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ORIGINAL ARTICLE

Evaluation of the existing triple point path models with new experimental data: proposal of an original empirical formulation

J. Boutillier[1] · **L. Ehrhardt**[2] · **S. De Mezzo**[2] · **C. Deck**[1] · **P. Magnan**[2] · **P. Naz**[2] · **R. Willinger**[1]

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Abstract With the increasing use of improvised explosive devices (IEDs), the need for better mitigation, either for building integrity or for personal security, increases in importance. Before focusing on the interaction of the shock wave with a target and the potential associated damage, knowledge must be acquired regarding the nature of the blast threat, i.e., the pressure–time history. This requirement motivates gaining further insight into the triple point (TP) path, in order to know precisely which regime the target will encounter (simple reflection or Mach reflection). Within this context, the purpose of this study is to evaluate three existing TP path empirical models, which in turn are used in other empirical models for the determination of the pressure profile. These three TP models are the empirical function of Kinney, the Unified Facilities Criteria (UFC) curves, and the model of the Natural Resources Defense Council (NRDC). As discrepancies are observed between these models, new experimental data were obtained to test their reliability and a new promising formulation is proposed for scaled heights of burst ranging from 24.6–172.9 cm/kg$^{1/3}$.

Keywords Blast waves · Triple point · Shock reflection · Empirical models

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✉ J. Boutillier
jboutillier@unistra.fr

[1] Multiscale Materials and Biomechanics, Strasbourg University, ICube, UMR 7357, 2 rue Boussingault, 67000 Strasbourg, France

[2] French-German Research Institute of Saint-Louis (ISL), 5 rue du Général Cassagnou, 68300 Saint-Louis, France

1 Introduction

The protection of buildings and humans from shock waves has been studied for decades in the defense and security fields. Since the number of exposures to shock waves continues to increase [1], the need for better protection, and therefore a better understanding of the physical phenomena related to blast, gains in importance. In particular, detonation in open space (without obstacles) above the ground must be fully understood before focusing on the understanding of the blast wave interaction with a target, since in such a scenario the target may face different loadings. Unfortunately, to this day no analytical solution is available and other approaches must be considered.

Empirical or semi-empirical tabulated values already exist to describe the pressure–time history, assuming that the waveform follows the classical Friedlander shape [2–7]. These values have been extensively determined for three scenarios: spherical blast in the free field, hemispherical blast, and above-ground detonation. In the latter case, the threat is more complex as at least three shocks are generated: the incident wave, the ground reflection wave, and the Mach stem. The Mach stem appears when the reflected wave catches up to and merges with the incident wave, due to a faster propagation in air that has been pre-shocked by the incident wave. Geometrically, the Mach stem occurs when the angle of incidence of the incident wave, β, is greater than a critical angle $\beta_{\max}$ [2]. As illustrated in Fig. 1, the intersection of the incident wave, the reflected wave, and the Mach stem is called the triple point (TP). The TP depends on both the height of burst (HoB) and the mass of explosive used [2], but also on the hardness and roughness of the reflecting surface [8]. An extensive series of experiments were performed in the 1970s by the US Ballistic Research Laboratory at Suffield

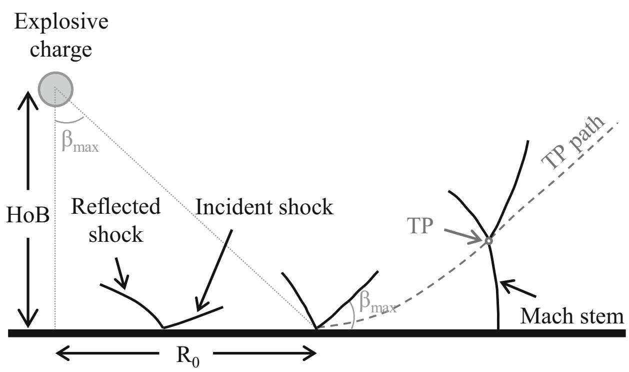

Fig. 1 Description of the shock configuration produced by a HoB explosion, and illustration of the determination of R_0 with the angle β_{max} and the HoB

(Alberta, Canada) to analyze the influence of the reflecting surface on the TP path. In addition, ideal shock-on-shock reflections have been studied, which avoids the energy loss due to the ground [8]. They noticed that, as the roughness increases, the transition from regular to Mach reflection is delayed and the triple point trajectory is lowered and flattened. If the roughness is great enough, the reflected shock is so segmented and weakened that it never overtakes the incident shock and transition to Mach reflection does not occur.

An important step toward the knowledge of the blast threat is the assessment of the TP trajectory. Indeed, above the TP, the pressure profile consists of the sum of the incident and reflected waves (two shocks), which is different from that of the Mach stem (only one shock). The three methods currently used to estimate the blast characteristics above the TP are: the method of images, which linearly adds a virtual ground-symmetrical blast source to the free-field blast; the use of the numerical *Load_Blast_Enhanced function from LS–DYNA [9]; and more recently the new formulation of Ehrhardt et al. [10] which gives a better prediction than the previous models. However, the use of these models is based on the knowledge of the TP path that can be obtained from three different empirical models: the model of Kinney, the Unified Facilities Criteria (UFC), or the Natural Resources Defense Council (NRDC) [2,7,11].

As little information is available concerning the formulation and reliability of these empirical models, an evaluation of these models is necessary. The aim of this study is to assess the existing procedures for obtaining the TP path. This paper is composed as follows. Section 2 presents three existing TP path prediction tools and describes the discrepancies between each model; the aim of Sect. 3 is to provide an extensive experimental dataset of TP paths in order to evaluate the reliability of the existing models, revealing the need for a new, more complete, and more simple tool presented and evaluated in Sect. 4.

