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Economic Theory

A general revealed preference theorem for stochastic demand behavior*

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Summary. We present a general revealed preference theorem concerning stochastic choice behavior by consumers. We show that, when the consumer spends her entire wealth, the Weak Axiom of Stochastic Revealed Preference due to Bandyopadhyay, Dasgupta, and Pattanaik (1999) is equivalent to a restriction on stochastic demand behavior that we call stochastic substitutability. We also show that the relationship between the Weak Axiom of Revealed Preference and Samuelson's inequality in the deterministic theory, and the main result of Bandyopadhyay, Dasgupta, and Pattanaik (1999) are both special cases of our result.

Keywords and Phrases: Stochastic choice, Weak axiom of revealed preference, Weak axiom of stochastic revealed preference, Stochastic substitutability, Stochastic substitution theorem.

JEL Classification Numbers: D11.

1 Introduction

The standard revealed preference theory of consumers' behavior assumes that a consumer's choices can be represented by a deterministic demand function. The central result in this theory is that, when the consumer spends her entire wealth, demand behavior in accordance with Samuelson's Weak Axiom of Revealed Preference (WARP) is equivalent to the satisfaction of a condition that we shall call

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Samuelson's inequality.[1] In this paper, we present a general revealed preference theorem concerning choice behavior by consumers, where consumers can choose in a stochastic fashion. This theorem provides an integrated and unified framework from which existing results in both deterministic demand theory and stochastic demand theory are shown to follow as special cases. Our result concerns the equivalence between the Weak Axiom of Stochastic Revealed Preference due to Bandyopadhyay, Dasgupta, and Pattanaik (1999) and a restriction on stochastic demand behavior that we call stochastic substitutability. We show that the central result in the classical deterministic theory of demand, as well as the main result of Bandyopadhyay, Dasgupta, and Pattanaik (1999) in the framework of stochastic choice, is a special case of this relationship.

Experimental evidence suggests that the choices of an individual may not always be open to explanation in terms of a deterministic objective function. In response to this, and, in contrast to the classical theory, a sizable literature has developed which attempts to model stochastic preference and/or stochastic choice behavior by individuals.[2] Intuitively, this literature seeks to analyze situations where, faced repeatedly with apparently the same feasible set, agents seem to choose some option some of the time, while rejecting that option in favor of other options the rest of the time, thus violating WARP. Such behavior may be observed either because the observer fails to notice changes in some aspects relevant for the agent's decision-making process, and, therefore, mis-specifies the feasible set, or because the agent's choice functions themselves are subject to random shocks.[3]

Earlier contributions to the literature on stochastic preference and stochastic choice paid little attention to the specific economic problem of choices made by a competitive consumer.[4] Recently Bandyopadhyay, Dasgupta, and Pattanaik (1999) studied this problem in a revealed preference framework with a relatively mild and intuitively appealing rationality postulate for individual stochastic choices, the Weak Axiom of Stochastic Revealed Preference (WASRP). However, in discussing the effect of compensated price changes, they confined themselves to the case where only one price could change at a time, since their focus was on the sign of the 'own-price effect' on the demand for a commodity. Thus, before one can have a revealed preference theory for stochastic consumer behavior that subsumes and extends the deterministic theory based on WARP, one needs to identify the implications of WASRP in a general setting, which permits many (possibly, all) prices to change simultaneously. Our paper resolves this issue, thus making the revealed-preference

1 See Section 3 below for a formal statement of this condition.

2 Contributions include Barbera and Pattanaik (1986), Becker et al. (1963), Block and Marschak (1960), Cohen (1980), Corbin and Marley (1974), Falmagne (1978), Fishburn (1973, 1977, 1978), Georgescu-Roegen (1936, 1950, 1958), Halldin (1974), Hey (1995), Loomes and Sugden (1995), Luce (1958, 1959, 1977), Luce and Suppes (1965), Machina (1985), Marschak (1960), Nandeibam (1999), Quandt (1956) and Sattath and Tversky (1976).

