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Shock Waves

Studies on bubble dynamics

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Abstract. In order to clarify the behavior of bubbles which is closely related to cavitation phenomena, the research of the dynamics of bubbles has been intensively conducted and established the research field of bubble dynamics. In this review paper it is intended to describe briefly studies on bubble dynamics including the history in conjunction with the shock wave dynamics.

Key words: Bubble, Cavitation, Impulsive pressure, Liquid micro jet, Shock wave, Damage pit

1 Introduction

In 1894 in England, the ship could not reach its design speed at the trial runs of a high speed marine propeller. A test was made to examine why it happened and cavitation phenomena were eventually discovered. Since then, studies of cavitation phenomena have been increasingly developed for the reason that cavitation phenomena are one of the important factors which prevent an improvement in the performance of hydraulic machinery where the working fluid is a liquid.

However, today in order to fundamentally understand the cavitation phenomena and the related topics, it began to recognized that the dynamics of bubbles should be investigated. The author devoted more than forty years for cavitation research and bubble dymanics and this review article briefly describes some studies of bubble dynamics and their relation to shock wave dynamics.

2 Cavitation and bubble nuclei

In flows of hydraulic machinery of water turbines, pumps, screws, various channels etc, when the relative velocity between the liquid and the solid surface of the hydrofoil or the channel wall becomes larger so that local static pressure in the flow decreases below a limiting pressure, a phenomenon called cavitation occurs and the limiting pressure is called a cavitation inception pressure.

Usually if minute bubbles known as bubble nuclei do not exist in water under conditions that do not solve in water, water can resist for very large negative pressure, and the cavitation never occurs easily.

However, usually water contains a total air content of several percent, and hence the conditions under which bubble nuclei grow to visible bubbles are readily observable by high speed photography (Knapp and Hollander 1948). This is the so-called cavitation.

Also, in the case that a bubble nucleus with radius R_o in a liquid grows following an isothermal change, the condition for a bubble to exist and to be stable is obtained by the formula derived by assuming a relation of static equilibrium (Daily and Johnson 1956).

$$\frac{4\sigma}{3(p_v - p)} \leq R_0 \leq \frac{2\sigma}{p_v - p}, \tag{1}$$

where, σ is the surface tension of the liquid, p_v the saturated vapor pressure of the liquid and p is the liquid pressure. When the value of R_0 in (1) exceeds or decreases the upper or lower limit of (1) a bubble nucleus begins to expand or shrink infinitely, respectively. Thus the bubbles which occur show complicated behavior depending upon the various hydrodynamic conditions in the surroundings. Since these conditions are in many cases related to the cavitation noise, cavitation damage etc, the research of bubble dynamics advanced for clarifying the mechanism of cavitation phenomena.

3 Behavior of bubbles in an infinite liquid

Besant (1859) presented, in the motion of a spherical bubble inside which is in vacuum in an infinite inviscid and incompressible liquid, a problem of predicting the pressure at any point in the liquid and its collapse time.

Rayleigh (1917) solved this problem theoretically and derived the analytical formula describing the bubble motion. His formulation of the motion of a single spherical bubble in an infinite, inviscid and imcompressible liquid is shown in Fig.1. The bubble surface velocity V was obtained by assuming that the work done by the liquid, when a bubble with an initial radius R_o collapses to R, equals the whole kinetic energy of the bubble motion.

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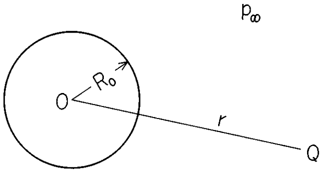


Fig. 1. Spherical bubble

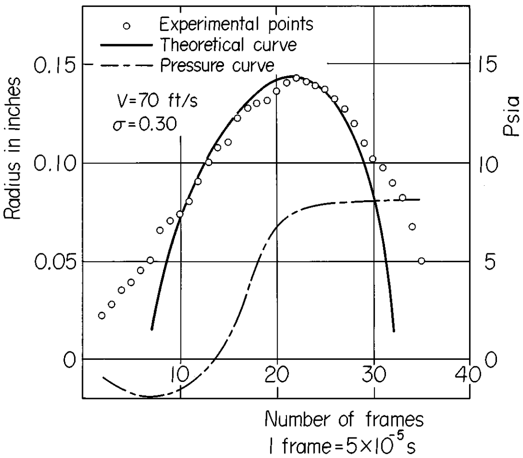


Fig. 2. Variation with time of the bubble radius (Plesset 1949)

$$V = \sqrt{\frac{2}{3}\frac{p_\infty}{\rho_l}\left[\left(\frac{R_0}{R}\right)^3 - 1\right]}, \tag{2}$$

where, p_∞ is the pressure at infinity in the liquid, ρ_l the liquid density. Also, by integrating from $R = R_0$ to $R = 0$ in (2) the complete collapse time τ is approximated as follows:

$$\tau = 0.915 R_0 \sqrt{\frac{\rho_l}{p_\infty}}, \tag{3}$$

Further, Plesset (1949) derived the equation for the motion of a bubble from the continuity equation and the equation of motion.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_l}\left[p(R) - p(t)\right], \tag{4}$$

where, $p(R)$ is the pressure at the bubble surface, $p(t)$ is the pressure in the liquid, t is the time, $\dot{R} = dR/dt$ and $\ddot{R} = d^2R/dt^2$. He calculated the $R - t$ curve by putting $p(R) = p_v - 2\sigma/R$ to the bubble which appears on the body of the revolution.

Comparing this analytical result with that obtained from high speed photographs, he ascertained that both of them agreed except the initial stage of the bubble growth and the last stage of the bubble collapse as seen in Fig. 2.

