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The evolutionary stability of perfectly competitive behavior*

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Summary. In a (generalized) symmetric aggregative game, payoffs depend only on individual strategy and an aggregate of all strategies. Players behaving as if they were negligible would optimize taking the aggregate as given. We provide evolutionary and dynamic foundations for such behavior when the game satisfies supermodularity conditions. The results obtained are also useful to characterize evolutionarily stable strategies in a finite population.

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1 Introduction

In perfectly competitive markets, price-taking behavior is often justified by assuming that agents are small relative to market size. The implication of this assumption is that prices are almost insensitive to individual actions. Hence, even if agents behave strategically, equilibrium behavior corresponds to price-taking optimization as the economy becomes large. The crucial axioms underlying this non-cooperative foundation of competitive equilibrium are *anonymity* – the names of the agents are

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irrelevant to the market – and *aggregation* – individual actions affect market price only through the average of all actions (Dubey et al. [8]).

Following Corchón [4], we say that a game is a (generalized) *aggregative* game if payoffs depend only on individual strategies and an aggregate of all strategies.[1] A prominent example is a Cournot oligopoly, where profits depend exclusively on individual and total output. If, additionally, payoffs do not depend on the names of the agents, the game is symmetric. Aggregate-taking optimization – the natural generalization of price-taking behavior – is then still well defined even if agents are not negligible, although it does not correspond to strategic, rational behavior. An optimal aggregate-taking strategy (ATS) is one that is individually optimal given the value of the aggregate that results when all players adopt it. In an ATS, players who are not negligible behave as if they were.

Instead of absolute payoffs, evolutionary game theory proposes relative performance as the important criterion for the survival of a strategy. The underlying assumption is that if a strategy earns higher payoffs than opponent strategies, it tends to be copied more frequently and propagates faster at the expense of worse performing strategies. We then say that a strategy is evolutionarily stable (ESS) if, once adopted by all players, it will not be discarded due to the appearance of a small fraction[2] of experimenters choosing a competing different strategy. If an ESS resists the appearance of *any* fraction of such experimenters, we say that it is *globally stable*. Evolutionary stability thus implies maximization of the difference between own and opponents' payoffs.[3]

In this context, Schaffer [20] observed that, in a Cournot duopoly, the output corresponding to a competitive equilibrium – the output level that maximizes profits at the market-clearing price – is evolutionarily stable. That is, a firm deviating from the competitive equilibrium will earn lower profits than its competitor after deviation.[4] This result was extended to a general oligopoly by Vega-Redondo [26], who additionally showed that the competitive equilibrium would be the only long-run outcome of a learning dynamics based on imitative behavior. The evolutionary approach, hence, provides foundations for competitive equilibrium dispensing with the assumption of negligible agents.

In the present work, we identify the structural characteristics of the Cournot oligopoly which underlie these results. The first is the fact that it is an aggregative game. The second is the strategic substitutability between individual and total out-

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[1] Games with an aggregative structure of this sort appear very often in economic models (cf. Section 2.3), although they are not always explicitly referred to as "aggregative games." Cornes and Hartley [5] also present examples of games which can be viewed as aggregative games after appropriate transformations of the strategy spaces.

[2] If the number of players is finite, the smallest fraction is one player (cf. Section 3).

[3] The concept of evolutionarily stable strategy used here, due to Schaffer [19], refers to a finite population and differs from the usual concept in evolutionary game theory for a continuum population (cf. Sect. 3). For an introduction to evolutionary game theory see e.g. Vega-Redondo [25], or Weibull [29].

[4] The key for the evolutionary success of the competitive firm is its *spiteful* behavior. Quoting Schaffer [20]: "When firms have market power, the potential for 'spiteful' behavior exists. A firm which forgoes the opportunity to maximise its absolute profit may still enjoy a selective advantage over its competitors if its 'spiteful' deviation from profit-maximisation harms its competitors more than itself."

put. Since the incentive to increase individual output *decreases* the higher the total output in the market, the Cournot oligopoly has a *submodular* structure.[5]

Indeed, we find that the results for the Cournot oligopoly are but an instance of a general phenomenon. An ATS is evolutionarily stable in any aggregative game with a submodular structure. This has a natural counterpart in the supermodular case, where any ESS corresponds to aggregate-taking optimization.

Possajennikov [18] already observed a relation between optimal aggregate-taking strategies and evolutionarily stable strategies in aggregative games. Under differentiability, he finds that the first-order conditions of their defining optimization problems are identical. Careful examination of the second-order conditions allows to determine conditions under which both concepts coincide. In contrast, our approach relies exclusively on the structure of the game and provides an intuitive and direct way of relating both concepts.

In the submodular case, we obtain even stronger results. Any ATS is *weakly* globally stable, i. e. weakly better in relative terms independently of the fraction of opponents behaving differently. If the game has a strict ATS, then this is *strictly* globally stable and the unique ESS.

Furthermore, we show that a strictly globally stable ESS is always the long-run outcome of a learning dynamics based on imitation and experimentation. This result, which is of independent interest, is proven for arbitrary (not necessarily aggregative) symmetric games. As a corollary, this will also hold for any strict ATS of a submodular aggregative game. In short, the dynamic stability result of price-taking behavior quoted above generalizes for aggregate-taking optimization to arbitrary submodular aggregative games.

In our view, these results might be taken to provide an alternative, evolutionary foundation for the perfect competition paradigm. In contrast to the large-population approach, this foundation does not rely on agents being negligible. In fact, the evolutionary success of behaving *as if* they were negligible is due precisely to the fact that they are not. When an agent optimizes assuming that she will not affect the aggregate, the latter will actually change, but in such a way that it is her opponents who will be more harmed. A key new insight is that this property derives directly from the supermodular or submodular structure of the game.

These results are also of interest for evolutionary game theory, since they provide either necessary or sufficient conditions to obtain ESS for a class of aggregative games. In the submodular case, we actually provide shortcuts for the computation of an ESS and the long-run outcomes of imitative learning dynamics. Further, our result on imitative dynamics is, to our knowledge, the first general result on the dynamic properties of finite-population ESS.

