Economic Theory **22**, 917–925 (2003)

Economic Theory

Optimal fiscal policy in the Uzawa-Lucas model with externalities*

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Received: March 21, 2002; revised version: September 4, 2002

Summary. This paper devises a fiscal policy by means of which the first-best optimum equilibrium is attained as a market equilibrium in the Uzawa-Lucas model when average human capital has an external effect on productivity. The optimal policy requires the use of a subsidy to investment in human capital which can be financed by a tax on labor income. Lump-sum taxation is not required to balance the government budget either in the steady state or in the transitional phase. Physical capital income should not be taxed. Alternatively, the optimal growth path can be attained by means of a subsidy to human capital.

Keywords and Phrases: Endogenous growth, Transitional dynamics, Optimal policy.

JEL Classification Numbers: O41, E62.

1 Introduction

The Uzawa (1965) and Lucas (1988) model has been the subject of active research in the past decade (e.g., Caballé and Santos, 1993; Chamley, 1993; Mulligan and Sala-i-Martín, 1993; Benhabib and Perli, 1994; Bond et al., 1996; Ladrón-de-Guevara et al., 1999; and Ortigueira, 2000). In the absence of externalities, the competitive equilibrium is optimal and government intervention is not justified. However, optimal growth paths and competitive equilibrium paths do not coincide if externalities are present. Lucas (1988) considers the case where average human capital has an external effect on the production of goods. Such externality causes the fraction of time devoted to human capital accumulation be inferior to the optimal. It could be

* Financial support from the Spanish Ministry of Science and Technology through PNICDYIT grant SEC2002-03663 is gratefully acknowledged.

argued that, because of the presence of this externality, there is no point in focusing on the optimal growth path of the Uzawa-Lucas model. However, an adequate government intervention can provide the required incentives to correct this market failure.

García-Castrillo and Sanso (2000) derive a fiscal policy that is capable to make the decentralized equilibrium with externalities be optimal in the Uzawa-Lucas model. However, the optimal policy must resort to lump-sum taxation to be feasible, at least in the transitory phase. More realistically, government spending would be financed by distortionary taxation and, thus, the government would have no access to lump-sum taxation.

The purpose of this paper is to devise a fiscal policy by means of which the first-best optimum equilibrium can be attained as a market equilibrium in this model. The government is one that taxes both physical and human capital income, and subsidizes investment in human capital. We shall assume that the government budget is balanced at any point in time, and the government cannot resort to lump-sum taxes. The optimal fiscal policy requires the use of a time-varying subsidy rate to investment in human capital. Public spending can be financed by means of a time-varying tax rate on labor income, without the necessity of resorting to lump-sum taxation to balance the government budget either in the steady state or in the transitional phase. Physical capital income should not be taxed. Alternatively, the optimal growth path can be attained by means of a subsidy to human capital, which can be fully financed by a constant tax on labor income. In this case, the optimal subsidy amounts to a constant share of output not only in the steady state but also in the transitory phase.

The remainder of this paper is organized as follows. Section 2 describes the decentralized economy, and Section 3, the centrally planned economy. Section 4 analyzes the optimal fiscal policy. Section 5 concludes.

2 The decentrally economy

Consider an economy populated by a large number of identical infinitely lived representative agents who derive utility from the consumption of a private consumption good, c. For simplicity, we assume that population is constant and normalized to one. The intertemporal utility derived by the agent is represented by the isoelastic utility function

$$W = \int_0^\infty \mathrm{e}^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt \qquad \rho > 0,\ \sigma > 0\ . \tag{1}$$

Here, ρ is the rate of time preference and σ is the inverse of the elasticity of intertemporal substitution. The endowment of time is normalized as a constant flow of one unit per period. A fraction u of time is allocated to work, and a fraction $1-u$ to learning. Human capital, h, is accumulated according to the dynamic equation

$$\dot{h} = B(1-u)h \qquad B > 0\ . \tag{2}$$

The rate of return on physical capital is denoted r, and the wage rate, w. The government taxes physical capital income at a rate τ_k and labor income at a rate