Poritsky (1952) was first to discuss the effect of liquid viscosity on bubble behavior. The equation of motion of a bubble in a viscous liquid is given by the following equation:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_l}\left[p_v - p_\infty - \frac{2\sigma}{R} - 4\mu\frac{\dot{R}}{R}\right], \tag{5}$$

where μ is the liquid viscosity.

The bubble behavior was characterized in terms of the dimensionless viscosity μ defined by:

$$\mu' = \frac{4\mu}{R_0\sqrt{\rho_l(p_\infty - p_v)}}. \tag{6}$$

In the case of sufficiently large μ', the motion of a bubble becomes smooth. In the case of the absence of the surface tension, the collapse time becomes infinity, that is, the bubble never collapses (Poritsky 1952). Also Shima et al (1973) found by solving (5) numerically that the effect of viscosity μ delays the bubble collapse. On the contrary, the surface tension σ shortens the bubble collapse time τ.

4 Effect of the liquid compressibility on bubble behavior

Gilmore (1952) theoretically clarified the effect of the compressibility of the liquid on the bubble motion. The following equation of motion of a spherical bubble was derived by introducing the hypothsis proposed by Kirkwood-Bethe (1942) that the quantity $r(h + u^2/2)$ propagates along the characteristic $c + u$, where c is the local sound speed of the liquid, u is the particle velocity and h the enthalpy of the liquid.

$$RV\frac{dV}{dR}(1 - \frac{V}{C}) + \frac{3}{2}V^2(1 - \frac{V}{3C})$$

$$= H(1 + \frac{V}{C}) + \frac{RV}{C}\frac{dH}{dR}(1 - \frac{V}{C}), \tag{7}$$

where $V = R$ the sound speed of the liquid at the bubble wall, $C = c_\infty(p_w + B/p_\infty + B)^{\frac{n-1}{2n}}$, c_∞ being the sound speed of the liquid at infinity.

Enthalpy at the bubble wall is defined by:

$$H = \frac{n(p_\infty + B)}{(n-1)\rho_l}\left[\left(\frac{p_w + B}{p_\infty + B}\right)^{\frac{n-1}{n}} - 1\right],$$

where $B \approx 300MPa$, and $n \approx 7.0$.

Also, the pressure in the liquid at the bubble wall p_w is given as follows.

$$p_w = p_i - \frac{2\sigma}{R} - 4\mu\frac{V}{R}, \tag{8}$$

where, p_i is the pressure exerted on the bubble wall by any interior gas or vapor.

In the case of $\mid H \mid \ll c^2$, by putting $\sigma = 0$ and $\mu = 0$ in (7) the following equation which is similar to an incompressible solution can be obtained as:

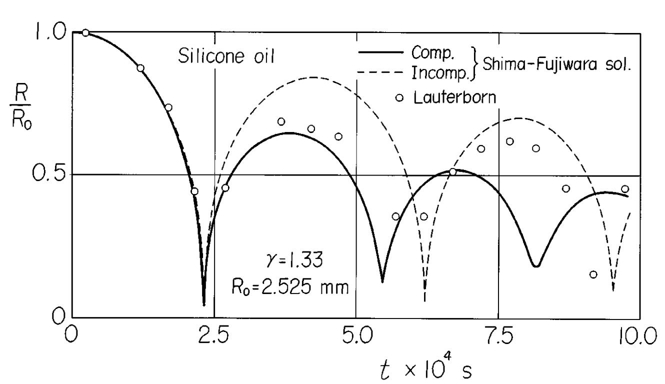


Fig. 3. Variation with time of the bubble radius: Comparison between Shima-Fujiwara solution and the experimental values obtained by Lauterborn (Shima and Fujiwara 1980)

$$
V=\sqrt{\frac{2}{3}\frac{(p_{\infty}-p_i)}{\rho_l}\left[\frac{(R_0/R)^3}{(1-V/3C)^4}-1\right]}. \tag{9}
$$

As $R \rightarrow 0$ it is found that $V \infty R^{-3/2}$ for an incompressible liquid, and $V \infty R^{-1/2}$ for a compressible liquid. In the latter case the collapse velocity is retarded significantly.

Tomita and Shima (1977) and Shima and Tomita (1979) derived the following equation for the motion of bubbles taking into consideration the second order term of liquid compressibility by applying the PLK method (Tsien 1956) to a nonlinear wave equation obtained by the derivation of a velocity potential.

$$
\begin{aligned}
&R\ddot{R}\left(1-\frac{2\dot{R}}{C_{\infty}}+\frac{23}{10}\cdot\frac{\dot{R}^2}{C_{\infty}^2}\right)+\frac{3}{2}\dot{R}^2\left(1-\frac{4}{3}\cdot\frac{\dot{R}}{C_{\infty}}+\frac{7}{5}\cdot\frac{\dot{R}^2}{C_{\infty}^2}\right)\\
&+\frac{1}{\rho_l}\left[p_{\infty}-p_{2.r=R}-\frac{\dot{R}p'_{1.r=R}}{C_{\infty}}+\frac{1}{C_{\infty}^2}\left\{2R\dot{R}\dot{p}_{1.r=R}\right.\right.\\
&\left.\left.+\frac{p_{\infty}-p_{1.r=R}}{2}\left[\dot{R}^2+\frac{3(p_{\infty}-p_{1.r=R})}{\rho_l}\right]\right\}\right]=0,
\end{aligned} \tag{10}
$$

where,

$$
p_{1.r=R}=p_o\left(\frac{R_0}{R}\right)-\frac{2\sigma}{R}-4\mu\frac{\dot{R}}{R}
$$

$$
\begin{aligned}
\dot{p}_{1.r=R}=&-3\gamma p_o\frac{\dot{R}}{R}\left(\frac{R_0}{R}\right)^{3\gamma}+\frac{2\sigma}{R^2}\dot{R}\\
&+\frac{4\mu}{R^2}\left(\frac{5}{2}\dot{R}^2+\frac{p_{\infty}-p_{1.r=R}}{\rho_l}\right)
\end{aligned}
$$

$$
p_{2.r=R}=p_{1.r=R}-\left(\frac{4\mu}{3}+\xi\right)\frac{\dot{p}_{1.r=R}}{\rho_l C_{\infty}^2}
$$

where, p_o is the initial gas pressure inside the bubble, γ a specific heats ratio of the gas inside the bubble, ξ the bulk viscosity of the liquid, suffix ∞ the value at infinity.