2 The existing TP path prediction models

The first TP prediction model developed is the empirical model of Kinney, which uses the geometrical aspect of the Mach stem formation [2]. It postulates that the Mach stem is created when the incident wave forms an angle with the reflecting surface greater than β_{max}, itself depending on the Mach number of the incident wave (M_i):

$$\beta_{max} = \frac{1.75}{M_i - 1} + 39. \tag{1}$$

where β_{max} is in degrees and M_i is defined as:

$$M_i = \frac{C_{wave}}{a_0}. \tag{2}$$

C_{wave} is the speed of the incident shock and a_0 is the speed of sound in the upstream medium. In Kinney's book [2], data are plotted for M_i from 1 to 3 and β from 0° to 90°.

Knowing M_i, the angle of formation of the Mach stem (β_{max}) is deduced. This leads to the determination of the distance R_0 at which it is formed, as illustrated in Fig. 1. Kinney and Graham determined R_0 from (3) [2]:

$$R_0 = \tan \beta_{max} \,\text{HoB}. \tag{3}$$

The TP path is finally determined as follows [2]:

$$H_{TP} = 0.07\,\text{HoB} \left(\frac{D}{R_0} - 1\right)^2. \tag{4}$$

where H_{TP} is the TP height and D is the ground distance.

Needham [12] proposed a different way to calculate the distance of appearance of the Mach stem (R_0) for nuclear explosions. For HoBs less than 99.25 m/kT$^{1/3}$, the Mach stem will be formed at a distance $R_0 = 0.825$ HoB. For higher HoBs, this distance (in cm) is defined as:

$$R_0 = \frac{170\,\text{HoB}}{1 + 25.505\,\text{HoB}^{0.25}} + 1.7176 \times 10^{-7}\,\text{HoB}^{2.5}. \tag{5}$$

Although these formulas were obtained from nuclear tests, they can be used for conventional explosives [12].

Besides the empirical method of Kinney [2], the UFC model provides ten TP paths for TNT explosive material as illustrated in Fig. 2 [7]. Each curve corresponds to a specific scaled HoB (SHoB) corresponding to the HoB divided by the cube root of the mass of the explosive charge. No extrapolation to other scenarios was proposed, and the distance R_0 is unknown with this chart. For this model, the SHoB range of validity extends from 39.7 to 277.6 cm/kg$^{1/3}$.

Another method for TP path prediction, which is effective for nuclear explosions, is the NRDC model [11]. It

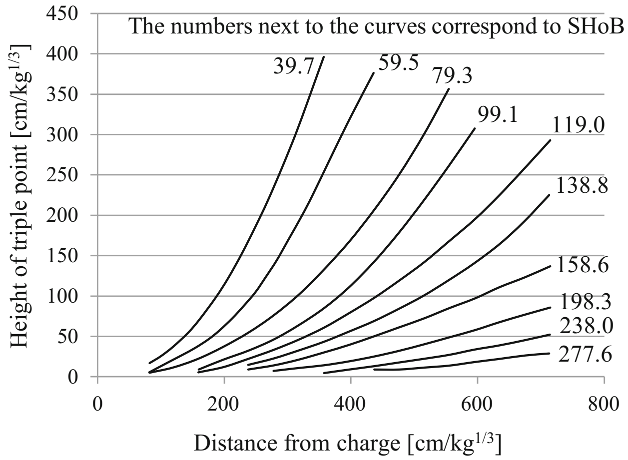

Fig. 2 Triple point paths for TNT explosive charge from the UFC model [7]. SHoB is the scaled HoB in $cm/kg^{1/3}$

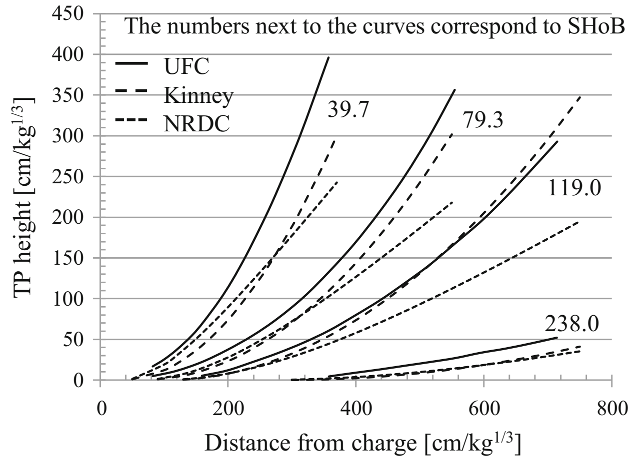

Fig. 3 Comparison of the three TP path empirical models (UFC, Kinney, and NRDC) for four different SHoBs (in $cm/kg^{1/3}$ with masses in TNT)

applies, among others, for masses of TNT ranging from 0.1 to 25,000 kT and is defined as follows:

$$\mathrm{SH_{TP}} = S\left(h + \left(h^2 + (\mathrm{SD} - 0.9\,\mathrm{SR_0})^2 - \frac{\mathrm{SR_0^2}}{100}\right)^{0.5}\right) \quad (6)$$

where:

$$S = \left(5.98 \times 10^{-5}\,\mathrm{SHoB}^2 + 3.8 \times 10^{-3}\,\mathrm{SHoB} + 0.766\right)^{-1} \quad (7)$$

$$h = 0.9\,\mathrm{SR_0} - 3.6\,\mathrm{SHoB} \quad (8)$$

$$\mathrm{SR_0} = \frac{\mathrm{SHoB}^{2.5}}{5822} + 2.09\,\mathrm{SHoB}^{0.75}. \quad (9)$$

The letter S before H_{TP}, D, and R_0 corresponds to the scaled versions of these parameters.

