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An evolutionary model of the size distribution of firms*

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Abstract. An analytical study of the evolution of the distribution of firm size in an industry is presented. A drift-diffusion model is proposed to express the time-evolution of density of firm size within the industry. The model blends the conventional, more or less static, determinants with the kinds of dynamic considerations introduced by stochastic processes of evolutionary dynamics. The steady-state distribution as well as the dynamic behavior of the model are derived. Parameters in the resulting analytical expressions are then fit to a population of firms in the non-manufacturing service sector. The empirical portion of the paper validates the proposed evolutionary model.

Key words: Industry evolution – Evolutionary dynamics – Size distribution of firms – Diffusion processes – Learning – Imitation – Incomplete information – Bounded rationality – Survival analysis

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1 Introduction

The economics literature is rich with theories in which firm size and the distribution of firm sizes are the determinate results of variables such as production technology and organization, transaction characteristics and institutional environment, and market power and market segmentation. This paper studies firm size distributions in the context of a different theoretical framework.

* This paper is based on work from a chapter of my Ph.D. dissertation (Hashemi 1995), completed at UCLA. I wish to thank my dissertation advisor, Jack Hirshleifer, for his invaluable support.

An analytical study of the evolution of firm size distribution in an industry is presented. In contrast to earlier literature on firm size distributions which focus on the evolution of size of an individual firm within an industry[1], the present paper develops a model to address the time evolution of density of firm size distribution, and postulates an evolutionary approach based on the theory of diffusion processes. The method illustrates how information on the time-evolution of the size distribution of firms over an extended period of time can be used to make inferences about an underlying process.

The study is motivated by the observation that the distribution of size of firms within an industry varies as a function of time. It would be reasonable to assume that there exists an equilibrium distribution of firm sizes with a certain mean and variance[2], towards which the ensemble of firms considered tend to converge. The study makes an attempt to model the time evolution of the size distribution of firms towards this equilibrium.

The dynamics of the proposed evolutionary model rely on two opposing forces: a mean reversion process which tends to concentrate the distribution, and a diffusion process which flattens it out. One of the parameters entering into diffusion is motivated by search and learning, using trial and error and imitation. Moreover, firm learning takes place in the presence of incomplete information and bounded rationality.

The question posed by this paper is an original one. While much attention has been given in the literature to the explanation of the shape of the size distribution at a given point in time by reference to steady state arguments, the question of how it changes over time has been relatively ignored. The main objective of this paper is to contribute to our understanding of some determinants of the dynamic process governing the size distribution of firms, and to build up a tractable structure for its analysis. This is achieved by deriving both the steady state distribution as well as the dynamic bahavior of the model. It is shown that interesting issues arise when one considers how firm size distributions evolve over time, rather than simply attending to equilibrium implications of processes. Moreover, the present paper is one of a rather small group of evolutionary studies which eschews simulation in favour of analytical derivations.

The model is applied to the evolution of the size distribution of a population of firms in the child care industry in Metropolitan Toronto, Canada, between the years of 1971–1987. The empirical portion of the paper illustrates that this population is well adapted to the model developed in this paper.

[1] These studies include Gibrat (1931), Simon (1955), Hart and Prais (1956), Simon and Bonini (1958), Ijiri and Simon (1977), Jovanovic (1982) and Hopenhayn (1992) among others.

[2] To illustrate, consider an industry in which average costs of producing an amount x of output are a non-increasing function of the size of the firm, for a given quality of output (each firm may have significant fixed costs, and marginal costs may essentially be constant). Furthermore, consumers prefer small firms for perceived higher quality of service. Under these assumptions, there would exist an equilibrium distribution of firm sizes with a certain mean and variance, determined by the tension between producer and consumer preferences regarding firm size. The equilibrium trades off productive efficiency against consumer preferences.

The paper is organized as follows. Section 2 develops and provides an analysis of the model. Section 3 reports some empirical results. Section 4 concludes and explores one way in which the model might fruitfully be elaborated.

2 Modeling the dynamics of firm size distribution

2.1 Basic assumptions

– Consider an industry consisting of N firms, identical, except with respect to size.
– There exists an equilibrium distribution of firm sizes with a certain unknown mean and variance.
– At any moment in time, a discrepancy exists between the current actual size distribution of firms and the equilibrium distribution. Firms find themselves in a dynamic adjustment process toward such an equilibrium, and the evolution of the size distribution reflects the convergence towards this equilibrium.