It is shown in (10) that the bubble motion is significantly damped due to the liquid compressibility (Tomita and Shima 1977; Shima and Tomita 1979; Shima and Fujiwara 1980).

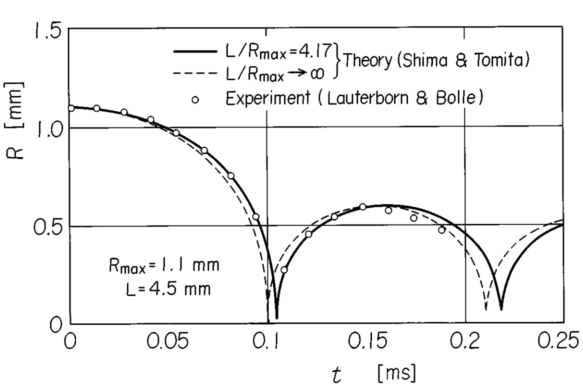


Fig. 4. Variation with time of the bubble radius: Comparison between Shima-Tomita theory and the experimental values obtained by Lauterborn and Bolle (Shima and Tomita 1981)

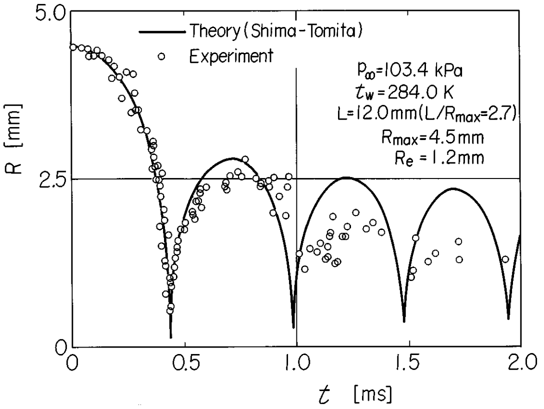


Fig. 5. Variation with time of the bubble radius: Comparison between Shima-Tomita theory and their experimental values (Shima and Tomita 1981)

Figure 3 shows the comparison the time variation of the bubble radius for silicone oil between the result obtained by Shima and Fujiwara (1980) and the experimental results obtained by Lauterborn (1974). It is found from Fig. 3 that the solution taking into account of the liquid compressibility coincides well with the experimental result up to the second rebound.

5 Behavior of bubbles near a solid wall

By paying attention to the fact that actual cavitation occurs in the vicinity of hydrofoils in water turbines, pumps, pump turbines and screws, many investigators theoretically analysed, in conjunction with the cavitation damage, the collapse of bubbles near or attached to a solid wall. The bubble behavior and the induced impulsive pressure have been clarified numerically (Rattray 1951; Shima 1968; 1971; Chapman and Plesset 1972; Lauterborn and Bolle 1975; Nakajima and Shima 1977; Shima and Nakajima 1977; Shima and Sato 1979; Sato and Shima 1980; Shima and Sato 1980; Shima et al. 1981; Shima and Sato 1981; Shima and Sato 1984; Dezhunov et al. 1980; Kuvshinov et al. 1982).