Figure 3 compares the TP path predictions from the three aforementioned empirical models (Kinney, UFC, and NRDC) for four SHoBs. For a quantitative comparison, the absolute relative error (in %) is calculated for different SD values. Compared with the UFC model, the predictions of the NRDC model show discrepancies from 15 to 80%. Errors from Kinney's model are of the same order of magnitude as those of NRDC—from 0 to 90%. Errors less than 10% for the latter model correspond to the SHoB $= 119\,cm/kg^{1/3}$ case, where the prediction crosses the UFC curve. These differences could be due to different reflecting surfaces (roughness and hardness) in the experimental tests used for the development of the TP models. However, the lack of information on these reflecting surfaces does not allow determination of which TP path model is the most reliable.

Moreover, the major drawback of the UFC model is the lack of a formula allowing extrapolation of the TP prediction for scenarios other than the ten illustrated previously. In addition, it does not allow determination of the ground distance where the TP initially appears. This implies that, outside the range of this model (ten curves), the empirical formula of Kinney or NRDC should be used, even if they can cause an underestimation or overestimation of the Mach stem height.

In order to evaluate the existing TP path models, an experimental campaign has been conducted with the aim to collect the TP paths for different scenarios.

3 Experimental data

3.1 Experimental setup

The scenario considered in our study is the detonation of a spherical composition-4 (C-4) charge over flat solid ground, as schematically illustrated in Fig. 1. This charge is initiated by a detonator located in its center and the typical blast propagation range we focus on is up to 6 m, as in this range the expected damage on humans is important but may be mitigated.

The first set of experiments was conducted in the *Polygone d'essais de Captieux*, in France where a reinforced concrete slab of 15 m × 15 m was chosen as the explosion location. A 4-cm-thick steel slab (1 m × 1 m) was placed directly below the charge (ground zero) to prevent the concrete from cratering, as explained in Ehrhardt et al. [10]. Four different masses (500 g, 1, 2, and 5 kg) and three different initial HoBs (33, 66, and 133 cm) were considered, leading to twelve different scenarios. Each scenario has been repeated twice, except for the scenarios with 5 kg of C-4 that were performed once. The charges were positioned on a cardboard tube to obtain the desired HoB.

The second set of experiments was conducted in the Baldersheim Proving Ground (France), where fifteen tests were performed with 0.2 kg of C-4 detonating at a HoB of 66 cm. In this experimental campaign, the explosive charges were suspended. To be more precise, the HoB defined previously is the distance between ground zero and the bottom of the spherical charge. To obtain the typical HoB, i.e., the distance between the center of the charge and ground zero, one should add the charge radius.

A digital high-speed video monochrome camera (Phantom V610) with 20,000 frames per second, image size of 896×800 pixels, and 1 µs exposure time was used to record the explosion events. During the event, the strong density gradients create a visual distortion of the observed background, which makes the shock front visible when the background is contrasted, without using any other optical technique such as interferometry (as in, e.g., [13,14]). The distance between the experimental setup and the camera was approximately 50 m.

As meteorological conditions affect the blast propagation, a weather station VAISALA WXT520 was used during the experiments. It recorded the ambient pressure, temperature, wind direction and speed, humidity, and rainfall.

3.2 Methodology

The TP paths were obtained experimentally by analyzing the monochrome camera records. The procedure is detailed here:

- A MATLAB program was created to derive the difference between two consecutive frames.
- A threshold, manually determined, was applied to the resulting image in order to see the shocks.
- The resulting image was recorded, and the TP position on the object plane was obtained via the open-source software *Tracker*. Tracking errors were introduced during this step due to difficulties in visualizing the shocks in some cases and to the pixels' size (6 mm $\times$ 6 mm).
- Finally, because the image of a three-dimensional shock surface seen against a background defines only the line of sight tangential to the shock, the distances obtained in the object plane (with *Tracker*) were not the real distances. An appropriate geometric correction was applied [15].

This process was repeated several times, enabling the determination of the TP path. Notice that the threshold was different for each image due to the fireball which increased over time. This process had to be used because, as illustrated in Fig. 4, while the shocks were almost invisible in a single high-speed camera frame (top), the difference (bottom) clearly shows curves which indicate the propagating shocks.

This method allowed easy tracking of the TP when the visible background was contrasted, otherwise the obtained TP paths were less precise. Moreover, when unexpected projection from the fireball of explosive products interfered with measurements of the blast front, the TP path was not considered. In addition, for the video tracking, the steel slab was not taken into account as the initial reflection occurred on the top of it.

Fig. 4 Focused camera view 7.3 ms after the detonation of the 500 g charge at 133 cm. (*Top*) raw camera data, i.e., single frame. (*Bottom*) difference between two consecutive frames

3.3 The triple point paths

The average TP paths and associated standard deviations obtained experimentally are plotted in Fig. 5. The general tendency of the TP evolution is visible and tends to follow a parabolic curve. The sensitivity toward the explosive charge is as follows: When the HoB increases, the TP path is located closer to the ground. This tendency is also observed when the charge mass decreases. These two sensitivities have already been observed and tabulated in the existing models [2,7,11].

