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Erratum

Gradients at a curved shock in reacting flow

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One of us (MJK) found some errors in Hornung (1998). A relatively minor one is that the term

$$(1 - \rho h_p)L$$

in Eq. (21) should be multiplied by v. More substantial errors result from several mistakes in the section on three–dimensional flows. None of the conclusions are affected, but Figs. 16 and 17 are substantially modified in the small–β range. Figure 18 is also changed somewhat. The simplest way to present the corrections is to re–present the whole of Sect. 8, which is done in the following:

8 Three–dimensional flows

Finally, consider the extension of these results to the more general case of three–dimensional flow. To this end, choose the $xy-$plane to be the plane of the free–stream direction and the local normal to the shock wave at the point of interest. With this choice, the velocity component in the third (z) direction and its gradients in the xy–plane are zero. Thus a suitable name for this plane is the "flow plane". By choosing x and y to lie in the flow plane, the derivatives of p, ρ and u with respect to z (the dimensionless coordinate normal to the flow plane) are zero, and the only non–zero gradient normal to the flow plane is

$$w_z = l \sin\beta,$$

where w is the dimensionless $z-$component of velocity and l is the shock curvature in the yz–plane. $k+l$ is the Gaussian curvature of the shock at the point considered.

If we write Eqs. (3) to (6) for $y = 0$, the only changes to these equations are that the term $-k\rho v$ in the continuity equation becomes $-(k + l)\rho v$, and a new term $\rho l \sin\beta$ is added. The equation (at $y = 0$) becomes

$$(\rho u)_x - (k + l)\rho v + \rho l \sin\beta + (\rho v)_y = 0. \quad (63)$$

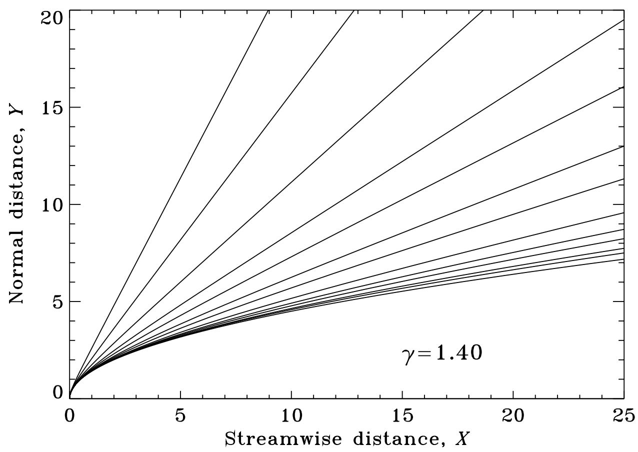

Fig. 15. Hyperbolic shock shapes, with finite curvature at the normal–shock point and asymptoting to a Mach wave at large X. Mach numbers are: 1.1, 1.2, 1.4, 1.7, 2, 2.5, 3, 4, 5, 6, 8, 10, 20

This causes additional terms proportional to l to appear in Eqs. (18), (20) and (21) of Hornung (1998), for the y–derivatives of p, v, and ρ as follows:

$$p_y F = \ldots + l\rho^2 h_\rho (1 - \sin\beta/v), \quad (64)$$

$$v v_y F = \ldots + l\rho h_\rho (\sin\beta/v - 1), \quad (65)$$

$$\rho_y F = \ldots + l\rho(1 - \rho h_p)(1 - \sin\beta/v). \quad (66)$$

Equation (19) for u_y remains unchanged.

A relatively simple example is that of axisymmetric flow. In this case, the flow plane is the meridional plane. Consider an axisymmetric shock wave of hyperbolic shape in the flow plane, such that the normal–shock point has finite curvature equal in both directions, and the shock is asymptotically conical with half–angle equal to the Mach angle far from this point. Defining the distance along the axis of symmetry, normalized by the radius of the shock at the nose, to be X, and the normal to it (similarly normalized) as Y, the shock shapes for a set of Mach number

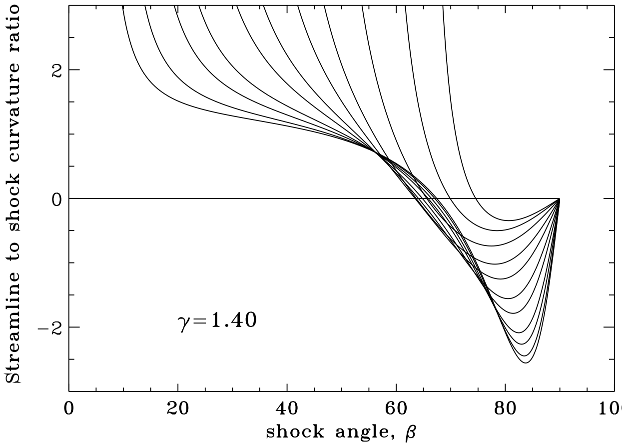

Fig. 16. Streamline–to–shock curvature ratio for plane curved shocks. Perfect gas, $\gamma = 1.4$, Mach numbers as in Fig. 15

