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REGULAR ARTICLE

Inequality and growth: the perverse relation between the productive and the non-productive assets of the economy

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Abstract The explosion generated by the global financial crisis in 2008 and its transmission to the real economies have been interpreted as calling for new kinds of regulation of the banking and the financial systems that would have allowed re-establishing a virtuous relation between the real and the financial sectors of the economy. In this paper we maintain a different view, that the financial crisis and the ensuing real crisis have roots in the strong increase in income inequality that has been taking place in the Western world in the last thirty years or so. This has created an all around aggregate demand deficiency crisis that has strongly reduced prospects and opportunities for investments in productive capacities and shifted resources toward other uses, thus feeding a perverse relation between the productive and the non-productive assets of the economy. In this context the way out of the crisis is re-establishing the right distributive conditions, which cannot be obtained by a policy aimed at relieving the weight of private or public debts but calls for a redistribution through taxes on the incomes of non-productive sectors, a fine tuning that should prevent excessive taxations transforming positive into negative effects.

Keywords Assets · Debt · Inequality · Taxation

JEL Classification D3 · E2

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1 Introduction

Money and financial assets have traditionally been regarded as allowing the real economy to run smoothly and sppedly. In this light, financial liberalization has been almost unconditionally welcomed as a good reform that would reduce rigidities that hamper growth. Thus in the last decades world economies have gone through a thorough financial liberalization that has transformed the international financial system from a government-led to a market-led one.

Experiences of low growth and financial mess, however, have shown a much less comforting reality. The explosion of the global financial crisis in 2008 and its transmission to the real economies, especially in the Western world, have been interpreted as calling for new kinds of regulation of the banking and the financial systems that would have allowed re-establishing a virtuous relation between the real and the financial sectors of the economy.

In this paper we maintain a different view, that the financial crisis and the ensuing real crisis have been essentially the result of a perverse relation that has its roots in the real economies: namely, in the strong increase in incomes inequality that, following fiscal, deregulation and privatization policies (Atkinson 1997, Levy and Temin 2007, Stiglitz 2012), has been taking place in the Western world in the last thirty years or so. As a matter of fact, the living conditions and real wages and salaries of both low and middle class workers have decreased substantially while profits and, in general, earnings of the top 1% have increased impressively, especially since the 2000s (Shupp 2002, Eckstein and Nagypál 2004, Atkinson 2008, Piketty and Saez 2006, Atkinson et al. 2011, Piketty and Zucman 2014, Piketty and Zucman 2014).

These issues have been analyzed in models making use of a DSGE framework (Kenc and Dibooglu 2007, Kumhof and Ranciere 2011, Kumhof et al. 2012). These models focus on a distributive shock that favours high-income households at the detriment of all other households, and affects negatively the performance of the economy.

Rather than describing an intertemporal equilibrium we intend instead focussing on an out-of-equilibrium path of the economy. In particular, we maintain that the excessive decrease of the median wage with respect to the average productivity has created an all-around *aggregate demand deficiency* crisis that has strongly reduced prospects and opportunities for investments in productive capacities and shifted resources toward other uses, thus feeding a perverse relation between the productive and the non-productive assets of the economy.

This paper is a first step in the analysis of this relation, in particular between finance and the real economy, trying to sketch out their interaction in the context of an economy where the increase in income inequalities and the resulting negative effects on final demand have substantially reduced growth rates or even brought the economies to stagnation, and where deflation rather than inflationary pressures appears as the main problem to be faced.

In our analysis, the economy consists of two sectors, which, to some extent, correspond to a 'real' and a 'financial' sector.[1] A productive sector is defined as dealing with assets and commodities that have to do with current or future production, including securities issued in a given period to finance real investments aimed at

[1] This definition draws on S. Bruno (2011).

creating productive capacity. A non-productive sector is defined as dealing with assets and commodities that already exist in a given period and that can be considered, and exchanged, as stores of values: such as residential houses, real estates, art objects, precious materials, oil, and so forth; as well as financial assets such as securities issued in the past the exchange of which has nothing to do with the creation of productive capacity and only implies a redistribution of property rights and hence of wealth (as it is also the case of most of the purchases of financial assets issued by purely financial corporations). This distinction is similar to the one stressed by Stiglitz (2014), according to which wealth and (productive) capital are markedly different objects, with the implication that wealth can go up as capital goes down.

The main difference between our analysis and the one developed by Piketty (2013), although sharing with the latter the focus on increases in the inequality of income distribution as the root of the dynamic processes characterizing capitalist economies. In the analysis of Piketty there is no distinction between productive and non-productive assets, between capital and wealth. Hence the rate of return of capital - its prevailing over time over the growth rate of the economy being stressed as the main reason of a trend interpreted as a secular stagnation - is calculated not on capital itself but on wealth as a whole, and, as a consequence, in accordance with neoclassical models but contrary to what happened with Ricardo and the classics, no distinction is made between profits and rents. This distinction, in our view, is instead essential not only to understand the origin and development of the ongoing crisis, but also the policies required to exit from it, in a perspective in which the evolution of the economy is not deterministically traced, as in Piketty, but is rather the result of the prevailing policies.

A main implication of the distinction just mentioned is that, since the transactions concerning the wealth assets constructed or issued in any period are a very small part of the transactions concerning similar assets produced or issued in the past, any change in the behavior of investors mainly implies price variations, likely resulting in capital gains and rents. As a matter of fact, an increase of the price of these assets is a strong incentive to the available financial resources toward the non-productive sector at the detriment of the productive sector: thus feeding a perverse relation whereby 'finance' in more general terms no longer sustains the growth of the real economy but rather hampers it.

This perverse relation appears as the engine of the crisis defined as a process of interacting disequilibria over time, stirred by a change in the income distribution that creates more inequality between capitalists and workers. This change results in a shrinking of final demand for goods produced in the productive sector, and hence in the reduction of the incentives to invest in activities that would allow the source of a significant productivity growth.

To carry out the analysis, three categories of income are considered: wages, profits and rents. The wages are fully absorbed by the consumption of goods produced by the productive sector, while the share of profits, which is not invested in productive capacities, is used for accumulating wealth or for consuming luxury goods, that is for buying goods of the so-called non-productive sector, and hence is in the nature of rents. We assume that higher inequality between wage earners and top earners (assimilated to rent earners) brings about an increase in the wage earners (bottom earners) debt-to-output ratio that helps alleviate the effect of this

change in income distribution on the final demand. As mentioned by Acemoglu (2011) and Rajan (2010), financial deregulation and political pressure are driving forces that encourage borrowing to keep stable the final demand despite diminishing revenues of bottom earners. Moreover, it appears that the wealth accumulated by rent earners may favor the credit supply to wage earners. This assumption is close to the one retained in DSGE models (Kumhof and Ranciere 2011; Kumhof et al. 2012; Kenc and Dibooglu 2007), in which a persistent shock to income distribution (i) increases the credit demand by households at the bottom of the income distribution due to consumption smoothing, and, at the same time, it increases the credit supply by richest households that exhibit a preference for wealth.[2]

Then, instead of focusing on the risk of default (Kumhof and Ranciere 2011; Kumhof et al. 2012), we will show that the main factor of the out-of-equilibrium process stirred by a reduction of the wage rate and an increase of rents, the one determining its path-dependence, is the existence of involuntary stocks, both real and financial (including unsustainable leverage), which allow fossilizing and transmitting the economic disequilibria over the successive steps that make up the process itself.