In order to evaluate the error of measurement per scenario, the following procedure has been followed:

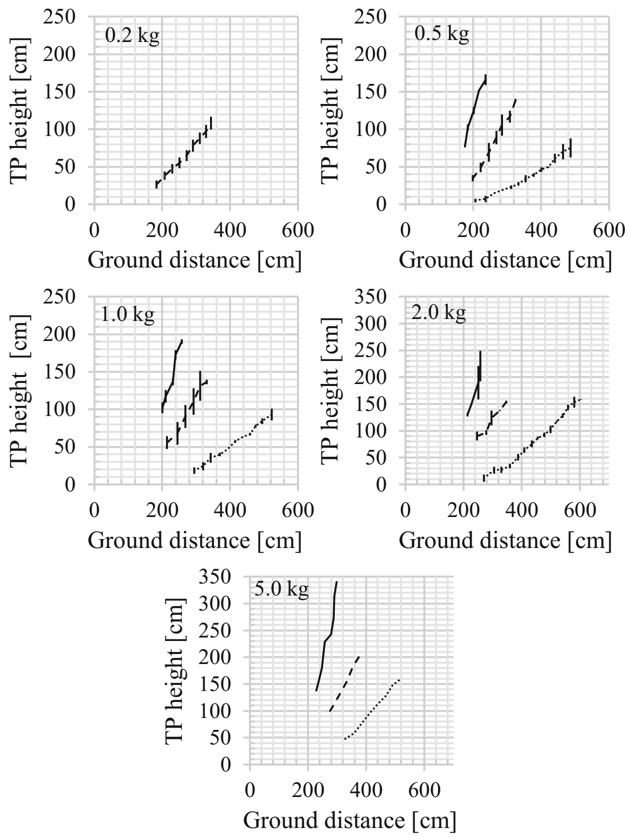

Fig. 5 TP paths obtained experimentally by video tracking. *Solid lines* HoB 33 cm; *dashed lines* HoB 66 cm and *dotted lines* HoB 133 cm

1. For each point of the TP trajectory, the coefficient of variation, CV, in % is calculated. For example, CV ranges from 7.8 to 20.8% for the scenario 0.2 kg at 66 cm, where nine points have been obtained by video tracking;
2. Then, an average CV is calculated per scenario. For the scenario 0.2 kg at 66 cm, the average CV for the 9 points is 12.1%;
3. Finally, in order to evaluate the spread of CV per scenario, a standard deviation s is calculated related to the set of CV previously calculated. For the scenario 0.2 kg at 66 cm, s is equal to 4.0 %. The error of measurement for this scenario is then $12.1 \pm 4.0\%$.

The error of measurement per scenario is summarized in Table 1. The calculated values show that the experimental TP paths are reproducible with most average coefficients of variation staying below 9%. These data can now be used to evaluate the existing TP path prediction models.

In order to perform the comparison between the new TP data and the existing empirical models, mathematical manipulations must be applied. First, the experimental scenarios (in C-4) must be converted into TNT scenarios using the TNT equivalent mass (Eq_{TNT}) which is taken equal to 1.1269 as proposed by UFC [7], and (10) must then be used. Using the definition of SHoB and (10), (11) is obtained. Second, the TP paths are finally scaled using the Sachs scaling to represent the values in an atmosphere at normal temperature and pressure [16] and then to the cube root of the TNT mass of the considered scenario.

$$m_{\mathrm{TNT}} = m_{\mathrm{C\text{-}4}} \mathrm{Eq}_{\mathrm{TNT}} \tag{10}$$

$$\mathrm{SHoB}_{\mathrm{TNT}} = \frac{1}{\sqrt[3]{\mathrm{Eq}_{\mathrm{TNT}}}} \mathrm{SHoB}_{\mathrm{C\text{-}4}} \tag{11}$$

The comparison of the existing empirical models with the new experimental data is shown in Fig. 6 for two scenarios. The predictions of the existing models are shown for SHoBs of 59.5 and 138.8 $\mathrm{cm/kg}^{1/3}$, while the experimental data are shown for slightly lower values (respectively, 55.5 and 132.9 $\mathrm{cm/kg}^{1/3}$). The experimental plots should then be slightly lower for the values of SHoBs used for the empirical models.

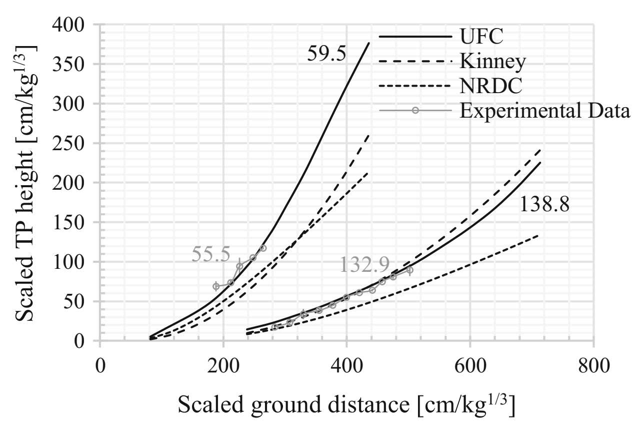

Fig. 6 Comparison of the UFC, Kinney, and NRDC models with new experimental data for two SHoBs. The value next to the curves corresponds to the associated SHoB (unit: $\mathrm{cm/kg}^{1/3}$)

Table 1 Error of measurement of the TP height per scenario, expressed through the average CV $\pm s$

	0.2 kg	0.5 kg (%)	1.0 kg (%)	2.0 kg (%)	5.0 kg
HoB 33 cm		2.7 ± 1.9	4.6 ± 2.5	8.1 ± 7.8	
HoB 66 cm	12.1 ± 4.0	10.7 ± 5.4	14.6 ± 6.6	5.5 ± 4.6	
HoB 133 cm		8.7 ± 5.0	8.4 ± 8.7	8.2 ± 6.7	

As only one test has been performed for the 5 kg of C-4 scenario, no errors were calculated