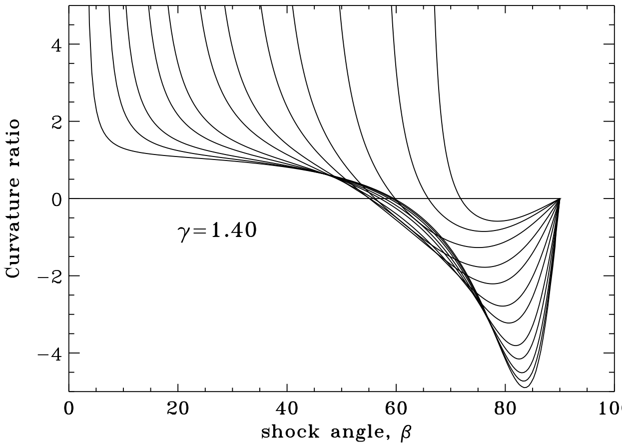

Fig. 17. Streamline–to–shock curvature ratio for axisymmetric shocks as shown in Fig. 15. Perfect gas, $\gamma = 1.4$, Mach numbers as in Fig. 15. The ratio is the streamline curvature divided by the shock curvature k in the xy–plane

values are as shown in Fig. 15. The equation of the shock shape is

$$Y = \tan\mu \sqrt{X\left(X + \frac{2}{\tan^2\mu}\right)} \quad (67)$$

where μ is the Mach angle $\arcsin(1/M)$. This gives a shock angle β that can be determined from

$$\tan\beta = \tan\mu \frac{1 + 1/(X\tan^2\mu)}{\sqrt{1 + 2/(X\tan^2\mu)}}. \quad (68)$$

Solving this for X,

$$X\tan^2\mu = \sqrt{\frac{\tan^2\beta}{\tan^2\beta - \tan^2\mu}} - 1. \quad (69)$$

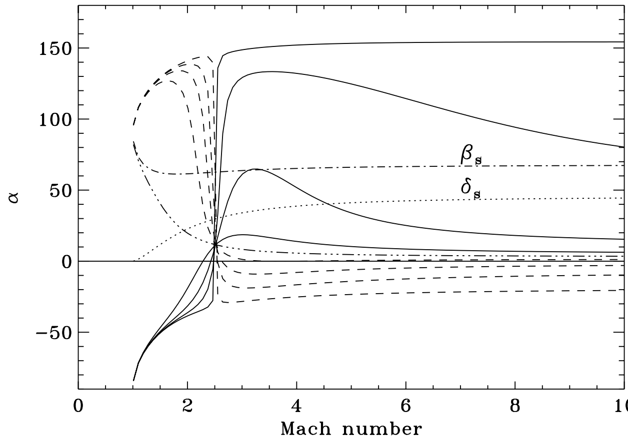

Fig. 18. Sonic line angle in axisymmetric flow. Shock shape as in Fig. 15, notation as in Fig. 13 of Hornung (1998). $\gamma = 1.4$, $\theta = 0.8$, $k\varepsilon = -0.0001, -0.001, -0.003, -0.01, 0.01, 0.003, 0.001, 0.0001$

With this, the shock curvature in the meridional plane becomes

$$k = (1 + 2X/\cos^2\mu + X^2\tan^2\mu/\cos^2\mu)^{-3/2}. \quad (70)$$

This gives an explicit relation between k and β with Mach number as a parameter. For an axisymmetric shock, the transverse curvature in the yz–plane is $l = \cos\beta/Y$.

The shape of the shock now permits the streamline to shock curvature ratio to be determined for the axisymmetric case as a function of the shock angle β. For plane flow the results are, of course, the same as those given in Fig. 2 of Hornung (1998), reproduced here as Fig. 16. For axisymmetric flow, the results are shown in Fig. 17. As may be seen, the transverse curvature causes no qualitative changes. Quantitative changes include slight changes in the Crocco points and a greater negative value of the curvature ratio for axisymmetric flow.

It is interesting to find the effect of the third dimension on the sonic line slope at the shock. This is not new in non–reacting flow (see Hayes and Probstein), but our results permit it to be obtained directly for reacting flow also. Figure 18 shows how α behaves in an axisymmetric flow with the same shock shapes as in Fig. 15. The effect of reactions is very similar to that for plane flow.

References

Hornung HG (1998) Gradients at a curved shock in reacting flow. Shock Waves 8: 11–21