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REGULAR ARTICLE

Spontaneous economic order

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Abstract This paper provides attempts to formalize Hayek's notion of spontaneous order within the framework of an Arrow-Debreu economy. Our study shows that, if a competitive economy is sufficiently fair and free, a spontaneous economic order will emerge in long-run competitive equilibria so that social members spontaneously occupy an unplanned distribution of income. Despite this, the spontaneous order may degenerate in the form of economic crises whenever an equilibrium economy approaches the extreme competition. Remarkably, such a theoretical framework of spontaneous order provides a bridge linking Austrian economics and neoclassical economics, where a truth begins to emerge: "Freedom promotes technological progress".

Keywords General equilibrium · Spontaneous order · Rawls' fairness · Freedom · Technological progress

JEL classifications D5 · D63 · B25

1 Introduction

Spontaneous order in economic interactions presented by Hayek (1948) is an important notion for economics. This notion originates from the interactions of members of society and is something to which everyone contributes, from which everyone benefits, which everyone normally takes for granted, but which individuals rarely understand (Witt 1997). Hayek believed that, if the degree of freedom within a society, under the constraint of limited resources, achieves a maximum, a spontaneous order would

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emerge so that social members occupy an optimal allocation of resources. Nevertheless, no one clarifies what the spontaneous order is; for instance, is it a natural law? We have known that, in his early years, Hayek focused his research on the theory of business cycles and later turned to the theory of a spontaneous economic order (Witt 1997). Regarding Hayek's change, one naturally wonders if there were certain relations between economic crisis and spontaneous order. Unfortunately, Hayek never reconsidered the business cycle theory, given his later beliefs.

We are all now witnesses to the huge financial crisis that began in 2008. Regarding this crisis, many people attribute the origin of it to the *laissez faire* policies that support free markets (Bouchaud 2008). Therefore, a question may arise: "Does the spontaneous order always benefit free economies?" Regrettably, we cannot answer this question because thus far there is no free market model that is truly based on the principle of spontaneous order.[1] For example, the mainstream model of modern economics is called the Dynamic Stochastic General Equilibrium (hereafter DSGE) in which there is not an interesting variable corresponding to spontaneous order or to a degree of freedom of economic systems. In particular, we often do not find a strict solution to DSGE; however, we can prove the existence of the equilibrium solution, which has certain fine properties such that economic crises are eliminated. This means that we cannot clarify the origin of economic crises solely via the general equilibrium theory. Consequently, certain scholars appealed to abandon DSGE and neoclassical economics to develop alternative economic theories for the early warning of economic crises, refer to Hodgson (2009), Leijonhufvud (2009) and Farmer and Foley (2009). However, it may be illogical to abandon neoclassical economics because it had attained great success in the past. This paper aims to formalize the notion of spontaneous order within the framework of neoclassical economics. Later, we will observe that introducing spontaneous order is inevitable if multiple competitive equilibria arise and that economic crisis is an unstable state of spontaneous order.

We first demonstrate why multiple competitive equilibria will occur in the neoclassical economics. As is well known, to guarantee that a competitive economy has a unique equilibrium outcome, we must assume that each consumer's preferences are strictly convex (Jehle and Reny 2001; Page 188) and that each firm's production possibility sets are strongly convex (Jehle and Reny 2001; Page 206). Indeed, strict convexity of preferences is necessary because it exhibits the principle of diminishing marginal rate of substitution in consumption (Jehle and Reny 2001; Page 12). However, strong convexity will eliminate constant returns to scale in production, and the latter is considerably important in the neoclassical production theory. If, instead, mere convexity of production possibility sets is assumed, the existence of an equilibrium outcome can still be guaranteed (Debreu 1971; Page 84). It is crucial that the convexity (rather than strong convexity) of production sets allows the possibility of constant returns to scale for firms (Jehle and Reny 2001; Page 216). Thus, the convexity of production possibility sets actually ensures the existence of long-run equilibrium outcome because

[1] It must be noticed that there have been much literature in which certain authors attempt to connect the principle of spontaneous order and the method of evolutionary game, e.g., refer to Schotter (1981), Sugden (1989) and Young (1993) (1996). Nonetheless, these excellent attempts pay more attentions to the order of social rules (e.g., conventions or institutions) rather than the order of economic rules (e.g., distribution of wealth or income). Obviously, the imbalance of the latter is more likely associated with economic crises. Additionally, the latter, in which we are chiefly concerned, is easier to be empirically tested.

constant returns technology is a (only sensible) long-run production technology (Varian 1992; Page 356). More importantly, the long-run level of profits for a competitive firm that has constant returns to scale is a zero level of profits (Varian 2003; Page 340); therefore, one must confront a curious equilibrium: each firm always gains zero economic profit regardless of how it behaves. This equilibrium strongly implies that the long-run competitive economy may have multiple (or indeterminate) equilibrium outcomes.

To strictly clarify whether a long-run competitive economy produces multiple equilibria, we need to introduce an exact definition for such an economy. In this paper, we will specify a long-run competitive economy using an Arrow-Debreu economy with additivity and publicly available technology. Traditional literature (Mas-Collel et al. 1995; Page 334) demonstrated that the long-run competitive economy is a situation of competitive economies when free entry is permitted. Generally, additivity means that there is free entry for firms into a possible industry (Debreu 1971; Page 41). Hence, additivity and publicly available technology combined would guarantee that there is free entry for firms into any (technology) industry. Moreover, publicly available technology (Mas-Collel et al. 1995; Page 653) implies that firms produce their products with a similar or an identical technology; therefore, monopolistic competition and perfect competition are allowed for as well. On the basis of these reasons above, we believe that the Arrow-Debreu economy with additivity and publicly available technology exactly describes the long-run competitive economy. Later, we will prove that such an economy indeed has many (infinitely many) equilibrium outcomes. In addition, according to the first fundamental theorem of welfare economics, these equilibrium outcomes should be all Pareto optimal. Because each equilibrium outcome is associated with a different social state, multiple equilibrium outcomes actually imply an uncertain economic world. To eliminate the uncertainty, welfare economists attempt to search the best outcome through an imaginary social welfare function. Unfortunately, *Arrow's Impossibility Theorem* refuted the existence of such a welfare function (Jehle and Reny 2001; Page 243).

In this paper, we propose a scheme for "eliminating" the uncertainty: we introduce the paradigm of natural selection into neoclassical economics. In our setting, the natural selection will obey "survival of the likeliest". The intuition behind our approach is as follows. Now, a long-run competitive economy produces multiple equilibrium outcomes, each of which is Pareto optimal. We may assume that all of these outcomes are equally likely to occur (or equivalently, to be selected with equal opportunities as collective decisions). Equal opportunities among equilibrium outcomes essentially imply an absolutely fair world in which there is no any difference between all of the outcomes. Thus, if there exists an economic order (or a convention) that contains the most equilibrium outcomes, it does occur with the highest probability (compared to other economic orders). We shall define such an economic order with the highest probability as the spontaneous economic order. Our definition is based on Hayek's core idea (Sugden 1989): Spontaneous order is a convention that is *most likely* to evolve and *survive*.

To formalize the intuition above, we may consider a long-run competitive economy that produces four equilibrium outcomes $\{B_1, B_2, B_3, B_4\}$, and each outcome is Pareto optimal. We have provided such an example in section 4 (refer to Example 4.1). If the competitive economy is sufficiently fair,[2] we can consider it as a fair procedure that would translate its fairness to the outcomes (Rawls 1999; Page 75) so that all of the social members would be indifferent between these equilibrium outcomes; that is,

$$B_1 \sim B_2 \sim B_3 \sim B_4. \tag{1.1}$$

By the indifference relation (1.1), every social member will have no desire to oppose or prefer a certain outcome. This means that, under the democratic circumstance, every equilibrium outcome should be selected with an equal opportunity as collective decisions (or equivalently, every equilibrium outcome should occur with an equal probability). From this meaning, (1.1) implies that every equilibrium outcome B_i should occur with the probability $\frac{1}{4}$, where i=1,2,3,4. Assume now that these four equilibrium outcomes could be divided into the following three economic orders[3] (or three conventions): $a_1=\{B_1\}$, $a_2=\{B_2,B_3\}$ and $a_3=\{B_4\}$. By such a division, we immediately understand that a_1 occurs with the probability $\frac{1}{4}$, a_2 occurs with the probability $\frac{1}{2}$, and a_3 occurs with the probability $\frac{1}{4}$. Therefore, a_2 will occur with the highest probability, and by our definition (i.e., survival of the likeliest) a_2 will be a spontaneous economic order. In addition, by Sen's argument (Sen 1993), more equilibrium outcomes imply more choice opportunities or greater opportunity-freedom. From this meaning, the spontaneous economic order a_2 not only obeys fairness but also owns the greatest opportunity-freedom.

Summarizing the analyses above, we are able to develop three steps for seeking the spontaneous economic order. First, we attempt to find all possible equilibrium outcomes of a competitive economy. Second, we divide all of these equilibrium outcomes into different economic orders. Finally, we find the economic order that contains the most equilibrium outcomes.

The main purpose of this paper is to seek the spontaneous order of a *long-run* competitive economy using the three steps above. To achieve this purpose, with each economic order we shall associate a possible individuals' revenue distribution. With this setting, we later show that the spontaneous order of a monopolistic-competitive economy will obey a stable rule: exponential distribution; in addition, the spontaneous order of a perfectly competitive economy will obey an unstable rule: Bose-Einstein distribution. Specifically, the instability of the latter may cause economic crises (Tao 2010). It is worth emphasizing that recent empirical investigations have supported that the individuals' revenue distribution of free economies (e.g., USA) during a stable economic period obeys exponential distribution, refer to Yakovenko and Rosser (2009), Clementi et al. (2012); and obeys Bose-Einstein distribution in the run-up to an economic crisis, refer to Kürten and Kusmartsev (2011), Kusmartsev (2011).

[2] It is worth emphasizing that there may be difficulty concerning the possibility of satisfying fairness and Pareto optimality objectives simultaneously when interpersonal comparisons of utility are allowed (Pazner and Schmeidler 1974). However, one can eliminate this difficulty by insisting on the perspective of ordinal utility (Pazner and Schmeidler 1978).

[3] $a_1=\{B_1\}$ represents an economic order or a convention that allows equilibrium outcome B_1 to occur. Similarly, $a_2=\{B_2,B_3\}$ allows B_2 and B_3; $a_3=\{B_4\}$ allows B_4.

Because our finding may appear moderately surprising, we attempt to convey an intuition for the result. As is well known, in microeconomics, there are four types of markets: perfectly competitive market, monopolistic-competitive market, oligopoly market and perfectly monopoly market. It is acknowledged, among these four types of markets, that the perfectly competitive market is most efficient. It is worth noting that, before every extremely serious economic crisis occurred, without exception, there appeared extremely prosperous economies, particularly in a financial market. [4] Therefore, a natural question arises: which type of the above four markets shall, most likely, cause the extremely prosperous economy?