Table 2 Average relative error (in absolute value and in %) of the TP path prediction from the existing models (UFC, Kinney, NRDC) compared with the new experimental data for two SHoBs: 59.5 and 138.8 cm/kg$^{1/3}$

	SHoB 59.5 cm/kg$^{1/3}$ (%)	SHoB 138.8 cm/kg$^{1/3}$ (%)
UFC	6.6	15.2
Kinney	37.8	6.2
NRDC	26.1	22.9

The average errors of the existing prediction models as compared to the experimental data are given in Table 2. For the smallest SHoB, the average error of the UFC model compared to the experimental data is around 7% as compared to 38 and 26% for the Kinney and NRDC models, respectively. This SHoB corresponds to the experimental scenario 2.0 kg at 66 cm where the average CV is 5.5%. For the highest SHoB, the average error is 15% for UFC, 6% for the Kinney model, and 23% for the NRDC model. For this SHoB, the experimental scenario is 2.0 kg at 133 cm and the average CV is 8.2%. This shows that the most reliable TP path prediction model depends on the considered scenario. Moreover, since these average errors should vary due to slight discrepancies in the compared SHoBs (comparison between 59.5 and 55.5 cm/kg$^{1/3}$, and between 138.8 and 132.9 cm/kg$^{1/3}$ for the models and the experimental data, respectively) and also with the use of a different $\mathrm{Eq}_{\mathrm{TNT}}$ value, it seems necessary to develop a new empirical formula to better determine the TP path for a large range of SHoBs of validity.

4 Proposal of a new empirical formulation

4.1 Development of an original formulation

In order to propose a simple and original formulation using the new experimental data, some working hypothesis must be defined. First, similar to the observation of Kinney [2], it seems that the evolution of the TP follows a parabolic-like shape. The TP evolution is then hypothesized to be:

$$\mathrm{SH_{TP}} = a\,\mathrm{SD}^2 + b\,\mathrm{SD} + c \qquad (12)$$

$\mathrm{SH_{TP}}$ is the scaled TP height (H_{TP}), SD is the scaled ground distance (D), and a, b, and c are parameters to be determined.

Second, for the determination of parameters a, b, and c, the following assumptions were made:

1. The tangent is zero when $\mathrm{SD} = \mathrm{SR_0}$, R_0 being the ground distance where the TP initially appears. It means that:

$$\left.\frac{\mathrm{d\,SH_{TP}}}{\mathrm{d\,SD}}\right|_{\mathrm{SR_0}} = 0. \qquad (13)$$

Combining (12) and (13), (14) is obtained, allowing the determination of the parameter b:

$$b = -2\,a\,\mathrm{SR_0}. \qquad (14)$$

2. When the triple point is created, $\mathrm{SH_{TP}} = 0$. Using (14) into (12) and the hypothesis mentioned previously, the parameter c can be determined as follows:

$$c = a\,\mathrm{SR_0^2}. \qquad (15)$$

As no physical statement was found to define the parameter a, it was taken equal to that of Kinney [2]:

$$a = 0.07\frac{\mathrm{SHoB}}{\mathrm{SR_0^2}}. \qquad (16)$$

The last unknown parameter is the distance at which the TP initially appears (R_0). Several $\mathrm{SR_0}$ were tested per scenario and iterations were stopped when the curve $\mathrm{SH_{TP}} = f\,(\mathrm{SD})$ visually fits the mean values of the experimental TP paths. Thirteen values of $\mathrm{SR_0}$ were obtained, each corresponding to a specific SHoB as illustrated in Fig. 7. A polynomial relation (order $n = 2$) has been chosen to fit these data ($R^2 = 0.997$), allowing the calculation of $\mathrm{SR_0}$ for SHoBs ranging from 24.6 to 172.9 cm/kg$^{1/3}$.

With the help of (12), (14), (15), (16), and finally (17) defined hereafter, the TP path can easily be determined for a scenario within the range of validity defined previously.

$$\mathrm{SR_0} = 1.99\times10^{-3}\,\mathrm{SHoB}^2 + 0.601\,\mathrm{SHoB}. \qquad (17)$$

This new TP path formulation must now be evaluated.

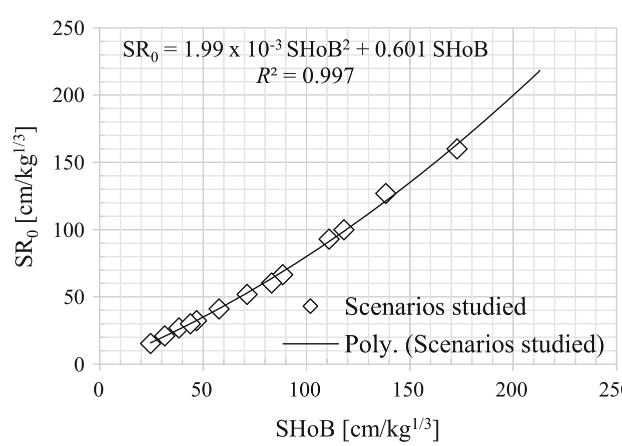

Fig. 7 Relation of $\mathrm{SR_0}$ against SHoB obtained from experimental data

4.2 Evaluation of the obtained formulation

First, to validate the proposed empirical formula, a comparison was performed with the data used for its calibration