The focus will be, in particular, on the accumulation of debt as the result of credit activity, first aimed at reducing the recessive effects of the ongoing crisis, and which might lead to a collapse of the economy. The analysis carried out proves that even transforming the private debt into a public debt could not be a solution to this problem.

We will finally show that the way out of the crisis is represented by measures that reverse the effects of the increase in the inequality of the distribution that stirred the crisis itself, such as a public intervention consisting in taxing rents.[3] Of course, our objective in the paper is not to discuss how public spending is chosen. As a matter of fact, the actual shaping of public policy, and its effective modelling, depends on the specific problems of a given economy at a certain time, e.g. whether public spending is required to deal with a deficit of final demand or aimed at increasing the productivity of the economy, through direct investments and/or creating infrastructures, financing education and health to induce private investments in the near future.

In section 2, we will present the baseline model. In section 3, we will analyze the properties of the initial equilibrium and the out-of-equilibrium dynamics of the baseline model in order to obtain a benchmark. In section 4 we will present all the equations of the complete model and the properties of the initial equilibrium. The numerical simulations of the transition dynamics of the complete model are shown and commented in section 5. To analyze the impact of the parameters of the complete model on system's stability we provide a robustness analysis in section 6.

[2] This hypothesis, descending from Weber's notion of the "spirit of capitalism", has been stressed in the more recent literature as the reason of the fault of both the life-time-cycle and the asset price theory in explaining, respectively, the rich saving behaviors and the asset price evidences, as the equity premium puzzle. See Menchik and David 1983, Bakshi and Chen 1996, Carroll 1998, Francis 2009.

[3] The macroeconomic effects on distribution are considered in a much more similar framework; see Patriarca and Vona 2013. The stability properties of redistribution policies are also analyzed in an intertemporal optimization framework; see Garratt and Goenka 1995.

2 The baseline model

The evolution of the economy is analyzed by means of a model derived from the one built by Amendola and Gaffard (1998, 2006, 2014).[4] This model stresses the time structure of production processes, and is characterized by an intra period and an inter period sequence that make it possible to sketch out the interaction over time of decisions and events in a process of restructuring of productive capacities. Coherently to the out-of-equilibrium literature, agents update consumption and production plans according to realized market disequilibria.[5]

The elementary period is defined as the length of time required to carry out a round of final output. It is also looked at as the decision period: thus not only investment and final output decisions but also those concerning price changes (the price of final output and the wage rate) can only be revised at the junction of one period to the next. This makes the model a discrete time model. This model will be used to simulate the dynamics involved by a change in the income distribution brought about by a decrease in wages. The economy may be stable, that is, will converge to a long-term equilibrium, or may be unstable, driving to a final crash, according to the value of some key parameters. Once the stability conditions ar obtained, we will analyze the long-term equilibrium relationships, although this equilibrium cannot be always fully characterized, in the sense that the relationships that make it up are not fully known ex ante. This is the case when, as will be shown in the next sections, a path dependency emerges, with the consequence that some of the parameters that define the equilibrium relationships may change as the result of the above-mentioned processes.

We consider a steady state, not steady growth as the benchmark. However, we will not carry out a comparative dynamic analysis but will see how a shock will lead from the original to the final level, if any. That is, if the economy does not collapse along the way) of the relevant variables. In particular we will investigate what will determine the amount of the fall in output and employment caused by an increase in income inequality that is the difference in the levels of these variables from the moment in which the shock takes place up to the time at which the specific change ensuing will be complete. The model is grounded on two general hypotheses: the first is that wage earners (and the public sector) demand the good produced in the productive sector, and capitalists demand both the good produced in the productive sector and that of the non productive sector. The second is that the good produced in the non productive sector has a fixed quantity and a flexible price. This assumption reflects the consideration made in the introduction, according to which in the non productive sector the transactions carried out in any period concern a very small part of the total amount of commodities and assets issued or constructed in the past, thus resulting in significant fluctuations in prices. We simplify, therefore, by assuming fixed -quantity and flex-price. The opposite characterizes the productive sector, where we assume flex-quantity and fixed-price.[6] These hypotheses imply that an increase in production and

[4] See also Nardini (1993), Amendola et al. (2004), Gaffard and Saraceno (2008) and (2012), Patriarca and Vona (2013), Attar and Campioni (2007).

[5] As a result, agents behaviours are not modelled in an optimization framework.

[6] The distinction between fixed-price and flex-price markets has been introduced by Hicks who underlines the notion that only speculative markets such as markets for staple commodties and markets for securities are flexprice markets (Hicks 1974, p.78).

employment can only occur by means of an expansion of sector 1, and thus that everything that directly or indirectly shifts the income towards the consumers of sector 1 is finally assumed to have a positive effect on the economy (and vice versa).

Although in the model a negative shock on real wages will always bring about a worsening of economic conditions, we shall be able to throw light on the following issues:

- Stability: what can drive the economy affected by a distributive shock on an explosive path;
- Path dependency: when cumulative processes take place (in particular in cases allowing for indebtedness), their final effects;
- Transition dynamics: the specific evolution of the main variables along the transition;
- Policies: what may be the effect of policies affecting the distribution of income.

2.1 Technology, final output, employment

The economy portrayed in the model is made up of two sectors (s = 1, 2), sector 1 (the fix price sector) and sector 2 (the fix quantity sector). In each sector, production is carried-out by means of fully vertically integrated processes of Neo-Austrian type (Hicks 1973).[7] Each production process, with labor as the only primary factor, goes through a construction phase of productive capacity ($i \leq z$), characterized by a constant labor input coefficient $l^s_i = l^s$ and a length of z periods, and an infinite utilization phase ($i > z$) in which the input coefficient l^i_u and the output coefficient b^i both decrease at a constant rate δ keeping fixed the ratio between input and output. In this context, the investment is identified with the wages fund that allows the labor force to be employed and thus production processes to be started and carried on.

Figure 1 shows the correspondent time profile of the output and input coefficients.

In each period, productive capacity depends on the number of processes in the utilization phase. Defining $x^s_t(i) = x^s_{t-i+1}(1)$ as the number of processes of age i at time t (activated at t – i + 1), the overall potential output is given by:

$$B^s_t = \sum_{i=z+1}^{\infty} x^s_t(i) b^s_t \tag{1}$$

where the sum starts at z + 1, since younger processes are not yet productive. At the same time the employment in each sector depends on the number of production processes, both in the construction and the utilization phase:

$$L^s_t = \sum_{i=z+1}^{\infty} x^s_t(i) l^s_i \tag{2}$$

[7] Such time to build hypothesis is consistent with the class of models stressing the role of the time profile of production. For an extended survey of such class of model, see Hagemann and Scazzieri (2009), Landesmann and Scazzieri (2009).