3 The actual underlying reason for randomness in an agent's observed choice behavior is not germane to our analysis. The problem is, of course, familiar to econometricians. While the predictions generated by estimated demand functions are probabilistic predictions, the standard economic theory against which these can be tested is a deterministic theory.

4 Some exceptions are Halldin (1974) and Quandt (1956), whose approaches are very different from the approach followed in Bandyopadhyay et al. (1999) and this paper.

theory, based on WASRP, of consumers' stochastic choices more extensive, both in empirical scope and logical completeness, than the standard revealed preference theory, based on WARP, for a consumer's deterministic choices.

We make the plausible and widely used assumption that demand functions satisfy the property of 'tightness', that is, the property that the consumer always spends her entire wealth. Given this assumption, we show that WASRP is equivalent to a restriction on stochastic demand behavior, which we call Stochastic Substitutability. This result subsumes the classical result regarding the equivalence between WARP and Samuelson's inequality. It also extends the main result of Bandyopadhyay, Dasgupta, and Pattanaik (1999) to a general framework where, in considering the effect of compensated changes in prices, one permits simultaneous changes in many prices.

Section 2 introduces the basic notation and definitions. Section 3 presents our results. We conclude in Section 4. All proofs are relegated to the Appendix.

2 Notation and definitions

Let m denote the number of commodities, $m \geq 2$, and let M denote the set $\{1, 2, \ldots, m\}$. R_+ and R_{++} will denote, respectively, the set of non-negative real numbers and the set of positive real numbers. We assume that R_+^m constitutes the consumer's consumption set. A price-wealth situation is an ordered pair (p, W), where $p \in R_{++}^m$ and $W \in R_+$. Let Z denote the set of all possible price-wealth situations. Given a price-wealth situation, (p, W), the *budget set* of the consumer is defined to be $\left\{x \in R_+^m | W \geq p \bullet x\right\}$ (here, as well as in the rest of the paper, we use $\bullet$ to denote the dot product). The budget sets corresponding to price-wealth situations (p, W), (p', W'), $\left(\tilde{p}, \tilde{W}\right)$, etc. will be denoted, respectively, by B, B', $\tilde{B}$, etc.

Definition 2.1.

(i) A *deterministic demand function* (DDF) is a rule d, which, for every $(p, W) \in Z$, specifies exactly one bundle x in B.

(ii) A *stochastic demand function* (SDF) is a rule D which, for every $(p, W) \in Z$, specifies exactly one finitely additive probability measure q over the class of all subsets of B.

Let $q = D(p, W)$, where D is an SDF. Then, for every subset A of B, $q(A)$ is to be interpreted as the probability that, given the price-wealth situation (p, W), the consumer's chosen bundle will belong to the set A. $D(p, W)$, $D(p', W')$, etc. will be denoted, respectively, by q, q', etc. The notion of a DDF is the same as the notion of a demand function used in the standard theory of consumers' choice. Intuitively, DDFs are degenerate SDFs.

Definition 2.2.

(i) A degenerate SDF, D, *induces* a DDF d iff, for every $(p, W) \in Z$, $d(p, W)$ is the consumption bundle $x_* \in B$ such that $q\left(\{x_*\}\right) = 1$, where $q = D(p, W)$.

(ii) A DDF d, *induces* a degenerate SDF, D, iff, for every $(p, W) \in Z$, $q(\{d(p, W)\}) = 1$, where $q = D(p, W)$.

It is easy to see that, if a degenerate SDF, D, induces a DDF, d, then the degenerate SDF induced by d must be D itself, and, if a DDF, d, induces a degenerate SDF, D, then the DDF induced by D must be d itself.

Definition 2.3.