Logically, the answer should be the most efficient market: a perfectly competitive market.

Let us recall that there was a common feature in the past three serious economic crises[5]; that is, before these crises occurred, without exception, extremely prosperous economies had appeared. Of course, for every past economic crisis, one always could find an explanation that appears appropriate for the origin of that crisis, e.g., asymmetric information, currency mismatch between assets and liabilities of firms (Deesomsak et al. 2009), or greed (selfishness), to explain the origin of economic crisis in 2008. However, we need to remember that the "selfishness" is one of several axioms of economics. Perfect competition is regarded as the extreme case of competitive economies. Logically, as the competition in a free economy increases, the economy shall naturally evolve toward extreme competition (i.e., perfect competition); however, according to our argument, perfect competition is unstable and may cause economic crisis. That is, a thing turns into its opposite if pushed too far.

The remainder of our paper is organized as follows. Section 2 introduces the definition of long-run competitive equilibrium within the framework of an Arrow-Debreu economy. Subsections 3.1 and 3.2 prove that a long-run competitive economy has at least an equilibrium outcome. Subsection 3.3 and section 4 show that, by a long-run equilibrium outcome, one can produce infinitely many long-run equilibrium outcomes. Section 5 introduces the concepts of economic order and spontaneous economic order, and we further show that all of the long-run equilibrium outcomes can be appropriately (non-repeated) divided into different economic orders. Section 6 shows that one can seek the spontaneous economic order through normative criteria regarding fairness and freedom. Section 7 investigates the possible link between spontaneous economic order and neoclassical macroeconomics, and later introduces certain empirical evidence that supports our results. Section 8 explores the relation between technological progress and social freedom. In section 9, our conclusion follows.

[4] Interestingly, compared to all the other real markets, the financial market is closest to a perfectly competitive market. This is the reason why the Black-Scholes equation of option pricing can be well applied in a financial market. The starting point of the Black-Scholes equation of option pricing is that the change in the price of stock obeys the law of Brownian movement. Only the perfectly competitive market, which is free of monopolization, is closest to such an ideal state. Minsky (1986) continuously claimed that the finance was the cause of the instability of capitalism. Now, according to our theory, it is because the financial market is closest to perfect competition.

[5] These three economic crises are, respectively, the Great Depression in 1929, the Asian financial crises in 1997, and the American subprime crisis in 2008.

2 Preliminaries

We begin by describing a competitive economy that is composed of a vast number of agents (consumers and firms) and diverse industries. In accordance with the standard framework of the neoclassical economics (Mas-Collel et al. 1995; Page 579), we assume that there are M consumers, N firms and L commodities. Every consumer $i=1,\ldots,M$ is specified by a consumption set $X_i \subset R^L$, a preference relation $\succsim_i$ on X_i, an initial endowment vector $\omega_i \in R^L$, and an ownership share $\theta_{ij} \geq 0$ of each firm $j=1,\ldots,N$ (where $\sum_{i=1}^{M} \theta_{ij} = 1$). Each firm j is characterized by a production set $Y_j \subset R^L$. All allocations for such an economy are a collection of consumption and production vectors:

$$((x),(y)) = (x_1,\ldots,x_M,y_1,\ldots,y_N) \in X_1 \times \ldots \times X_M \times Y_1 \times \ldots \times Y_N,$$

where $x_i=(x_{1i},\ldots,x_{Li})$ and $y_j=(y_{1j},\ldots,y_{Lj})$.

2.1 Arrow-Debreu economy and competitive equilibrium

A well-known definition for competitive equilibrium is introduced as follows.

Definition 2.1 (Mas-Collel et al. 1995; Page 579): An allocation $((x^c),(y^c))$ and a price vector $p=(p_1,\ldots,p_L)$ constitute a competitive (or a Walrasian) equilibrium if the following three conditions are satisfied:

(1). Profit maximization: For every firm j, $y_j^c \in Y_j$ maximizes profits in Y_j; that is,

$$p \cdot y_j \leq p \cdot y_j^c \text{ for all } y_j \in Y_j$$

(2). Utility maximization: For every consumer i, $x_i^c \in X_i$ is maximal for $\succsim_i$ in the budget set:

$$\left\{ x_i \in X_i : p \cdot x_i \leq p \cdot \omega_i + \sum_{j=1}^{N} \theta_{ij} p \cdot y_j^c \right\}.$$

(3). Market clearing: $\sum_{i=1}^{M} x_i^c = \sum_{i=1}^{M} \omega_i + \sum_{j=1}^{N} y_j^c$.

One can verify that the equilibrium allocation $((x^c),(y^c))$ does exist if the following nine conditions are satisfied (Debreu 1971; page 84).

For every consumer i:

(a) Each consumer's consumption set X_i is closed, convex, and bounded below;
(b) There is no satiation consumption bundle for any consumer;

(c) For each consumer $i=1,\ldots,M$, the sets $\left\{x_i \in X_i \,\middle|\, x_i \succsim_i x_i'\right\}$ and $\left\{x_i \in X_i \,\middle|\, x_i' \succsim_i x_i\right\}$ are closed;
(d) If x_i^1 and x_i^2 are two points of X_i and if t is a real number in $(0,1)$, then $x_i^2 \succsim_i x_i^1$ implies $tx_i^2 + (1-t)x_i^1 \succ_i x_i^1$;
(e) There is x_i^0 in X_i, such that $x_i^0 << \omega_i$;

For every firm j:

(f) $0 \in Y_j$;
(g) $Y = \sum_{j=1}^{N} Y_j$ is closed and convex;
(h) $-Y \cap Y = \{0\}$;
(i) $-R_+^L \subset Y$.

If a competitive economy satisfies (a)-(i), it is called an Arrow-Debreu economy (Arrow and Debreu 1954). In particular, (d) will guarantee that each consumer's preferences are *strictly convex*. It should be noted that Debreu did not restrict his discussion on the strict convexity of preferences. Instead, mere convexity of preferences was assumed in his famous book (Debreu, 1971; Page 84). However, strict convexity of preferences is necessary for neoclassical economics because it exhibits the principle of diminishing marginal rate of substitution in consumption (Jehle and Reny 2001; Page 12). Technically, strict convexity of preferences will guarantee that $(x_1^c,\ldots,x_M^c)$ is a unique equilibrium consumption allocation.

Here, we do not assume the strong convexity of production possibility sets because it will eliminate constant returns to scale in production (Jehle and Reny 2001; Page 206). In section 3, we will observe that constant returns technology is inevitable when long-run competition is considered and that (f)-(i) allow the possibility of constant returns to scale for firms.

2.2 Long-run competitive equilibrium

In accordance with Mas-Collel et al. (1995; Page 334), we consider the case in which "an infinite number of firms can potentially be formed"; that is, $N \to \infty$. Moreover, each firm has access to the publicly available technology; thus, it may enter and exit an industry in response to profit opportunities. "This scenario, known as a situation of free entry, is often a reasonable approximation when we think of long-run outcomes" in an industry (or a market). Under such a scenario, Mas-Collel et al. deduced (1995; Page 335): "A firm will enter the market if it can earn positive profits at the going market price and will exit if it can make only negative profits at any positive production level given this price. If all firms, active and potential, take prices as unaffected by their own actions, this implies that active firms must earn exactly *zero profits* in any long-run competitive equilibrium; otherwise, we would have either no firms willing to be active in the market (if profits were negative) or an infinite number of firms entering the market (if profits were positive)". Thus, if all of the industries (or markets) remain at long-run competitive equilibria, we have:

$$p \cdot y_j^c = 0, \quad j = 1, \ldots, N \tag{2.1}$$

By substituting (2.1) into Definition 2.1, we can present a natural definition for long-run competitive equilibrium as follows.

Definition 2.2: An allocation $((x^*),(y^*))$ and a price vector $p=(p_1,\ldots,p_L)$ constitute a long-run competitive equilibrium if the following three conditions are satisfied:

(1). For every firm j, there exists $y_j^* \in Y_j$ such that $p \cdot y_j \leq p \cdot y_j^* = 0$ for all $y_j \in Y_j$.
(2). For every consumer i, $x_i^* \in X_i$ is maximal for $\underset{i}{\succsim}$ in the budget set:

$$\{x_i \in X_i : p \cdot x_i \leq p \cdot \omega_i\}.$$

(3). $\sum_{i=1}^{M} x_i^* = \sum_{i=1}^{M} \omega_i + \sum_{j=1}^{N} y_j^*.$

In contrast to Definition 2.1, Definition 2.2 has a stronger constraint; that is, the maximum profit of every firm j, $p \cdot y_j^*$, is restricted to be null. Because of this constraint, we cannot guarantee that $((x^*),(y^*))$ does exist although (a)-(i) are satisfied. In the next section, our focus will be on the existence of the long-run equilibrium allocation $((x^*),(y^*))$. To avoid confusion, when we note an equilibrium allocation in the remainder of this paper, we always mean that it denotes a long-run equilibrium allocation.

3 Long-run competitive economy

In this section, we will verify the existence of long-run competitive equilibrium. Before proceeding, let us first explore what conditions will restrict the maximum profit of every firm j to be null within the framework of the Arrow-Debreu economy.

3.1 Assumptions

Assumption 3.1 (additivity): $Y_j + Y_j \subset Y_j$ for every j.

If the production set of the jth firm, Y_j, can be interpreted as an industry, the Assumption 3.1 means that there is free entry for firms into that industry (Debreu 1971; Page 41). More importantly, we have the result as below:

Theorem 3.1: If Assumption 3.1 and (f) are satisfied, the maximum profit of every firm j is zero; that is, $p \cdot y_j^* = 0$ for $j=1,\ldots,N$.

Proof. Refer to page 45 in Debreu (1971). □

Theorem 3.1 demonstrates that the Arrow-Debreu economy under Assumption 3.1 will restrict the maximum profit of every firm to be null (if the maximum profit exists). Unfortunately, Assumption 3.1 does not really imply free entry because Y_j represents a private production set (of the jth firm) rather than a public industry. However, if the following assumption is satisfied, Assumption 3.1 will imply free entry.

Assumption 3.2 (publicly available technology[6]): $Y_1 = Y_2 = \ldots = Y_N$.

Assumption 3.2 implies that every firm has free access to one another's technology. Then, every Y_j represents a public production set, and thus can be interpreted as a public (open) industry. Therefore, Assumptions 3.1 and 3.2 combined imply free entry. Furthermore, we have the following proposition:

Proposition 3.1: Assumptions 3.1 and 3.2 combined guarantee that $Y_j = Y$ for $j=1,2,\ldots,N$ and $Y+Y \subset Y$, where $Y = \sum_{j=1}^{N} Y_j$.