The paper is organized as follows. Section 2 introduces the notion of (generalized) aggregative games and presents examples beyond the Cournot oligopoly. Section 3 presents the concepts of evolutionary and global stability for n-player games and particularizes them for aggregative games. Section 4 discusses aggregate-taking

[5] We refer here to n-firm Cournot oligopolies with homogeneous product. Certain Cournot oligopolies are supermodular, or can be seen as such through suitable changes of variable. On this see e.g. Amir [1], Amir and Lambson [2], Vives [27], which introduced supermodularity techniques in Economics, or Vives [28].

behavior. Section 5 presents the results relating aggregate-taking behavior and evolutionary stability. Section 6 contains the dynamic results. Section 7 concludes.

2 Generalized symmetric aggregative games

A game is called aggregative if the payoffs to any player depend only on that player's strategy and the sum of all strategies chosen. If the sum is replaced by an arbitrary aggregate g, we refer to a generalized aggregative game (Corchón [4]).

In the present work we will consider symmetric games with a strategy space S common to all players, assumed to be a subset of a totally ordered space X. For our purposes it will be enough to let $S \subseteq X = \mathbb{R}$. Further we will assume the aggregate g to be a symmetric and monotone increasing function.[6] For the sake of expositional simplicity we will drop the qualifiers generalized, symmetric, and monotone, referring to such games simply as aggregative games.

Definition 1 *A (generalized)* symmetric aggregative game *with aggregate g is a tuple $\Gamma \equiv (N, S, \pi)$ where N is the number of players, the strategy set S, common to all players, is a subset of a totally ordered space X, $\pi : S \times X \to \mathbb{R}$ is a real-valued function, and $g : S^N \to X$ is a symmetric and monotone increasing function, such that individual payoff functions are given by $\pi_i(\mathbf{s}) \equiv \pi(s_i, g(\mathbf{s}))$ for all $\mathbf{s} = (s_1, \ldots, s_N) \in S^N$ and $i = 1, \ldots, N$.*

2.1 Families of aggregative games

Existence of a monotone aggregate function is the only requirement for a game to be representable as an aggregative game. Hence, this class of games may be rather large. Actually, in the examples we consider the aggregate is a functional form that can be extended to any number of players as captured by the following definition.

Definition 2 *A* family of symmetric aggregative games *is a collection of games $\{\Gamma^n\}_{n=1}^{\infty}$ where $\Gamma^n \equiv (n, S, \pi)$ is a (generalized) symmetric aggregative game with aggregate g^n such that $g^1(s) = s$ for all $s \in S$ and there exists a function $g : X \times S \to X$ such that*

$$g^{n+1}(s_1, \ldots, s_n, s_{n+1}) = g\left(g^n(s_1, \ldots, s_n), s_{n+1}\right) \quad (1)$$

for all $s_1, \ldots, s_{n+1} \in S$, and all $n \geq 1$.

Note that the construction of an aggregate in Definition 2 follows an inductive scheme. The condition that $g^1(s) = s$ strikes us as natural, although it is not necessary for our analysis. This condition implies that the restriction of g to $S \times S$ coincides with g^2 and is, hence, symmetric. Constructing the aggregate in an inductive way has two advantages. First, it allows us to speak of families of games with a variable number of players but the same strategic structure. This will be useful

[6] The analysis could be analogously performed for the case of decreasing aggregates.

to perform comparative statics with respect to the number of players. Second, it allows to formulate the payoffs of the game depending only on individual strategy and either an aggregate of all strategies, or an aggregate of the strategies of the other players. Indeed, consider a family of symmetric aggregative games $\{\Gamma^n\}_{n=1}^{\infty}$ with $\Gamma^n \equiv (n, S, \pi)$. Define $\widetilde{\pi} : S \times X \to \mathbb{R}$ by

$$\widetilde{\pi}(s, x) = \pi(s, g(x, s)).$$

Now, using (1), we can view the payoffs of the game Γ^n as a function of individual strategy and an aggregate (namely g^{n-1}) of the strategies of the other players as follows.

$$\pi_i(s_i, s_{-i}) = \pi(s_i, g^n(s_i, s_{-i})) = \widetilde{\pi}(s_i, g^{n-1}(s_{-i}))$$

In the literature, the dependence of the payoff function on an aggregate of the opponents' strategies is exploited to simplify the analysis of best reply correspondences (see e.g. Vives [28]).

2.2 Super- and submodularity in aggregative games

In this section we adapt the concepts of super- and submodular games (see e.g. Topkis [24]) to the case of aggregative games.

Definition 3 *We say that an aggregative game* $\Gamma \equiv (N, S, \pi)$ *is* supermodular *(resp.* submodular*) in individual strategy and the aggregate if* π *has* increasing *(resp.* decreasing*)* differences*; i. e. if* $\pi(s'', x) - \pi(s', x)$ *is increasing (resp. decreasing) in* $x \in X$ *for all* $s'' > s' \in S$.

If $X = \mathbb{R}$ and $\pi(s, x)$ is continuously twice differentiable, then π has increasing (resp. decreasing) differences if and only if

$$\frac{\partial^2 \pi(s, x)}{\partial x \partial s} \geq \text{(resp.} \leq \text{)}\ 0$$

The concept of increasing differences captures the notion of *complementarity* – the incentive to increase s increases with the level of the aggregate x. Respectively, the concept of decreasing differences captures the notion of *substitutability* – the incentive to increase s decreases with the level of the aggregate x.

Definition 4 *We say that an aggregative game* $\Gamma \equiv (N, S, \pi)$ *is* quasisupermodular *in individual strategy and the aggregate if* π *satisfies the* single-crossing property *in* $(s, x) \in S \times X$*; i. e. if, for all* $s'' > s'$ *and* $x'' > x'$

$$\pi(s'', x') \geq \pi(s', x') \Rightarrow \pi(s'', x'') \geq \pi(s', x'')$$
$$\pi(s'', x') > \pi(s', x') \Rightarrow \pi(s'', x'') > \pi(s', x'')$$

We say that Γ *is* quasisubmodular *in individual strategy and the aggregate if* π *satisfies the* dual single crossing property *in* (s, x)*; i. e. if the conditions above hold with the reversed inequalities.*

The single-crossing property (SCP) is an *ordinal* version of complementarity weaker than increasing differences. If s'' is preferred to s' given $x = x'$, then s'' is preferred to s' given a higher $x = x''$, although we cannot say whether the incentive to replace s' with s'' has increased. Thus, increasing differences implies the SCP, but not vice versa. An analogous remark can be made for the dual SCP.