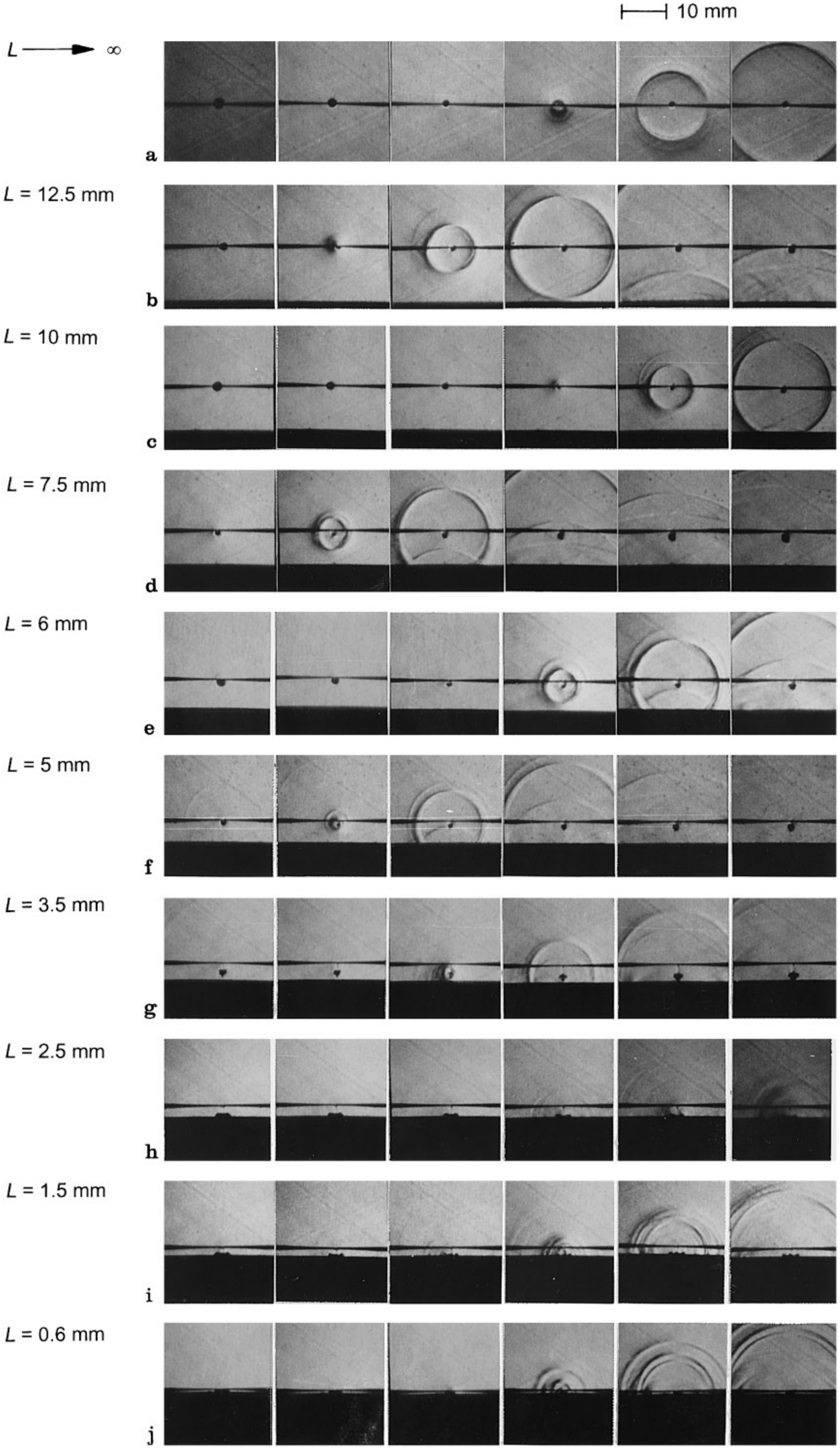
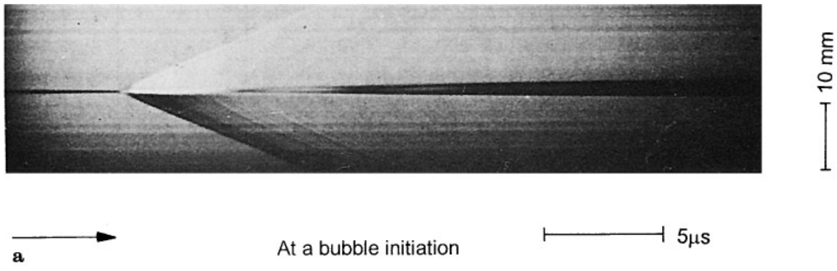
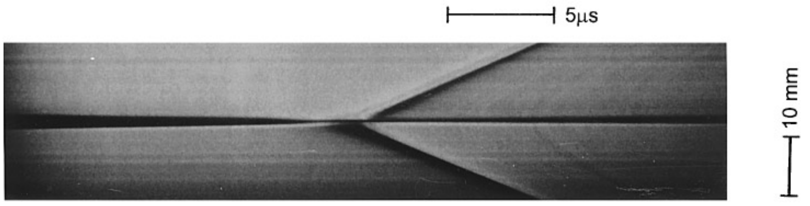


Fig. 6. Framing schlieren photographs of the flow-field around the collapsing bubble for each L (Distance between the electrodes and a solid wall): 200,000 frames/s, Frame interval 5μs, Exposure 1μs/frame (Shima et al. 1981)



At a bubble initiation



At a first collapse

Fig. 7a,b. Streak schlieren photographs **a** at a bubble initiation, **b** at a first collapse, $L \rightarrow \infty$, Streaking rate 0.473 $\sim$ 0.499 μs/mm, streaking direction from left to right (Shima et al. 1981)

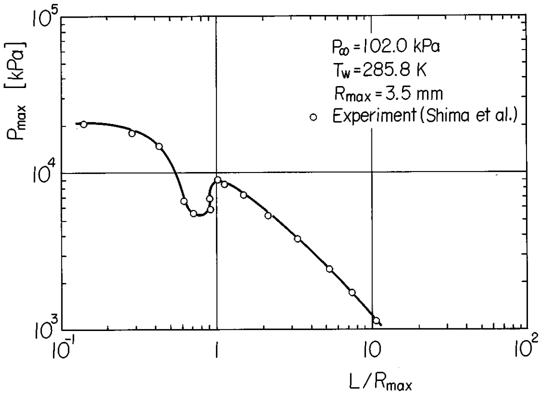


Fig. 8. Relationship between the maximum impact wall pressure P_{max} and the dimensionless distance L/R_{max} (Shima at al. 1983)

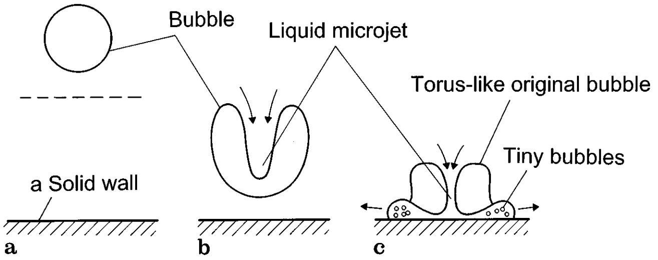


Fig. 9. Modes of the bubble collapse (Shima and Tomita 1987)

In this section particularly the collapse of a spherical bubble separated by some degrees from a solid wall is described. Rattray (1951) and Shima (1968; 1971; 1968/1969a; 1968/1969b) theoretically analysed the collapse of an initial spherical bubble in an inviscid, inompressible liquid near a solid wall. In addition, Shima and Tomita(1981) uniquely took into account the effect of the liquid compressibility.