Proof. To verify this proposition, we need to prove that, for any $y \in Y$ there must be $y \in Y_j$. Because $y \in Y$, we have $y = \sum_{k=1}^{N} y_k$, where $y_k \in Y_k$. Then, by Assumption 3.2, we immediately obtain $y_k \in Y_j$ for $k=1,2,\ldots N$, where $j=1,2,\ldots,N$. Finally, by Assumption 3.1, we have $y = \sum_{k=1}^{N} y_k \in Y_j$. □

3.2 Existence of long-run competitive equilibrium

Because the Arrow-Debreu economy under Assumption 3.1 will restrict the maximum profit of every firm to be null (if the maximum profit exists), and because Assumptions 3.1 and 3.2 combined imply free entry, we note that, if the Arrow-Debreu economy under Assumptions 3.1 and 3.2 has an equilibrium allocation $((x'),(y'))$, $((x'),(y'))$ does satisfy the Definition 2.2. Therefore, we can introduce an exact definition for long-run competitive economy as follows.

Definition 3.1: A competitive economy is a long-run competitive economy (hereafter LRCE) if and only if:

(1). (a)-(i) are satisfied;
(2). Assumptions 3.1 and 3.2 hold.

To verify that a LRCE has at least an equilibrium allocation, we solely need to prove that (a)-(i) are compatible with Assumptions 3.1 and 3.2. This is because (a)-(i) would ensure the existence of an equilibrium allocation (Debreu 1971; page 84). Before proceeding, let us introduce two lemmas.

Lemma 3.1: If (f)-(g) are satisfied, and if Assumptions 3.1 and 3.2 hold, Y is a cone with vertex 0, i.e., $y \in Y$ implies $ty \in Y$ for any scalar $t \geq 0$.

Proof. First, by (f), one has $0 \in Y$, and by (g), Y satisfies convexity; therefore, for any $y \in Y$ and any $c \in [0,1]$, one has $cy = cy + (1-c) \cdot 0 \in Y$. Second, by proposition 3.1 (because Assumptions 3.1 and 3.2 hold) Y satisfies additivity; that is, for any non-negative integer k, one has $ky \in Y$. Let t be any non-negative number satisfying $t \leq k$, then the two results above imply $ty = \frac{t}{k} \cdot ky \in Y$. □

Because a cone with vertex 0 implies constant returns to scale (Debreu 1971; page 46), we immediately have two corollaries:

Corollary 3.1: The production set Y exhibits constant returns to scale.

[6] Publicly available technology coincides with Rawls' principle of fair equality of opportunity (Rawls 1999; Page 63)

Corollary 3.2: The production set of each firm, Y_j, exhibits constant returns to scale.

Lemma 3.2: If Y is a cone with vertex 0, and if Y is closed and convex, Y must be a closed, convex cone with vertex 0.

Proof. Refer to page 42 in Debreu (1971) □

Combining Lemmas 3.1 and 3.2, we have the following theorem:

Theorem 3.2: A LRCE has at least an equilibrium allocation $((x^*),(y^*))$.

Proof. We start to prove that (a)-(i) are compatible with Assumptions 3.1 and 3.2. By Lemma 3.1, (f)-(g) together with Assumptions 3.1 and 3.2 guarantee that Y is a cone with vertex 0. Thus, by (g) and Lemma 3.2, Y is further a closed, convex cone with vertex 0. Such a result does not contradict (f)-(g). This means that (a)-(i) still hold although Assumptions 3.1 and 3.2 arise. □

Now that the LRCE satisfies (a)-(i) (refer to the proof above), we immediately obtain two corollaries as below:

Corollary 3.3: The LRCE is an Arrow-Debreu economy.

Corollary 3.4: Any long-run equilibrium allocation $((x^*),(y^*))$ is a competitive (or a Walrasian) equilibrium.

3.3 Multiplicity of long-run competitive equilibria

In subsection 3.2, we have proved that the LRCE has at least an equilibrium allocation $((x^*),(y^*))=(x_1^*,\ldots,x_M^*,y_1^*,\ldots,y_N^*)$. Next, we show that by $((x^*),(y^*))$ one can produce infinitely many equilibrium allocations.

Let $z\,(p)=\sum\limits_{j=1}^{N} y_j^*$ denote the aggregate production vector, then we have three results as below:

Proposition 3.2: $z(p)\in Y_j$ for $j=1,\ldots,N$.

Proof. By Proposition 3.1, one has $z(p)\in Y=Y_j$ for $j=1,\ldots N$. □

Lemma 3.3: $tz(p)\in Y_j$ for $j=1,\ldots,N$, where $t\geq 0$.

Proof. By Lemma 3.1 and Proposition 3.2, one concludes $tz(p)\in Y_j$. □

Lemma 3.4: $p\cdot z(p)=0$.

Proof. By Definition 2.2, one has $p\cdot y_j^*=0$ for $j=1,\ldots,N$. □

Let us consider a sequence of numbers, $\{t_j\}_{j=1}^{N}$, satisfying:

$$\begin{cases} t_j\geq 0 \quad for \quad j=1,2,\ldots,N \\ \sum\limits_{j=1}^{N} t_j=1 \end{cases}. \tag{3.1}$$

Then, by Lemmas 3.3 and 3.4, we can prove the following proposition.

Proposition 3.3: Let

$$y_j^e(t_j)=t_j z(p) \tag{3.2}$$

for $j=1,2,\ldots,N$, then $(x_1^*,\ldots,x_M^*,y_1^e(t_1),\ldots,y_N^e(t_N))$ constitutes a long-run equilibrium allocation.

Proof. Because $\sum_{j=1}^{N} y_j^e(t_j) = z(p) = \sum_{j=1}^{N} y_j^*$, we only need to verify that each $y_j^e(t_j)$ satisfies the condition (1) in Definition 2.2. Thus, by Lemma 3.3 $y_j^e(t_j) \in Y_j$ and by Lemma 3.4 $p \cdot y_j^e(t_j) = 0$, where $j=1,\ldots,N$. □

The proof above implies two corollaries as below:

Corollary 3.5: $\sum_{j=1}^{N} y_j^e(t_j) = z(p)$.

Corollary 3.6: $p \cdot y_j^e(t_j) = 0$ for $j=1,\ldots,N$.

Proposition 3.3 indicates that each sequence $\{t_j\}_{j=1}^N$ satisfying (3.1) will correspondingly produce a different long-run equilibrium allocation. Undoubtedly, there are infinitely many possible sequences $\{t_j\}_{j=1}^N$ satisfying (3.1); therefore, there will be infinitely many possible long-run equilibrium allocations, too.

Lemma 3.5 (First fundamental theorem of welfare economics): Any Walrasian equilibrium allocation is Pareto optimal.

Proof. Refer to page 549 in Mas-Collel et al. (1995). □

Using Corollary 3.4 and Lemma 3.5, we can prove an important proposition.

Proposition 3.4: Any equilibrium allocation $(x_1^*,\ldots,x_M^*,y_1^e(t_1),\ldots,y_N^e(t_N))$ obeying (3.1) is Pareto optimal.

Proof. By Proposition 3.3 and Corollary 3.4 any $(x_1^*,\ldots,x_M^*,y_1^e(t_1),\ldots,y_N^e(t_N))$ obeying (3.1) is Walrasian equilibrium. Then, by Lemma 3.5, we complete this proof. □

4 Uncertainty of social choice

Propositions 3.3 and 3.4 combined demonstrate that the LRCE will produce uncertain equilibrium outcomes $(x_1^*,\ldots,x_M^*,y_1^e(t_1),\ldots,y_N^e(t_N))$, each of which is Pareto optimal. Additionally, the following proposition will further reveal that the uncertainty of equilibrium outcomes is due to production rather than consumption.

Proposition 4.1: $(x_1^*,x_2^*,\ldots,x_M^*)$ is a unique equilibrium consumption allocation.

Proof. If there were another equilibrium consumption allocation $(x_1',x_2',\ldots,x_M')$ satisfying $x_i' \succsim_i x_i^*$ for $i=1,\ldots,M$, by the condition (d) we do have $x_i''=tx_i'+(1-t)x_i^* \in X_i$ so that $x_i'' \succ_i x_i^*$ contradicting the condition (2) of Definition 2.2, where $0<t<1$. □

Using the Proposition 4.1 and the condition (3) of Definition 2.2 we immediately arrive at two corollaries:

Corollary 4.1: $(x_1^*,\ldots,x_M^*,y_1^e(t_1),\ldots,y_N^e(t_N))$ can be reduced to $(y_1^e(t_1)\ldots,y_N^e(t_N))$.

Corollary 4.2: $z(p)$ is a fixed vector.

Proposition 4.1 and Corollary 4.1 imply that all equilibria involve the same consumption vector; hence, all consumers are indifferent between all equilibria, and the multiplicity arises solely from the distribution of production. Then, a doubt may occur; because the publicly available technology is assumed (refer to Assumption 3.2), the multiplicity is perhaps a meaningless (or spurious) multiplicity. However, the multiplicity of equilibria must be admitted because the *economic crises* are hidden in such a multiplicity. To observe this, we consider a possible *long-run* equilibrium outcome $(x_1^*, \ldots,x_M^*,z(p),0,\ldots,0)$, where $y_1^e(t_1)=z(p)$ and $y_j^e(t_j)=0$ for $j=2,\ldots,N$. This equilibrium strongly indicates an economic crisis: only one firm survives (wins), and others all go bankrupt because they cease production for a long time. Later, we shall observe that such an equilibrium involving economic crisis will not occur in a monopolistic-competitive economy.[7] Unfortunately, we cannot eliminate it in a perfectly competitive economy.[8]

Moreover, the Corollary 4.1 reminds us: to describe the uncertainty of equilibrium outcomes, we only need to consider the equilibrium production allocation $(y_1^e(t_1),\ldots, y_N^e(t_N))$. Thus, for convenience, we may denote by $(y_1^e(t_1),\ldots,y_N^e(t_N))$ the long-run equilibrium allocation.

4.1 Equilibrium revenue allocation

Without loss of generality, we assume that $z(p)$ has at most one positive component (namely, a single output[9]).

Assumption 4.1:

$$\begin{cases} z(p) = (z_1(p),\ldots,z_m(p),\ldots,z_L(p)) \\ z_m(p) \geq 0 \\ z_l(p) \leq 0 \quad l = 1,\ldots,m-1,m+1,\ldots,L \end{cases} \tag{4.1}$$

where, $z_m(p)$ represents the outputs' amount, and $z_l(p)$ represents the inputs' amount.

Substituting (4.1) into (3.2), we understand that the equilibrium outputs' amount of the jth firm is specified by $t_j z_m(p)$. Because the equilibrium price of the mth commodity (i.e., output) is denoted by p_m, the jth firm will obtain $t_j p_m z_m(p)$ units of revenue.

If we refer to $\varepsilon_j(t_j)$ as the *equilibrium revenue* of the jth firm, and refer to Π as the *equilibrium total revenue* of all firms, we have:

$$\varepsilon_j(t_j) = t_j p_m z_m(p), \tag{4.2}$$

$$\sum_{j=1}^{N} \varepsilon_j(t_j) = \Pi. \tag{4.3}$$

Substituting (4.2) into (4.3) we obtain:

$$\Pi = p_m z_m(p). \tag{4.4}$$

[7] The formula (7.9) shows that firms' revenue (or equivalently "output value") distribution in a monopolistic-competitive economy obeys the exponential law. Then there is no possibility that one firm's output value is positive, and others' all are null.