τ_h, and subsidizes investment in education at a rate s_h. In this model, the sole cost of education is foregone earnings, $w(1-u)h$, a fraction s_h of which is therefore financed by the government. In absence of depreciation of physical capital, the household's budget constraint is, then,

$$\dot{k} = (1-\tau_k)rk + (1-\tau_h)wuh - c + s_h w(1-u)h \ . \tag{3}$$

The representative agent maximizes (1) subject to the constraints (2) and (3). To simplify the subsequent exposition we shall slightly change the budget constraint (3), and express it equivalently as

$$\dot{k} = (1-\tau_k)rk + (1-\hat{\tau}_h)wuh - c + s_h wh \ , \tag{4}$$

where

$$\hat{\tau}_h = \tau_h + s_h \ . \tag{5}$$

Output, y, is a function of the stocks of physical and human capital, the time individuals supply as labor, u, and the average human capital of the economy, h_a:

$$y = Ak^{\beta}(uh)^{1-\beta}h_a^{\psi} \quad A > 0,\ 0 < \beta < 1,\ \psi > 0 \ .$$

In the market solution, the atomistic agents treat h_a as given. By symmetry, the value of h_a is equal to h in equilibrium. Because of the externality, the competitive solution differs from the planner's solution.

Profit maximization implies that labor and capital are used up to the point at which marginal product equates marginal cost: $r = \beta y/k$, and $w = (1-\beta)y/(uh)$. We shall assume that the government runs a balanced-budget and has no access to lump-sum taxation, $\tau_k rk + \tau_h wuh = s_h w(1-u)h$, which can be equivalently expressed, using (5), as

$$\tau_k rk + \hat{\tau}_h wuh = s_h wh \ . \tag{6}$$

A consumption tax is not included since it acts as a lump-sum tax in this framework.

Let J be the current value Hamiltonian of the household's utility maximization problem, and let λ and μ be the multipliers for the constraints (4) and (2), respectively:

$$J = (c^{1-\sigma}-1)/(1-\sigma) + \lambda[(1-\tau_k)rk + (1-\hat{\tau}_h)wuh - c + s_h wh] + \mu[B(1-u)h] \ .$$

The first order necessary conditions for an interior solution are

$$c^{-\sigma} = \lambda \ , \tag{7a}$$

$$\lambda(1-\hat{\tau}_h)wh = \mu Bh \ , \tag{7b}$$

$$\dot{\lambda} = (\rho - (1-\tau_k)r)\lambda \ , \tag{7c}$$

$$\dot{\mu} = (\rho - B(1-u))\mu - \lambda((1-\hat{\tau}_h)wu + s_h w) \ , \tag{7d}$$

plus the usual transversality conditions. Hereafter, let $\gamma_z = \dot{z}/z$ denote the growth rate of the variable z. In what follows, the equilibrium condition $h = h_a$ will be imposed, and the expressions for r and w will be taken into account. From (7a) and (7c) we obtain

$$\gamma_c = (1/\sigma)((1-\tau_k)\beta y/k - \rho)\ . \tag{8}$$

Using the government's budget constraint (6), Eq. (4) can be expressed as:

$$\gamma_k = y/k - c/k\ . \tag{9}$$

Substituting the expression for λ from (7b) into (7d) yields $\gamma_\mu = \rho - B(1-\hat{\tau}_h + s_h)/(1-\hat{\tau}_h)$. Log-differentiating (7b) with respect to time, and taking into account the previous expression for γ_μ, Eq. (7c), and the growth rates of h and y, we can obtain the growth rate of u as

$$\begin{aligned}\gamma_u = {} & \tau_k y/k - c/k + B(1-\beta-\psi)/\beta + B(\beta-\psi)u/\beta \\ & + Bs_h/((1-\hat{\tau}_h)\beta) - \dot{\hat{\tau}}_h/((1-\hat{\tau}_h)\beta)\ .\end{aligned} \tag{10}$$