(i) A DDF d satisfies the *weak axiom of revealed preference* (WARP) iff, for all $(p, W), (p', W') \in Z$, $[d\,(p, W) \neq d\,(p', W')$ and $d\,(p', W') \in \quad B]$ implies $p' \bullet d\,(p, W) > W'$.
(ii) An SDF D satisfies the *weak axiom of stochastic revealed preference* (WASRP) iff, for all (p, W), $(p', W') \in Z$, and, for every $A \subseteq B \cap B'$, $q\,(B \backslash B') \geq q'\,(A) - q\,(A)$.

Clearly, if an SDF, D, is degenerate and satisfies WASRP, then the DDF induced by D satisfies WARP, and, conversely, if a DDF d satisfies WARP, then the degenerate SDF induced by d must satisfy WASRP.

Definition 2.4.

(i) A DDF d is *tight* iff, for every $(p, W) \in Z$, $p \bullet d(p, W) = W$.
(ii) An SDF D is *tight* iff, for every $(p, W) \in Z$, $q\,(\{x \in B | p \bullet x = W\}) = 1$.

If the SDF is tight, then, for every price-wealth situation, the consumer spends her entire wealth with probability 1, and similarly in the case of a DDF. In all our formal results, we shall assume that the SDFs and/or the DDFs under consideration satisfy tightness.

We now introduce some notation, which will help us to keep several of our proofs compact.

Notation 2.5. Given two price – wealth situations (p, W) and (p', W'), let

$$I = \{x \in R_+^m | p \bullet x = W \ and \ p' \bullet x = W'\}$$
$$G = \{x \in R_+^m | p \bullet x = W \ and \ p' \bullet x > W'\}$$
$$H = \{x \in R_+^m | p \bullet x = W \ and \ p' \bullet x < W'\}$$
$$G' = \{x \in R_+^m | p \bullet x > W \ and \ p' \bullet x = W'\}$$

and

$$H' = \{x \in R_+^m | p \bullet x < W \ and \ p' \bullet x = W'\}.$$

Figure 1 illustrates these sets for the special case where we have exactly two commodities, $\frac{W}{p_2} - \frac{W'}{p'_2} > 0$, and $\frac{W}{p_1} - \frac{W'}{p'_1} < 0$, and $(p_1'/p_2') < (p_1/p_2)$.

Remark 2.6. The sets I, G, H, G' and H' are pairwise disjoint. G is a subset of $B \backslash B'$; G' is a subset of $B' \backslash B$; H, H' and I are all subsets of $B \cap B'$.

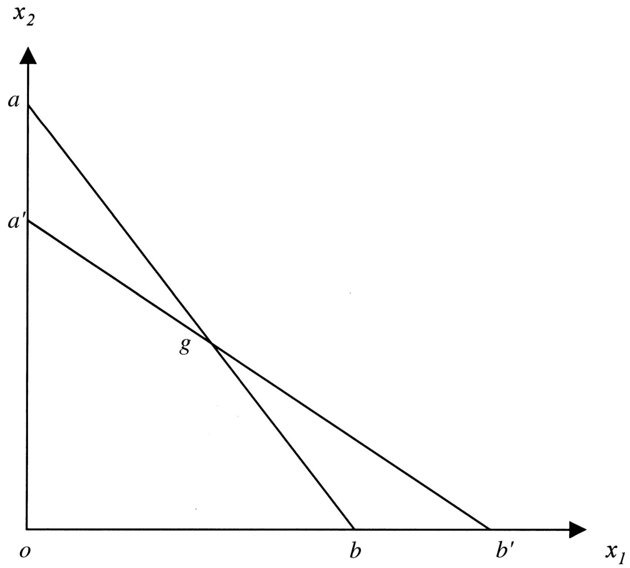

Fig. 1. oab is the budget set, B, corresponding to the price-wealth situation (p, W). $oa'b'$ is the budget set, B', corresponding to the price-wealth situation (p', W'). I is the singleton set containing g. G is ag excluding the point g. H is gb excluding the point g. G' is gb' excluding the point g. H' is $a'g$ excluding the point g

3 The stochastic substitution theorem

Using a general framework that permits simultaneous changes in several prices, in this section we explore the implications, for stochastic demand behavior of the consumer, of the weak axiom of stochastic revealed preference. We first introduce a restriction on the stochastic demand function that we term stochastic substitutability. We show that, for tight SDFs, WASRP is equivalent to stochastic substitutability. We then show that the central results in both deterministic and stochastic revealed preference theories of demand are special cases of this relationship.