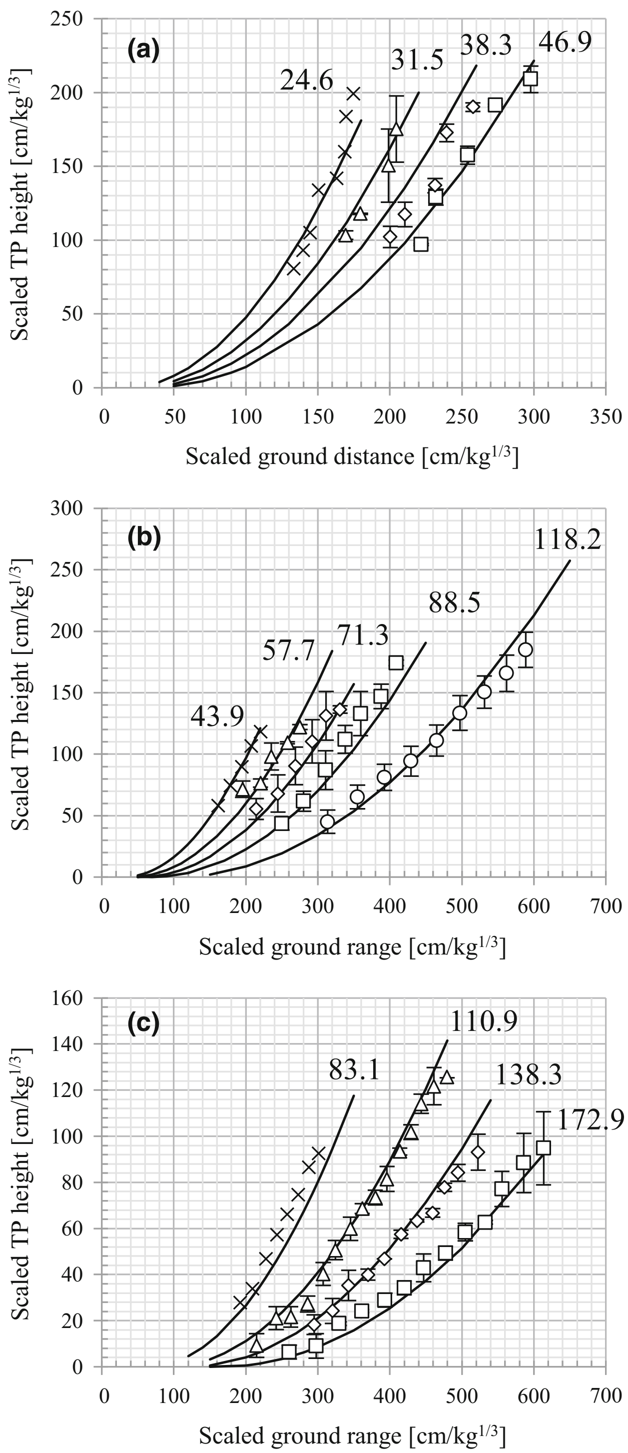

Fig. 8 Comparison of the new empirical formulation with the experimental data used for its calibration. **a** HoB 33 cm; **b** HoB 66 cm; **c** HoB 133 cm. The value near the curves corresponds to the SHoB (unit: cm/kg$^{1/3}$)

as illustrated in Fig. 8. Qualitatively, it can be noticed that the TP paths from the new formulation stay within the standard deviations of the experimental data. For a quantitative assessment, Table 3, which summarizes the average errors between the new curves and the experimental data, must be related to Table 1, which summarizes the experimental standard deviations. These tables denote an excellent correlation between the new formulation and the experimental data used for its calibration. Indeed, they reveal, for example, that compared with the experimental data, the new formulation induces 5.7% of error for the 0.2 kg at 66 cm scenario, whereas the experimental average CV is 12.1%. It shows that the calculated TP path is within the experimental standard deviation, which is the case for 50% of the scenarios tested. For the other scenarios, the error is around 5% regarding the experimental standard deviation.

This new formulation must now be evaluated with other data not used for its calibration. To do so, the comparison was performed with three scenarios within the range of validity of the proposed empirical model and a fourth one outside this range. The first three scenarios were the following:

- Detonation of a suspended C-4 charge of 0.2 kg at 88 cm, corresponding to a SHoB of 155.8 cm/kg$^{1/3}$.
- Detonation of 0.3 kg at a HoB of 44 cm, corresponding to a SHoB of 71.0 cm/kg$^{1/3}$. The explosive charge was positioned on a cardboard tube.
- Detonation of 0.3 kg at a HoB of 88 cm, corresponding to a SHoB of 136.8 cm/kg$^{1/3}$. The explosive charge was positioned on a cardboard tube.

The last scenario used for the evaluation was outside the range of validity of the proposed formulation with a SHoB of 194.9 cm/kg$^{1/3}$, corresponding to the detonation of a suspended charge of 0.1 kg at 88 cm.

A test matrix of these scenarios is presented in Table 4. For these tests, the TP paths have again been obtained from video tracking (data from experiments presented in Boutillier et al. [17]).

Table 3 Average relative error (in absolute value and in %) of the TP path prediction from the proposed empirical formula compared with the experimental data

	HoB 33 cm (%)	HoB 66 cm (%)	HoB 133 cm (%)
0.2 kg		5.7	
0.5 kg	6.6	10.4	14.6
1.0 kg	15	7.8	7.0
2.0 kg	6.2	7.7	7.5
5.0 kg	9.2	1.2	15.6

Table 4 Test matrix of the data used for the evaluation of the new formulation

	0.1 kg at 88 cm	0.2 kg at 88 cm	0.3 kg at 44 cm	0.3 kg at 88 cm
Number of test	4	4	7	8

