $g : S \times \Omega \to \ell_+^\infty$, with $g(t, \cdot) \in \mathcal{X}_i$ for every $t \in S_i$, such that

(i) $\int_S g(t, \cdot) d\mu(t) \leq \int_S e(t, \cdot) d\mu(t) = \sum_{i=1}^n \mu(S_i) e_i$ and
(ii) $V_i(g(t, \cdot)) > V_i(x_i)$ for every $t \in S_i$ and for every $i = 1, \ldots, n$.

Consider the allocation $y : S \times \Omega \to \ell_+^\infty$ given by

$$y(t, \cdot) = y_i = \frac{1}{\mu(S_i)} \int_{S_i} g(t, \cdot) d\mu(t) \quad \text{for every } t \in S_i.$$

Observe that $y_i \in \mathcal{X}_i$ because $g(t, \cdot) \in \mathcal{X}_i$ for every $t \in S_i$. Then, taking $\alpha_i = n\mu(S_i) \in (0, 1]$ for every $i = 1, \ldots, n$, we have that

(i) $\sum_{i=1}^n \alpha_i y_i \leq \sum_{i=1}^n \alpha_i e_i$ and
(ii) $V_i(y_i) > V_i(x_i)$ for every $i = 1, \ldots, n$.

Condition *(i)* comes from the construction of the allocation y whereas condition *(ii)* is a consequence of convexity of preferences. Therefore, we conclude that x is privately dominated in the sense of Aubin. □

Remark. If we interpret that the participation of an agent i in the grand coalition is close to the total or complete participation when the corresponding coefficient α_i is close to one ($\alpha_i > 1 - \delta$, for any small δ), we will show that in Theorem 4.2 the participation of each agent can actually be required to be close to the total participation:

Given a positive real number $\delta < 1$, by Theorem 3.3, we can take the coalition S blocking the allocation f such that $\mu(S) > 1 - \frac{\delta}{n}$. Therefore, the coefficient $\alpha_i = n\mu(S_i) = n\mu(S \bigcap I_i) > 1 - \delta$ for every $i = 1, \ldots, n$.

Note that as an immediate consequence of the equivalence result above and the characterization stated in Theorem 4.1, we obtain the following corollary.

Corollary 4.1 *Let $\mathcal{E}$ be an economy under assumptions (A.1)–(A.4) and let x be a feasible allocation in $\mathcal{E}$. The following statements are equivalent:*

1. *The allocation x is a Radner equilibrium allocation.*
2. *The allocation x is not privately blocked in the sense of Aubin.*
3. *The allocation x is not privately blocked in the sense of Aubin by the grand coalition.*
4. *The allocation x is not privately blocked in the sense of Aubin by the grand coalition with a participation of each agent as close as the total participation as one wants.*
5. *The allocation x is a non-dominated allocation in every economy $\mathcal{E}(a, x)$.*
6. *The allocation x is not dominated in any economy $\mathcal{E}(a, x)$ with coefficients a_i as close to the unit as one wants.*

5 Radner equilibrium and Bayesian incentive compatibility

Consider the differential information economy $\mathcal{E}$ described in Section 2:

$$\mathcal{E} = \{((\Omega, \mathcal{F}), \ell^{\infty}_{+}, \mathcal{F}_i, U_i, e_i, q) : i = 1, \ldots, n\}$$

Definition 5.1 *A no-free disposal Radner equilibrium for the economy $\mathcal{E}$ is a pair (p, x), where $p \in \ell_1, p \neq 0$ is a price system and $x = (x_1, \ldots, x_n) \in \mathcal{X}$ is an allocation, such that*

(i) for all i the consumption function x_i maximizes V_i on $B_i(p)$,
(ii) $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} e_i$ (no-free disposal).

Denote by $E_i(\omega)$ the event of agent i which contains the realized state of nature $\omega \in \Omega$. Obviously, $E_i(\omega)$ is an element of $\mathcal{F}_i$.

Definition 5.2 *An allocation $x \in \mathcal{X} = \prod_{i=1}^{n} \mathcal{X}_i$ is said to be Coalitional Bayesian Incentive Compatible (CBIC) if the following is not true:*

There exists a coalition $S \subset \{1, \ldots, n\}$ and states ω, ω', $\omega \neq \omega'$ with $\omega' \in E_i(\omega)$ for all $i \notin S$, such that

$$U_i(\omega, e_i(\omega) + x_i(\omega') - e_i(\omega')) > U_i(\omega, x_i(\omega)) \text{ for every } i \in S.$$

The above definition of CBIC is related to the one in Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994), but we don't need to assume that the event $E_i(\omega)$ is an element of the[1] $\wedge_{C \in S} \mathcal{F}_i$, i.e., the event $E_i(\omega)$ is known to every member of the coalition S. Thus, our concept is slightly stronger than the one in the above papers. In essence, this notion of CBIC states that it is not possible for a coalition of agents S to benefit by announcing to the members of the complementary coalition $I \setminus S$, a false state that all members in $I \setminus S$ cannot distinguish from the true trade of

[1] The symbol $\wedge$ denotes the "meet" and $\wedge_{i \in S} \mathcal{F}_i$. is the finest partition contained in each $\mathcal{F}_i$

nature. Since the Radner equilibrium allows for multilateral contracts we insist on a coalitional notion of incentive compatibility since a contract which is individual Bayesian incentive compatible may not be CBIC; of course the reverse is always true.