2.2 Development of the model

Let $n(s,t)$ be the measure of firms of size s at time t, where s measures some relevant aspect of size in logarithms. The total number of firms between s_1 and s_2 is

$$N\left(s_1, s_2, t\right) = \int_{s_1}^{s_2} n\left(s, t\right)\, ds \tag{1}$$

It is assumed that

$$N_0 = \int_{-\infty}^{\infty} n(s,t)\, ds < \infty$$

where N_0 is the total number of firms which is constant, so firms are neither created nor destroyed and

$$\frac{dN}{dt}\left(s_1, s_2, t\right) = Q\left(s_1, t\right) - Q\left(s_2, t\right) \tag{2}$$

where $Q\left(s, t\right)$ is the flux of firms of size s at time t. However it follows from (1) that

$$\frac{dN}{dt}\left(s_1, s_2, t\right) = \int_{s_1}^{s_2} \frac{\partial n}{\partial t}\left(s, t\right)\, ds \tag{3}$$

Thus we get

$$\int_{s_1}^{s_2} \frac{\partial n}{\partial t}\left(s, t\right)\, ds + Q\left(s_2, t\right) - Q\left(s_1, t\right) = 0 \tag{4}$$

It follows from the fundamental theorem of calculus that

$$Q\left(s_2, t\right) - Q\left(s_1, t\right) = \int_{s_1}^{s_2} \frac{\partial Q}{\partial s}\left(s, t\right)\, ds \tag{5}$$

Thus

$$\int_{s_1}^{s_2} \left(\frac{\partial n}{\partial t} + \frac{\partial Q}{\partial s}\right) ds = 0\,. \tag{6}$$

Since this equation is true for all s_1 and s_2 it follows

$$\frac{\partial n}{\partial t} + \frac{\partial Q}{\partial s} = 0\,.$$

So let

$$f(s,t) = \frac{n(s,t)}{N_0}$$

denote the probability density of firms. It then follows that

$$\frac{\partial f}{\partial t} + \frac{\partial q}{\partial s} = 0 \text{ where } q = \frac{Q}{N_0}. \tag{7}$$

This is the basic conservation law and q is a flux of probability (which can be interpreted as the variation, or number of firms entering and number of firms exiting a size interval).

In this paper it is assumed that the flux is made of two different parts: a *drift* q_c and some *diffusion (dispersion)* q_d. *Drift* describes supply and demand forces at work, and *dispersion* describes random processes. Thus,

$$q = q_c + q_d\,. \tag{8}$$

Drift: There exists some equilibrium distribution of firm sizes with a certain mean and variance, towards which the ensemble of firms tend. i. e., a flux towards the equilibrium distribution. Let $u > 0$ be the mean of this distribution.

The term q_c measures the portion of the function f transported by the drift velocity

$$v(s) = \lambda(u - s) \tag{9}$$

where $\lambda > 0$ is a fixed parameter.

Thus,

$$q_c = vf \tag{10}$$

i. e.

$$q_c(s,t) = \lambda(u - s)f(s,t)\,. \tag{11}$$

Diffusion: Although an equilibrium distribution exists, this equilibrium is assumed uncertain from the point of view of the firm. Moreover, owing to individuals' imperfect information and limited computational capacity, bounded rationality is assumed (Simon, 1947). Boundedly rational firms use various processes to arrive at the optimal distribution, and they may switch between processes over time. Firms follow no precise law to arrive at this optimum, they search and learn by means of trial and error and imitation[3]. This search process generates randomness in the system[4].

Random effects therefore tend to cause a flux from regions of low probability to high probability. The simplest choice is

$$q_d(s,t) = -\epsilon \frac{\partial f}{\partial s}(s,t) \tag{12}$$

where ϵ is a constant diffusion parameter. Thus we have

$$q = \lambda(u - s)f - \epsilon \frac{\partial f}{\partial s} \tag{13}$$

so

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial s}(\lambda(u - s)f) = \epsilon \frac{\partial^2 f}{\partial s^2} . \tag{14}$$

This is our equation for the density of firm size distribution, where f measures density and s measures some relevant aspect of size in logarithms.

Initial conditions in time and boundary conditions with respect to s are needed to completely formulate the model. In order to allow an analytic solution to the model equation, I impose the initial condition:

$$f(s,0) = f_0(s) = Ne^{\frac{-(s-u_0)^2}{2\sigma_0^2}}$$

on $s \in (-\infty, \infty)$, where N is the normalization constant, $N = 1/\sigma_0\sqrt{2\pi}$.