For a given policy path, the system (2), (8), (9) and (10) characterizes the dynamics of the decentralized economy. This system can be reformulated in terms of variables that are constant in the steady state, defining $x = kh^{(1-\beta+\psi)/(\beta-1)}$ and $q = c/k$. Then, we obtain

$$\gamma_q = ((1-\tau_k)\beta - \sigma)Ax^{\beta-1}u^{1-\beta}/\sigma + q - \rho/\sigma\ , \tag{11a}$$

$$\gamma_x = Ax^{\beta-1}u^{1-\beta} - (1-\beta+\psi)B(1-u)/(1-\beta) - q\ , \tag{11b}$$

$$\begin{aligned}\gamma_u = {} & \tau_k Ax^{\beta-1} - q + B(1-\beta+\psi)/\beta + B(\beta-\psi)u/\beta \\ & + Bs_h/((1-\hat{\tau}_h)\beta) - \dot{\hat{\tau}}_h/((1-\hat{\tau}_h)\beta)\ .\end{aligned} \tag{11c}$$

If $\tau_k = \hat{\tau}_h = s_h = 0$, we obtain the system derived by Benhabib and Perli (1994) that describes the dynamics of the market economy in absence of government intervention.

3 The centrally planned economy

The central planner possesses complete information and chooses all quantities directly, taking all the relevant information into account. She maximizes (1) subject to (2) and

$$\dot{k} = Ak^\beta u^{1-\beta} h^{1-\beta+\psi} - c\ . \tag{12}$$

Let J be the current value Hamiltonian of the planner's maximization problem, and let λ and μ be the multipliers for the constraints (12) and (2), respectively:

$$J = (c^{1-\sigma}-1)/(1-\sigma) + \lambda[Ak^\beta u^{1-\beta}h^{1-\beta+\psi} - c] + \mu[B(1-u)hJ\ .$$

The first order necessary conditions for an interior solution are

$$c^{-\sigma} = \lambda\ , \tag{13a}$$

$$\lambda(1-\beta)Ak^{\beta}u^{-\beta}h^{1-\beta+\psi} = \mu Bh \,, \tag{13b}$$

$$\dot{\lambda} = (\rho - \beta Ak^{\beta-1}u^{1-\beta}h^{1-\beta+\psi})\lambda \,, \tag{13c}$$

$$\dot{\mu} = (\rho - B(1-u))\mu - \lambda((1-\beta+\psi)Ak^{\beta}u^{1-\beta}h^{-\beta+\psi}) \,, \tag{13d}$$

plus the usual transversality conditions. There are two main qualitative differences between the decentralized and the centrally planned economies. First, the tax rate on physical capital income influences the return to physical capital in the market economy but not the implicit interest rate used by the planner. Second, the productivity elasticity determining the planner's accumulation of human capital is the social productivity of human capital, $1-\beta+\psi$, rather than the private productivity, $1-\beta$, relevant for the representative agent in the market economy.

In the same manner as in the case of the market economy, we can obtain the following system of equations which characterizes the dynamics of the centrally planned economy:

$$\gamma_q = (\beta-\sigma)Ax^{\beta-1}u^{1-\beta}/\sigma + q - \rho/\sigma \,, \tag{14a}$$

$$\gamma_x = Ax^{\beta-1}u^{1-\beta} - (1-\beta+\psi)B(1-u)/(1-\beta) - q \,, \tag{14b}$$

$$\gamma_u = B(1-\beta+\psi)/\beta + B(1-\beta+\psi)u/(1-\beta) - q \,. \tag{14c}$$

Equating the growth rates to zero yields the steady state values:

$$u^* = \frac{\rho(1-\beta) + B(1-\beta+\psi)(\sigma-1)}{B(1-\beta+\psi)\sigma} \,,$$

$$q^* = B(1-\beta+\psi)/\beta + B(1-\beta+\psi)u^*/(1-\beta) \,,$$

$$x^* = u^*\left(\frac{A\beta(1-\beta)}{B(1-\beta+\psi)}\right)^{1/(1-\beta)} \,.$$

The condition $0 < u^* < 1$ holds if and only if $B(1-\beta+\psi) > \rho(1-\beta) > B(1-\beta+\psi)(1-\sigma)$. Notice that $q^* > 0$ and $x^* > 0$ if $0 < u^* < 1$. The transversality conditions can be easily shown to be satisfied if $0 < u^* < 1$.