2.3 Examples of aggregative games

Example 1
Cournot oligopoly. Consider an oligopolistic market for a homogeneous good with quantity-setting firms. Let $q_i \in \mathbb{R}_+$ be the quantity supplied by firm $i = 1, \ldots, n$. Inverse demand is given by a strictly decreasing function $P(\cdot)$ that depends on the aggregate output level $Q = \sum_i q_i$. All firms face the same increasing cost function $C(q)$. The profit to firm i is then given by

$$\pi_i(\mathbf{q}) = \pi(q_i, g^n(\mathbf{q})) = P\left(g^n(\mathbf{q})\right) q_i - C(q_i)$$

with $\mathbf{q} \in \mathbb{R}^n_+$ and $g^n(\mathbf{q}) = \sum_{j=1}^n q_j$ increasing. This defines a family of aggregative games in the sense of Definition 2, with aggregate equal to the sum of all quantities.[7]

The Cournot game is submodular in own (q_i) and total (Q) output. To see this, let $q_i'' > q_i'$, and note that

$$\pi(q_i'', Q) - \pi(q_i', Q) = P(Q)(q_i'' - q_i') - (C(q_i'') - C(q_i'))$$

is decreasing in Q for P decreasing.

No further assumptions are required for the Cournot oligopoly to be submodular in individual strategy and the aggregate. If, alternatively, we conceive the payoffs of this game as a function of individual strategy and an aggregate of the opponents' strategies, the corresponding submodularity is obtained only under the additional assumption of decreasing marginal revenues. Particular instances of the Cournot game are usually analyzed in the literature as supermodular in own output and the opponents' total output through convenient changes of variable (see Amir [1], or Vives [28, Ch.4]).

Example 2
Rent-seeking. There is a rent V to be obtained – e.g. rent derived from monopoly power, a prize, some commonly valued good (auction). Players compete for this rent by investing some effort or income, $s_i \in \mathbb{R}_+$, $i = 1, \ldots, n$. Only the player that wins the contest obtains the rent, while all other expenditures are lost. The higher the expenditure of a player, s_i, the higher the probability that i obtains the rent, given by

$$\text{Prob}\{i \text{ gets } V \mid s_1, \ldots, s_n\} = \frac{s_i^r}{\sum_{j=1}^n s_j^r}$$

[7] Alternatively, we could have chosen the inverse demand function itself as a (decreasing) aggregate. As noted above, our results could be rewritten for such aggregates.

The parameter r models a technology that turns expenditures or efforts into probabilities of winning. If $r < 1$ there are decreasing returns to these efforts. If $r > 1$ there are increasing returns. The borderline case $r = 1$ corresponds to constant returns.

In a Nash equilibrium total expenditure is always lower than V. In particular, if the number of players is $n \leq r/(r-1)$, there is a symmetric Nash equilibrium of this game with $\hat{s} = \frac{n-1}{n^2} rV$ (see e.g. Lockard and Tullock (eds.) [13]).

Rent-seeking corresponds to a family of aggregative games with payoff function

$$\pi_i(\mathbf{s}) = \pi(s_i, g^n(\mathbf{s})) = \left(\frac{s_i}{g^n(\mathbf{s})}\right)^r V - s_i$$

with $g^n(\mathbf{s}) = \left(\sum_{j=1}^n s_j^r\right)^{1/r}$ and $r > 0$.

Note that rent-seeking games are submodular in individual strategy and the aggregate, since

$$\frac{\partial^2 \pi}{\partial x \partial s} = -r^2 \frac{s^{r-1}}{x^{r+1}} V \leq 0.$$

Alternatively, we could have defined the aggregate to be $g(\mathbf{s}) = \sum_{j=1}^n s_j^r$. The payoff function would then be

$$\pi_i(\mathbf{s}) = \frac{s_i^r}{g(\mathbf{s})} V - s_i$$

This, however, would not fulfill Definition 2.

Example 3

Tragedy of the commons. Consider the following version of the problem of the commons. A set of agents operate a commonly owned production process with decreasing returns to scale. Agents choose their input contributions and total output is distributed in proportion to individual contributions. This results in an *average return game* as defined by Moulin and Watts [16]. Let $s_i \in \mathbb{R}_+$ denote the individual contribution of agent $i = 1, \ldots, n$, and let $g^n(\mathbf{s}) = \sum_i s_i$ be the aggregate input. Output is produced with a technology given by $y = f(g^n(\mathbf{s}))$, with $f(0) = 0$ and f concave.[8] Payoffs are given by

$$\pi_i(\mathbf{s}) = \pi(s_i, g^n(\mathbf{s})) = \frac{s_i}{g^n(\mathbf{s})} \cdot f(g^n(\mathbf{s})) - s_i$$

A Nash equilibrium of this game involves an overutilization of the technology due to the presence of a negative externality which is not taken into account by individual agents.[9]

[8] The production function f need not be differentiable. E. g. $f(x) = ax$ for all $x \leq \bar{x}$ and $f(x) = b_0 + b_1 x$ for all $x \geq \bar{x}$, with $b_1 < a < 1$ and $b_0 = (a - b_1)\bar{x}$.

[9] Moulin and Watts [16] show this in a general framework where agents are endowed with convex preferences on output share and input consumption, and both goods are normal. The version presented here is akin to the *common pool resource extraction* game in Sethi and Somanathan [22].

Let $A(x) = f(x)/x$ denote the average output. Set $A(0) = \lim_{x\to 0} f(x)/x$, i. e. the slope of f at zero, and assume $A(0) > 1$. The function A is decreasing by concavity of f. Note that payoffs can be written as $\pi(s, x) = s[A(x) - 1]$.

The game is submodular in own contribution and the aggregate. To see this, let $s'' > s'$ and note that

$$\pi(s'', x) - \pi(s', x) = (s'' - s')[A(x) - 1]$$

is decreasing in x.