The dynamics of the model rely on two opposing forces: a mean reversion process which tends to concentrate the distribution towards an equilibrium one, and a diffusion (disruptive) process which flattens it out. The disruptive component counteracts the shrinkage of the variance, with ε representing a measure of perturbation. The stabilizing component however provides a competing force to offset this diffusion; it shrinks the variance of the distribution, with λ measuring the speed of convergence. In this model there exists an equilibrium distribution of firm sizes with mean u and variance σ^2, determined as the stabilizing forces that tend to eliminate diversity come into balance with disruptive (mutation) forces that constantly renew it.

[3] Following Alchian (1950), I propose that modes of behavior replace optimum equilibrium conditions as guiding rules of action, and emphasize a form of conscious adaptive behavior on the part of the individual firm, akin to *imitation* and *trial and error*.

[4] Although the present paper builds on previous literature such as Jovanovic (1982) and Hopenhayn (1992), it takes a slightly different approach. Jovanovic (1982) models the evolution of size of an individual firm over time, where noise is generated by firm specific shocks independent over time and across firms. Hopenhayn (1992) likewise models the time-evolution of firm size, where noise is generated by shocks. The model developed in this paper is concerned with the smooth evolution of density of firm size distribution, and noise in the present model is generated by the process of search and learning on the part of boundedly rational firms.

2.3 Analysis of the model

Given the prescribed initial and boundary conditions, it can be shown that the model contains a steady-state solution. Setting $\frac{\partial f}{\partial t}$ in equation (14) equal to 0, the steady-state behaviour can be expressed by the following equation:

$$f(s) = Ce^{-\frac{\lambda}{2\varepsilon}(s-u)^2} \tag{15}$$

In order to describe the time-behavior of the model, the model is solved for its analytic solution $f(s,t)$, describing the evolution of the distribution through time. There exists a unique analytic solution for this model, expressed by the following equation:

$$f(s,t) = Ne^{\lambda t}\sqrt{\frac{a}{a+\beta}}e^{-\frac{((s-u)e^{\lambda t}+u-u_0)^2}{2\sigma_0^2+\frac{2\varepsilon}{\lambda}(e^{2\lambda t}-1)}} = Ne^{\lambda t}\sqrt{\frac{a}{a+\beta}}e^{-\frac{(s-u_t)^2}{2\sigma_t^2}} \tag{16}$$

where

$$\begin{aligned}
a &= \frac{\sigma_0^2}{2}\\
\beta &= \frac{\varepsilon}{2\lambda}(e^{2\lambda t}-1)\\
u_t &= E[f]_t = u(1-e^{-\lambda t}) + u_0 e^{-\lambda t}\\
\sigma_t^2 &= \sigma_0^2 e^{-2\lambda t} + \frac{\varepsilon}{\lambda}(1-e^{-2\lambda t})
\end{aligned}$$

u_0 represents the initial mean size, σ_0^2 represents the initial variance of the distribution, and N is the normalization constant: $N = 1/\sigma_0\sqrt{2\pi}$.

The process derived from the model is size distributions of the population at chosen sequence of times through the observation period. From the analytic solution to this model, the dynamics of the size distribution can be followed through time given our initial distribution function f_0.

The expectation of the distribution shown in equation (16) is the first moment, expressed by:

$$u_t = u(1-e^{-\lambda t}) + u_0 e^{-\lambda t} \tag{17}$$

Moreover, using the expression for the second moment of the distribution, the variance of the distribution, σ_t^2, can be expressed by[5]:

$$\sigma_t^2 = \sigma_0^2 e^{-2\lambda t} + \frac{\varepsilon}{\lambda}(1-e^{-2\lambda t}) \tag{18}$$

To examine the behaviour of f in equation (16) as $t \to \infty$, one observes that $f(s,t) \to f_\infty(s)$ as $t \to \infty$, where

[5] Alternatively, a more straight-forward way to derive an expression for the mean and variance of the distribution $f(s,t)$ shown in equation (16), is to mathematically express this solution in the form $c_1 e^{\frac{-(s-c_2)^2}{2c_3}}$. The first and second moments can then be extracted from this expression.

$$f(s,t) \rightarrow_{t\rightarrow\infty} N\sqrt{\frac{2a\lambda}{\varepsilon}}e^{\frac{(s-u)^2}{2\frac{\varepsilon}{\lambda}}} = f_\infty(s)$$

and that $\lim_{t\rightarrow\infty} E[f]_t = u$. Moreover, the diffusive limit, i.e., the limit as $t \rightarrow \infty$ of the variance is[6]: $\lim_{t\rightarrow\infty} \sigma_t^2 = \varepsilon/\lambda$.