Definition 3.1. A tight SDF D satisfies *stochastic substitutability* (SS) iff, for every ordered pair of price-wealth situations $\langle (p, W), (p', W') \rangle$, and, for all $A \subseteq I$, we have:

$$q'(G') + q'(A) \geq q(H) + q(A). \tag{3.1}$$

See Notation 2.5 and Figure 1.

The intuition of SS may be explained as follows. Consider two price-wealth situations (p, W) and (p', W'). Taking (p, W) to be the 'initial' price-wealth situation, one can think of $W - W'$ as an arbitrary level of 'wealth compensation' when the price vector changes from p to p'. First, if we take the set A figuring in the definition of SS to be the empty set, then SS can be seen to imply that the probability that the consumer will choose some bundle from G' in the new situation is no less than the probability that the consumer chooses some bundle in H in the original situation (see Figure 1). Now consider the case where the set A figuring in the definition of SS is some non-empty subset of I. SS then implies that, if the probability of choosing from A is lower in the new price-wealth situation (p', W'), then the probability of choosing from G' in the new situation must be higher than that of choosing from H in the original situation by at least the magnitude of this decline. If the probability of choosing from A is higher in the new situation, then the probability of choosing from G' in the new situation can be higher, lower, or identical to that of choosing from H in the original situation. However, in this case, the probability of choosing from G' in the new situation cannot be lower than that of choosing from H in the original situation by a magnitude greater than that of the increase in the probability of choosing from A.

Note that, if q (resp. q') is atomless, then $q(A) = 0$ (resp. $q'(A) = 0$) for every subset A of I (since the Lebesgue measure of A on the budget plane is 0).

Definition 3.2. A DDF d satisfies *Samuelson's Inequality (SI)* iff, for every ordered pair of price-wealth situations $\langle (p, W), (p', W') \rangle$ such that $W' = p' \bullet d(p, W)$, we have:

$$0 \geq (p - p') \bullet (x_* - x'_*), \tag{3.2}$$

and

$$0 > (p - p') \bullet (x_* - x'_*) \text{ if } x_* \neq x'_* \tag{3.3}$$

where $x_* = d(p, W)$ and $x'_* = d(p', W')$.

Claim 3.3. A tight DDF d satisfies Samuelson's Inequality if and only if the degenerate and tight SDF induced by d satisfies stochastic substitutability.

Proof. See the Appendix.

We now present our main result, namely, that WASRP is equivalent to SS for tight SDFs.[5]

Proposition 3.4 (Stochastic Substitution Theorem). *A tight SDF satisfies WASRP if and only if it also satisfies SS.*

Proof. See the Appendix.

Remark 3.5. Recall that, if a tight DDF d satisfies WARP, then the degenerate and tight SDF, D, induced by it must satisfy WASRP. Hence, by Proposition 3.4, D must also satisfy stochastic substitutability. However, we know that, in that case, the original tight DDF d must satisfy SI (see Claim 3.3). Now note that, by Claim 3.3, if d satisfies SI, then D must satisfy stochastic substitutability. It then

[5] In establishing that WASRP implies SS, we assume tightness only for convenience of exposition. Discarding the assumption of tightness, one can establish a stronger version of this result, at the cost of a major increase in notational complexity. However, the converse result, that SS implies WASRP, depends in a substantive way on the SDF satisfying tightness. It is possible to construct examples to show that SDFs violating tightness can violate WASRP while satisfying SS (redefined for the general case where the SDF is not necessarily tight).

follows from Proposition 3.4 that D must also satisfy WASRP. Consequently, d must satisfy WARP. Thus, the central result in deterministic revealed preference theory of demand[6] follows from our Stochastic Substitution Theorem.