Example 4
Diamond's search. Milgrom and Roberts [14] present a simplified version of Diamond's search model (Diamond [7]) of an economy where production results from a technology with specialized labor, modelled through an individual level of effort, $s_i \in \mathbb{R}_+$. In order to consume, each individual must first produce a good at cost $C(s_i)$, increasing with s_i, that must be exchanged for another individual's good. Success in finding a trading partner – and thus in consumption of produced goods – depends proportionally on the own effort and the total level of effort in the economy. The latter is then interpreted as employment. The point was to show that there may be multiple equilibria, i. e., multiple natural rates of unemployment. This is captured by a family of aggregative games with payoff function

$$\pi_i(\mathbf{s}) = \pi(s_i, g^n(\mathbf{s})) = \alpha s_i g^n(\mathbf{s}) - C(s_i)$$

with $g^n(\mathbf{s}) = \sum_{j=1}^n s_j$ and $\alpha > 0$.

This game is supermodular, since for $s'' > s'$

$$\pi(s'', x) - \pi(s', x) = \alpha(s'' - s')x - (C(s'') - C(s'))$$

is increasing in the aggregate x.

Example 5
Minimum effort. The minimum-effort game can be used to model a Stag-Hunt production game where the inputs are n different types of specialized labor, all of them perfect complements for the production of the output (see e.g. Bryant [3]). Individual level of effort is denoted $s_i \in \mathbb{R}_+$ and production costs are linear. This can be seen as a family of aggregative games with payoff function

$$\pi_i(\mathbf{s}) = \pi(s_i, g^n(\mathbf{s})) = a g^n(\mathbf{s}) - b s_i$$

aggregate $g^n(\mathbf{s}) = \min_i\{s_i\}$, and $a > b \geq 0$.

This game is simultaneously super- and submodular, since for $s'' > s'$

$$\pi(s'', x) - \pi(s', x) = -b(s'' - s')$$

is constant in x.[10]

The focus of this paper is on symmetric games. Classic examples of aggregative games include, however, models of Bertrand competition with differentiated products and monopolistic competition. We refer to Cornes and Hartley (2001) for further examples of asymmetric games which can be seen as symmetric aggregative games through suitable transformations.

[10] This holds true for any *separable* payoff function $\pi(s, x) = h_1(s) + h_2(x)$.

3 Evolutionary stability in a finite population

Standard evolutionary game theory considers random, pairwise contests between individuals drawn from an infinite population – two individuals are repeatedly chosen at random to play a given two-player game. In that context, a strategy is an evolutionarily stable strategy (ESS) if, once adopted by the whole population, it cannot be invaded by a small *mass* of mutants, that is, individuals displaying different behavior (see e.g. Weibull [29]).

To apply the principle of natural selection to, say, firms in an industry, we need a definition of an ESS for a finite population of players which "play the field", that is all compete with each other simultaneously (Schaffer [19]). This will differ from the analogous concept for an infinite population. In a small population with mutants coming in one at a time, the single mutant will not face other mutants.

Let $\Gamma \equiv (N, S, \Pi)$ be a symmetric N-player game. That is, S is the common strategy set for all players, $\Pi : S \times S^{N-1} \to \mathbb{R}$, and the individual payoff functions are given by $\pi_i(\mathbf{s}) \equiv \Pi(s_i|s_{-i})$ for all $\mathbf{s} \in S^N$ and $i = 1, \ldots, N$, where $\Pi(s_i|s_{-i}) = \Pi(s_i|s'_{-i})$ if s'_{-i} is a permutation of s_{-i}.

Definition 5 *We say that $s \in S$ is an ESS of a symmetric game $\Gamma \equiv (N, S, \Pi)$ if for all $s' \in S$,*

$$\Pi(s|s', s, \ldots, s) \geq \Pi(s'|s, s, \ldots, s).$$

An ESS is strict if the inequality holds strictly for all $s' \neq s$.

In a finite population, an ESS strategist does not maximize own payoffs in general; rather, it is *relative* payoffs that are maximized – the difference between own and opponents' payoffs. A deviation to an ESS may decrease own survival probability, but in that case it will decrease the opponents' probability of survival even more. This is called *spiteful behavior* (Hamilton [10]). As observed by Schaffer [19], an ESS is a strategy s such that

$$s \in \arg\max_{s'} \left[\Pi(s'|s, s, \ldots, s) - \Pi(s|s', s, \ldots, s)\right]$$

Thus, an ESS corresponds to a symmetric Nash equilibrium of the game with relative payoffs. In general, however, a finite-population ESS does not necessarily correspond to a Nash equilibrium of the original game in stark contrast to the standard ESS concept for an infinite population.

Allowing for the appearance of mutants in groups results in a more stringent concept of stability of a finite-population ESS.

Definition 6 *Let s be an ESS of a symmetric game $\Gamma \equiv (N, S, \Pi)$. We say that s is weakly (strictly) globally stable if for all $s' \in S$, $s' \neq s$*

$$\Pi(s|s', \overset{m}{\ldots}, s', s, \ldots, s) \geq (>) \Pi(s'|s', \overset{m-1}{\ldots}, s', s, \ldots, s)$$

for all $1 \leq m \leq N - 1$.

Note that in a finite population of N players with m mutants, players choosing the incumbent strategy face m mutants, while mutants face only $m-1$ other mutants, since the mutant never faces herself.

Definition 6 differs slightly from the one by Schaffer [19], who calls an ESS globally stable if it fulfills the strict inequality in Definition 6 for $m \geq 2$ (see Crawford [6] and Tanaka [23] for closely related concepts).