[8] The formula (7.9) shows that firms' revenue (or equivalently "output value") distribution in a perfectly competitive economy is unstable, because the denominator of (7.9) corresponding to $I=1$ may equal zero. Then, there is indeed a possibility that one firm's output value is positive, and others' all are null. For more details, refer to Tao (2010).

[9] The assumption regarding single output appears very restrictive; however, it does not affect our final results. This assumption is made solely to keep our writing to follow succinct.

Using (4.4), the formula (4.2) can be rewritten as:

$$\varepsilon_j\left(t_j\right) = t_j \Pi. \tag{4.5}$$

Definition 4.1: An equilibrium revenue allocation is a collection of firms' revenue scalars:

$$(\varepsilon_1(t_1), \ldots, \varepsilon_N(t_N)). \tag{4.6}$$

The equilibrium revenue allocation $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ shows revenue allocations among N firms when the economy achieves long-run competitive equilibria; therefore, there is no essential distinction[10] in denoting the long-run equilibrium allocation either by $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ or by $(y_1^e(t_1),\ldots,y_N^e(t_N))$. However, compared to a production allocation $(y_1^e(t_1),\ldots,y_N^e(t_N))$, a revenue allocation $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ is often easier to be empirically tested. Therefore, we are more interested in exploring the revenue allocation of an economy. In the remainder of this paper, we always denote the long-run equilibrium allocation (or equilibrium outcome) by $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$.

4.2 Uncertain equilibrium outcomes

Combining (3.1) and (4.5), one easily notes that any revenue allocation $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ satisfying the following requirements

$$\begin{cases} \varepsilon_j\left(t_j\right) \geq 0 \quad for \quad j = 1, 2, \ldots, N \\ \sum_{j=1}^{N} \varepsilon_j\left(t_j\right) = \Pi \end{cases} \tag{4.7}$$

is a long-run equilibrium allocation.

Therefore, by Corollary 3.4 and Lemma 3.5, we immediately arrive at[11]:

Corollary 4.3: Any revenue allocation $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ satisfying (4.7) is Pareto optimal.

Undoubtedly, (4.7) implies that there are infinitely many possible equilibrium outcomes. We soon show that these equilibrium outcomes $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ can be simply depicted as different figures. To help readers follow our idea more easily, the constraint $\sum_{j=1}^{N} \varepsilon_j\left(t_j\right) = \Pi$ in (4.7) will be *temporarily* abandoned (specifically, we

[10] With each equilibrium revenue allocation one may associate several or many equilibrium production allocations. For example, we cannot eliminate a possibility that there were another equilibrium production allocation $(y_1^a,\ldots,y_N^a)$ whose every vector y_j^a has two positive components: y_{1j}^a and y_{2j}^a, which are defined by $\varepsilon_j(t_j)=p_1 y_{1j}^a+p_2 y_{2j}^a$ for $j=1,\ldots,N$, where $z\,(p) = \sum_{j=1}^{N} y_j^a$.

[11] When we here say that an equilibrium revenue allocation is Pareto optimal, we actually mean that the corresponding equilibrium production allocation is Pareto optimal. In this case, $(\varepsilon_1(t_1),\ldots,\varepsilon_N(t_N))$ corresponds to $(y_1^e(t_1),\ldots,y_N^e(t_N))$ at least, refer to (3.2) and (4.2).

abandon this constraint in subsection 4.2, sections 5 and subsections 6.1-6.2). However, we will resume the constraint $\sum_{j=1}^{N} \varepsilon_j(t_j) = \Pi$ starting from subsection 6.3.

We now consider a simple LRCE with two firms as below:

Example 4.1: Assume that there exists an LRCE in which there are a total of two firms and two industries. Moreover, assume that, if a firm enters industry 1, it will obtain ε_1 units of revenue; if a firm enters industry 2, it will obtain ε_2 units of revenue. In addition, assume that $\varepsilon_1<\varepsilon_2$.

Let us first explore how many equilibrium outcomes Example 4.1 has. Because there are a total of two firms, we need to count all possible revenue allocations $(\varepsilon_1(t_1),\varepsilon_2(t_2))$ that satisfy (4.7). However, because the constraint $\sum_{j=1}^{2} \varepsilon_j(t_j) = \Pi$ has been temporarily abandoned, we only need to count all possible revenue allocations $(\varepsilon_1(t_1),\varepsilon_2(t_2))$ satisfying $\varepsilon_j(t_j)\geq 0$ for $j=1,2$. Consequently, there are a total of four equilibrium outcomes, which are, respectively, as follows:

$$A_1 = (\varepsilon_2, \varepsilon_2), A_2 = (\varepsilon_1, \varepsilon_2), A_3 = (\varepsilon_2, \varepsilon_1), A_4 = (\varepsilon_1, \varepsilon_1).$$

If we denote a firm by a ball and denote an industry by a box, each equilibrium outcome $A_i (i=1,2,3,4)$ can be depicted as a different figure, refer to Figs. 1, 2 and 3. For example, Fig. 1 depicts the equilibrium outcome A_1 in which firms 1 and 2 both occupy industry 2, and each thereby obtains ε_2 units of revenue, where ball 1 represents firm 1 and box 1 represents industry 1, and so forth.

Although Example 4.1 merely describes a simple situation of (4.7) when $N=2$ and $\varepsilon_j(t_j)$ takes two possible values: ε_1 or ε_2, four equilibrium outcomes remain. Additionally, because each equilibrium outcome is Pareto optimal, we are not able to clarify which equilibrium outcome is best for society so that all of the social members want to select it. In accordance with the conventional economic analysis, welfare economists believe that one can find the best equilibrium outcome by taking advantage of an imaginary social welfare function. Unfortunately, Arrow's Impossibility Theorem has refuted the existence of such a social welfare function in the framework of ordinal utility (Jehle and Reny 2001; Page 243). Consequently, one must confront an *uncertain* economic world that exhibits four possible social states.[12] This fact is well known as the "dilemma of social choice".

In fact, evolutionary economists have already been aware that neoclassical economics lacks a body of economic analysis that could address choice under uncertainty. These economists further argued that what is missing in conventional economic analysis is a treatment of "economic emergence" (Foster and Metcalfe 2012). Therefore, it is considerably significant to materialize "economic emergence" in neoclassical economics. Our plan is to introduce Hayek's principle of spontaneous order into neoclassical economics (From an evolutionary economic perspective, order and emergence are inseparable). We hope that such an attempt will facilitate eliminating the "dilemma of social choice". We next introduce the concept of economic order.

[12] From the perspective of empirical observation, there must be one and only one equilibrium outcome (or social state), which would occur (at a given time, although we do not know which equilibrium outcome would occur).

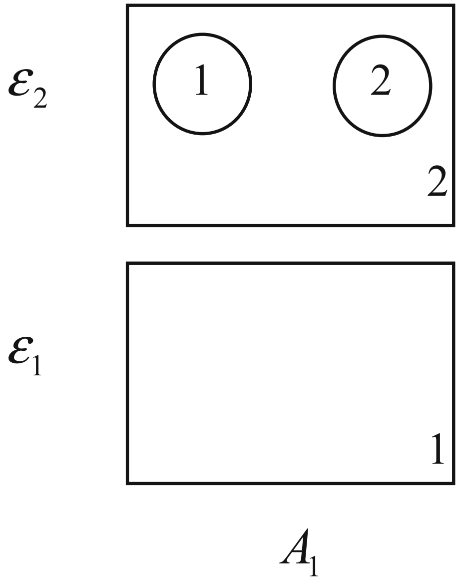

Fig. 1 In the equilibrium outcome A_1, firms 1 and 2 both occupy industry 2, and each obtains ε_2 units of revenue

5 Economic order

To simplify the analysis, we begin to introduce the concept of economic order by investigating the four equilibrium outcomes of the Example 4.1.

5.1 Definition

As noted in subsection 4.2, the LRCE described by Example 4.1 has four possible equilibrium outcomes: A_1, A_2, A_3 and A_4; each of these can be associated with a figure. If one observes Figs. 1, 2 and 3 carefully, one may find that these four

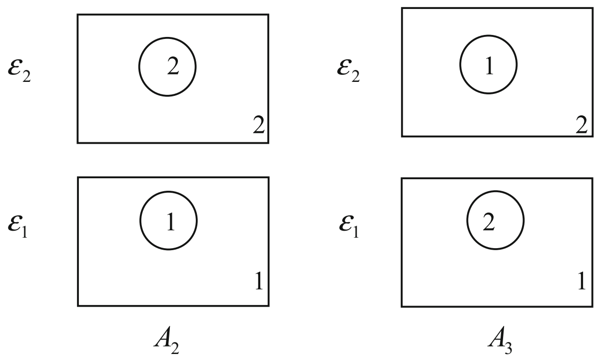

Fig. 2 In the equilibrium outcome A_2, firm 1 occupies industry 1 and obtains ε_1 units of revenue; firm 2 occupies industry 2 and obtains ε_2 units of revenue. In the equilibrium outcome A_3, firm 1 occupies industry 2 and obtains ε_2 units of revenue; firm 2 occupies industry 1 and obtains ε_1 units of revenue

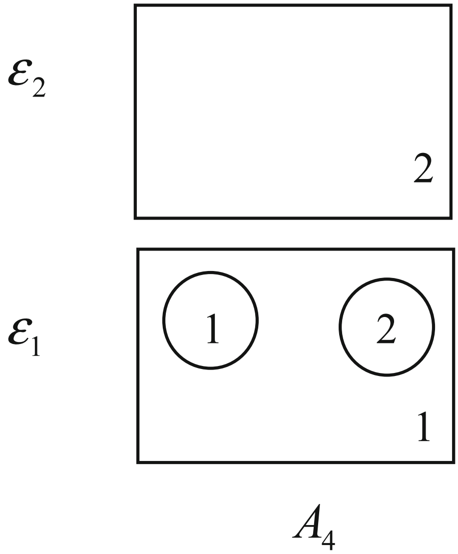

Fig. 3 In the equilibrium outcome A_4, firms 1 and 2 both occupy industry 1, and each obtains ε_1 units of revenue

outcomes can be divided into three different groups. To observe this, we consider an ordered pair $\{a_1,a_2\}$, where a_1 represents that there are a_1 firms, each of which obtains ε_1 units of revenue; a_2 represents that there are a_2 firms, each of which obtains ε_2 units of revenue. Adopting this notion, one easily finds that Figs. 1, 2 and 3 can be denoted by $\{a_1=0,a_2=2\}$, $\{a_1=1,a_2=1\}$ and $\{a_1=2,a_2=0\}$, respectively.