Corollary 3.6. *A tight DDF satisfies WARP if and only if it satisfies Samuelson's Inequality.*

The stochastic counterpart of the familiar non-positivity property of the own-price substitution effect for deterministic demand functions, presented in Bandyopadhyay, Dasgupta and Pattanaik (1999), also follows from the Stochastic Substitution Theorem.

Corollary 3.7 (Bandyopadhyay, Dasgupta, and Pattanaik (1999)). *Let D be a tight SDF satisfying WASRP. Let $i \in M, (p, W), (p', W') \in Z$, and $b \in [0, W/p_i]$ be such that:*

$$p_i > p'_i; p_k = p'_k \text{ for all } k \in (M \backslash \{i\}); \text{ and } W' = W - (p_i - p'_i)\, b.$$

Then:

$$q'(\{x \in B' | x_i \geq b\}) \geq q(\{x \in B | x_i \geq b\}); \tag{3.4}$$

and

$$q'(\{x \in B' | x_i > b\}) \geq q(\{x \in B | x_i > b\}). \tag{3.5}$$

Proof. See the Appendix.

Non-positivity of the deterministic own substitution effect follows directly from Corollary 3.7 as well as from Corollary 3.6.[7]

4 Conclusion

In this paper, we have presented a revealed preference theorem for consumers' stochastic demand behavior based on WASRP that integrates and extends existing results, both in the standard revealed preference theory of demand based on Samuelson's WARP and in the stochastic revealed preference theory of demand based on WASRP. Our result is that, when consumers spend their entire wealth, WASRP is equivalent to a restriction that we call stochastic substitutability. This result subsumes the classical result regarding the equivalence between WARP and Samuelson's inequality, and extends the main result of Bandyopadhyay, Dasgupta and Pattanaik (1999) to a general framework where, in considering the effect of compensated changes in prices, one permits simultaneous changes in many prices.

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[6] See Mas-Colell, Whinston and Green (1995, pp. 28–32) and Samuelson (1947).

[7] Strictly speaking, Corollary 3.7 and the version of the non-positivity of the deterministic substitution effect that follows from Corollary 3.7 as well as from Corollary 3.6 are both weaker than their original versions, since we assume tightness, while both Samuelson (1947) and Bandyopadhyay, Dasgupta and Pattanaik (1999) derived their results without assuming this property. However, as discussed in footnote 5, one can establish a stronger version of Proposition 3.4 after discarding the assumption of tightness; the two above-mentioned results can be shown to follow from this stronger version of Proposition 3.4.

In some ways, the analysis in this paper completes the specific line of investigation initiated in Bandyopadhyay et al. (1999). There are, however, several interesting and as yet unexplored problems in this general area. For instance, it is not at all obvious what would be a natural stochastic translation of the familiar strong axiom of revealed preference and what would be the implications of such a 'strong axiom of stochastic revealed preference'. Nor is it clear how one should reformulate, in stochastic terms, the important property of N-monotonicity due to Nachbar (1999) and Quah (2001). These are just two examples of the issues that remain to be studied in the theory of stochastic choice of consumers. These issues need separate and detailed investigation.

Appendix

The proofs below often refer to the sets I, G, G', H, and H', introduced in Notation 2.5; Figure 1 can be helpful on such occasions.

To prove Claim 3.3, we shall use Lemmas A1 and A2 below.