Both ESS and global stability constitute a stability check against a *single* competing strategy. An ESS is robust against all possible mutants coming in small fractions; i. e. in a finite population only one at a time. A globally stable strategy is robust against all possible mutant strategies independently of the fraction of mutants.[11]

3.1 ESS in an aggregative game

Let $\Gamma \equiv (N, S, \pi)$ be a symmetric aggregative game with aggregate g. Then, $s \in S$ is an ESS if, for all $s' \in S$,

$$\pi(s, g(s', s, \ldots, s)) \geq \pi(s', g(s', s, \ldots, s)).$$

That is, s performs better than the mutant strategy s' in the post-mutation strategy profile with aggregate $g(s', s, \ldots, s)$. Thus, an ESS solves

$$s \in \arg\max_{s'} \left[\pi(s', g(s', s, \ldots, s)) - \pi(s, g(s', s, \ldots, s))\right] \tag{2}$$

An ESS, s, is weakly (strictly) globally stable if, for all $s' \neq s$ and all $1 \leq m \leq N-1$

$$\pi(s, g(s', \overset{m}{\ldots}, s', s, \ldots, s)) \geq (>) \pi(s', g(s', \overset{m}{\ldots}, s', s, \ldots, s)). \tag{3}$$

Example 1
Cournot oligopoly (continued). Denote by q^w the output level corresponding to a Walrasian equilibrium, which satisfies

$$P(n \cdot q^w)\, q^w - C(q^w) \geq P(n \cdot q^w)\, q - C(q)$$

for all $q \neq q^w$. In words, q^w maximizes profits given the price. Vega-Redondo [26] shows that for all $q \neq q^w$ and $1 \leq k \leq n$

$$\pi(q^w, g(q, \overset{n-k}{\ldots}, q, q^w, \overset{k}{\ldots}, q^w)) = P((n-k)q + kq^w)q^w - C(q^w) >$$
$$P((n-k)q + kq^w)q - C(q) = \pi(q, g(q, \overset{n-k}{\ldots}, q, q^w, \overset{k}{\ldots}, q^w))$$

which implies that q^w is a strictly globally stable ESS. To see this, note that it follows from $P(.)$ strictly decreasing that

$$\left[P(nq^w) - P((n-k)q + kq^w)\right](q^w - q) < 0$$

[11] In Section 6 we will postulate a dynamic model where simultaneous mutations to different strategies are allowed.

Subtracting $C(q) + C(q^w)$ and rearranging we obtain

$$[P((n-k)q + kq^w) q^w - C(q^w)] - [P((n-k)q + kq^w) q - C(q)] > [P(nq^w) q^w - C(q^w)] - [P(nq^w) q - C(q)]$$

It suffices to notice that the right-hand side of the previous inequality is non-negative by definition of q^w.

Remark 1 In general, the output corresponding to a competitive equilibrium is larger than the output corresponding to a Cournot equilibrium. It is worth noting that this fact generalizes as follows. For any aggregative game with strictly increasing aggregate g and payoff function $\pi(s, x)$ strictly decreasing in x, a globally stable ESS, s^*, will always be larger than the strategy corresponding to a symmetric Nash equilibrium, $\tilde{s}$. For

$$\pi(\tilde{s}, g(\tilde{s}, \ldots, \tilde{s})) \geq \pi(s^*, g(s^*, \tilde{s}, \ldots, \tilde{s})) \geq \pi(\tilde{s}, g(s^*, \tilde{s}, \ldots, \tilde{s})),$$

but $\tilde{s} > s^*$ would imply $g(s^*, \tilde{s}, \ldots, \tilde{s}) < g(\tilde{s}, \ldots, \tilde{s})$ and $\pi(\tilde{s}, g(s^*, \tilde{s}, \ldots, \tilde{s})) > \pi(\tilde{s}, g(\tilde{s}, \ldots, \tilde{s}))$, a contradiction.

4 Aggregate-taking behavior

We have just seen in Example 1 that the outcome of price-taking behavior corresponds to a finite population ESS. By price-taking behavior it is meant that agents ignore the effect of their individual decisions on the market price. The generalization of this idea to an arbitrary aggregative game results in the concept of aggregate-taking behavior.

Definition 7 *Let $\Gamma \equiv (N, S, \pi)$ be a symmetric aggregative game. We say that $s^* \in S$ is an* optimal aggregate-taking strategy *(ATS) if*

$$s^* \in \arg\max_s \pi(s, g(s^*, \ldots, s^*)) \tag{4}$$

A strict ATS *is an ATS which is a strict maximizer of this problem.*

Example 2
Rent-seeking (continued). The first order condition of problem (4) for this case yields

$$\left.\frac{\partial \pi(s_i, g(s^*, \ldots, s^*))}{\partial s_i}\right|_{s_i = s^*} = \frac{r}{ns^*} \cdot V - 1 = 0.$$

Moreover, since

$$\frac{\partial^2 \pi(s_i, g(s^*, \ldots, s^*))}{\partial s_i^2} = \frac{r(r-1)s_i^{r-2}}{n(s^*)^r} \cdot V$$

it follows that $\pi(s_i, g(s^*, \ldots, s^*))$ is strictly concave in s_i if $r < 1$. Thus, $s^* = \frac{r}{n} \cdot V$ is a strict maximum and, hence, a strict ATS in that case. Note that total investment is $n \cdot s^* = r \cdot V < V$; i. e., there is no overdissipation of rent. The Nash equilibrium of the game, however, is given by $\hat{s} = \frac{n-1}{n^2} \cdot r \cdot V \neq s^*$.

Hehenkamp et al. [11] find that s^* is an ESS of this game for $r \leq 1 + \frac{1}{n-1}$. This is a second example where ATS and ESS coincide, for a certain range of parameters. The ESS problem in this example captures the tradeoff between increasing the relative probability of winning the prize and the additional relative per unit investment necessary to do so, where relative here means in comparison with the opponents. The fact that s^* is an ESS means that ignoring the effect of individual investments on the aggregate level of investment is a shortcut to solve that problem. In a sense, an ATS maximizes the relative probability of winning the prize taking the cost into account.

4.1 Existence of ATS

Existence of a solution to problem (4) is guaranteed by Kakutani's fixed point theorem if the strategy set S is a compact, convex subset of $\mathbb{R}$ and the payoff function $\pi(s, x)$ is continuous in (s, x) and quasiconcave in s. Here we provide alternative conditions based on supermodularity for the existence of an ATS.