It is here worth noting that, although Fig. 2 depicts two equilibrium outcomes (thereby two figures): A_2 and A_3, we can still use a unique ordered pair $\{a_1=1,a_2=1\}$ to denote it. This is because A_2 and A_3 obey a unified rule (or convention): one firm obtains ε_1 units of revenue, and another obtains ε_2 units of revenue. Thus, the ordered pair $\{a_1,a_2\}$ can be considered as a 'set' whose elements are equilibrium outcomes. For example, A_2 and A_3 obey the rule $\{a_1=1,a_2=1\}$; therefore, we obtain:

$$\{a_1 = 1, a_2 = 1\} = \{A_2, A_3\}. \tag{5.1}$$

Similarly, we have:

$$\{a_1 = 0, a_2 = 2\} = \{A_1\}. \tag{5.2}$$

$$\{a_1 = 2, a_2 = 0\} = \{A_4\}. \tag{5.3}$$

If we extend the analysis regarding the two firms above to N firms, we have the following definition regarding economic order.

Definition 5.1: Let W denote the set of all possible equilibrium outcomes satisfying (4.7). A sequence of non-negative numbers, $\{a_k\}_{k=1}^{n}=\{a_1,a_2,\ldots,a_n\}$, is called an economic order if and only if it denotes a subset of W, obeying the following four conventions:

(1). There are a total of n possible revenue levels[13]: $\varepsilon_1<\varepsilon_2<\ldots<\varepsilon_n$;
(2). There are a_k firms each of which obtains ε_k units of revenue, and k runs from 1 to n;
(3). These a_k firms are distributed among g_k industries[14];
(4). $\sum\limits_{k=1}^{n} a_k = N$.

It is easy to observe that, with every economic order $\{a_k\}_{k=1}^{n}$, one can associate a different revenue distribution as follows: There are a_1 firms each of which obtains ε_1 units of revenue; there are a_2 firms each of which obtains ε_2 units of revenue, and so on. From this meaning, an economic order actually denotes an ordered distribution rule of society's wealth. In general, any distribution rule is always due to certain social institutions or conventions. Thus, we are eager to clarify what distribution rule (or economic order) a free economy would obey. Hayek believed that, if a competitive economy is sufficiently free, a spontaneous economic order will arise. According to Hayek's belief, a striking feature of the spontaneous economic order is that it is *more likely* to emerge or more able to *survive* than other economic orders (Sugden 1989). With this belief, we can present a concrete definition for spontaneous economic order as below:

Definition 5.2: Among all possible economic orders $\{a_k\}_{k=1}^{n}$ satisfying Definition 5.1, if there exists an economic order $\{a_k^*\}_{k=1}^{n}$ which would occur with the highest probability, $\{a_k^*\}_{k=1}^{n}$ is called a spontaneous economic order.

To understand Definition 5.2, we can informally adopt the following statistical notion: One considers W as a sample space in which each equilibrium outcome is regarded as a sample outcome (or an outcome of 'experiment'), and one considers an economic order as a random event that is identified with a collection of sample outcomes. Adopting such a notion, the spontaneous economic order is of course the most likely event (this is why it can spontaneously arise). In section 6, we will formalize this notion and further show how to seek the spontaneous economic order from among all possible economic orders.

Before proceeding, we are particularly interested in counting how many equilibrium outcomes a given economic order would contain. Let us next attempt to accomplish this task.

[13] To guarantee that all possible equilibrium outcomes satisfying (4.7) can be, without loss of any outcomes, divided into different economic orders fulfilling Definition 5.1, we may require that $n\to\infty$ and $\varepsilon_{l+1}-\varepsilon_l\to 0$, where $l=1,2,\ldots,n-1$.

[14] It must be noted that we cannot prevent the possibility that $g_k>1$. To observe this, suppose that there were an equilibrium production allocation which contains several different equilibrium production vectors each of which generates a same revenue level. These different equilibrium production vectors (any two vectors must be linearly independent with each other and otherwise should be considered as an industry) can be considered as different industries. However, (3.2) and (4.1) together imply $g_k=1$ for $k=1,2,\ldots,n$.

5.2 Monopolistic competition and perfect competition

For convenience, we denote by[15] $\Omega(\{a_k\}_{k=1}^n)$ the number of elements in a given economic order $\{a_k\}_{k=1}^n$. In other words, the economic order $\{a_k\}_{k=1}^n$ contains $\Omega(\{a_k\}_{k=1}^n)$ equilibrium outcomes. In microeconomics, we have clarified that there are two types of competitive structures, that is, perfect competition and monopolistic competition. Hence, we need to find $\Omega(\{a_k\}_{k=1}^n)$ in terms of these two types of economic structures, respectively.

Definition 5.3: A monopolistic-competitive economy is an Arrow-Debreu economy in which firms are completely distinguishable[16] (or heterogeneous).

In fact, targeting the LRCE described by Example 4.1, Figs. 1, 2 and 3 have exhibited the situation of monopolistic competition where two balls (firms) are marked by serial numbers so that we can distinguish which is firm 1 and which is firm 2. For monopolistic-competitive LRCE, Tao (2010) has computed the number of elements in a given economic order $\{a_k\}_{k=1}^n$ in the form:

$$\Omega\left(\{a_k\}_{k=1}^n\right)_{mon} = \frac{N!}{\prod_{k=1}^n a_k!} \prod_{k=1}^n g_k^{a_k}. \tag{5.4}$$

For instance, the LRCE described by Example 4.1 requires that $N=2$, $n=2$ and $g_1=g_2=1$. Thus, using the formula (5.4) we can compute the number of elements in each economic order as follows:

$$\Omega(\{a_1 = 0, a_2 = 2\})_{mon} = \frac{2!}{0! \times 2!} \times 1^0 \times 1^2 = 1, \tag{5.5}$$

$$\Omega(\{a_1 = 1, a_2 = 1\})_{mon} = \frac{2!}{1! \times 1!} \times 1^1 \times 1^1 = 2, \tag{5.6}$$

$$\Omega(\{a_1 = 2, a_2 = 0\})_{mon} = \frac{2!}{2! \times 0!} \times 1^2 \times 1^0 = 1. \tag{5.7}$$

Clearly, the results (5.5)–(5.7) are consistent with the numbers of equilibrium outcomes listed by Figs. 1, 2 and 3, respectively.

Definition 5.4: A perfectly competitive economy is an Arrow-Debreu economy in which firms are indistinguishable[17] (or identical).

[15] Adopting this notation, $\Omega(\{a_k\}_{k=1}^n)$ should be a function of a_k, where $k=1,2,\ldots,n$.

[16] Namely, every firm corresponds to a different brand (Varian 2003; Page 453)

[17] Namely, firms produce homogeneous products (Varian 2003; Page 380); thus, the notion of brand does not exist. Perhaps, certain people may argue that homogeneous products, generally, solely hold in one industry. However, in the long run, if a firm exits an industry, then it can enter an arbitrary industry in which there should not be differentiated products; otherwise there exists monopoly. Consequently, homogeneous products, in the long run, hold in all industries; this case can be understood as products without brands (or equivalently, firms without brands).

To observe the difference between perfect competition and monopolistic competition, we continue to concentrate on the LRCE described by Example 4.1. Because firms are indistinguishable in a perfectly competitive economy, we use the following three figures (refer to Figs. 4, 5 and 6) to list all possible equilibrium outcomes and the corresponding economic orders.

Typically, for the case of perfect competition, two balls (firms) are not marked by serial number (refer to Figs. 4, 5 and 6); thus, we cannot distinguish which is firm 1 or which is firm 2. Therefore, the economic order obeying perfect competition, $\{a_1{=}1, a_2{=}1\}$, contains only one equilibrium outcome, refer to Fig. 5. This is clearly different from the case of monopolistic competition (comparing Figs 2 and 5).

For a perfectly competitive LRCE, Tao (2010) has computed the number of elements in a given economic order $\{a_k\}_{k=1}^n$ in the form:

$$\Omega\left(\{a_k\}_{k=1}^n\right)_{per} = \prod_{k=1}^{n} \frac{(a_k + g_k - 1)!}{a_k!(g_k - 1)!}. \tag{5.8}$$

Similarly, targeting the LRCE described by Example 4.1, using the formula (5.8) we can compute the number of elements in each economic order as follows:

$$\Omega(\{a_1 = 0, a_2 = 2\})_{per} = \frac{(0 + 1 - 1)!}{0! \times (1 - 1)!} \times \frac{(2 + 1 - 1)!}{2! \times (1 - 1)!} = 1, \tag{5.9}$$

$$\Omega(\{a_1 = 1, a_2 = 1\})_{per} = \frac{(1 + 1 - 1)!}{1! \times (1 - 1)!} \times \frac{(1 + 1 - 1)!}{1! \times (1 - 1)!} = 1, \tag{5.10}$$

$$\Omega(\{a_1 = 2, a_2 = 0\})_{per} = \frac{(2 + 1 - 1)!}{2! \times (1 - 1)!} \times \frac{(0 + 1 - 1)!}{0! \times (1 - 1)!} = 1. \tag{5.11}$$

The results (5.9)–(5.11) are consistent with the numbers of equilibrium outcomes listed by Figs. 4, 5 and 6, respectively.

In summary, we have:

$$\Omega\left(\{a_k\}_{k=1}^n\right) = \begin{cases} \displaystyle\prod_{k=1}^{n} \frac{(a_k + g_k - 1)!}{a_k!(g_k - 1)!} & (perfect \;\; competition) \\ \displaystyle\frac{N!}{\prod_{k=1}^{n} a_k!} \prod_{k=1}^{n} {g_k}^{a_k} & (monopolistic \;\; competition) \end{cases}. \tag{5.12}$$

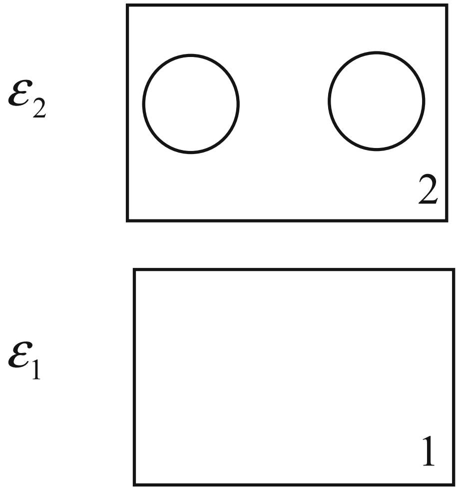

Fig. 4 The economic order $\{a_1=0, a_2=2\}$ allows a single equilibrium outcome in which two indistinguishable firms occupy industry 2, and each obtains ε_2 units of revenue

6 Fairness, freedom and spontaneous economic order

In this section, we show how to seek the spontaneous economic order from among all possible economic orders by using the normative criteria of fairness and freedom.