Since an assumption that the spherical symmetry for the bubble shape is satisfied approximately in the case where a bubble moves to a position relatively far from a solid wall, without migrating toward it, Shima and Tomita (1981) obtained the following equation which described the bubble motion with the first order correction for the effects of the liquid compressibility and the solid wall.

$$R\ddot{R}\left(1+\frac{R}{2L}-\frac{2\dot{R}}{C_\infty}\right)+\frac{3}{2}\dot{R}^2\left(1+\frac{2R}{3L}-\frac{4\dot{R}}{3_{C_\infty}}\right)$$

$$+\frac{1}{\rho_l}\left(p_\infty-p_{r=R}-\frac{R\dot{p}_{r=R}}{C_\infty}\right)=0, \qquad (11)$$

where,

$$p_{r=R}=p_o\left(\frac{R_{max}}{R}\right)^{3Kvg}-\frac{2\sigma}{R}$$

$$\dot{p}_{r=R}=-3\gamma p_o\frac{\dot{R}}{R}\left(\frac{R_{max}}{R}\right)^{3Kvg}+\frac{2\sigma}{R^2}\dot{R}$$

p_o is the initial pressure inside the bubble, R_{max} the maximum bubble radius, $p_{r=R}$ the pressure of the liquid at the bubble wall, C_∞ the velocity of sound in the liquid at infinity, L the distance between the center of a bubble and the solid wall, K_{vg} the specific heats ratio of the mixture consisting of the vapor and the noncondensable gas. Equation (11) is identical with that obtained by Trilling (1952) in the case of $L \rightarrow \infty$ and furthermore with the result obtained by Shima (1971) in the case of $C_\infty \rightarrow \infty$.

Figures 4 and 5 show the comparison between the result of $R-t$ curves predicted by (11) and the experimental result obtained by Lauterborn and Bolle (1975) and Shima and Tomita (1981), respectively. It can be seen from Fig.4 that the theory (Shima and Tomita 1981) indicated by a solid line coincides satisfactorily with the experiment. In Fig.5, the theoretical and experimental results agree well up to the second rebound.

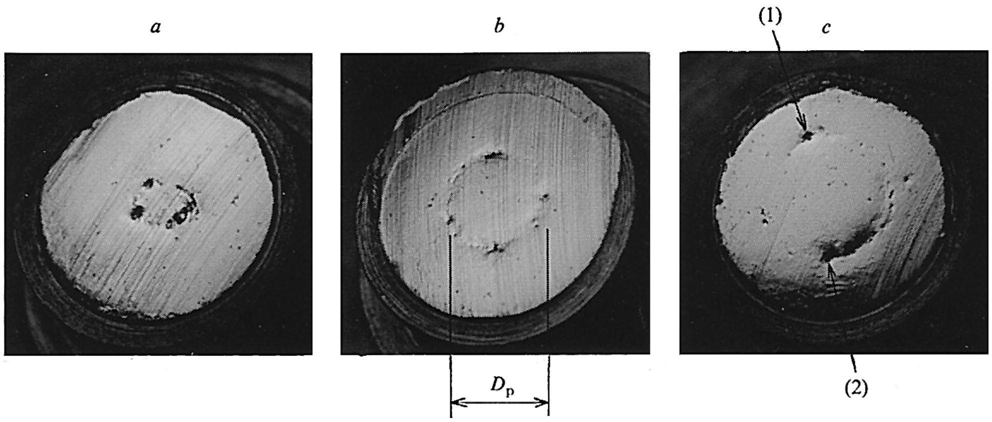
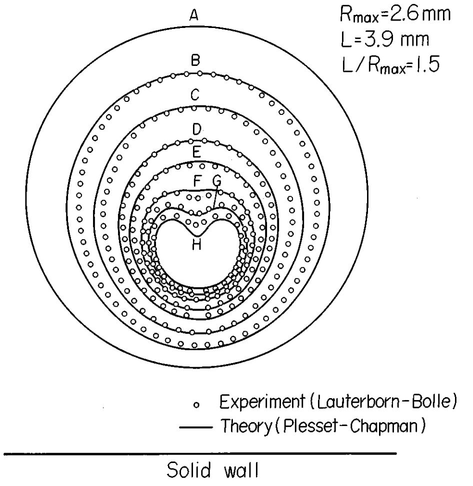


Fig. 10. Damage patterns produced on indium specimen: R_{max} = 5.1mm; (a) L/R_{max} = 0.82, (b) 0.23; (c) 0.32 D_p: Average diameter of the damage pattern (Tomita and Shima 1986)



Curve	A	B	C	D	E	F	G	H
Time, $R_{max}(\rho/\Delta P)^{\frac{1}{2}}$	0	0.725	0.825	0.961	0.991	1.016	1.028	1.036

Fig. 11. Collapse of a spherical bubble near a solid wall; Comparison between the experimantal values obtained by Lauterborn-Bolle and the theoretical curves taken from Plesset-Chapman (Lauterborn-Bolle 1975)

6 The effects of a solid wall on the bubble behavior and shock wave induced from the collapsing bubble

Two theories existed to explain the cavitation damage. One theory concerned the impulsive pressures or shock waves caused by the collapse of bubbles (Hickling and Plesset 1964; Fujikawa and Akamatsu 1980) and a second theory concerned the liquid microjet which was induced from the non-spherical collapse of the bubbles (Kornfeld and Suvorov 1944; Naudé and Ellis 1961; Plesset and Chapman 1971). It was still unknown whether either of these theories were correct.