Proposition 1 *Let $\Gamma \equiv (N, S, \pi)$ be a symmetric, quasisupermodular aggregative game. If $S \subset \mathbb{R}$ is compact and $\pi(s, x)$ is upper semicontinuous in s for each x, then an ATS exists.*

Proof. The result follows as an application of Lemma 1 in the Appendix to the function $F(s, t) = \pi(s, g(t, \dots, t))$. The function F satisfies the single-crossing property by quasisupermodularity of Γ and the fact that g is increasing. □

Existence of an ATS for a quasisubmodular game cannot be directly established. For the case of a Cournot oligopoly, Amir and Lambson [2] observe that payoff functions can be rewritten to depend only on total output and the sum of the opponents' output levels. Under mild, additional assumptions, the game is supermodular in these two variables, a fact that can be used to show existence of Cournot-Nash equilibria. This approach can be generalized to show existence of Nash equilibrium in families of aggregative games, for which the aggregate of the opponents' strategies is well defined by g^{n-1}. It can be shown by means of counterexamples, however, that this method fails to provide an existence result for ATS.

5 ESS, ATS, and supermodularity

In Examples 1 and 2 we saw that ESS and ATS coincide at least for certain parameter ranges. We also saw that both are examples of submodular aggregative games. In the present section, we explore the relation between ATS and ESS in the framework of a general super- or submodular aggregative game.

Proposition 2 *Let $\Gamma \equiv (N, S, \pi)$ be a symmetric aggregative game. Suppose Γ is quasisupermodular in individual strategy and the aggregate. If $s^* \in S$ is an ESS, then s^* is also an ATS. If s^* is a strict ESS, then s^* is also a strict ATS.*

Proof. Let s^* be an ESS. Consider a mutation to a strategy $s < s^*$. By monotonicity of the aggregate,

$$g(s, s^*, \ldots, s^*) \leq g(s^*, s^*, \ldots, s^*). \tag{5}$$

Since s^* is an ESS, we have that

$$\pi(s, g(s, s^*, \ldots, s^*)) \leq \pi(s^*, g(s, s^*, \ldots, s^*)). \tag{6}$$

Since π satisfies the SCP, (5) and (6) imply that

$$\pi(s, g(s^*, \ldots, s^*)) \leq \pi(s^*, g(s^*, \ldots, s^*)), \tag{7}$$

verifying the ATS property for s.

Consider now a mutation to $s > s^*$. By monotonicity of the aggregate,

$$g(s, s^*, \ldots, s^*) \geq g(s^*, s^*, \ldots, s^*). \tag{8}$$

By contradiction, suppose that the ATS property is not fulfilled:

$$\pi(s^*, g(s^*, \ldots, s^*)) < \pi(s, g(s^*, \ldots, s^*)). \tag{9}$$

By the SCP, (8) and (9) imply that

$$\pi(s^*, g(s, s^*, \ldots, s^*)) < \pi(s, g(s, s^*, \ldots, s^*)), \tag{10}$$

which contradicts that s^* is an ESS.

The proof that strict ESS implies strict ATS follows analogously, with strict inequalities in (6) and (7), and weak inequalities in (9) and (10). □

Proposition 3 *Let $\Gamma \equiv (N, S, \pi)$ be a symmetric aggregative game. Suppose Γ is quasisubmodular in individual strategy and the aggregate. If $s^* \in S$ is an ATS, then s^* is also an ESS and it is weakly globally stable. If s^* is a strict ATS, then s^* is the unique ESS (and hence also the unique ATS) and it is strictly globally stable.*

Proof. Let s^* be an ATS. To check weak global stability and, in particular, the ESS property, we consider first m mutations to the same strategy $s > s^*$, with $1 \leq m \leq N - 1$. By monotonicity of the aggregate,

$$g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*) \geq g(s^*, s^*, \ldots, s^*). \tag{11}$$

Since s^* is an ATS, we have that

$$\pi(s, g(s^*, \ldots, s^*)) \leq \pi(s^*, g(s^*, \ldots, s^*)). \tag{12}$$

Since π satisfies the dual SCP, (11) and (12) imply that

$$\pi(s, g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*)) \leq \pi(s^*, g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*)), \tag{13}$$

verifying the ESS property for s.

Consider now m mutations to $s < s^*$. By monotonicity of the aggregate,

$$g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*) \leq g(s^*, s^*, \ldots, s^*). \tag{14}$$

By contradiction, suppose that the weak global stability property is not fulfilled:

$$\pi(s^*, g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*)) < \pi(s, g(s, \overset{m}{\ldots}, s, s^*, \ldots, s^*)). \quad (15)$$

By the dual SCP, (14) and (15) imply that

$$\pi(s^*, g(s^*, \ldots, s^*)) < \pi(s, g(s^*, \ldots, s^*)), \quad (16)$$

which contradicts that s^* is an ATS.

The proof that strict ATS implies strict global stability and, in particular strict ESS follows analogously, with strict inequalities in (12) and (13), and weak inequalities in (15) and (16). To see uniqueness, suppose there is a different ESS $\tilde{s} \neq s^*$. Applying strict global stability of s^* for $m = N - 1$, we obtain

$$\pi(s^*, g(s^*, \tilde{s}, \ldots, \tilde{s})) > \pi(\tilde{s}, g(s^*, \tilde{s}, \ldots, \tilde{s})),$$

in contradiction with $\tilde{s}$ being an ESS. □

Summarizing, the last two propositions show that ESS implies ATS in the supermodular case, and the reverse implication is true in the submodular case.[12] For instance, the Cournot oligopoly of Example 1 is submodular in own and aggregate output. Hence, the individual output level of a Walrasian equilibrium (by definition, an ATS) is an ESS by Proposition 3.

To get an intuition for these results, consider an ATS s^* and an arbitrary strategy $s > s^*$ in the quasisubmodular case. By definition of ATS, there is no incentive to switch from s^* to s given the value of the aggregate. Mutations to s will increase the value of the aggregate. Quasisubmodularity implies that there are no gains in relative terms from playing s rather than s^* in the post-mutation profile.

Note that our results for the submodular case are stronger than those for supermodularity. This is due to an asymmetry in the concepts of ATS and ESS. In particular, Proposition 3 will be more useful than Proposition 2, as we will illustrate in examples below. Recall an ESS solves the maximization problem (2) and an ATS solves the maximization problem (4). In general, the latter is much easier to solve than the former. In the supermodular case, Proposition 2 implies that solving (4) yields a necessary condition for an ESS. In that case, sufficient conditions for ESS need still be checked. In the submodular case, though, solving (4) is sufficient to find an ESS by Proposition 3. Moreover, in this case, strict ATS will always be strictly globally stable, a fact that will have strong implications for dynamic stability (see Sect. 6).