6.1 Social fairness

Undoubtedly, an ideal framework of considering social fairness or equity is the theory of social choice. To keep the following analysis simple, we proceed to investigate the Example 4.1. As noted in subsection 4.2, the Example 4.1 has four possible equilibrium

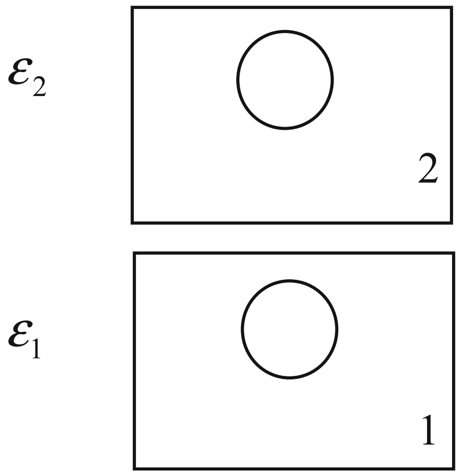

Fig. 5 The economic order $\{a_1=1, a_2=1\}$ allows a single equilibrium outcome in which one firm occupies industry 1 (hence, obtains ε_1 units of revenue), and another firm occupies industry 2 (hence, obtains ε_2 units of revenue)

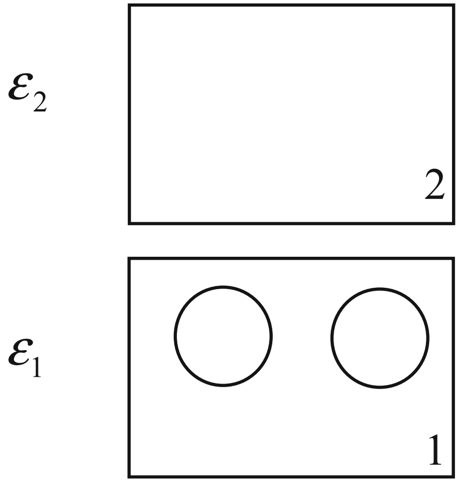

Fig. 6 The economic order $\{a_1=2, a_2=0\}$ allows a single equilibrium outcome in which two indistinguishable firms occupy industry 1, and each obtains ε_1 units of revenue

outcomes: A_1,A_2,A_3 and A_4; each of which is Pareto optimal. In this case, the task of the theory of social choice is to answer: Which of these four equilibrium outcomes is best for society. In accordance with the welfare economists' convention, one may denote the set of equilibrium outcomes by $A=\{A_1,A_2,A_3,A_4\}$. Then, if one can find a ranking of the equilibrium outcomes in A that reflects 'society's' preferences, one would capture the best social choice. Unfortunately, Arrow's Impossibility Theorem has refuted the existence of such 'society's' preferences (Jehle and Reny 2001; Page 243). Hence, one is not able to compare any two alternatives in A from a perspective that is individually and socially consistent; otherwise, the social choice will be unfair. To ensure fairness, a wise treatment is to abandon comparing any two equilibrium outcomes in A, and furthermore, to admit equality between all of these equilibrium outcomes; that is,

$$A_1 \sim A_2 \sim A_3 \sim A_4, \tag{6.1}$$

where, the symbol ~ represents the indifference relation.

Such a treatment exhibits Leibniz's *principle of the identity of indiscernibles* (Arrow 1963; Page 109). Now that social members are indifferent between all equilibrium outcomes, we cannot ensure which outcome will be selected as a collective decision. This means that collective choices should be completely random.

Because of the randomness of the collective choices, we are very interested in exploring the probability that a certain equilibrium outcome will be selected as a collective decision in a just society. Thus, let us concentrate on Rawls' *pure procedural justice* (Rawls 1999; Page 74) which aims to design the social system (or economic institutions) so that the outcome is just whatever it happens to be, at least so long as it is within a certain range. With this idea, a just economy can be regarded as a fair procedure that will translate its fairness to the (equilibrium) outcomes; thus, every social member would have no desire to oppose or prefer a certain outcome. That is, (6.1) holds. Technically, to ensure that the economy is one of pure procedural justice, Rawls suggested considering the *principle of fair equality of opportunity* (Rawls 1999;

Page 76). In accordance with this principle, a fair economy implies that each outcome should be selected with equal opportunities[18]; in other words, each outcome will then occur with an equal probability. Based on the analyses above, we can present the following axiom for social fairness.

Axiom 6.1: If a competitive economy produces ω equilibrium outcomes, and if this economy is absolutely fair, each equilibrium outcome will occur with an equal probability $\frac{1}{\omega}$.

For instance, if the LRCE described by Example 4.1 is absolutely fair, Axiom 6.1 implies:

$$P[A_1] = P[A_2] = P[A_3] = P[A_4] = \frac{1}{4}, \tag{6.2}$$

where, we denote by $P[X]$ the probability that an equilibrium outcome X occurs.

Due to Axiom 6.1, we may apply the concept of classical probability (refer to page 21 in Larsen and Marx (2001)) to the set of equilibrium outcomes of the LRCE. Specifically, we adopt the following three conventions:

(i). *The set of all possible equilibrium outcomes satisfying* (4.7), W, *is referred to as the sample space.*
(ii). *Each element* (*or equilibrium outcome*) *of the sample space* W *is referred to as a sample outcome.*
(iii). *Each economic order that is identified with a collection of sample outcomes is referred to as a random event.*

Adopting the conventions (i)–(iii), we are able to compute the probability that any economic order occurs, provided that all possible equilibrium outcomes had been found.

For example, by (6.2) and (5.1) one has

$$P[\{a_1 = 1, a_2 = 1\}] = P[\{A_2, A_3\}] = P[A_2] + P[A_3] = \frac{1}{2},$$

and by (6.2) and (5.2) one has $P\ [\{a_1 = 0, a_2 = 2\}] = P\ [\{A_1\}] = P\ [A_1] = \frac{1}{4}$.

6.2 Social freedom

The concept of freedom is very complex, and every attempt to formalize it must neglect important aspects (Puppe 1996). As most authors have done (Sen 1993) (Pattanaik and Xu 1998), this paper concentrates on the opportunity aspect of freedom. In this case, if social members are indifferent between alternatives, the extent of freedom offered to the social members is entirely determined by the size of the set of alternatives (i.e., opportunity set), refer to Sen (1993).

[18] In accordance with Rawls (1999; Page 134), fairness here has been modeled as a demand for uncertainty. For more investigations concentrating on the relation between random choice and fairness, refer to Broome (1984).

In subsection 5.2, we have known that a given economic order $\{a_k\}_{k=1}^n$ contains $\Omega(\{a_k\}_{k=1}^n)$ equilibrium outcomes. Therefore, if an economy obeys the economic order $\{a_k\}_{k=1}^n$, the social members will encounter $\Omega(\{a_k\}_{k=1}^n)$ possible choices. In the spirit of the opportunity-freedom (Sen 1993), one can refer to an economic order as an opportunity set. Because social members are indifferent between equilibrium outcomes (refer to subsection 6.1), the degree of freedom of an economic order may be denoted by its size (i.e., number of elements in it). Thus, we have the following axiom:

Axiom 6.2: If a competitive economy obeys an economic order $\{a_k\}_{k=1}^n$, which contains $\Omega(\{a_k\}_{k=1}^n)$ equilibrium outcomes, the degree of freedom of this economy is denoted by $\Omega(\{a_k\}_{k=1}^n)$.

Because valuing freedom of choice may involve psychology (Verme 2009), we will not discuss the relation between freedom and preference.[19] In addition, we must emphasize that[20] the degree of freedom defined by the Axiom 6.2 has no ethical standard regarding 'good' or 'bad'. The larger degree of freedom merely implies more possible choices. For example, by (5.1), the degree of freedom of $\{a_1=1,a_2=1\}$ equals 2; by (5.2), the degree of freedom of $\{a_1=0,a_2=2\}$ equals 1. Then, we do not mean that $\{a_1=1,a_2=1\}$ is better than $\{a_1=0,a_2=2\}$.

In subsection 5.1, we have defined the spontaneous economic order $\{a_k^*\}_{k=1}^n$ as an economic order with the highest probability, refer to Definition 5.2. In accordance with this definition, we will now show that, if a competitive economy is not only absolutely fair but also has the largest degree of freedom, it would obey the spontaneous economic order $\{a_k^*\}_{k=1}^n$.

Lemma 6.1: If the competitive economy is absolutely fair, the probability of the economic order $\{a_k\}_{k=1}^n$ occurring is given by:

$$P\left[\{a_k\}_{k=1}^n\right] = \frac{\Omega\left(\{a_k\}_{k=1}^n\right)}{\sum\limits_{\left\{a_k'\right\}_{k=1}^n} \Omega\left(\left\{a_k'\right\}_{k=1}^n\right)}, \tag{6.3}$$

where, $\sum\limits_{\{a_k'\}}{}_{k=1}^n \Omega\left(\left\{a_k'\right\}_{k=1}^n\right)$ denotes the sum of the numbers of equilibrium outcomes over all possible economic orders.

Proof. Because an economic order $\{a_k\}_{k=1}^n$ contains $\Omega(\{a_k\}_{k=1}^n)$ equilibrium outcomes, one easily counts that the competitive economy totally produces $\sum\limits_{\left\{a_k'\right\}_{k=1}^n} \Omega\left(\left\{a_k'\right\}_{k=1}^n\right)$ equilibrium outcomes. According to Axiom 6.1, if the competitive economy is absolutely fair, each equilibrium outcome will occur with an equal probability $\frac{1}{\sum\limits_{\left\{a_k'\right\}_{k=1}^n} \Omega\left(\left\{a_k'\right\}_{k=1}^n\right)}$. In accordance with the

[19] However, certain authors believe that judgments regarding the degree of freedom offered to an agent by different opportunity sets must consider the agent's preferences over alternatives, refer to Sen (1993), Kreps (1979) and Koopmans (1964).

[20] Sudgen (1998) also emphasized this point, and he further noted that the problem of measuring opportunity has many similarities with the familiar preference-aggregation problems of welfare economics and social choice theory.

conventions (i)-(iii), the probability of the economic order $\{a_k\}_{k=1}^n$ occurring is denoted by (6.3). □

Using the Lemma 6.1, we can prove the following important result.

Proposition 6.1: If a competitive economy is absolutely fair, and if it obeys the spontaneous economic order $\{a_k^*\}_{k=1}^n$, it would have the largest degree of freedom; that is,

$$\Omega\left(\left\{a_k^*\right\}_{k=1}^n\right) = \max_{\{a_k\}_{k=1}^n} \Omega\left(\{a_k\}_{k=1}^n\right). \tag{6.4}$$

***Proof*.** According to Definition 5.2, the spontaneous economic order $\{a_k^*\}_{k=1}^n$ is the most likely economic order; therefore, we have:

$$P\left[\left\{a_k^*\right\}_{k=1}^n\right] = \max_{\{a_k\}_{k=1}^n} P\left[\{a_k\}_{k=1}^n\right]. \tag{6.5}$$

Substituting (6.3) into (6.5) yields (6.4). □

The proof above implies a corollary as below:

Corollary 6.1: If a competitive economy is not only absolutely fair but also has the largest degree of freedom, it would obey a spontaneous economic order.