Lemma A1. *A tight DDF d satisfies SI iff, for every ordered pair of price-wealth situations $\langle (p, W), (p', W') \rangle$ such that $W' = p' \bullet d(p, W)$, the degenerate SDF, D, induced by d satisfies the following:*

$$\text{for all } A \subseteq I, q'(G') + q'(A) \geq q(H) + q(A).$$

Proof of Lemma A1. Let d be a tight DDF. Consider an ordered pair of price-wealth situations $\langle (p, W), (p', W') \rangle$ such that $W' = p' \bullet x_*$, where $x_* = d(p, W)$. Let $x'_* = d(p', W')$. Let D be the tight SDF induced by d. Given the assumption that d is tight, which implies $p \bullet x_* = W, p' \bullet x'_* = W'$, and the assumption that $p' \bullet x_* = W'$, we have:

$$\text{(3.2) holds iff } x'_* \in G' \cup I, \tag{A.1}$$

and

$$\text{(3.3) holds iff } (\text{for } x'_* \neq x_*, \quad x'_* \in G') \tag{A.2}$$

Noting $x_* \in I$, and G', H, and I are pairwise disjoint, Lemma A1 follows from (A.1) and (A.2). □

Lemma A2. *If a tight DDF d satisfies SI, then, for all ordered pairs of price-wealth situations $\langle (p, W), (p', W') \rangle$ such that $W' > p' \bullet d(p, W), \quad p \bullet d(p', W') > W$.*

Proof of Lemma A2. Let $\langle (p, W), (p', W') \rangle$ be any ordered pair of price-wealth situations such that $W' > p' \bullet d(p, W)$. Let $x_* = d(p, W)$ and let $x'_* = d(p', W')$. Clearly, $x_* \neq x'_*$. Hence, if $p \bullet x'_* = W$, d must violate SI. Thus, to establish Lemma A2, we only need to rule out $p \bullet x'_* < W$. Suppose $p \bullet x'_* < W$. Then, since $p \bullet x'_* < W = p \bullet x_*$, and $p' \bullet x'_* = W' > p' \bullet x_*$, there exists $\alpha \in (0, 1)$ such that:

$$[\alpha p' + (1 - \alpha) p] \bullet x'_* = [\alpha p' + (1 - \alpha) p] \bullet x_*.$$

Let $\tilde{p} = \alpha p' + (1-\alpha) p$, and let $\tilde{W} = \tilde{p} \bullet x'_* = \tilde{p} \bullet x_*$. Then, noting that, by tightness, $W' = p' \bullet x'_*$, and, by assumption, $W > p \bullet x'_*$, we get:

$$\alpha W' + (1-\alpha) W > \tilde{W}. \tag{A.3}$$

Now let $\tilde{x} = d\left(\tilde{p}, \tilde{W}\right)$ and consider the ordered pairs

$$\left\langle (p, W), \left(\tilde{p}, \tilde{W}\right)\right\rangle, \left\langle (p', W'), \left(\tilde{p}, \tilde{W}\right)\right\rangle.$$

Since d is tight and satisfies SI, noting that, by construction, $\tilde{W} = \tilde{p} \bullet x'_* = \tilde{p} \bullet x_*$, we then get:

$$p \bullet \tilde{x} \geq W; \tag{A.4}$$
$$p' \bullet \tilde{x} \geq W'. \tag{A.5}$$

(A.4) and (A.5), together, contradict (A.3). □

Proof of Claim 3.3. First, suppose the degenerate and tight SDF, D, induced by a tight DDF, d, satisfies SS. Then Lemma A1 implies that d itself must satisfy SI.

Now suppose a tight DDF d satisfies SI. Let D be the tight degenerate SDF induced by d. Consider any ordered pair of price-wealth situations $\langle (p, W), (p', W')\rangle$. We need to show that:

$$\text{for all A } \subseteq I, q'(G') + q'(A) \geq q(H) + q(A). \tag{A.6}$$

If $W' > p' \bullet x_*$, then (A.6) follows from Lemma A2. If $W' = p' \bullet x_*$, then (A.6) follows from Lemma A1. If $W' < p' \bullet x_*$, then, since $x_* \in G$, (A.6) must hold trivially. □

Proof of Proposition 3.4. Let the SDF, D, be tight.