5.1 The differentiable case

Propositions 2 and 3 do not require any differentiability assumptions on the considered aggregative game, relying only on sub- or supermodularity. For specific examples, however, differentiability helps to establish the equivalence of ESS and

[12] If we allow for decreasing aggregates in Definition 1, we obtain the dual results, i. e., ESS implies ATS if π satisfies the dual SCP, and ATS implies ESS if π satisfies the SCP.

ATS (or to identify the parameter range where this equivalence holds). Possajennikov [18] observes that under differentiability, the first order conditions of problems (2) and (4) are identical. He then finds sufficient conditions for (interior) ESS and ATS to coincide. These conditions can be summarized as follows. If relative payoffs (the argument in problem (2)) are quasiconcave in the mutant's strategy (s') – and hence the second-order condition for a global maximum of (2) is fulfilled – then ATS implies ESS; conversely, if the function π (the argument in problem (4)) is quasiconcave in individual strategy – the second-order condition for a global maximum of (4) is fulfilled – then ESS implies ATS. The difference between these and our results is illustrated in Example 2 below.

5.2 Examples

Example 2
Rent-seeking (continued). We saw that this game is submodular in individual strategy and the aggregate, and that $s^* = \frac{r}{n} \cdot V$ is a strict ATS for $0 < r < 1$. By Proposition 3, it follows that s^* is the *unique* ESS. Hence, ATS implies ESS, and vice versa (by uniqueness). Therefore, ATS and ESS coincide for $0 < r < 1$.

In order to apply the approach in Possajennikov [18] the second-order conditions of both problems must be carefully examined to reach the previous conclusion. The point here is that examination of the second-order condition for problem (2) is more cumbersome than the direct application of Proposition 3.

For $r > 1$ there is no ATS, so neither Proposition 3 nor the results in Possajennikov [18] can be applied. Hehenkamp et al. [11] show, however, that s^* is an ESS for $r \leq 1 + \frac{1}{n-1}$. For $1 < r < 1 + \frac{1}{n-1}$, s^* is an ESS but not an ATS.

Example 3
Tragedy of the Commons (continued). We saw that this game is submodular in individual strategy and the aggregate. An interior ATS is given by the condition $A(ns^*) = 1$.[13] By Proposition 3, it follows that every ATS is a globally stable ESS. By Remark 1, in a globally stable ESS input contributions are larger than in a Nash equilibrium, and the tragedy of the commons is exacerbated. The intuition is straightforward. If selfish agents act strategically, they neglect to consider the negative externality that increasing their contribution imposes on the other agents. Under aggregate-taking behavior, they further neglect to consider the negative effect that an increase of their input has on their own payoff. This resembles the case of a Cournot oligopoly with constant returns to scale. From the firms' point of view, the Cournot-Nash equilibrium is strictly worse than the "efficient" collusive outcome, and the Walrasian outcome (which is an ATS) is even worse.

Example 4
Diamond's search (continued). We saw that this game is supermodular in individual strategy and the aggregate. In this case, by Proposition 2, it follows that every ESS is an ATS. If $C'' > 0$, an ATS is given by the first-order condition for problem

[13] If $S = [0, K]$ and $A(nK) > 1$, the ATS is given by $s^* = K$.

(4), $\alpha n s^* - C'(s^*) = 0$. Hence, this is also a necessary conditions for an ESS.[14] As in Possajennikov [18], here we must check the second-order condition for problem (2). Direct computations show that if $C'' > 2\alpha$, then the condition above is also sufficient for ESS. Therefore, ESS and ATS coincide for $C'' > 2\alpha$, but it is easy to construct examples (with $C'' > 0$ but $C'' \not> 2\alpha$) where there is no ESS but there is an ATS.

Example 5
Minimum effort (continued). In this case, since the aggregate is a minimum function, the individual payoff functions are not differentiable and the analysis based on first- and second-order conditions does not apply. The game, though, is both super- and submodular in individual strategy and the aggregate. By Propositions 2 and 3, every ESS is an ATS and vice versa. Since π is decreasing in s_i the only ATS (hence, the only ESS) is $s^* = 0$. Note that all symmetric profiles $(s, \dots, s)$ with $s \in \mathbb{R}_+$ are Nash equilibria. Thus, in this case the finite-population ESS is a Nash equilibrium.

6 Stochastic stability of an ESS

Vega-Redondo [26] considers a discrete-time dynamic model of a Cournot oligopoly where firms choose quantities from a finite grid.[15] Each period, imperfectly informed, boundedly rational firms imitate the output level of any firm with highest profits in the previous period. Occasionally, with an exogenous probability $\varepsilon > 0$, firms experiment with an arbitrary output level. The prediction of the model is that, for small ε, the system spends most of the time at the state where all firms produce the output corresponding to the Walrasian equilibrium – strict ATS (hence, strictly globally stable ESS) of the Cournot game with strictly decreasing demand. Formally, this state is *stochastically stable*.[16] Using recent results on stochastic stability from Ellison [9], it is easy to show that the former conclusion generalizes to any strictly globally stable ESS. This result is of independent interest and can be stated for symmetric games in general, and not only for aggregative games. To our knowledge, this is the first result on dynamic stability of a finite-population ESS.

Let $\Gamma \equiv (N, S, \Pi)$ be any symmetric game with finite S. Assume players choose strategies from S in discrete time $t = 0, 1, \dots$ according to the following two rules:

(i) *Imitation:* Each period $t \geq 1$, players mimic one of the strategies that gave highest payoffs in the previous period.
(ii) *Experimentation:* With independent probability $\varepsilon > 0$, players ignore the prescription of imitation, and choose a strategy from S according to a probability distribution with full support.

[14] In contrast, the necessary condition for a symmetric Nash equilibrium is $\alpha(n+1) \cdot s^N - C'(s^N) = 0$.

[15] This requirement is for tractability. For a discussion of this model with a continuum of strategies see K.R.Schenk-Hoppé [12].

[16] A state is stochastically stable if it is in the support of the limit invariant distribution of the process as $\varepsilon \to 0$.