Proposition 6.1 is a central result of this paper because it not only tells us that a spontaneous economic order is completely determined by fairness, freedom and competition but also implies a method of seeking the spontaneous economic order. By (6.4), seeking the spontaneous economic order $\{a_k^*\}_{k=1}^n$ is equivalent to solving an extremum problem $\max\limits_{\{a_k\}_{k=1}^n} \Omega\left(\{a_k\}_{k=1}^n\right)$. This is what we will accomplish in the next subsection.

6.3 Spontaneous economic order

When we introduced the concept of economic order in section 5, we abandoned the constraint $\sum\limits_{j=1}^{N} \varepsilon_j\left(t_j\right) = \Pi$ in (4.7). Such a treatment is incorrect. To observe this, let us now return to Example 4.1. It is easy to observe that, if one assumes $\varepsilon_1+\varepsilon_2=\Pi$, one does have $\varepsilon_1+\varepsilon_1<\Pi$ and $\varepsilon_2+\varepsilon_2>\Pi$; therefore, the economic orders $\{a_1=2, a_2=0\}$ and $\{a_1=0, a_2=2\}$ do not satisfy (4.7). To eliminate the economic orders transgressing (4.7), we now resume the constraint $\sum\limits_{j=1}^{N} \varepsilon_j\left(t_j\right) = \Pi$.

Without loss of generality, all of the economic orders obeying (4.7) must satisfy the following two constraints:

$$\sum_{k=1}^{n} a_k = N, \tag{6.6}$$

$$\sum_{k=1}^{n} a_k \varepsilon_k = \Pi. \tag{6.7}$$

It is easy to observe that (6.7) is the constraint $\sum_{j=1}^{N} \varepsilon_j(t_j) = \Pi$. Thus, seeking the spontaneous economic order $\{a_k^*\}_{k=1}^n$ from among all of the possible economic orders, obeying (4.7) is equivalent to solving the extremum problem as below:

$$\begin{cases} \max\limits_{\{a_k\}_{k=1}^n} \Omega\left(\{a_k\}_{k=1}^n\right) \\ s.t. \quad N = \sum\limits_{k=1}^{n} a_k \\ \Pi = \sum\limits_{k=1}^{n} a_k \varepsilon_k \end{cases} \tag{6.8}$$

where, $\Omega(\{a_k\}_{k=1}^n)$ is denoted by (5.12).

To solve the extremum problem (6.8), we need to introduce a lemma.

Lemma 6.2: Let $U[\Omega]=\ln\Omega(\{a_k\}_{k=1}^n)$. If $U[\Omega]$ achieves the maximum value at $\{a_k^*\}_{k=1}^n$, then $\Omega(\{a_k\}_{k=1}^n)$ achieves the maximum value at $\{a_k^*\}_{k=1}^n$ as well.

Proof. If one observes that $U[\Omega]$ is a monotonically increasing function of $\Omega(\{a_k\}_{k=1}^n)$, one easily completes the proof. □

Using the Lemma 6.2, the extremum problem (6.8) is equivalent to the following extremum problem:

$$\begin{cases} \max\limits_{\{a_k\}_{k=1}^n} \ln\Omega\left(\{a_k\}_{k=1}^n\right) \\ s.t. \quad N = \sum\limits_{k=1}^{n} a_k \\ \Pi = \sum\limits_{k=1}^{n} a_k \varepsilon_k \end{cases}. \tag{6.9}$$

Substituting (5.12) into (6.9), we obtain the spontaneous economic order of the LRCE in the form:

$$a_k^*(I) = \frac{g_k - I}{e^{\alpha+\beta\varepsilon_k} - I} \begin{cases} I = 1 & (perfect\ \ competition) \\ I = 0 & (monopolistic\ \ competition) \end{cases}, \tag{6.10}$$

$$k = 1, 2, \ldots, n$$

where, α and β are Lagrange multipliers. For detailed calculations, refer to Appendix A and B.

The spontaneous economic order (6.10) is a central result of this paper; it is called the *Bose-Einstein distribution* whenever I=1 and is called the *Boltzmann distribution* whenever I=0 (Carter 2001). Such an economic order determines the following rule of revenue distribution:

There are $a_k^*(I)$ firms each of which obtains ε_k units of revenue, and k runs from 1 to n.

It is easy to observe that $a_k^*(I)$ will decrease as k grows. This result strongly implies revenue inequality. However, revenue inequality does not contradict our definition for social fairness (refer to Axiom 6.1). In fact, Axiom 6.1 merely indicates that each firm has an equal chance of occupying any possible revenue level. In this case, to obtain a higher revenue, luck and effort are similarly important (Alesina and Angeletos 2005) (Alesina et al. 2012).

It must be noticed that α and β in (6.10) are two indeterminate multipliers, and the principle of spontaneous order cannot determine them.

7 Empirical investigation to spontaneous economic order

To clarify the economic meanings of α and β, we need to introduce the neoclassical macroeconomics (Romer 2000; Page 120) in which the aggregate revenue[21] Π is completely determined by labor L, capital K and technological progress T; that is,

$$\Pi = L^x K^y T^z. \tag{7.1}$$

As is well known, an important role of a firm is to collect labor and capital. Naturally, a firm can be considered as composed of labor and capital (Williamson and Winter 1993), e.g., a unit of a firm corresponds to a unit of labor and capital. Hence, the total number of firms, N, can be written as a function with respect to labor L and capital K; that is,

$$N = N(L^x K^y). \tag{7.2}$$

Using (7.2), the function (7.1) can be rewritten in the form:

$$\Pi = \Pi(N, T). \tag{7.3}$$

The complete differential of (7.3) yields:

$$d\Pi(N, T) = \mu dN + \theta dT, \tag{7.4}$$

where, $\mu = \frac{\partial \Pi}{\partial N}$ and $\theta = \frac{\partial \Pi}{\partial T}$.

μ and θ denote the marginal labor-capital return and the marginal technology return of an economy, respectively.

[21] In fact, we should here consider the aggregate production function $z_m(p)$ rather than the aggregate revenue function Π. However, (4.4) implies that there is no essential difference between $z_m(p)$ and Π (except a constant factor p_m).

In addition, using (6.6), (6.7) and (6.10), Tao (2010) obtained:

$$d\Pi = -\frac{\alpha}{\beta}dN + \frac{1}{\beta}d\left(\ln W - \alpha\frac{\partial \ln W}{\partial \alpha} - \beta\frac{\partial \ln W}{\partial \beta}\right), \tag{7.5}$$

where $W = W\ (\alpha, \beta) = \prod_{k=1}^{n}\left(1 - Ie^{-\alpha-\beta\varepsilon_k}\right)^{-\frac{g_k - I}{I}}$.

Remarkably, the differential aggregate revenue (7.4) (from neoclassical economics) and the differential aggregate revenue (7.5) (from spontaneous order theory) share the same functional form. This means that neoclassical economics and Austrian economics will be compatible with each other as long as (7.4) equals (7.5).

In accordance with the belief of unifying neoclassical and Austrian theories, by comparing (7.4) and (7.5), we obtain:

$$\alpha = -\frac{\mu}{\lambda\theta}, \tag{7.6}$$

$$\beta = \frac{1}{\lambda\theta}, \tag{7.7}$$

$$T = \lambda\left(\ln W - \alpha\frac{\partial \ln W}{\partial \alpha} - \beta\frac{\partial \ln W}{\partial \beta}\right), \tag{7.8}$$

where λ is a positive constant.

Substituting (7.6) and (7.7) into (6.10) yields a definite form:

$$a_k^*(I) = \frac{g_k - I}{e^{\frac{\varepsilon_k - \mu}{\lambda\theta}} - I}\begin{cases} I = 1 & (perfect\ \ competition) \\ I = 0 & (monopolistic\ \ competition) \end{cases}, \tag{7.9}$$

$$k = 1, 2, \ldots, n.$$

The formula (7.9) has earlier been obtained by Tao (2010). The formula describes the firms' revenue distribution in an economy. If we associate each firm with a different agent, the formula (7.9) may describe the income distribution of the society. An attractive idea is to test (7.9) by collecting firms' revenue data or individuals' income data. It is worth emphasizing that there had been empirical evidence supporting (7.9), refer to Yakovenko and Rosser (2009), Kürten and Kusmartsev (2011), Kusmartsev (2011), Clementi et al. (2012). Let us next demonstrate how these empirical investigations support (7.9).

It is easy to observe that $\{a_k^*(I=0)\}_{k=1}^n$ is an exponential distribution. Such a distribution is associated with the monopolistic-competitive economy. As noted in microeconomics, monopolistic competition is a common competitive mode, and most real economies usually obey this mode. Yakovenko and Rosser (2009) have confirmed that the income distribution in the USA from 1983 to 2000 obeys the exponential distribution well; refer to Fig. 7. Later, Clementi et al. (2012) also confirmed this point. More specifically, Yakovenko and Rosser (2009) show that approximately 3 % of the population obey Pareto distribution (i.e., power-law distribution), and 97 % obey exponential distribution. This fact that income distribution consists of two distinct parts reveals the two-class structure of the American society.[22]

In contrast, $\{a_k^*(I=1)\}_{k=1}^n$ is an unstable distribution. To observe this, let us observe that there may be a singularity $\varepsilon_k=\mu$ so that the denominator of (7.9) corresponding to $I=1$ equals zero. Therefore, there may be a great many firms (or agents) occupying a very low revenue level (or income level) μ, for details observe the section IV in Tao (2010). Such an unstable distribution is associated with the case of extreme competition, i.e., perfect competition. Recently, Kürten and Kusmartsev (2011) have confirmed that the income distribution in the USA from 1996–2008 obeys the Bose-Einstein distribution well; in addition, the financial crisis in 2008 is due to the instability of the Bose-Einstein distribution. Refer to Fig. 8.

8 Technological progress and freedom

From a neoclassical economic perspective, the technological progress T is mysterious, and no one clarifies what the origin of it is. Of course, there had been certain excellent economic models, e.g., Romer (1990), in which the technological progress is interpreted as an endogenous variable, whereas it is artificially taken into these models. Remarkably, soon we shall observe that, if one treats neoclassical economics and Austrian economics in a unified perspective, one will decipher the profound origin of technological progress.

Using (5.12) and (6.10), Tao (2010) obtained:

$$\ln\Omega\left(\left\{a_k^*\right\}\right) = \ln W - \alpha\frac{\partial \ln W}{\partial \alpha} - \beta\frac{\partial \ln W}{\partial \beta}. \tag{8.1}$$

Substituting (8.1) into (7.8) yields a refined form[23]:

$$T = \lambda \ln\Omega. \tag{8.2}$$

From (8.2), we surprisingly find that the technological progress T is exactly proportional to $\ln\Omega$. In addition, by Axiom 6.2, the variable Ω (or equivalently $\ln\Omega$) represents the degree of freedom of an economy; from this, we conclude: The more freedom, the higher the technological progress.