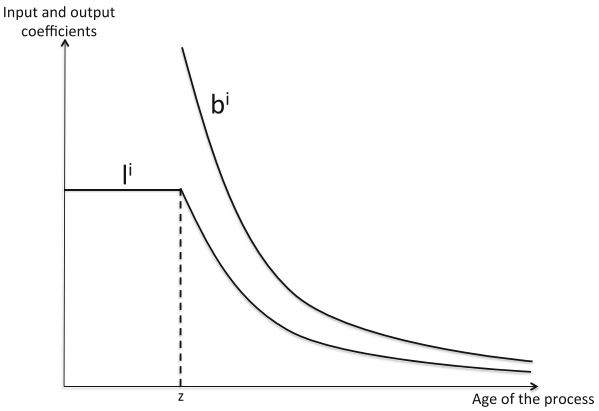

Fig. 1 Time profile of a production process

The macroeconomic equilibrium is given by:

$$p_t^s B_t^s = wL_t^s + \Pi_t^s \tag{3}$$

where w is the rate of wage, wL the wage fund, and Π the take-out (consumption out of profits), which can be assimilated to a rent.

In this framework, the wage fund is the amount of financial resources devoted to the construction and the utilization of production processes, and the capitalists' revenues are divided between the part devoted to the net investment in productive capacity (by paying wages) and the take-out. Here, as we shall see, the take-out (consumption out of profits) is integrally used to buy goods produced by the sector 2, and can be defined as a rent.

In what follows and in the simulations we will consider the two sectors to have the same technological parameters. Indeed, the sector distinction that we want to analyze is embodied in the respective fixed or flex price and flex or fixed quantity hypothesis described above; hence, the specific technological differences are not addressed directly.

2.2 Production and investment decisions

Given the potential output B^s_t, in each sector the decisions concerning how many already existing production processes to use and how many new processes to activate, are taken coherently with the expectations about final demand. These expectations are formed according to an adaptive rule:

$$eD_t^s = \alpha D_{t-1}^s + (1-\alpha) eD_{t-1}^s \tag{4}$$

If the expected demand is lower than the potential output B^s_t, fewer of the processes in utilization phase will be used: $x^s_t(j) < x^s_{t-1}(j-1)$ for some stages $j > z$.[8] Investment decisions, concerning the starting new production processes $x^s_t(1)$, intend to fill the gap between effective potential output and expected real demand[9]:

$$x^s_t(1) = \frac{\frac{ED^s_{t-1}}{p^s_t}}{b^s_{z+1}} + \sum_{j=z+1}^{\infty} x^s_t(j)\delta^{j-z-1} \tag{5}$$

where p^s_t is the price of sector s output as defined below. The second term is equal to the depreciation of capital such that, when productive capacity is at the desired level, it is maintained over time at this level (net investments are null and gross investments are equal to obsolescence). The first term is equal to the amount of investments needed to fill the gap between potential output and expected demand when the new process will arrive at the utilization phase.

Once the number of processes in all stages is determined,[10] employment in each sector is determined according to Eq. 2 and, given the wage rate w, the wage fund $W_t = w(L^1_t + L^2_t)$ is determined too. The sector 2 fix quantity hypothesis implies that the production in sector 2 is bounded to the initial equilibrium level of production B^2. Along an out-of-equilibrium path, once the population of production processes allows for a potential output equal to B^2, only replacement investments are realized and there are no further net investments.

2.3 Final demand

The sequence of events in each period takes place as follows: production decisions are taken; wages are paid; final markets open. At the end of the period, expectations and prices change according to the current disequilibria. According to this sequence, workers (W) and capitalists (K) face the respective budget constraints[11]:

$$D^W_t = W_t + H_t \tag{6}$$

$$D^K_t = p^1_{t-1}Y^1_{t-1} + p^2_{t-1}Y^2_{t-1} - W_t + F_t \tag{7}$$

where W_t is the wage fund, $p^s_{t-1}Y^s_{t-1}$ the proceeds of the previous period, and H_t and F_t are the money balances accumulated by wage earners and capitalists, respectively, that

[8] We assume that the processes possibly not activated are the older ones.

[9] We assume that the processes in the utilization phase not activated are not truncated but put aside to be possibly used in the future. Truncation would speed up a downward adjustment of productive capacity.

[10] We further make the hypothesis that all processes in the construction phase ($x^s_t(j)$ with $1 < j < z + 1$) keep being carried out until they reach the utilization phase.

[11] Since there is no heterogeneity within workers and within capitalists, we can consider the aggregate budget constraint as in a case of representative agents.

may result both from the accumulation of idle balances (if any) along the out-of-equilibrium path, and from other income sources such as bank credit and transfers from the private to the public sector. The functional distinction of income sources would determine the structure of final demand in case the two classes of income earners had different preferences. Here we consider the extreme case in which there is a complete polarization of consumption, that is, the wage earners only demand the good of sector 1 and the capitalists only demand the good of sector 2:

$$D_t^1 = D_t^W; \;\; D_t^2 = D_t^K \tag{8}$$

2.4 Market disequilibria and money balances

The outcome of each market Y_t^i is the lowest value between the demand in real terms D_t^i/p_t^i and the production B_t^i. While quantity excesses are supposed not to be storable, demand excesses are transferred to the next period, resulting in the idle balances F_{t+1} (demand excesses in the market of sector 2) and H_{t+1} (demand excesses in the market of sector 1) as additional demand sources. Thus:

$$\begin{aligned} H_t &= ED_{t-1}^1 = \max\left(0; \frac{D_{t-1}^1}{p_{t-1}^1} - B_{t-1}^1\right) \\ F_t &= ED_{t-1}^2 = \max\left(0; \frac{ED_{t-1}^2}{p_{t-1}^2} - B_{t-1}^2\right) \end{aligned} \tag{9}$$

Market disequilibria have an impact not only on expectations but also on prices. While we assume fix prices in sector 1, we make the hypothesis that in sector 2 the price changes from one period to the next in reaction to the disequilibria in the final market, with a given elasticity $ß^2$:

$$\frac{p_t^2}{p_{t-1}^2} = \beta^2 \frac{D_{t-1}^2}{p_{t-1}^2 Y_{t-1}^2} \tag{10}$$

3 Equilibrium and dynamic properties of the baseline model

3.1 Equilibrium conditions

We consider as a benchmark the steady state of the economy in which expectations are realized and the markets are in equilibrium. All variables are constant and the number of production processes $x_t^i(j)$ is constant at the equilibrium level x_E^i in every stage.

Thus, for a given level of x^1_E and x^2_E, (defining the real scale of the economy), the hypotheses on the technical coefficients imply[12]:

$$B^s_E = x^s_E \frac{b}{\delta} \quad L^s_E = x^s_E \left(zl + \frac{l}{\delta} \right) \tag{11}$$

$$\frac{B^s_E}{L^s_E} = \frac{b}{l(z\delta + 1)} = \frac{1}{\gamma} \tag{12}$$

where $1/\gamma$ is the productivity of labor (and γ the unitary labor costs) defined by the technical coefficients b, l, z and δ in case of a constant age distribution of the productive processes.