Shima et al. (1981) observed, by using a high speed camera operated in framing and streaking modes, the motion of a collapsing bubble and the generation of shock waves at the moment of the bubble collapse. They obtained the following results. A collapsing bubble always generates a spherical shock wave at the moment of the re-expansion phase wherever the bubble is located from a solid wall. The shock wave strength is weakened more when a bubble collapes at a closer distance from a solid wall. Eventually as an extreme case, when it collapses at the stand-off diatance very close to the wall, an asymmetric collapse appears which induces several wave sources so that multiple shock waves are observable. An interesting result is found that the jet and the shock wave co-exist at a very short time interval at its maximum expansion for a bubble nearly touching a solid wall. When a bubble, attached to a solid wall, collapses, it is found that if the bubble shape, at its maximum expansion, is prolate spheroidal with an acute contact angle, the contribution from the jet to the impulsive force is predominant, where as if it is flat spheroidal with an obtuse contact angle, the contribution from the shock wave is predominant as seen in Figs. 6 and 7.

7 Impulsive pressure produced from the collapsing bubbles

Shima et al. (1983) obtained the following conclusions from the high speed photography of bubbles and the measurement of the impact wall pressure histories based on the impulsive pressure produced from the collapse of a single spark-generated bubble near a solid wall. (1) The source of the impact pressure at the solid wall was classified into three types, depending on L/R_{max} where L is the distance between the electrodes and the solid wall and R_{max} is the maximum bubble radius as seen in Fig.8.

The type and the region of their existance are: i) the region where a shock wave is dominant at $L/R_{max} \lesssim 0.3$ and $\gtrsim 1.5$; b) the region where a liquid jet is dominant at $0.6 \lesssim L/R_{max} \gtrsim 0.8$; and c) the region where a shock wave and a liquid jet co-exist at $0.3 \lesssim L/R_{max} \lesssim 0.6$ and $0.8 \lesssim L/R_{max} \lesssim 1.5$. Tsuda et al. (1982) also found the area in which both shock wave and liquid jet co-exist.

(2) The impulse pressure at a solid wall is larger for a bubble position closer or attached to the wall. However, for

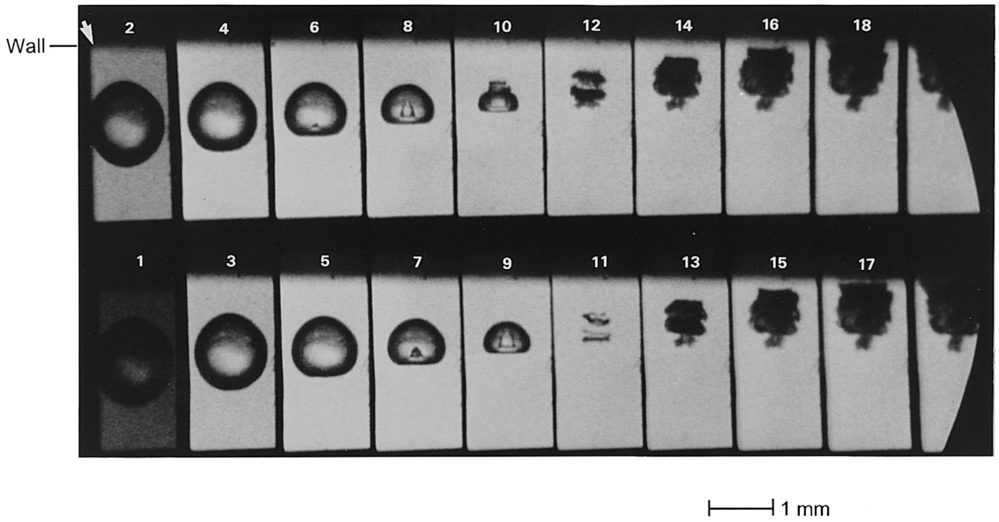


Fig. 12. Microjet formation inside a bubble generated in the vicinity of a plane rigid boundary; Frame interval $2\mu s$ (Tomita and Shima 1990)

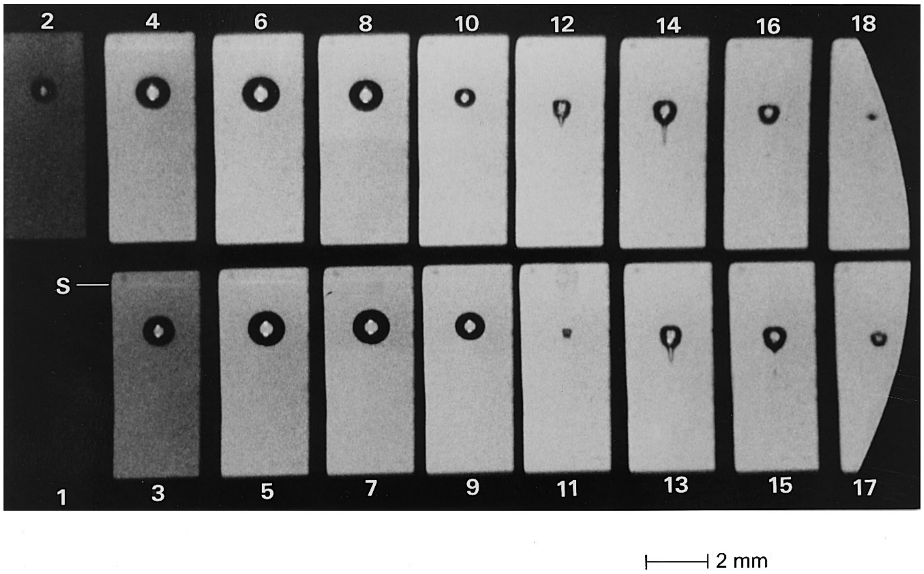


Fig. 13. Behavior of a bubble near a free surface; Frame interval $10\mu s$, R_{max} = $0.61mm$, L/R_{max} = 2.61 (Tomita and Shima 1990)

clarifying the cavitation damage, it is necessary to know the probability for the existance of bubbles at each position.