If we denote $r = \beta y/k$, the system (14) can be expressed equivalently as

$$\gamma_r = -(1-\beta)r/\beta + B(1-\beta+\psi)/\beta \,, \tag{15a}$$

$$\gamma_q = (\beta-\sigma)r/(\beta\sigma) + q - \rho/\sigma \,, \tag{15b}$$

$$\gamma_u = B(1-\beta+\psi)/\beta + B(1-\beta+\psi)u/(1-\beta) - q \,. \tag{15c}$$

This system is accessible to a phase diagram analysis similar to that performed by Barro and Sala-i-Martín (1995, Sect. 5.2.2) and Arnold (2000) in the model without externalities. The top left panel of Figure 1 is a phase diagram in the (r, q)-space when $\sigma > \beta$. From Eq. (15a) the $\gamma_r = 0$-locus is vertical and stable. From

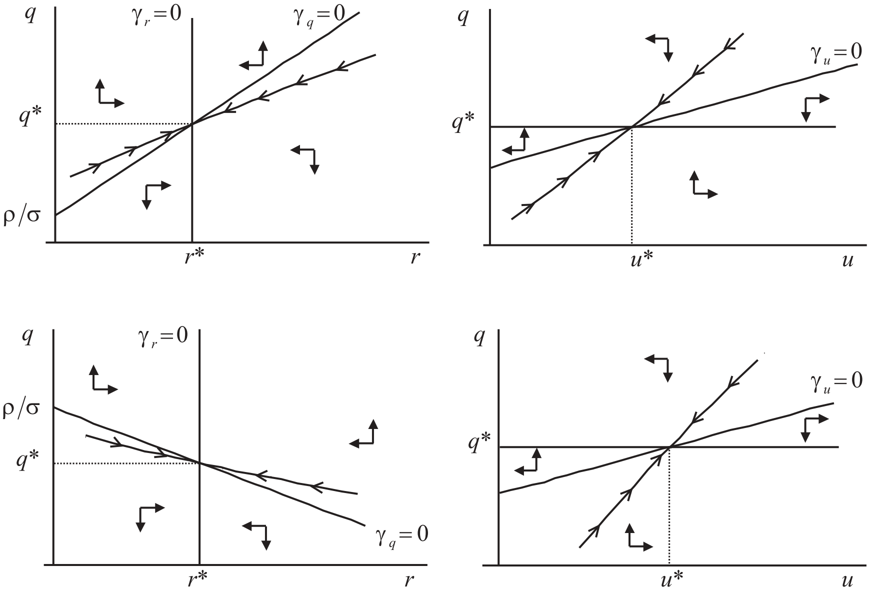

Figure 1. Phase diagram

Eq. (15b), the $\gamma_q = 0$-locus is increasing and unstable. Then, there is a unique and saddle point steady state (r^*, q^*). The top right panel of Figure 1 depicts a phase diagram in the (u, q)-space. Given that the economy is on its saddle path in the (r, q)-space, q converges monotonically. Thus, the $\gamma_q = 0$-locus is horizontal and stable in the (u, q)-space. The $\gamma_u = 0$-locus is increasing and unstable. Then, there exists a unique and saddle-point steady state (u^*, q^*). The configuration of the two loci in the top panels of Figure 1 implies that the stable saddle-paths $q(r)$ and $u(q)$ are increasing.

The bottom left and right panels of Figure 1 are phase diagrams in the (r, q)-space and the (u, q)-space, respectively, when $\sigma < \beta$. A symmetrical analysis to the one performed above leads to the conclusion that there exists a unique and saddle-point steady state, and the stable saddle-paths $q(r)$ and $u(q)$ are now decreasing and increasing, respectively. If $\sigma = \beta$, then $q = q^*$ and $u = u^*$, and the variables q and u remain fixed at their steady-state values along the transitional phase. Hence, we can state the following proposition.

Proposition 1 *The steady state of the optimal-growth problem in the Uzawa-Lucas model when average human capital has an external* effect *on productivity is a saddle-point.*

The local stability analysis confirms the saddle-point property of the steady state. Linearizing the system (15) around the steady state (r^*, q^*, u^*) yields:

$$\begin{pmatrix} \dot{r} \\ \dot{q} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} -(1-\beta)r^*/\beta & 0 & 0 \\ (\beta-\sigma)q^*/(\beta\sigma) & q^* & 0 \\ 0 & -u^* & B(1-\beta+\psi)u^*/(1-\beta) \end{pmatrix} \begin{pmatrix} r - r^* \\ q - q^* \\ u - u^* \end{pmatrix} .$$

As the coefficient matrix is triangular, the characteristic roots are its diagonal elements. Two roots are positive and one is negative, which proves that the steady state is a saddle-point.