The model in equation (14) says that if we start with a normal distribution, and let the model drive the distribution, the distribution mean will converge to u, and variance to a constant, ε/λ. With the parameter estimates and the initial size distribution specified, the time dependence of the firm size distribution characteristics for a specific population of firms will next be found, from the model equations developed.

3 Empirical application

3.1 Data

The empirical analysis applies the model to the log size distribution of a population of child care service organizations in Metropolitan Toronto, Canada, between the years of 1971–1987. The data set consists of a total of 2214 firm sizes coming from 290 organizations, and observations were available annually[7].

Figure 1 provides a description of the evolution of the distribution of firm size in the data between the years of 1971–1987. The horizontal axis on each histogram measures firm size in logarithms, and the histograms are scaled such that the area under each histogram equals 1.

3.2 Method of estimation

A second order partial differential equation model has been proposed to express the dynamics of firm size distribution. The model has five parameters: $u_0, u, \epsilon, \sigma_0^2$, and$\lambda$. u_0 denotes the initial mean of the size distribution (1971), and u denotes where the initial mean is heading. σ_0 is the standard deviation at time zero (1971), ϵ represents the diffusion parameter, and λ determines the rate of convergence.

The statistical analysis is done in a Bayesian way (Zellner, 1975; Gelman et al., 1995) with non-informative prior distributions for all model parameters. The analysis can be divided into the following four main steps:

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[6] The process derived from the diffusion model evolves according to an Ornstein-Uhlenbeck, but with a transition, such that the mean tends to u, instead of 0 (Feller, 1966). The Ornstein-Uhlenbeck process is the most general normal stationary Markovian process with zero expectations. For $t > T$, the transition density from (T,s) to (t,y) is normal with expectation $e^{-\lambda(r-t)}s$ and variance $\sigma^2(1 - e^{-2\lambda(r-t)})$. As $t \rightarrow \infty$, the expectation tends to 0 and the variance to σ^2. The analytic solution derived for our diffusion equation is a normal distribution for all t. There is the $se^{\lambda t}$ factor; with a change of variables, it can be shown that the solution is normal with a constant multiplied by it.

[7] I am grateful to Joel Baum for his generous provision of the data set.

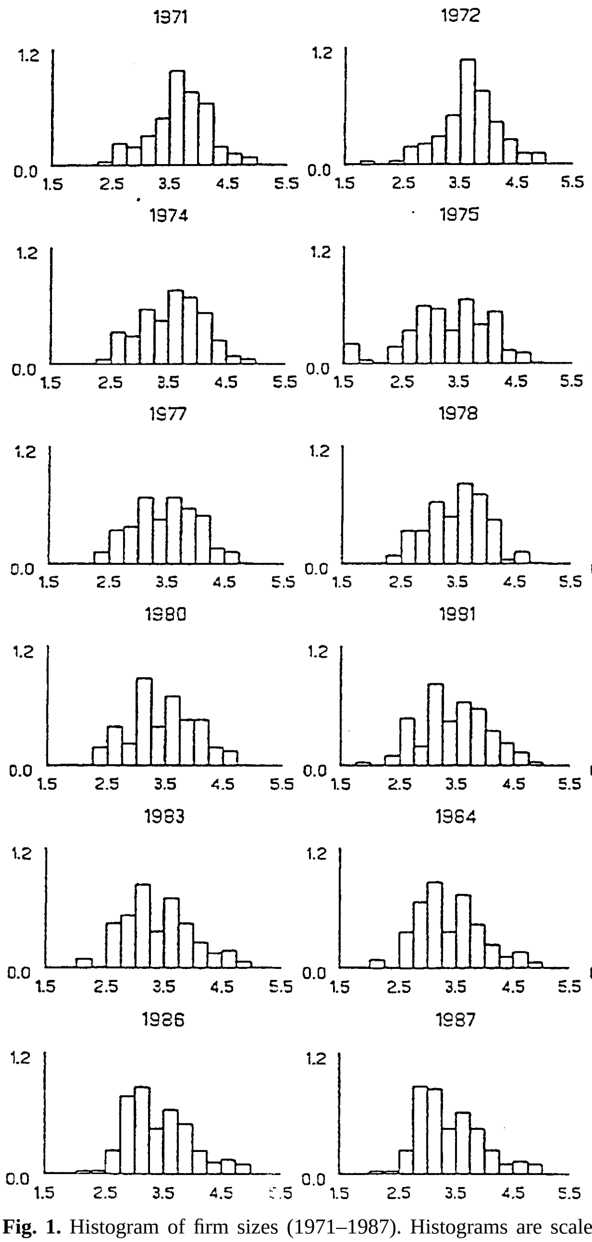

Fig. 1. Histogram of firm sizes (1971–1987). Histograms are scaled such that the area under each histogram equals 1