8 Mechanisms of the bubble collapse and damage pit formation

The relationship between the impulsive pressure resulting from bubble collapse and the damage pit on material has been clarified experimentally by Tomita and Shima (1986). They used spark generated bubbles and observed circular damage pit patterns on indium as a soft material when the bubbles collapsed on it.

(1) The modes of bubble collapse depend upon their proximity to a solid wall as shown in Fig. 9. A dotted line in Fig. 9 indicates a limit which represent the solid wall effect. At the first collapse of a bubble positioned either very close or attached to a solid wall, the following impulsive pressure is created for an extremely short time duration: (i) the pressure pulse during bubble collapse; (ii) the impact pressure from a liquid jet formed inside the original bubble; (iii) the impulsive pressure caused by collapses of many minute bubbles resulting from the interaction between the outward radial flow following the liquid jet impact and the contracting bubble surface; and (iv) the impact pressure from a shock wave radiated from the torus-like original bubble at its rebound as shown in Fig. 9.

(2) For a bubble placed very close to a solid wall, high impulsive pressures occur not only at the first collapse but also at the socond collapse.

(3) The circular damage pattern on indum caused by spark induced bubble collapse results from the impulsive pressures as mentioned in (i), (ii) and (iii) as shown in Fig. 10.

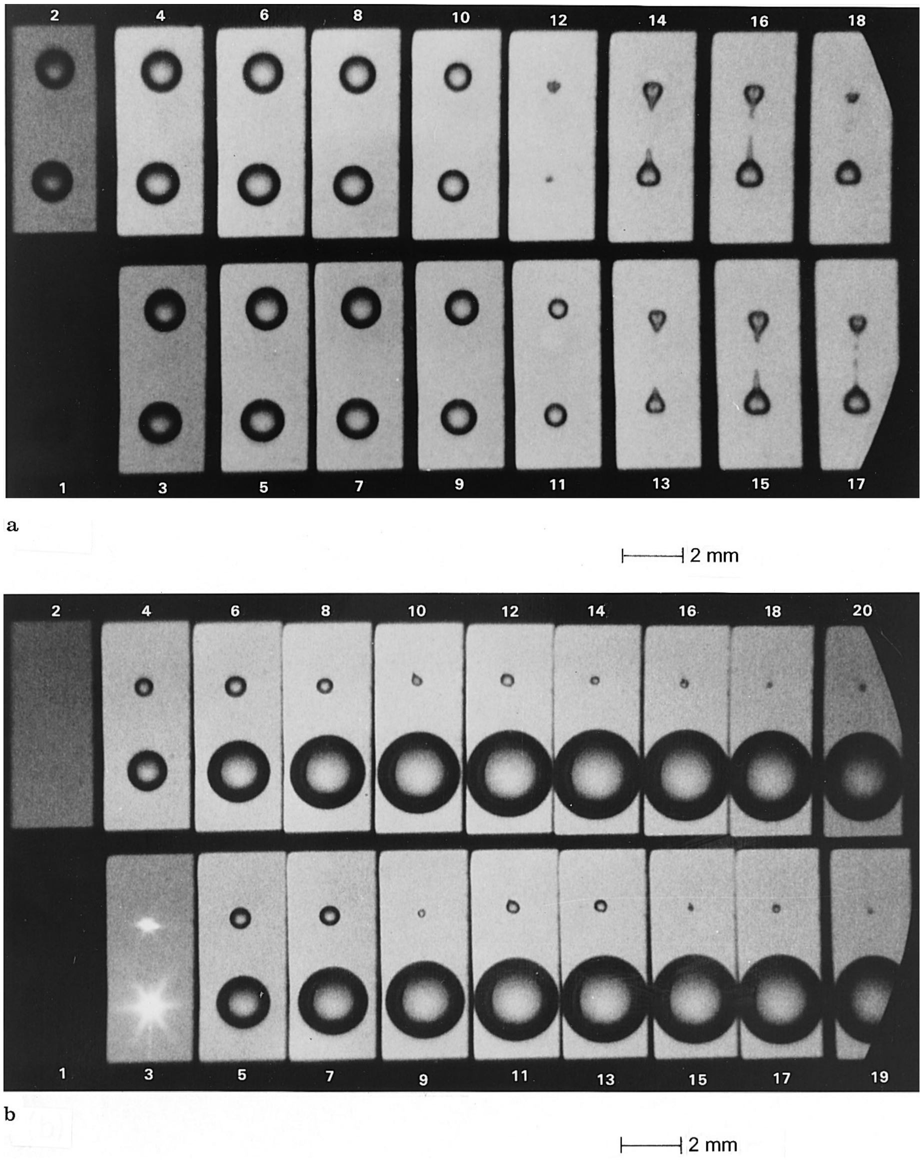


Fig. 14a,b. Behavior of two bubbles in phase; Frame interval $10\mu s$ (Tomita and Shima 1990)