Since all markets are at equilibrium, prices are constant, investments equal capital depreciation, additional funds F and H are null. As to the nominal variables, equilibrium implies a given relationships between the sector distribution of production and the income distribution given by prices and wages. Indeed, as shown in the Appendix, the model outlined above is compatible with an equilibrium if:

$$\frac{Y^1_E}{Y^2_E} = \frac{\gamma}{\frac{p^1}{w} - \gamma} \tag{13}$$

Note that since workers consume only the good of sector 1, p^1/w is the inverse of the real wage. As a result, a higher distribution to workers is compatible with an equilibrium with a higher share of production of sector 1. The hypothesis of complete polarization of consumption also implies that p^2 will not influence the equilibrium share between the two sectors, since the price of good 2 only determines the distribution of income among the capitalists of the two sectors.

3.2 Change in income distribution

The full model presented below will be used to analyze the process stirred by a negative shock on wages, maintaining the hypotheses of fixed price p^1 in sector 1 and fixed quantity B^2 in sector 2. As a result, a shock considered will suddenly and permanently decrease the real wage w/p^1. At the same time employment, investments and production in sector 2 will be fixed at the initial level.

Whatever the dynamics involved by this shock, we have two general possible outcomes: explosive dynamics or the convergence to a new equilibrium. In this baseline version of the model, whenever the dynamics are stable, the hypotheses on

[12] According to Eqs. 1 and 2 and the hypotheses on the parameters, we have:

$$B^s_E = \sum_{i=z+1}^{\infty} x^s_E(i) b^s_i = \sum_{i=z+1}^{\infty} x^s_E(i) b(1-\delta)^{i-z-1} = x^s_E \frac{b}{\delta}$$

$$L^s_E = \sum_{i=1}^{\infty} x^s_E(i) l^s_i = \sum_{i=1}^{z} x^s_E(i) l + \sum_{i=z+1}^{\infty} x^s_E(i) l(1-\delta)^{i-z-1} = x^s_E \left(zl + \frac{l}{\delta} \right).$$

p^1 and B^2, together with the condition in Eq. 13 allow us to characterize fully the real side of the final equilibrium. Indeed, using $Y^2 = B^2$, p^1 and the new level of wages w in Eq. 13 we can obtain the new level of production Y^1 and then with Eq. 1 and 2 obtain all the other real variables.

As to the nominal variables, while p^1 doesn't change, the final level of p^2 cannot be analytically derived.

Besides, the equilibrium conditions states that for the given level of Y^2, the production of Y^1 is increasing in the real wage. Thus, after the negative shock on wages, if the system converges to a new equilibrium, this equilibrium will be characterized by a lower level of overall production and employment. As a result, from a comparative dynamic perspective we can state that the economy portrayed by the model corresponds to what in post-Keynesian frameworks is labelled as a "wage-led economy"[13] since higher real wages entail higher production and employment. Indeed, coherently with the purposes of the analysis discussed in the introduction, the baseline model is compatible with the general framework of crises primed by a fall of the final demand induced by an increase in income inequality.

3.3 Robustness

While the final equilibrium conditions can be established without running simulations, since the model includes non-linear behavior and binding constraints, it is not possible to prove analytically whether the economy will convergence to this equilibrium. We assess such stability properties by running a robustness analysis. We random select 100.000 different vectors of all the parameters. Then we choose one parameter and, by varying the value in its relevant range (for each level of the parameter), we trace the dynamics involved by a 5% negative shock on wages for all the combination of the other parameters in the random set. Then we observe the ratio of non-convergent cases as a function of the parameter. We repeat the analysis for all the parameters.[14] The results are shown in Figs. 2 and 3 where we plot the share of unstable transitions as a function of each parameter when all the other parameters vary in the 100.000 random sets.

The parameters that affect more the stability of the system are the two technical parameters related to the time dimension of the production process, z and δ. When the construction phase is reduced to one period (z = 1), the ratio of unstable paths is close to zero (0.6%). When z increases up to three periods the ratio of unstable transitions is close to one half. Higher instability is also related to higher levels of δ. This is due to the fact that the higher the depreciation rate, the stronger the link between the overall productive capacity and new investments.

[13] See Bhaduri (2008).

[14] We random select a vector of the relevant parameters in each respective significant interval: $\alpha \in (0;1)$; $\beta \in (0;1)$; $\sigma \in (0;1)$; $\tau \in (0;1)$; $r \in (0;0.2)$; $\delta \in (0;0.3)$; $z = (1,2,3)$; $p/w \in (\gamma; 1.3\gamma)$.

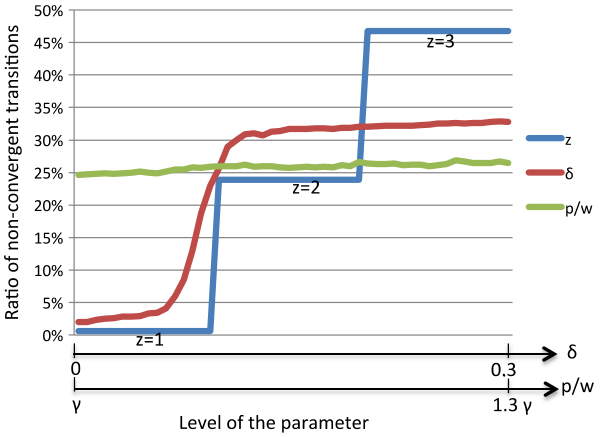

Fig. 2 Ratio of non-convergent transitions as a function of the parameters defining the initial conditions – benchmark case

The initial distribution of income between capitalists and workers, represented by p/w,[15] is less important though low levels of profits reduce the impact of the shock triggering a slightly lower stability. At the same time, the comparative dynamics tells us that low levels of p/w are associated with higher employment and overall production (Eq. 13).

The parameter β measures the price elasticity to market disequilibria in sector 2 and the parameter α the speed of adaptation of demand expectations. Since in this benchmark case the price in sector 2 does not affect the rest of the economy (it is also a matter of redistribution within capitalists), the rate of adjustment of the price in sector 2 (β) does not affect the stability of the system but only has an impact on the speed of the transition. Besides, a slow adaptation of the expectations and hence a higher persistence of investment decisions is the only key for the system to ensure stability, since only for low levels of (α) can we avoid unstable paths (0% of non-convergent cases).

All these results are strongly consistent with the out-of-equilibrium literature, which advocates a certain stickiness of agents' behavior to dampen the volatility of the system.[16]

4 The complete model

Having discussed the properties of the baseline model that embodies the main characteristics of out-of-equilibrium frameworks, we move now to the complete model and the evidences discussed in the introduction. The extensions of the model move in three different directions: consider the option of workers'

[15] The prices of the two goods are set equal at the initial equilibrium. Given the technological parameters that defines γ, p/w in a range (γ; 1.3γ) and thus corresponding to a profit margin spanning up to 30%.
[16] See Amendola and Gaffard (1998, 2003), Amendola and Vona (2012), Patriarca and Vona (2013).