1. Specification of the statistical model: A statistical model, given by a family of probability distributions $f(D|\theta)$, needs to be specified. D denotes the (observed) data, and θ is a vector of unobservable or unobserved quantities (model parameters or missing data). $f(D|\theta)$ is the *likelihood function*.
   For the model of firm sizes we have $\theta = (u_0, u, \sigma_0^2, \epsilon, \lambda)$. The likelihood function is given by the product of normal densities with means $u_i(t)$ and $\sigma_i(t)$, where $u_i(t)$ and $\sigma_i(t)$ refer to firm size i $(i = 1, \ldots, n)$. $u_i(t)$ and $\sigma_i(t)$ follow from the normal diffusion parametrization (refer to equations (13) and (14)).
2. Specification of the *prior distribution*: Information about θ is incorporated into the *prior distribution* $\pi(\theta)$. $\pi(\theta)$ reflects what is known about θ before the data were observed. I used non-informative (flat) prior distributions for all model parameters.
3. Computation and interpretation of the *posterior distribution*, i. e., the conditional probability distribution of the parameters θ, given the observed data: The *posterior distribution* of θ, $g(\theta|D)$, is given by *Bayes theorem,* i. e.,

$$g(\theta|D) = \frac{f(D|\theta)\pi(\theta)}{\int f(D|\theta)\pi(\theta)d\theta}.$$

   The posterior distribution contains all the information about θ after having observed the data D. Computing the posterior distribution can be a difficult task. However, recent computational and methodological advances have made these computations feasible. In particular, Markov Chain Monte Carlo techniques have been the most popular tool to tackle problems in computing posterior distributions. Typically, after having obtained a sample from the posterior distribution, a posterior summary of θ consists of the posterior means (or medians) and posterior standard deviations (or posterior credibility intervals) for each parameter in θ.
   Markov Chain Monte Carlo (Gilks et al. 1996) was used for computing the posterior distribution; in particular, a random walk Metropolis algorithm (Tanner et al., 1993) with a burn-in of 50000 iterations followed by 6 Mio. iterations was used. Every 3000th iteration was stored, such that a posterior sample of size 2000 was used for the final results. Convergence checks of the Metropolis sampler revealed no problems regarding the stability of the estimates.
4. Evaluating the fit of the model: *Posterior predictive model checks* were used to check whether a specific feature of the observed data, say $T(D)$, was compatible with the model prediction. That is, $T(D)$ was compared to the posterior predictive distribution $h(T(D_{new})|D)$, where D_{new} denotes a new data set generated from the posterior distribution. The posterior predictive distribution was given by