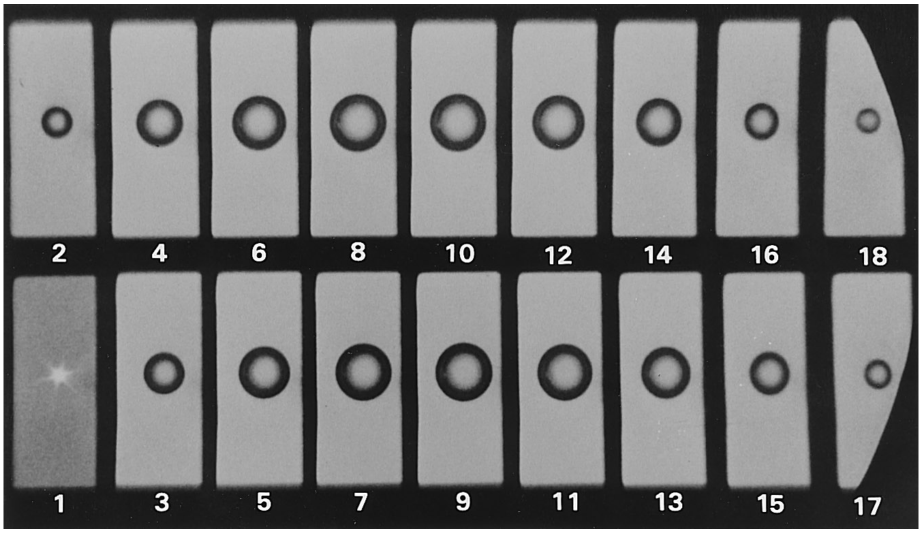


Fig. 15. Behavior of a laser-induced bubble in liquid nitrogen; Frame interval $10\mu s$ (Tsubota et al. 1996)

(4) Local high pressure may occur as a result of the interaction of a small bubble with either a shock wave with a sufficiently large pressure increase.

(5) The damage pit was caused by the interaction of an attached air bubble with a shock wave. In this case, pit formation results from the impact pressure of a liquid microjet.

However, the damage pit seems to be caused by the collapses of bubbles near a solid wall.

9 The behavior of laser-induced bubbles

When a laser light with an intensity beyond the liquid dielectric breakdown threshold is focused into a transparent liquid, an optical breakdown occurs. Then a bubble is generated. Lauterborn (1974) developed this technique to the field of cavitation. He also compared the laser induced bubble near a solid wall with the analytical result proposed by Plesset and Chapman (1971) and showed a good agreement between them except at the final stage of the bubble collapse (Lauterborn and Bolle 1975).

Figure 11 shows thier comparison. Laser-induced bubbles have attracted many investigator and the interesting bubble dynamic phenomena have been clarified one after another (Teslenko 1980; Lauterborn 1982; Hentschel and Lauterborn 1982; Giovanneschi and Dufresne 1985; Sanada et al. 1987).

Tomita and Shima (1990) succeeded in the creation of an ideal bubble with the high degree of sphericity by the laser beam focusing. Laser-induced bubbles generated under various conditions were observed by a high speed photography. Figures 12, 13 and 14 show four typical examples of high speed photographs. Figure 12 shows the first collapse of a bubble near a solid wall. In this case due to the high inertia the liquid jet is ejected to the solid surface from the main troidal shaped collapsing bubble in which two vortex rings are generated.

Figure 13 shows the bubble motion near a free surface. When a bubble collapses in the distance relatively far from the boundary, the free surface slightly deforms and only a small repulsion takes place even at the final stages of bubble collapse. A fully developed liquid jet is clearly observable at the rebound stage. Figure 14 shows typical examples of two bubbles. In Fig. 14(a) the two bubbles have almost the same radii and their stand-off distance $l_o = 3{,}72$mm and move in phase. This is the identical case in which a single bubble moves near a solid wall at the distance of $L = l_o/2$ from the bubble centre.

In Fig. 14(b) the two bubbles having extremely different radii move eventually out of phase. It is clearly seen that the smaller bubble is significantly influenced by the pressure field resulting from the motion of its neighbouring larger bubble. The small one collapses and is repelled from the large one.

10 The behavior of bubbles in a low temprature liquid

Recently cryogenic fluids are being widely applied to the coolant of a superconducting machinery, nuclear fusion, etc. In the high speed flow of these liquids, the flow easily becomes a gas-liquid two phase flow and the cavitation phenomena are an important subject to be solved. The motion of cavitation bubbles in a cryogenic liquid was studied: the growth of a bubble in the liquid when the pressure is reduced and the bubble collapse at subcooled area (Hewitt and Parker 1968); the dynamics of a laser-induced bubble in liquid nitrogen (Maeno et al. 1991); and the dynamics of bubbles in cryogenic liquids (Tsubota and Shima 1993). There are many unsolved questions which need to be clarified.

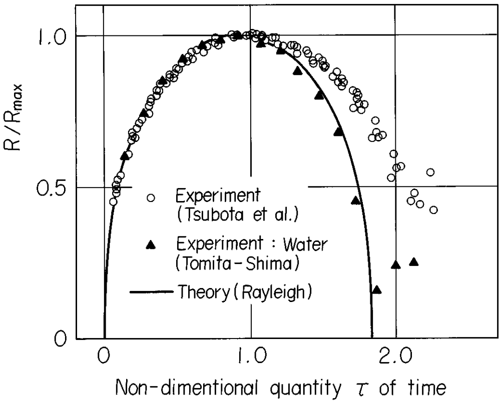


Fig. 16. Variation with time of the bubble radius (Tsubota et al. 1996)

Tsubota et al. (1996) established a technique for the generation of an almost spherical bubble by focusing a pulse ruby laser in liquid nitrogen, as a typical cryogenic liquid, and succeeded in the observation of its behavior by using a high speed camera. An example of the results is shown in Fig. 15. The time variation of the bubble radius is shown in Fig. 16.

In Fig. 16, the experimental result of a single bubble in water at room temperature (Tomita and Shima 1990) and the analytical results based on the Rayleigh solution are also compared. In liquid nitrogen, the growth and collapse time of a bubble is found to be longer than that in water.

11 Conclusions

In this review the dynamics of a mostly single bubble appearing in various liquids are described. It is necessary to clarify the dynamics of multiple bubbles in the flow field. It is expected that these bubble dynamics will be studied in the future.

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