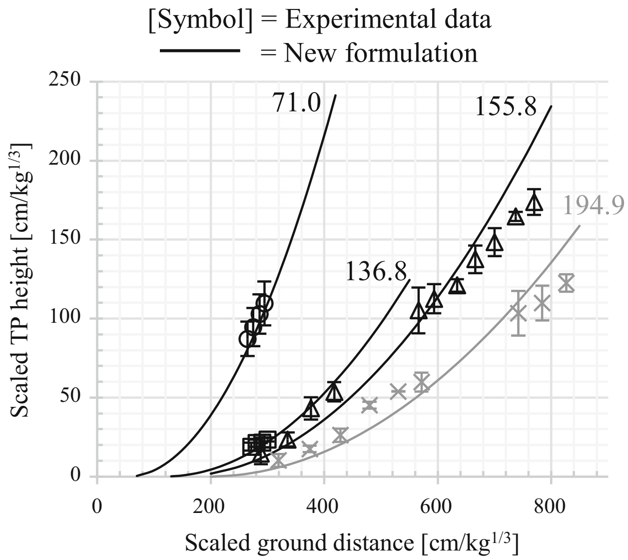

Fig. 9 Comparison of the TP path obtained with the proposed empirical formulation with new experimental data [17]. The value near the curves corresponds to the SHoB in cm/kg$^{1/3}$. The *black curves* with symbols are the data from scenarios within the range of validity of the new formulation, and the *gray* one is from the scenario outside the range of validity

The comparison of the proposed formulation with the experimental data is illustrated in Fig. 9, on which the prediction of the new formulation for the two smallest SHoBs is within the standard deviation of the experimental data. The third scenario (within the range of validity of the model) matches the experimental data up to a scaled ground distance of 700 cm/kg$^{1/3}$. Actually, below that value the model is in the standard deviation of the experimental data or generates errors less than 10%. Above this scaled ground distance, the error is between 15 and 20% compared with the experimental data. Using these few experimental data, it is shown that the new formulation allows the TP trajectory to be obtained precisely within the SHoB's range of validity specified previously. However, the use of the proposed prediction model outside its range of validity induces significant differences. Indeed, for the SHoB of 194.9 cm/kg$^{1/3}$, with the exception of some values, errors are greater than 15%. However, data from this scenario were obtained only from two tests where the video tracking was difficult due to a lack of contrast in the background of the high-speed camera records. For these reasons, it is difficult to make a conclusion regarding the possibility of extending the range of validity of the proposed formulation. Further comparisons should be made to answer this question.

5 Discussion

After evaluating the existing TP path prediction models with new experimental data, the need for a better formulation has been shown. An original and simple formulation has been proposed using working assumptions and the new experimental data giving TP paths. The range of validity for values of SHoB extends from 24.6 to 172.9 cm/kg$^{1/3}$, bearing in mind the ground distance we focus on.

While these evaluations led to the conclusion that the proposed formulation is the most reliable model to catch the evolution of the TP, it is mitigated by the fact that the model was optimized with the data used for the first evaluation and evaluated a second time with few other data. A possible perspective would be to evaluate this new model with external data. However, such experimental data are scarce in the open literature. Another solution could be to use finite element software such as LS-DYNA, but they must be first validated using experimental data. LS–DYNA provides several approaches to study blast, e.g., the ALE approach and the empirical function *Load_Blast_Enhanced (LBE). As the LBE function uses the TP prediction from the UFC model, it cannot be used for an in-depth evaluation. The ALE approach has already been evaluated regarding the experimental data presented in the current study [18], showing good correlation in terms of TP path (errors below 10%). This tool could then be used in further studies to strengthen the proposed formulation. In addition, to reinforce the reliability of the new model, it would also be interesting to complete this study by evaluating the TP path from detonations over different reflecting surfaces, as in [8], but with masses and HoBs we focus on in order to propose different formulations for different reflecting surfaces. Figure 10 shows the TP paths of shocks reflecting from real surfaces (smooth–hard and soft–rough) and ideal reflecting planes from [8] compared with the predictions from Kinney, NRDC, and the proposed formulation. As the compared scenarios (SHoB 54.4 and 90.7 cm/kg$^{1/3}$) are not available with the UFC model, the comparison with that model was not possible. The comparison shows that the reflecting surface used for the experiments constitute a hard and highly reflective surface, as the TP paths from the proposed formulation are between data for smooth–hard surface and ideal surface (no loss of energy). It also reveals that the models of Kinney and NRDC seem to be valid for surfaces between the smooth–hard and the soft–rough one. This difference in reflecting surfaces account for some or all of the differences observed in Sect. 3 between the new experimental data and the existing TP prediction models.

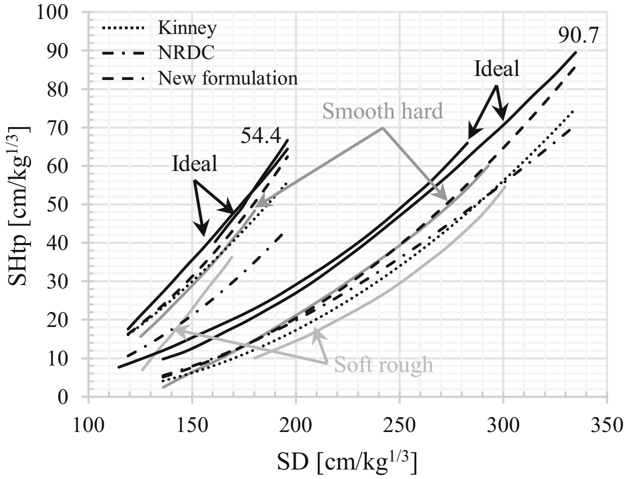

Fig. 10 TP paths of shocks reflecting from real surfaces (smooth–hard/soft–rough) and ideal reflecting planes [8], and comparison with the predictions from Kinney, NRDC, and the proposed formulation. The value near the curves corresponds to the SHoB in cm/kg$^{1/3}$