[22] It is worth noting that (7.9) is due to the *Axiom 6.1*, which arises because the society is assumed to be absolutely fair. However, human society *cannot* be absolutely fair; therefore, this (7.9) may be only suitable for a segment of the population. Therefore, we can conclude that approximately 97 % of the population in the American society obeys fair behavior rules; however, the remaining fraction may involve unfair behaviors.

[23] (8.2) implies that technological progress T appears similar to the entropy in physics (Tao 2010). The latter is often related to "information" (or "knowledge").

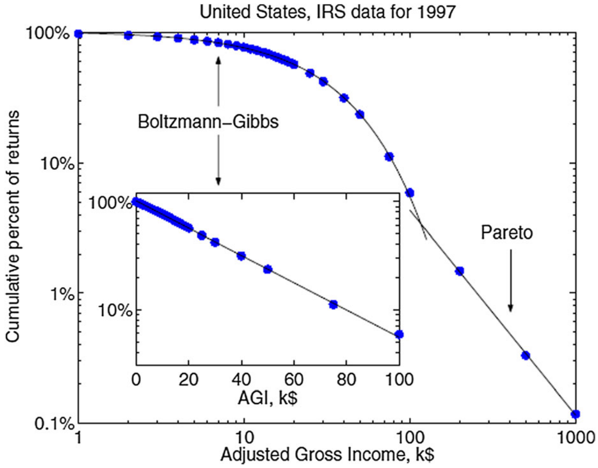

Fig. 7 Reprinted from Yakovenko and Rosser (2009). Points represent the Internal Revenue Service data, and solid lines are fits to Boltzmann (exponential) and Pareto distributions

We attempt to convey an intuition for the result above. To simplify things, we examine Figs 1 and 2. Figure 2 depicts the economic order $\{a_1=1, a_2=1\}$ whose degree of freedom is denoted by 2, then firm 1 not only may enter industry 1 but also may enter industry 2. In contrast, the degree of freedom of $\{a_1=0, a_2=2\}$ is denoted by 1, then firm 1 is confined within industry 2 (refer to Fig. 1). Logically, if a firm has the chance of entering two industries, the probability of causing innovation should increase relative to being solely confined within one industry because the genius distribution among agents is heterogeneous. That is, the more freedom (namely, the larger Ω), the greater probability of causing innovation.

As is well known, Schumpeter (1934) emphasized that innovation is a main driving force of promoting economic development. In contrast, Hayek (1948) believed truly that freedom will induce a spontaneous economic order in which economic development is most efficient. Interestingly, (8.2) undoubtedly indicates that the innovation

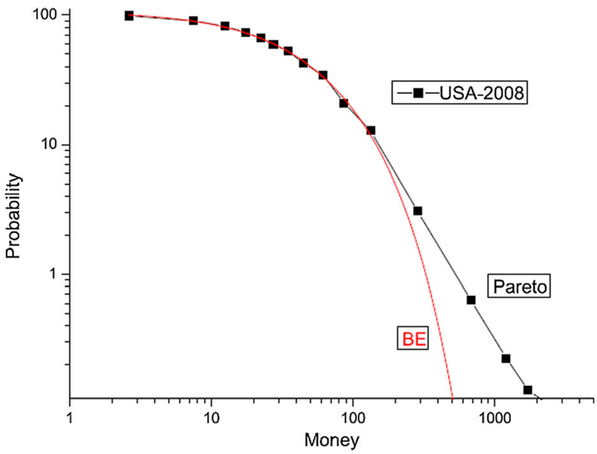

Fig. 8 Reprinted from Kusmartsev (2011). Solid squares represent the Internal Revenue Service data in 2008. The *red curve* and the *black straight line* are fits to Bose-Einstein (*BE*) and Pareto distributions, respectively

emphasized by Schumpeter is essentially equivalent to the freedom highlighted by Hayek. In other words, our theory may unify the ideas of Schumpeter and Hayek, which appear independent of each other. Thus, when an economy favors a state with more freedom, it essentially favors a state (or evolutionary direction) with higher technology level as well. By (5.12) the "freedom" Ω is an endogenous variable in our theory.[24] Substituting (8.2) into (7.3) we obtain:

$$\Pi = \hat{\Pi}(N, \Omega). \tag{8.3}$$

From (8.3), we observe that the freedom (Ω), as an equivalent replacement of technological progress (T), will become an important driving force of promoting economic growth. Actually, certain empirical studies have found a non-linear relation between economic freedom and growth, refer to Barro (1996).

Thus far, we have presented a complete theoretical framework for spontaneous economic order. At the moment, it is the proper time to summarize the logical establishments of this theoretical framework. First, we prove that a LRCE will produce infinitely many equilibrium outcomes. Second, we show that all of these equilibrium outcomes can be appropriately (non-repeated) divided into different economic orders. Third, according to normative criteria of fairness and freedom, one can determine an economic order that will occur with the highest probability, and we dub such an economic order the spontaneous economic order. Finally, we verify that Austrian economics and neoclassical economics will become compatible with each other within the framework of spontaneous economic order, provided that the technological progress T is proportional to the freedom variable $\ln \Omega$. Because of these, we believe that our spontaneous order theory presents a possible link between Austrian economics and neoclassical economics.

9 Conclusion

This paper presents a theoretical framework for spontaneous economic order in which the union of Austrian economics and neoclassical economics lies at the heart of our attempt. Our study shows that, if a competitive economy is sufficiently fair and free, an unplanned economic order will spontaneously emerge. It must be noticed that such an economic order is not the result of any process of collective choice (in contrast to that expected by many welfare economists), but is an unplanned and spontaneous consequence (as expected by Hayek).

In general, we cannot guarantee that an Arrow-Debreu economy has one sole equilibrium outcome. If an Arrow-Debreu economy (e.g., long-run competitive situation) produces multiple equilibrium outcomes, according to the first fundamental theorem of welfare economics, each outcome will be Pareto optimal. Consequently, social members must confront a problem of social choice: They must choose an equilibrium outcome that is best for society. For this problem, the proposal of welfare economists is to seek the best outcome through an imaginary social welfare function.

[24] Therefore, technological progress T is also an endogenous variable.

Unfortunately, Arrow's Impossibility Theorem warns us: There will be no such social welfare function if we insist on the perspective of ordinal utility. This means, a plan of seeking the best equilibrium outcome through a social welfare function is doomed to failure.

Our proposal here is to abandon seeking the best equilibrium outcome and furthermore, admit the equality between all possible equilibrium outcomes. In this case, we attempt to divide all of these equilibrium outcomes into different groups, where each group exhibits different convention. These conventions are called the economic orders by us. Based on these preparations, we further show that, if one adds normative criteria regarding fairness and freedom into the Arrow-Debreu economy, one will find that there is an economic order with the highest probability, which we call the spontaneous economic order. In accordance with the Darwinian spirit of natural selection, an economic order with the highest probability would *most likely* occur; this is why we call it the *spontaneous* order. Thus, we can say that the economic world does change in the manner it does because it seeks an economic order of higher probability. Our attempt has very strong theoretical and practical significance. The goal of human society should be to insist on the criteria regarding fairness and freedom (similar to Axioms 6.1–6.2). In accordance with these criteria, the competitive society will spontaneously obey an economic order. Specifically, we conclude that the spontaneous order of a monopolistic-competitive economy will obey a stable rule: exponential distribution; in addition, the spontaneous order of a perfectly competitive economy will obey an unstable rule: Bose-Einstein distribution. Additionally, the instability of the latter may cause economic crises. These conclusions have been supported by recent empirical investigations.

Obviously, our spontaneous order theory is, in principle, based on the theoretical framework of an Arrow-Debreu economy; therefore, it may present a bridge linking Austrian economics and neoclassical economics. Then, an interesting conjecture arises: Could Austrian economics and neoclassical economics yield to a unified framework? Our spontaneous order theory strongly supports this conjecture. As a possible result of unifying these two types of economics, we should comprehend a truth: "Freedom promotes technological progress". This is because, within the framework of our spontaneous order theory, "technological progress" and "freedom" will become equivalent to each other.

A. Spontaneous economic order of perfectly competitive economy

Allowing for the number of firms $N \to \infty$ in a long-run competitive economy, we assume that every a_k is a sufficiently large number.

If one considers the perfect competition, using (5.12), the function $U[\Omega]$ can be written in the form:

$$U\left[\Omega_{per}\right] = \sum_{k=1}^{n} \ln(a_k + g_k - 1)! - \sum_{k=1}^{n} \ln a_k! - \sum_{k=1}^{n} \ln(g_k - 1)!. \tag{A.1}$$

Because the value of a_k is sufficiently large, using the Stirling's formula (Carter 2001; Page 218)

$$\ln m! = m(\ln m - 1), (m >> 1) \tag{A.2}$$

(A.1) can be rewritten in the form:

$$U\left[\Omega_{per}\right] = \sum_{k=1}^{n} [(a_k + g_k - 1)\ln(a_k + g_k - 1) - a_k \ln a_k - g_k - \ln (g_k - 1)! + 1]. \tag{A.3}$$

The method of Lagrange multiplier for the optimal problem (6.9) gives

$$\frac{\partial\{U[\Omega]\}}{\partial a_k} - \alpha\frac{\partial N}{\partial a_k} - \beta\frac{\partial \prod}{\partial a_k} = 0, k = 1, 2, \ldots n \tag{A.4}$$

where, α and β are Lagrange multipliers.

Substituting (6.6), (6.7) and (A.3) into (A.4) yields

$$\ln\left(\frac{a_k + g_k - 1}{a_k} - \alpha - \beta\varepsilon_k\right) = 0, \tag{A.5}$$

$$k = 1, 2, \ldots, n.$$

which is the spontaneous economic order of a perfectly competitive economy:

$$a_k = \frac{g_k - 1}{e^{\alpha+\beta\varepsilon_k} - 1}, \tag{A.6}$$

$$k = 1, 2, \ldots, n.$$

B. Spontaneous economic order of monopolistic-competitive economy

If one considers the monopolistic competition, using (5.12) the function $U[\Omega]$ can be written in the form:

$$U[\Omega_{mon}] = \ln N! + \sum_{k=1}^{n} a_k \ln g_k - \sum_{k=1}^{n} \ln a_k!. \tag{B.1}$$

Using the Stirling's formula (A.2), (B.1) can be rewritten in the form:

$$U[\Omega_{mon}] = \ln N! + \sum_{k=1}^{n} a_k \ln g_k - \sum_{k=1}^{n} a_k \ln a_k + \sum_{k=1}^{n} a_k. \tag{B.2}$$

Substituting (6.6), (6.7) and (B.2) into (A.4) gives the spontaneous economic order of a monopolistic-competitive economy:

$$a_k = \frac{g_k}{e^{\alpha+\beta\varepsilon_k}}, \tag{B.3}$$

$$k = 1, 2, \ldots, n.$$

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