$$h(T(D_{new})|D) = \int f(T(D_{new})|\theta)g(\theta|D)d\theta.$$

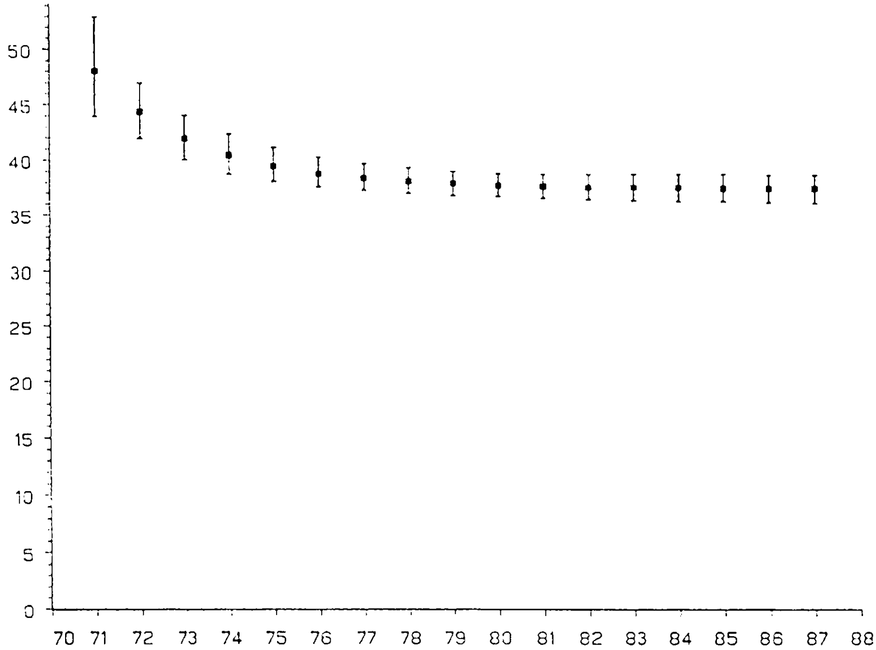

Fig. 2. Posterior means and 95% credibility intervals of means of log-normal distributions (1971–1987)

As Markov Chain Monte Carlo was used for computing the posterior distribution, the simulation of samples from the posterior predictive distribution was straighforward (Sect. 3.3 elaborates further).

3.3 Estimation results and model checks

The results obtained from the Bayesian analysis are reported in Tables 1 and 2, and in Figures 2 and 3. Table 1 reports the estimates obtained for the model parameters u_0, u, ϵ, σ_0, and λ, where for each parameter, posterior means and posterior 95% credibility intervals (in parantheses) are given. Table 2 reports the mean and standard deviation of log-normal distributions of firm sizes for each year, where the posterior means and posterior 95% credibility intervals are given. The means and standard deviations of the log-normal distributions follow from

$$X \sim N(u, \sigma^2) \implies Y = \exp(X) \sim LN(u, \sigma^2),$$

Table 1. Parameter estimates: posterior means and posterior 95% credibility intervals (in parentheses)

μ_0	μ	σ_0	ϵ	λ
3.73	3.44	0.53	0.15	0.44
(3.64, 3.82)	(3.40, 3.48)	(0.47, 0.60)	(0.07, 0.27)	(0.20, 0.78)

Table 2. Yearly means and standard deviations of log-normal distributions: posterior means and posterior 95% credibility intervals (in parantheses)

Year	Mean		Standard deviation	
1971	48.0	(43.9, 52.9)	27.2	(22.6, 33.3)
1972	44.3	(41.9, 46.9)	24.7	(24.7, 29.6)
1973	41.9	(40.0, 44.0)	26.3	(24.8, 28.1)
1974	40.4	(38.7, 42.3)	25.6	(24.2, 27.2)
1975	39.4	(38.0, 41.1)	25.1	(23.8, 26.6)
1976	38.7	(37.5, 40.2)	24.8	(23.5, 26.3)
1977	38.3	(37.2, 39.6)	24.5	(23.2, 26.0)
1978	38.0	(36.9, 39.2)	24.3	(23.1, 25.8)
1979	37.8	(36.7, 38.9)	24.2	(22.9, 25.7)
1980	37.6	(36.6, 38.7)	24.1	(22.8, 25.6)
1981	37.5	(36.4, 38.6)	24.1	(22.7, 25.5)
1982	37.4	(36.3, 38.6)	24.0	(22.7, 25.5)
1983	37.4	(36.2, 38.6)	24.0	(22.7, 25.4)
1984	37.4	(36.1, 38.6)	23.9	(22.6, 25.4)
1985	37.3	(36.1, 38.6)	23.9	(22.6, 25.4)
1986	37.3	(36.0, 38.5)	23.9	(22.5, 25.3)
1987	37.3	(35.9, 38.5)	23.9	(22.5, 25.3)

$$\text{with} \quad E[Y] = \exp(u)\exp(\sigma^2/2), \quad \text{Var}[Y] = \exp(2u)\exp(\sigma^2)(\exp(\sigma^2) - 1).$$

The results show a tendency towards a concentration of firm sizes: the mean of firm sizes decreases from 48.0 in 1971 to 37.3 in 1987, as shown in Figure 2, while the standard deviation decreases from 27.2 to 23.9 during the same time period. Figure 3 graphically illustrates the evolution of the firm size distribution (log-normals) over time, superimposed on the histograms of Figure 1 which describe the time evolution of the distribution of firm sizes in the data. This figure illustrates that the nice pattern which we see in the fitted log normals is being pulled out of a set of histograms whose shape is irregular.