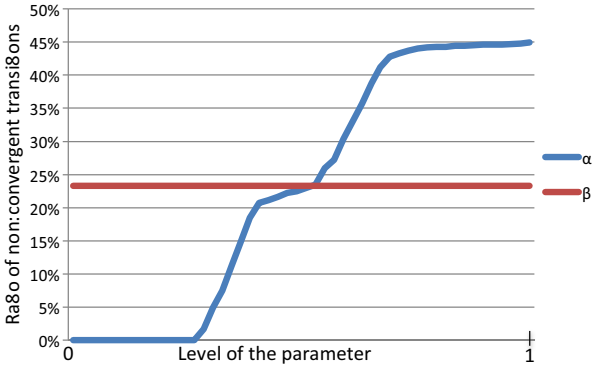

Fig. 3 Ratio of non-convergent transitions as a function of the parameters regulating the reaction to disequilibria – benchmark case

indebtedness, model a tax on capitalists' revenues and allow for a price reallocation effect on investments driven by the dynamics of relative prices in the two sectors.

4.1 Credit

Coherently with the "love for wealth" framework introduced in the previous section, we assume now that, for capitalists, a positive credit position is a substitute for their consumption. Thus, when capitalists' demand is rationed, a share σ of their demand excess is transformed into credit to workers and only the residual is transferred to the next period[17]:

$$NC_t = \sigma DE^C_{t-1} \tag{14}$$

where DE^C_t is the possible demand excess of sector 2, and NC_t the new credit issued. Considering the budget constraint in Eq. 5, we have now to include in H_t all the non-wage workers' funds given by:

$$H^W_t = DE^W_{t-1} + NC_t - rC_t \tag{15}$$

where DE^W_{t-1} are the eventual demand excesses of workers that may have occurred out-of-equilibrium, NC_t is the new credit issued to the workers, and rC_t is the interest paid on the stock of debt C_t.

[17] For a deeper assessment of credit markets in an out-of-equilibrium framework, see Attar and Campioni 2007, while for the modelling of the "love for wealth" in general equilibrium frameworks, see Kenc and Dibooglu (2007).

4.2 Taxation

Let us assume now that the Government obtains resources to provide for productive needs (infrastructures, education, health,…) by raising taxes on the rents of capitalists, that is

$$T_t = \tau\left(p^1_{t-1}Y^1_{t-1} + p^2_{t-1}Y^2_{t-1} - W_{t-1} + rC_{t-1}\right) = G_t \tag{16}$$

where T is the amount of taxes raised, τ is the tax rate and G the Government expenditure.

The proceeds from the tax are added to the demand of the good of sector 1[18]:

$$D^1_t = D^W_t + G_t \tag{17}$$

At the same time, when considering the capitalists' budget constraint' we have to take into account in the definition of F_t both the interest on the debt and the tax paid:

$$F_t = (1-\sigma)DE^2_{t-1} + rC_{t-1} - T_{t-1} \tag{18}$$

and Eq. (7) will still holds.

4.3 The investment reallocation effect

An investment reallocation effect between the two sectors can take place due to the expectations of capital gains resulting from the variations of relative prices:

$$x^i_t(1) = \frac{\frac{ED^i_{t-1}}{p^s_t} - B^i_t}{b^s_{z+1}} + \sum_{j=z+1}^{\infty} x^s_t(j)\delta^{j-z-1}\zeta\frac{p^{-s}_t - p^{-s}_{t-1}}{p^{-s}_{t-1}}\frac{p^s_{t-1}}{p^s_t - p^s_{t-1}} \tag{19}$$

Note that our hypotheses of fix quantity/or price that distinguish the two sectors imply that only the price of sector 2 changes and that it can only affect the investments of sector 1 since net investments in sector 2 are null.

4.4 Equilibrium properties of the complete model

In the Appendix we derive the conditions for which the economic system represented by the complete model is compatible with a steady state equilibrium where no demand excesses occur, indebtedness is constant, prices are fixed and the amount of production processes of each age is constant. At the same time, as in the case of the baseline model, the issue of whether the out-of-equilibrium paths involved by a shock converge or not to such an equilibrium is not analytically solvable given the structure of the model out

[18] Our objective in the paper is not to discuss how public spending is chosen. Were the public spending non productive, the public policy would be inefficient with regard to the problem dealt with.

of equilibrium. Before analyzing the dynamics, it is worthwhile to analyze the equilibrium conditions.

For a given level of the parameters, the equilibrium implies a relation between prices, and the ratio between the productions of the two sectors Y^1_e/Y^2_e.[19] The equilibrium condition is:

$$\frac{Y^1_e}{Y^2_e} = \frac{\gamma + \tau\left(\frac{p^2}{w} - \gamma\right) - (1-\tau) r d_e \gamma}{(1-\tau)\left(\frac{p^1}{w} - \gamma + r d_e \gamma\right)} \tag{20}$$

where d_e is the workers' debt to income ratio C/W. Note that, as for Eq. 12, the hypothesis of fox quantity in sector 2 implies that both Y^1 and total employment are increasing in the ratio defined in the r.s.h. of Eq. 19, and thus they increase in the parameter γ, in τ and in p^2/w, while they decrease in p^1/w, r and d_e. As a result, from a comparative dynamics perspective, we can state that, for the complete model, higher real wages (w/p^1) are associated with higher production and employment, and that the debt conditions affect negatively the equilibrium level of overall output and employment. From the same analytical perspective a higher public intervention (τ) sustains employment and overall production.

Once the system is shocked we cannot know ex-ante whether the out-of-equilibrium path leads to an equilibrium and, in this case, to which one. Indeed, the new debt ratio d_e and the price p^2 depend on what happens along the transition and can not be predicted ex-ante. As a consequence, the results stated in the comparative dynamic perspective could not completely hold whenever we move to a truly dynamic context. In particular, along the transition the parameters α, β and σ, which are not present in the equilibrium condition, will also play a role. If we consider a setting without taxation and without credit, Eq. 20 reduces to the equilibrium condition of the baseline model in Eq. 13. All the other parameters, including the new parameter ζ of the price reallocation effects are not directly involved in the equilibrium condition. However they impact the out-of-equilibrium transition and thus have an indirect impact allowing or not for the convergence to an equilibrium and determining the final values of Y^1_e, d_e and p^2.

5 Numerical simulations of transition dynamics

We shall now analyze the transition dynamics of the economy involved by an increase in the inequality of the distribution of income due to a reduction of the wage rate.

5.1 The benchmark case

As discussed in section 3, in the case of the baseline model, the effect of the shock in terms of conditions of the finale equilibrium can be obtained analytically. Thus, simulations have mostly the illustrative character of a benchmark case.

[19] In the simulations,we consider the same initial level of prices in the two sectors.