First, suppose D satisfies WASRP. Let $A \subseteq I$. Then $H' \cup (I - A)$ is a subset of $B \cap B'$. By WASRP,

$$q'(H' \cup (I \backslash A)) - q(H' \cup (I \backslash A)) \leq q(B \backslash B'). \tag{A.7}$$

Since D is tight, $q(B \backslash B') = q(G)$ and $q(H' \cup (I \backslash A)) = q(I \backslash A)$; further, since H' and $I \backslash A$ are disjoint, $q'(H' \cup (I \backslash A)) = q'(H') + q'(I \backslash A)$. Therefore, (A.7) implies

$$q'(H') + q'(I \backslash A) - q(I \backslash A) \leq q(G). \tag{A.8}$$

By the tightness of D, $q'(H') + q'(I \backslash A) + q'(A) + q'(G') = 1 = q(G) + q(I \backslash A) + q(A) + q(H)$. Hence, from (A.8), we have $1 - q'(G') - q'(A) \leq 1 - q(H) - q(A)$, which implies $q'(G') + q'(A) \geq q(H) + q(A)$, as required by SS.

Now suppose D satisfies SS. Let K be any subset of $B \cap B'$. We define

$$\begin{aligned}
K_0 &= K \cap I, \\
K_1 &= K \cap H', \\
K_2 &= K \cap H, \text{ and} \\
K_3 &= \{x \in K | p \bullet x < W \; and \; p' \bullet x < W'\}.
\end{aligned}$$

Note that K_0, K_1, K_2, and K_3 are pairwise disjoint and their union is K. By SS, $q(G) + q(K_0) \geq q'(H') + q'(K_0)$. Noting $K_1 \subseteq H'$, it follows that $q(G) + q(K_0) \geq q'(K_1) + q'(K_0)$, and, hence,

$$q(G) \geq q'(K_1) + q'(K_0) - q(K_0). \tag{A.9}$$

By the tightness of D, $q(G) = q(B \backslash B')$ and $q'(K) = q'(K_0) + q'(K_1)$. Further, since $K_0 \subseteq K,\ q(K) \geq q(K_0)$. Hence, from (A.9), we have $q(B \backslash B') \geq q'(K) - q(K)$, as required by WASRP. □

Proof of Corollary 3.7. Let D be a tight SDF satisfying WASRP. Consider $i \in M$, $(p, W), (p', W') \in Z$ and $b \in [0, W/p_i]$, as specified in the statement of Corollary 3.7.

By assumption, $p_i > p'_i$, $p_k = p'_k$ for all $k \in (M \backslash \{i\})$, and $W - W' = (p_i - p'_i)\, b$. Therefore,

$$\begin{aligned}
&\text{for all } x \in I,\ x_i = b;\\
&\text{for all } x \in G,\ x_i < b;\\
&\text{for all } x \in G',\ x_i > b;\\
&\text{for all } x \in H,\ x_i > b;\ \text{and,}\\
&\text{for all } x \in H',\ x_i < b.
\end{aligned} \tag{A.10}$$

By the tightness of D, $q'(G') + q'(I) + q'(H') = 1 = q(G) + q(I) + q(H)$. Hence, noting (A.10), it follows that

$$\begin{aligned}
q'(\{x \in B' | x_i > b\}) &= q'(G');\ q'(\{x \in B' | x_i = b\}) = q'(I);\\
q(\{x \in B | x_i > b\}) &= q(H);\ \textit{and}\ q(\{x \in B | x_i = b\}) = q(I).
\end{aligned} \tag{A.11}$$

Since D satisfies WASRP, it follows from Proposition 3.4 that it must satisfy SS, i.e.,

$$\text{for all } A \subseteq I, q'(G') + q'(A) \geq q(H) + q(A).$$

Taking A to be I and $\emptyset$, respectively, and noting (A.11), (3.4) and (3.5) follow immediately. □

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