Finally, I checked for possible violations of the diffusion model by comparing the empirical distribution of firm sizes with the model predictions. This was achieved by dividing yearly firm sizes into 10 intervals ($1-10, 11-20, 21-30, 31-40, 41-60, 61-80, 81-100, 101-150, 151-200$), and checking whether the observed frequencies were within their 95% posterior predictive credibility intervals. This is shown in Figure 4. The squares in this figure denote the observed frequencies and the vertical bars denote their 95% predictive credibility intervals. The results show no problems regarding the fit of the model, since among the 17×10 frequencies, only 5 (2.9%) fall outside their predictive credibility intervals.

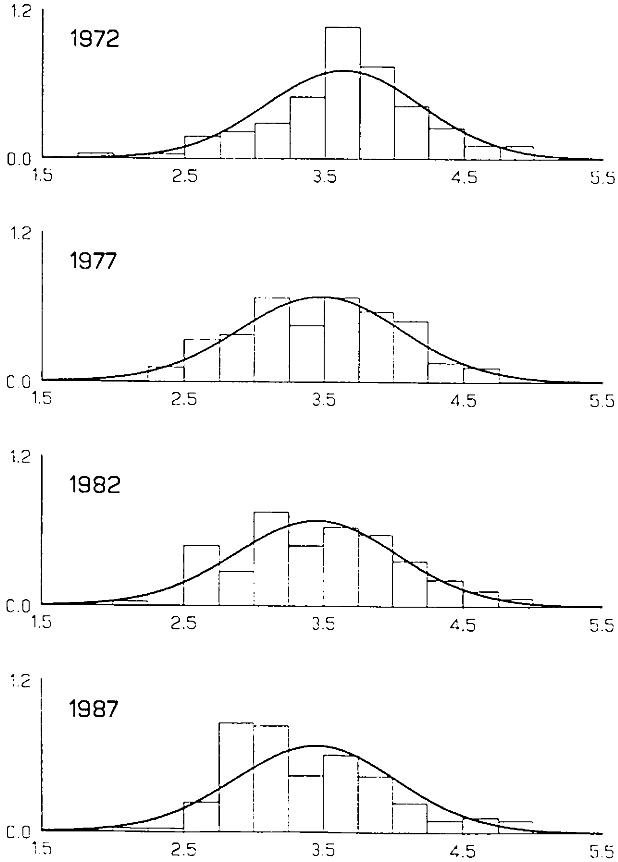

Fig. 3. Time evolution of the size distribution of firms (log-normals), superimposed on histograms of Figure 1, describing the time evolution of the distribution of firm sizes in the data (selected years)

4 Concluding remarks

This paper presents an analytical study of the evolution of the distribution of firm size in an industry. The dynamics of the model rely on two opposing forces: a mean reversion process, and a diffusion process. The steady state distribution of the model as well as its dynamic behavior are derived. Paramters in the resulting analytical expressions were fitted to data on the child care industry in Metropolitan Toronto, Canada.

4.1 Implications

The model developed in this paper connects up well with the literature on the *survivor principle* (Stigler, 1958), which has been widely used to determine the equilibrium distribution of firm sizes in industries (Saving, 1961; Weiss, 1964; Frech and Ginsberg, 1974; Keeler, 1989, among others). While the survivor