A lower wage rate with a fixed-price p^1 is associated with a lower real wage. In sector 1 we shall, hence, have a smaller wage bill and a lower demand for good 1. The resulting excess supply of good 1, given adaptive expectations, will lead to less output and less employment, further falls in the wage bill and hence a negative evolution in sector 1. The fall in output decelerates, though, because the dynamics of the demand for good 1, on which production plans depend, puts a brake on this fall. As a matter of fact, only one component of this demand, that depending on the sector bill of sector 1, falls in accordance with output. The demand coming from sector 2, after the first reduction in the wage rate, remains constant, as the output and hence employment and the wage bill of sector 2 are fixed. Thus, on the whole, the demand for good 1 falls less rapidly than its production, putting an increasing brake on the fall of the last. The system will thus converge to a new equilibrium, worse than the initial one.

At the same time, the money saved from wages will be used by the capitalists to increase the demand of good 2, which, given the fixed supply, will result in an excess demand and an increase in the price p^2. However, as the output of good 1 keeps falling, total profits, notwithstanding the lower wages, decrease, and the dynamics of p^2 decelerates: the price system as well will then converge to a new equilibrium.

Figure 4 shows the dynamics of the output of sector 1 and the price of sector 2 for a given set of parameters.

5.2 Credit

Consider now the possibility of capitalists making loans to wage earners, to alleviate the effects of the assumed wages reduction.

In each period only the interest on the debt is repaid, so that there is an accumulation leading to the appearance of a *stock of debts* variable $\boldsymbol{C}$.

In this case, although there are some short term advantages in that the fall of output and employment is slowed with respect to the case without credit, the accumulation of a stock of debts leads in the long run to completely different results: a worse equilibrium level in the case of a low interest rate (green lines in Fig. 5), but a collapse of the economy when the interest rate is higher (red lines). This latter case happens because the interest over the cumulated stock of debts absorbs the whole wage bill. There is a cut off value of r that can be determined.

It is the existence of stocks that determines the path-dependence of the out-of-equilibrium process stirred by the initial change in the distribution and that results in

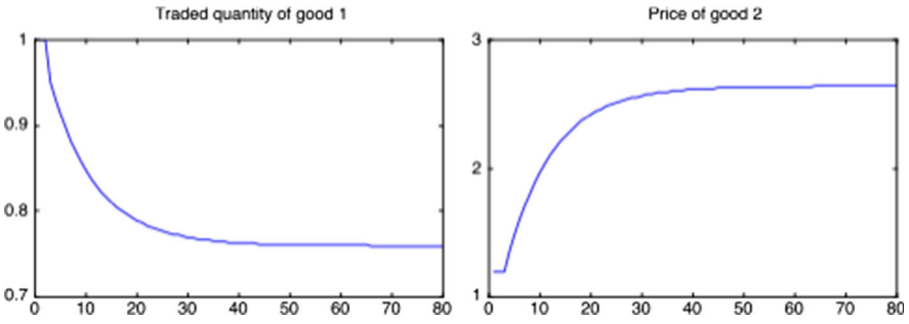

Fig. 4 The benchmark case (dynamics over 80 iterations)

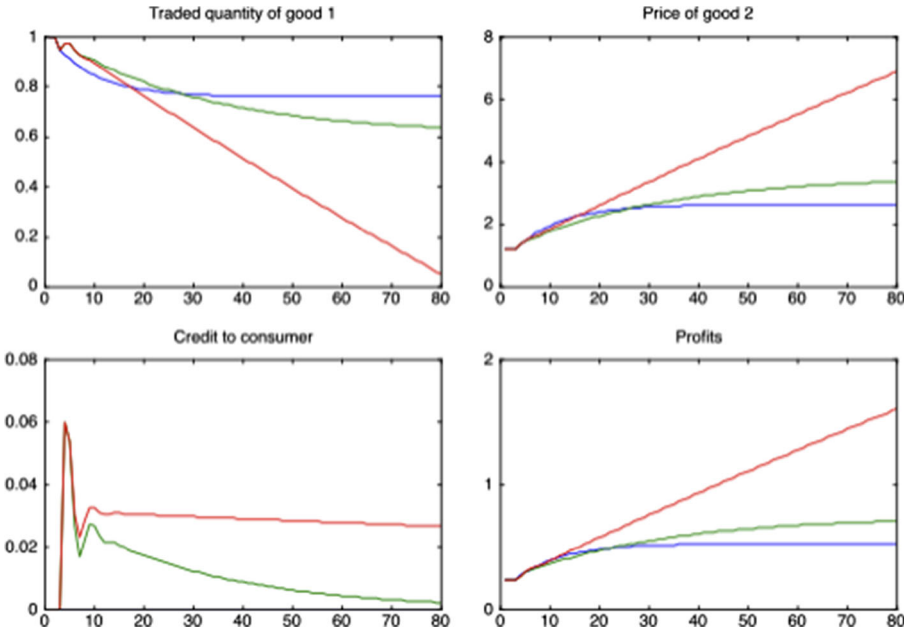

Fig. 5 Dynamics with credit*. *The blue line is the benchmark case, the green and the red lines correspond, respectively, to the cases with $r = 0{,}02$ and $r = 0{,}15$

further changes in the distribution (increases in the inequality), aggravating the results of the crisis or even leading to the collapse of the economy.

5.3 Investment reallocation effect

A further negative effect that substantiates the perverse relation between the two sectors of the economy is the crowding out of resources, subtracted from the investment in sector 1 to be employed in sector 2, as the result of the higher rewards due to the increasing prices in that sector.

The negative effect, present more or less strongly in the short run according to the lower (green lines) or the stronger (red lines) shift of resources due to a lower or stronger sensibility to price changes, does not affect the final equilibrium level of output and employment in sector 1, as shown in Fig. 6.

5.4 Effects of taxation

Taxes on rents shift incomes from the consumers of good 2 towards the consumers of good 1, thus affecting the evolution of the economy.

As a matter of fact, taxes on rents have a double effect. On the one hand, they bring about an increase in the demand for the productive good, and hence in its production and in the amount of rents realized in the productive sector. On the other, they reduce the rents realized in the non-productive sector, and hence the demand of the non-productive goods, slowing down the inflation in this sector.

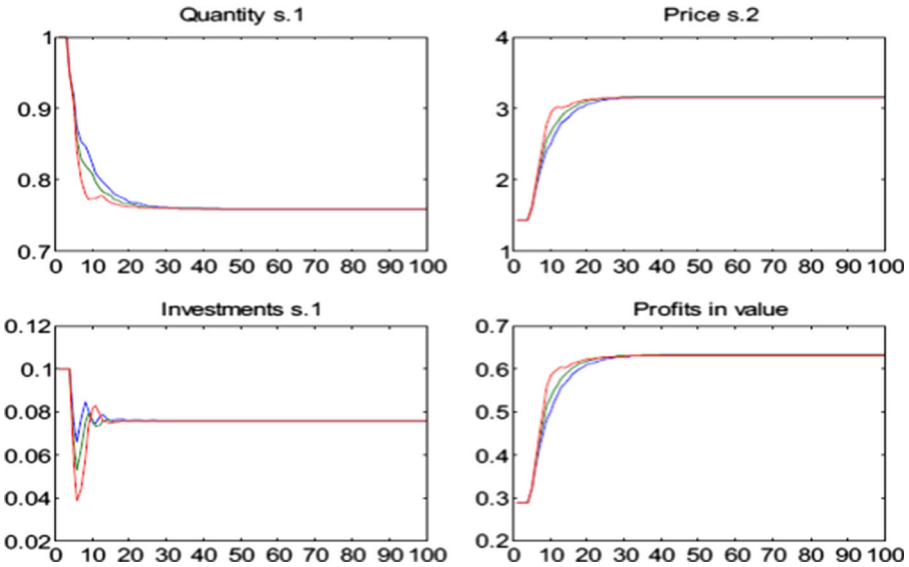

Fig. 6 Dynamics with price reallocation effects*.*The blue line is the case $\zeta = 0{,}1$, the green and the red lines correspond, respectively, to the cases with $\zeta = 0{,}5$ and $\zeta = 1$