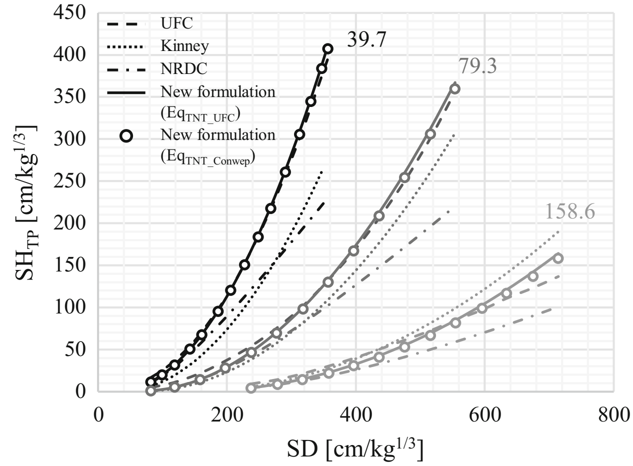

Fig. 11 Comparison of the TP path from the proposed formulation with the existing prediction models (UFC; Kinney; NRDC) for three scenarios. The TNT equivalent used to convert the new formulation to TNT scenarios is 1.1269 (*solid line*) or 1.28 (*circle*). The value near the curves corresponds to the SHoB in cm/kg$^{1/3}$

Nevertheless, more data are needed to validate this explanation.

Since the proposed formulation is reliable within its range of validity, it can be used to perform a comparison with the existing models. It implies that the proposed empirical model, made for C-4 explosive charges, must be modified for TNT, the explosive material used in the existing prediction models. Using the TNT equivalency, the new formulation for the calculation of SR_0 is:

$$SR_0 = 1.99.10^{-3} \sqrt[3]{Eq_{TNT}}\ SHoB^2 + 0.601\ SHoB. \quad (18)$$

where SR_0 and SHoB are for TNT masses.

The comparison is illustrated in Fig. 11, and the errors of the existing models compared with the new formulation are summarized in Table 5. It can be seen that the model of Kinney is in poor agreement with the proposed model, with errors greater than 20%. These deviations from the experimental data are due to the way SR_0 is calculated in the Kinney formulation. Similarly to Kinney, the formulation of NRDC induces errors around 25% compared with the new formulation. Unlike these two models, UFC predicted more accurately the TP paths, but only on a specific reduced scaled ground distance depending on the scenario. Examining more closely the performed comparison, it appears that the UFC model does not predict accurately the evolution of the TP for SH_{TP} below 50 cm/kg$^{1/3}$, corresponding to, for example, a TP height of 63 cm for 2 kg of TNT regardless of the value of the HoB.

A limitation of this comparison arises from the choice of the TNT equivalent mass (i.e., $Eq_{TNT} = 1.1269$ [7]). However, if the TNT equivalent mass from ConWep [4] had been used ($Eq_{TNT} = 1.28$), very few variations would have

Table 5 Relative errors (*e* in absolute value and in %) of the TP path prediction from existing models (UFC, Kinney, and NRDC) compared with the proposed formulation ($Eq_{TNT} = 1.1269$), where SD is the scaled ground distance

SHoB (cm/kg$^{1/3}$)	UFC	Kinney (%)	NRDC (%)
39.7	SD < 120:	$e \approx 35$	SD < 160:
	$e > 20\%$		$e < 15\%$
	SD > 120:		SD > 160:
	$e < 15\%$		$e > 20\%$
79.3	SD < 240:	$e \approx 22$	SD < 280:
	$e > 20\%$		$e < 15\%$
	SD > 240:		SD > 280:
	$e < 15\%$		$e > 20\%$
158.6	SD < 440:	$e \approx 20$	SD < 350:
	$e > 20\%$		$e < 15\%$
	SD > 440:		SD > 350:
	$e < 15\%$		$e > 20\%$

been noticed, as illustrated qualitatively in Fig. 11. Quantitatively, mean errors induced by the choice of the Conwep TNT equivalent regarding the prediction with the UFC one is 1.2, 2.8, and 4.5% for SHoB 39.7, 79.3, and 158.6 cm/kg$^{1/3}$, respectively. It means that the previous conclusions remain unchanged.

Even if the simple statistics presented indicate that the TP paths from UFC match correctly with the new formulation in a specific scaled ground distance, Fig. 11 shows that the discrepancies between these models should increase for higher SH_{TP} and SD (data not available for the UFC model). Adding to these conclusions that the UFC model does not predict either the trajectory of the TP outside of the ten curves proposed or the distance where the TP initially appears, these results show that the proposed formulation can be used as a simple and efficient tool for the prediction of the TP path for a wide range of scenarios. This new formulation can be used, for example:

– In numerical tools, such as the in the *Load_Blast_Enhanced function under LS-DYNA, which currently uses the UFC model
– With pressure prediction models to fully characterize the threat.

6 Conclusion

In order to mitigate the blast threat, either for building integrity or for personal security, empirical models describing the blast threat can be employed. Two issues are important: the determination of the TP path, in order to know which regime the target will encounter and the pressure–time histories below or above that path.

Three existing TP path models (UFC, Kinney, and NRDC), showing large internal deviations, have been evaluated with regard to new experimental data from the detonation of C-4 charges at different HoBs. The evaluation revealed that the existing TP path models do not accurately fit the experimental data, leading to the proposition of a new, simple, and efficient formulation based on these new experimental data. This new model is valid for SHoBs ranging from 24.6 to 172.9 $cm/kg^{1/3}$.

Combining this new TP path formulation with the determination of the pressure–time history from an existing model [9], the full blast characteristics in the location of interest can now be completely known.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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