4 The optimal fiscal policy

The key question to be addressed in this section is what fiscal policy is capable to make the decentralized economy replicate the first-best optimum attainable by a central planner and described by system (14). First, note that equations (11b) and (14b), which describe the dynamics of x in the decentralized and centrally planned economies, respectively, coincide. Comparing Eq. (11a) with Eq. (14a), we see that the decentralized economy will fully replicate the dynamic time path of q in the centrally planned economy only if the tax rate on physical capital income is zero, $\tau_k = 0$. Equating the right hand sides of Eqs. (11c) and (14c), after substituting τ_k for 0, yields the following relationship:

$$s_h = \psi(1-\hat{\tau}_h)u/(1-\beta) + \dot{\hat{\tau}}_h/B \; . \tag{16}$$

García-Castrillo and Sanso (2000) show that the decentralized economy cannot attain the optimal growth path by using simply a positive or a negative tax rate on human capital income. Suppose that the only policy instrument is the labor income tax, $\hat{\tau}_h$, that is, $s_h = 0$. If $\hat{\tau}_h$ were constant, Eq. (11c) would simplify to $\gamma_u = B(1-\beta+\psi)/\beta + B(\beta-\psi)u/\beta - q$ which, in the presence of externalities $(\psi > 0)$, is different to its counterpart in the centrally planned economy (14c). Therefore, $\dot{\hat{\tau}}_h$ must be different from zero to reach the optimal path. Now, Eq. (16) reduces to $\dot{\hat{\tau}}_h = -B\psi(1-\hat{\tau}_h)u/(1-\beta)$. This dynamic behavior is not feasible since, when the economy has reached the steady-state value $u^* \in (0,1)$, the solution to this differential equation implies that the government share in output will be growing indefinitely.

As $\dot{\hat{\tau}}_h$ must be null in the steady state, we guess an optimal tax on labor income that is constant both in the steady state and in the transitional phase. Hence, Eq. (16) reduces to

$$s_h = \psi(1-\hat{\tau}_h)u/(1-\beta) \; . \tag{17}$$

Simultaneously solving the system (6) and (17), and substituting the optimal tax rate on physical capital income τ_k for 0, we obtain the optimal tax rate on human capital income, $\hat{\tau}_h$:

$$\hat{\tau}_h = \psi/(1-\beta+\psi) \; , \tag{18a}$$

which is effectively constant as guessed, and $0 < \hat{\tau}_h < 1$. The optimal subsidy rate, s_h, is then

$$s_h = \psi u/(1-\beta+\psi) \; , \tag{18b}$$

which satisfies that $0 \leq s_h < 1$. The optimal subsidy rate is not constant in time, but converges to $s_h^* = \psi u^*/(1-\beta+\psi)$. The government size, measured as the subsidy (or taxes) share of output, ϕ, is constant at any time, since $\phi = s_h wh/y = \psi(1-\beta)/(1-\beta+\psi)$. The condition that the government size must be less than one is therefore satisfied at any point in time.

If we turn to the initial specification and consider a subsidy to investment in human capital, Eqs. (5) and (18) yield the time-varying tax rate on labor income as

$$\tau_h = \hat{\tau}_h - s_h = \psi(1-u)/(1-\beta+\psi) \ ,$$

which satisfies that $0 \leq \tau_h < 1$. Thus, the optimal tax rate on labor income is not constant but converges to $\tau_h^* = \psi(1-u^*)/(1-\beta+\psi)$. The government size is not constant but the condition that must be less than one, $\phi = s_h w(1-u)h/y = \psi(1-\beta)(1-u)/(1-\beta+\psi) < 1$, is satisfied. The following proposition summarizes the former findings.