As long as the first effect prevails over the second, the amount of resources obtained by the government from taxation, and hence its demand for the productive good, increases, thus counteracting the negative effect on the economy of the initial reduction in wages: as shown in Fig. 7 by the evolution of the output and of the employment in

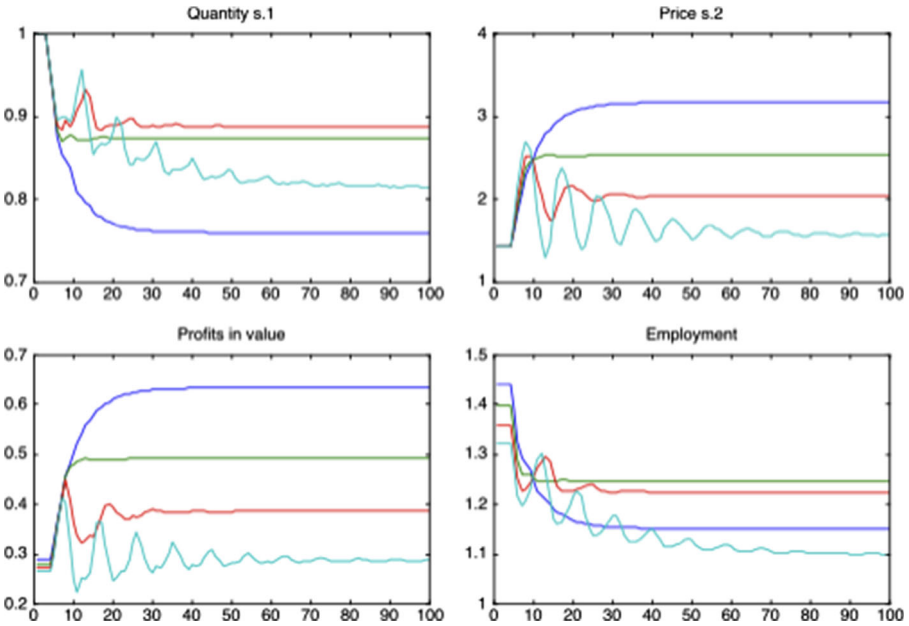

Fig. 7 Dynamics with a tax on rents*. *The blue line is the benchmark case the green lines is associated to a tax rate of 15%, the red lines to one of 30% and the light blue lines with one of 40%

the sector 1, as traced by the green line (case with a tax on rents) with respect to the blue line (the benchmark case without a tax on rents).

Whether the first effect prevails over the second depends on the tax rate. Beyond a certain value, in fact, it reduces the total amount of rents to tax, and hence the demand for the productive good financed by this tax. There is a cut off value of this rate: up to this value an increase in the tax rate goes on reducing the negative effect on output and employment of the initial wages reduction. A higher tax rate reduces this positive effect. In Fig. 5 this cut off value is the one associated with the evolution of the economy traced by the red line, as concerns the effect on employment, while a still higher tax rate affects the dynamics of production, as is the case of the evolution of the economy traced by the light blue line.

A tax on rents may also affect the viability of the economy in the presence of a crowding-out effect, that is, a shift of investments from sector 1 to sector 2 due to expectations of capital gains resulting from variations of relative prices. In Fig. 8, different crowding-out effects on the evolution of output due to different sensitivity to prices changes are shown by the different colours of the functions: from the blue – the benchmark case without crowding-out – to the violet - the higher effect.

Up to a certain value of the tax rate on rents, measured along the horizontal axis, this tax has no effect on the spreading of the crowding out effect, but from a certain value on (around 10% in Fig. 8) increasing tax rates widen this spread, casting increasing doubt on the viability of the economy.

6 Robustness and stability

We move now to the analysis of the impact of the parameters of the complete model on system's stability by the mean of the same analysis done for the benchmark case.

Figure 9 is the analogue of Fig. 2 displaying the ratio of unstable transitions of our set of 100.000 random sets of parameters as a function of each single parameter. We note that the impact of the technological parameters defining the time dimension of technology is the same as in the benchmark case. However, in this case the initial distribution of income embodied in the parameter p/w is much more important.

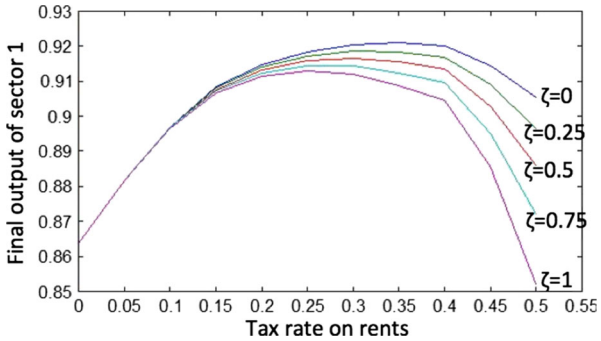

Fig. 8 Final output of sector 1 as a function of the tax rate on rents for different levels of crowding-out

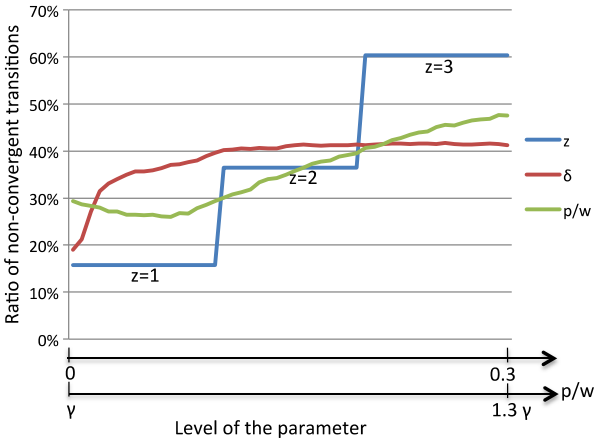

Fig. 9 Ratio of non-convergent transitions as a function of the parameters defining the initial conditions – full model

As noted in the previous section, moving to the complete model, we add a further and important root of instability confirmed by the higher average ratio of unstable paths: the risk of running a debt crisis. Accordingly, since high values of distribution to capital allow for a higher base of credit creation along all the transition, this entails a greater instability. At the same time for very low values of p/w, the relation slightly changes sign. This is mostly a scale effect since a 5% negative shock on wages produces a correspondent positive shock on p/w and then for low levels of this parameter, the revenue margins of capitalists are more then doubled, bringing a sudden and strong credit supply at the beginning of the transition. Confronting Figs. 10 and 3 we can see that, while the speed of adjustment of expectations α has the same impact as in the baseline case, allowing for indebtedness entails an important role also for the parameter β. A very slow adaptation of the price in sector 2 lengthens the demand excesses of the capitalists favoring the creation of higher credit and thus favoring the roots of instability involved by worker indebtedness. The relationship is not linear since for