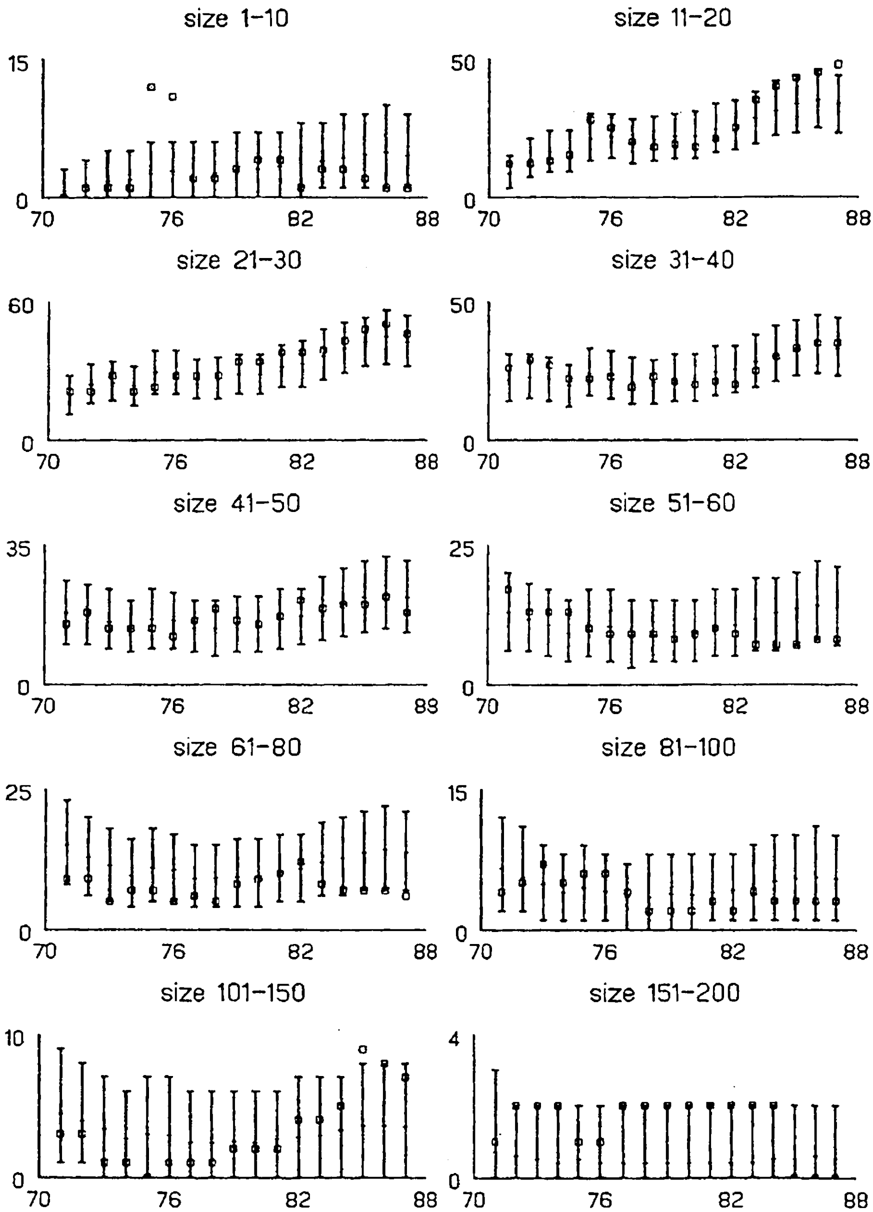

Fig. 4. Observed frequencies of the size intervals and posterior predictive distributions (means and 95% credibility intervals), 1971–1987

principle can be employed to draw inferences about the equilibrium distribution of firm sizes, it does not deal with the processes responsible for bringing about the equilibrium. The analysis in this paper goes beyond devoting attention to the final equilibrium, and focuses on the disequilibrium process, in light of the theories arising out of the evolutionary literature. It is shown that interesting issues arise when one considers how empirical size distributions evolve over time rather than simply attending to equilibrium implications of processes. The model developed in this paper further allows the estimation of both the final equilibrium distribution, and the rate of progress toward equilibrium.

4.2 Model limitations

By considerations of analytic tractability, the model developed in this paper constitutes a considerable simplification. The forces determining the distribution of firm sizes within an industry are so varied however, that any theoretical attempt to describe the effects of their interactions must be either simplified or else hopelessly complicated. Moreover, since a significant part of the Industrial organization literature is devoted to the study of industry evolution, even a simplified model for examining the disequilibrium process governing industry dynamics may not be without interest. In what follows, I will elaborate on two limitations in the present analysis.

One limitation owes to the assumption regarding the initial condition. In Section 2, the initial size distribution of the population was approximated to be distributed normally about some average value $[u_0]$: $f(s,0) = f_0(s)$ on $s \in [-\infty, \infty]$. This assumption, although limiting, was chosen on grounds that it allowed an analytic solution to the diffusion model. It can be removed using numerical methods.

A second limitation of the model developed in this paper, owes to the fact that it does not allow entry into and exit out of the population (births and deaths). As a result, the analysis does not allow for the determination of the relative importance of entry/exit versus growth of existing firms, in bringing about the evolution in the size distribution of firms.

Generally however, the diffusion model developed in this paper is capable of extrapolation to new and different situations. The model can be solved analytically for a variety of realistic assumptions pertaining to the rates of entry and exit. These extensions would no doubt enrich the model and should prove insightful but only at the expense of considerable complexity. I hope this paper is sufficient to indicate that the endeavor shows promise.

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