Proposition 4 *Let $\Gamma \equiv (N, S, \Pi)$ be a symmetric N-player game with finite S. Let s^* be a strictly globally stable ESS. Then, the profile $(s^*, \ldots, s^*)$ is the unique stochastically stable state of the imitation dynamics with experimentation.*

Proof. s^* is a strictly globally stable ESS; i. e., it is resistant to any number of simultaneous experiments (mutations) with the same strategy. Taking $m = 1$ and $m = N - 1$ in Definition 6, we obtain that

(a) starting at s^*, an 'experimenter' choosing any other $s \neq s^*$ performs strictly worse, and
(b) starting at any $s \neq s^*$, an 'experimenter' with s^* performs strictly better.

Ellison [9, Theorem 1] provides the following result for stochastic stability of a state ω. Let the *radius* of the state, $R(\omega)$, be the minimum number of experiments necessary to leave ω. Let the coradius of the state, $CR(\omega)$, be the maximum number of experiments necessary to reach ω from any other state. If $R(\omega) > CR(\omega)$, then ω is the only stochastically stable state.

For our particular imitation dynamics with experimentation, (a) above implies that $R(s^*, \ldots, s^*) > 1$ and $CR(\omega) > 1$ for any other state. By (b), $CR(s^*, \ldots, s^*) = 1$ and $R(\omega) = 1$ for any other state. In particular, $R(s^*, \ldots, s^*) > CR(s^*, \ldots, s^*)$, implying that $(s^*, \ldots, s^*)$ is the only stochastically stable state.[17] Intuitively, this state is harder to destabilize through experimentation than any other state. □

We mentioned in Section 3 that a finite-population ESS is not necessarily a Nash equilibrium of the game. This implies that there may be incentives to deviate from an ESS. By definition, though, starting at a population profile where all players are choosing an ESS, *any* experimenter would be worse in relative terms after deviation. We should stress the fact that the latter holds even if the 'experimenter' cleverly chooses a best response to her opponents' strategies. Note that by allowing experimentation with full support we allow, among others, also 'clever' experimentation with best replies.

Corollary 1 *Let $\Gamma \equiv (N, S, \pi)$ be a quasisubmodular aggregative game with finite S. Let s^* be a strict ATS. Then the profile $(s^*, \ldots, s^*)$ is the unique stochastically stable state of the imitation dynamics with experimentation.*

Corollary 1 follows from Propositions 3 and 4.[18] It provides a link between the ATS concept in submodular aggregative games and the long-run outcome of dynamical models based on imitative behavior. Applied to a Cournot oligopoly as in Example 1, it yields the result in Vega-Redondo [26]. Applied to a rent-seeking game as in Example 2, it implies stochastic stability of the profile where each player invests $s^* = \frac{r}{n} \cdot V$ when $r < 1$. This can be seen as an efficient outcome since it avoids overdissipation of rent.

[17] Moreover, the expected waiting time until this state is first reached is of order ε^{-1}. In particular, the order of convergence is independent of population size.

[18] It has come to our attention after circulating our paper that Corollary 1 has independently been shown by Schipper [21], using the concept of recurrent set introduced by Nöldeke and Samuelson [17].

7 Conclusions

The present work deals with the class of (generalized) symmetric aggregative games, whose payoff function may be written to depend only on individual strategy and an aggregate of all strategies. If players were negligible, in a Nash equilibrium of such games their behavior would correspond to optimization given the value of the aggregate. If players are not negligible, this kind of aggregate-taking behavior is still well defined, although it does not correspond to rational behavior. We refer to an optimal aggregate-taking strategy (ATS) as an optimizing strategy given the value of the aggregate, when all players choose that strategy. This is a generalization of the concept of competitive equilibrium.

We consider two dual cases. Under submodularity of the payoff function, which includes the case of Cournot oligopoly, an ATS satisfies an evolutionary stability criterion. Specifically, any deviation from an ATS in that case leaves the deviator worse off in relative terms. A strategy verifying this property is called a finite-population ESS. Under supermodularity of the payoff function, the converse result obtains; i. e. aggregate-taking behavior is a necessary condition for evolutionary stability.

Moreover, in the submodular case, we show that a *strict* ATS is also the long-run outcome of a learning dynamics based on imitation and experimentation. This provides dynamic foundation for aggregate-taking behavior in such settings.

In other words, in the supermodular case we find that ATS is a necessary condition for ESS, while in the submodular case it is a sufficient condition for globally stable ESS. In the latter case, this provides a shortcut for the computation of an ESS and the long-run outcomes of imitative learning dynamics. Of course, these findings are useful provided an ATS exists. Existence is guaranteed if the payoff function of the game is quasiconcave in individual strategy. It turns out that this requirement is easier to verify than the conditions required to find an ESS directly, due to the complexity of the objective function of the associated optimization problem.

Appendix

We say that $F : \mathbb{R}^2 \to \mathbb{R}$ satisfies the *single-crossing property* in $(s, x) \in \mathbb{R}^2$ if, for all $s'' > s'$ and $x'' > x'$

$$F(s'', x') \geq F(s', x') \Rightarrow F(s'', x'') \geq F(s', x'')$$
$$F(s'', x') > F(s', x') \Rightarrow F(s'', x'') > F(s', x'')$$

The following result is an application of well known lattice programming results. We refer the reader to Topkis [24] for further details.

Lemma 1 *Let $S \subset \mathbb{R}$ be compact. Suppose $F : \mathbb{R}^2 \to \mathbb{R}$ satisfies the single-crossing property and $F(s, x)$ is upper semicontinuous in s for each value of x. Then there exists $s^* \in S$ such that*

$$s^* \in \arg\max_{s \in S} F(s, s^*)$$

Proof. Upper-semicontinuity of F and compactness of S guarantee that $\arg\max_{s \in S} F(s,x)$ is non-empty for each x. By Topkis [24, Theorem 2.8.6] (due to Milgrom and Shannon [15]) and Topkis [24, Corollary 2.7.1 and Theorem 2.4.3] the maximum and minimum selections of $\arg\max_{s \in S} F(s,x)$ are increasing. By Tarski's fixed point theorem (see e.g. Topkis [24, Corollary 2.5.1]) these selections have a fixed point. □

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