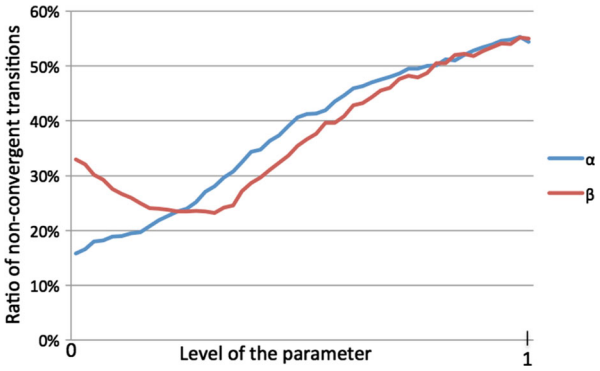

Fig. 10 Ratio of non-convergent transitions as a function of the parameters regulating the reaction to disequilibria – full model

intermediate and high values of the price flexibility: investments reallocation effects prevail accelerating the crisis of sector 1.

Figure 11 shows the robustness results for the parameters which were not present in the benchmark case. As noticed above, the interest rate plays an important role in the risk of a final debt crisis. The parameter σ has a similar role of r since it determines the extent to which capitalists' excesses are transformed into credit. A very similar impact is the one concerning the investments reallocation effect represented by the parameter ζ confirming it as being a source of instability.

Finally, we can take into consideration the role of the tax rate. In addition to the analysis done above, in the case of stable transitions, intermediate values of τ bring about final equilibria with higher production and employment, that is, we can say, with a higher stability of the system.

7 Conclusion

We have shown how an increase in income inequality may substantially reduce the output of the economy and its employment rate due to an aggregate demand deficiency that reduces prospects and opportunities for investments in productive capacities and shifts resources toward what we have defined as the non-productive sector of the economy. This takes place through an out-of-equilibrium process, the profile of which depends on the emergence of involuntary stocks, both real and financial (including unsustainable leverage), which allow for fossilizing and transmitting the economic disequilibria over the successive steps of the process itself.

The focus has been put in particular on the accumulation of stocks of debt, as the result of a private credit activity first aimed at reducing the recessive effects of the crisis stirred by the original change in the income distribution. The option of indebtedness has been proved to slow down the effects of the crisis in the short run, but has a permanent negative effect due to the implicit further redistribution of incomes represented by the interest to be paid on the debts, the stronger the higher the interest rate. The analysis

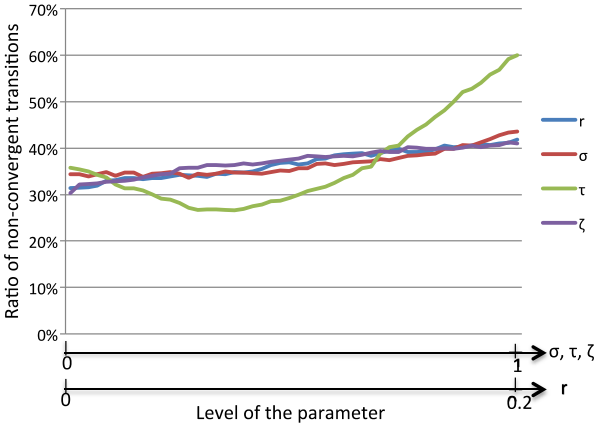

Fig. 11 Ratio of non-convergent transitions as a function of the parameters regulating credit and the tax rate – full model

carried out also proves that transforming the private indebtedness into a public indebtedness does not change the results obtained.

The role of the Government is then not to be a substitute for the private sector but to restore the right distributive conditions. A powerful tool is a tax on rents, standing for all incomes originating a demand for the goods of the non-productive sector. As a matter of fact, taxing wages to finance the public sector does not change the evolution of the economy, as it represents just a shift of resources from the private to the public sector that leaves unchanged the amount of the demand for the goods of the productive sector, which is the factor actually determining both real production and employment. It would even have a negative effect if the proceeds of the tax were spent on goods of sector 2, that is, if the Government behaved as a rent earner.

Tax on rents instead shift incomes from the consumers of non-productive goods towards the consumers of productive goods, to whom the government is assimilated, thus actually affecting the evolution of the economy. However, the final result depends on the level of the tax. Beyond a certain value, we have seen it reduces the total amount of rents to tax, and hence the demand of the productive good financed by this tax.

In a crisis due to an increasing income inequality, and hence to a shift of demand from the productive to the non-productive sector, the way out is re-establishing the right distributive conditions, which cannot be obtained by a policy aimed at relieving the weight of private debts but calls for a redistribution through taxes on the incomes of non-productive sectors according to a fine tuning that should prevent excessive taxations transforming positive into negative effects.

Appendix

In this section we check whether the benchmark and the complete models described in Eqs. 1–10 and 14–19 are compatible with a steady state equilibrium and prove the results in Eq. 13 and 20.

By definition, at a steady state equilibrium, demand excesses are null, all variables including the indebtedness are constant (no new debt is issued), demand expectations are realized and the value of production is constant and equals the demand in each sector. Considering Eq. 6–7 and 16–18 we have:

$$p_e^1 Y_e^1 = D_e^1 + G_e = W_e - rC_e + \tau\left(p_e^1 Y_e^1 + p_e^2 Y_e^2 - W_e + rC_e\right) \tag{21}$$

$$p_e^2 Y_e^2 = D_e^2 = F_e - W_e = p_e^1 Y_e^1 + p_e^2 Y_e^2 - T_e - W_e + rC_e \tag{22}$$

These two conditions are equivalent and since we have defined de = Ce/We, they are satisfied if and only if:

$$(1-\tau)p_e^1 Y_e^1 - (1-\tau)(1-rd_e)W_e^1 = \tau p_e^2 Y_e^2 + (1-\tau)(1-rd_e)W_e^2 \tag{23}$$

where W^s_e is the wage fund paid in sector s.

Furthermore, the steady state implies that the number of processes of each age is the same and constant ($x^s(i) = x^s$). Considering Eq. 11 the ratio W^s/B^s is:

$$\frac{W_e^s}{B_s^e} = w\gamma \tag{24}$$

Using this definition in the equilibrium condition in 23 and rearranging we obtain Eq. 20 and, taking $\tau = 0$ and $d_e = 0$, we obtain 13 as a particular case. Note that, since the firms would not produce with negative profits ($p^sB^s \geq W^s$), we must have $p^s/w \geq \gamma$ and thus a sufficient condition for the ratio in the right hand side of the equilibrium condition in Eq. 20 to be positive is that $rd_e \leq 1$, that is, it is verified whenever the interest on their debt paid by the workers are lower than their labor income.

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