Proposition 2 *The decentralized economy can attain the first-best equilibrium solution if physical capital income is not taxed and investment in human capital is subsidized at a rate $s_h = \psi u/(1-\beta+\psi)$. The subsidy can be financed by taxing human capital income at a rate $\tau_h = \psi(1-u)/(1-\beta+\psi)$. Lump-sum taxation is not required to balance the government budget either in the steady state or in the transitory phase.*

Alternatively, let us suppose that the government subsidizes human capital, instead of investment in human capital, at a rate $\hat{s}_h$. In this case, the household's budget constraint is

$$\dot{k} = (1-\tau_k)rk + (1-\hat{\tau}_h)wuh - c + \hat{s}_h h \ . \tag{19}$$

Notice that the household's budget constraint (19) coincides with that of the case of the subsidy to investment in human capital (4) if we make $\hat{s}_h = s_h w$. Handling the necessary conditions in a way analogous to that in Section 2, it can be readily shown that the relationship $\hat{s}_h = s_h w$ carries over all the calculations. The dynamics of the decentralized economy in this case is also summarized by the system (11), after being s_h substituted by $\hat{s}_h/w$. The optimal tax rate on physical capital income is then zero, the optimal subsidy is obtained by substituting (18b) into $\hat{s}_h = s_h w$ as $\hat{s}_h = \psi(1-\beta)y/((1-\beta+\psi)h)$, and the optimal tax rate on human capital income is also given by (18a). The size of the government is constant and the condition that must be less than one at any point in time is fullfilled since $\phi = \hat{s}_h h/y = \psi(1-\beta)/(1-\beta+\psi) < 1$. Thus, we can state the following proposition.

Proposition 3 *The decentralized economy can attain the first-best equilibrium solution if physical capital income is not taxed and human capital is subsidized at a rate $\hat{s}_h = \psi(1-\beta)y/((1-\beta+\psi)h)$. The subsidy can be financed by taxing labor income at a rate $\hat{\tau}_h = \psi/(1-\beta+\psi)$. Lump-sum taxation is not required to balance the government budget either in the steady state or in the transitory phase. The size of the government, measured as the subsidy (or taxes) share of output, is constant and equal to $\phi = \psi(1-\beta)/(1-\beta+\psi)$.*

5 Conclusions

In this paper we devised a fiscal policy capable to make the decentralized economy achieve the first-best equilibrium in the Uzawa-Lucas model with externalities. The optimal policy requires making use of a subsidy to investment in human capital, which can be financed by a tax on labor income. Alternatively, the optimal growth path can be attained by means of a subsidy to human capital. In this case, the subsidy can be financed by a constant rate tax on human capital income, and government size is constant at any time. In any case, the return on physical capital must be free of taxes, and resorting to lump-sum taxation is not needed to balance the government budget either in the steady state or in the transitional phase.

References

Arnold, L. G.: Endogenous technological change: a note on stability. Economic Theory **16**, 219–226 (2000)

Barro, R. J., Sala-i-Martín, X.: Economic growth. New York: McGraw-Hill 1995

Benhabib, J., Perli, R.: Uniqueness and indeterminacy: on the dynamics of endogenous growth. Journal of Economic Theory **63**, 113–142 (1994)

Bond, R. W., Wang, P., Yip, C. K.: A general two-sector model of endogenous growth with human and physical capital: balanced growth and transitional dynamics. Journal of Economic Theory **68**, 149–173 (1996)

Caballé, J., Santos, M.: On endogenous growth with physical and human capital. Journal of Political Economy **101**, 1042–1067 (1993)

Chamley, C. P.: Externalities and dynamics in models of "learning or doing". International Economic Review **34**, 583–610 (1993)

García-Castrillo, P., Sanso, M.: Human capital and optimal policy in a Lucas-type model. Review of Economic Dynamics **3**, 757–770 (2000)

Ladrón-de-Guevara, A., Ortigueira, S., Santos, M. S.: A two-sector model of endogenous growth with leisure. Review of Economic Studies **66**, 609–631 (1999)

Lucas, R. E., Jr.: On the mechanics of economic development. Journal of Monetary Economics **22**, 3–42 (1988)

Mulligan, C. B., Sala-i-Martín, X.: Transitional dynamics in two-sector models of endogenous growth. Quarterly Journal of Economics **108**, 739–773 (1993)

Ortigueira, S.: A dynamic analysis of an endogenous growth model with leisure. Economic Theory **16**, 46–62 (2000)

Uzawa, H.: Optimum technical change in an aggregative model of economic growth. International Economic Review **6**, 18–31 (1965)