We remark (see also Glycopantis et al., 2002) that the Radner equilibrium with free disposal is not CBIC, as the next example shows:

Example 5.1 Let $\Omega = \{a, b, c\},\ N = \{1, 2\},\ U_i(\omega, x) = x^{1/2}$ for every $x \in \mathbb{R}_+$, for each state of nature $\omega \in \Omega$ and for every agent $i = 1, 2$; $q(a) = q(b) = q(c) = 1/3$; $\mathcal{F}_1 = \{\{a, b\}, \{c\}\}$, $\mathcal{F}_2 = \{\{a, c\}, \{b\}\}$ and $e_1 = (15, 15, 0),\ e_2 = (15, 0, 15)$.

One can compute the Radner equilibrium for the economy above and find that $x_1 = (12, 12, 3)$ and $x_2 = (12, 3, 12)$ is an equilibrium allocation (with free disposal). Notice that this allocation is not CBIC because if a is the realized state of nature, $c \in E_2(a)$ (i.e., agent 2 can not distinguish a from c), then agent 1 reports c and if agent 2 believes her, agent 1 is better off, that is,

$$U_1(a, e_1(a) + x_1(c) - e_1(c)) = U_1(a, 12 + 3 + 0) > U_1(a, x_1(a)) = U_1(a, 12).$$

In other words, state a has occurred and agent 1 reports that it is state c. Thus, agent 1 keeps the initial endowment in state a (notice that she can even consume 15 units instead of 12 because nobody can verify that she wasted 3 units) and adds the 3 units she received in state c from agent 2 who believes that c has occurred and gives agent 1, 3 units.

The theorem below shows that if in the Radner equilibrium allocation we do not allow for free disposal, then it is always CBIC. Notice that without free disposal the Radner equilibrium in the example above is no trade and thus it is CBIC.

Theorem 5.1 *Let $\mathcal{E}$ be a differential information economy satisfying the assumptions (A.1) and (A.2). Then, any no free disposal Radner equilibrium allocation is Coalitional Bayesian Incentive Compatible.*

Proof. Let $x \in \mathcal{X}$ be a Radner equilibrium allocation and by way of contradiction, suppose that x is not CBIC. Then, there exist $S,\ \omega, \omega',\ \omega \neq \omega'$, with $\omega \in E_i(\omega')$ for every $i \notin S$, such that

$$U_i(\omega, e_i(\omega) + x_i(\omega') - e_i(\omega')) > U_i(\omega, x_i(\omega)) \text{ for every } i \in S. \tag{1}$$

Since for all i net trades are $\mathcal{F}_i$-measurable and $\omega \in E_i(\omega')$, for every $i \notin S$, it follows that $x_i(\omega) - e_i(\omega) = z_i(\omega) = x_i(\omega') - e_i(\omega') = z_i(\omega')$ for all $i \notin S$.

Hence,

$$U_i(\omega, e_i(\omega) + z_i(\omega)) = U_i(\omega, x_i(\omega)) \text{ for all } i \notin S. \tag{2}$$

It follows from (1) and the continuity of U_i that there exists a positive $\varepsilon \in \ell_+^\infty$ such that

$$U_i(\omega, e_i(\omega) + z_i(\omega) - \varepsilon) > U_i(\omega, x_i(\omega)) \text{ for every } i \in S. \tag{3}$$

Define for each agent i the function $y_i : \Omega \to \ell^\infty_+$ as

$$y_i(\omega) = \begin{cases} e_i(\omega) + z_i(\omega') - \varepsilon & \text{for } i \in S \\ e_i(\omega) + z_i(\omega') + \frac{|S|}{|N|-|S|}\,\varepsilon & \text{for } i \notin S, \end{cases}$$

where $\mid S \mid$ denotes the cardinality of the set S.

It can be easily checked that $y = (y_1, \dots, y_n)$ is feasible and also $\mathcal{F}_i$-measurable for every i.

It follows from (3) and from the definition of y that

$$U_i(\omega, y_i(\omega)) > U_i(\omega, x_i(\omega)) \text{ for every } i \in S. \tag{4}$$

From (2) and taking into account monotonicity we obtain that

$$\begin{aligned} U_i(\omega, y_i(\omega)) &= U_i(\omega, e_i(\omega) + z_i(\omega') + \frac{\mid S \mid}{\mid N \mid - \mid S \mid}\,\varepsilon) \\ &> U_i(\omega, x_i(\omega)) \text{ for all } i \notin S. \end{aligned} \tag{5}$$

Thus, (4) and (5) imply that $U_i(\omega, y_i(\omega)) > U_i(\omega, x_i(\omega))$ for every agent $i, (i = 1, \dots, n)$ and consequently for every i

$$V_i(y_i) = \sum_{\omega \in \Omega} U_i(\omega, y_i(\omega)) > \sum_{\omega \in \Omega} U_i(\omega, x_i(\omega)) = V_i(x_i).$$

Since $\sum_{i=1}^n y_i = \sum_{i=1}^n e_i = \sum_{i=1}^n x_i$, one has that $p \cdot \sum_{i=1}^n y_i = p \cdot \sum_{i=1}^n e_i$ for any $p \neq 0$ and consequently $p \cdot y_j \leq p \cdot e_j$ for some agent j. Thus, y_j is $\mathcal{F}_i$-measurable, it belongs to the budget set for j and it yields higher expected utility to agent j than $V_j(x_j)$, a contradiction to the fact that x is a Radner equilibrium allocation. □

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