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A consumer-based model of competitive diffusion: the multiplicative effects of global and local network externalities*

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Abstract. Competitive diffusion of two incompatible technologies, such as PC vs. Macintosh, VHS vs. Betamax and so on, is studied under the framework of a spatial game in which consumers are distributed on a two-dimensional square lattice network. The consumers play coordination-like games with their nearest neighbors and imitate the most successful strategy in their neighborhood in terms of aggregated payoffs after each round. The effects of global network externality are realized in the dynamic payoff matrix of the game, and the framework of spatial game provides the model with the effects of local network externality. These two types of externalities are set as multiplicative, that is, as nonlinear. Both simulations and mean-field approximation show that not only total but also partial standardization (robust polymorphic equilibrium) occurs depending upon the parameters and initial configurations, even when there are positive effects of both global and local network externalities. Moreover, effects of innovation factors that alter paths toward a lock-in situation are studied. It is shown that both the timing and the size of the innovation factors matter for a disadvantaged technology in order to overwhelm a market.

Keywords: Competitive diffusion – Technological Standardization – Global and local network externalities – Spatial coordination game – Dynamic payoffs

JEL Classification: C7, D8, O3

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1 Introduction

When it comes to the competitive diffusion of technologies that are distinguished by compatibility, most studies are characterized by positive effects of the only global network externality. The positive effects of the global network externality exist if consumers' utilities obtained by adopting a technology are an increasing function of the number of consumers adopting a technology compatible with it in the global network (whole market). There exists an extensive literature of the effects of the only global network externality (Arthur, 1989; Besen and Farrell, 1994; David, 1985; Farrell and Saloner, 1988; Farrell, and Shapiro, 1988; Katz and Shapiro, 1985, 1992, 1994). The standard result is that global coordination, that is, total standardization, occurs due to the global network externality, which is a global feedback by its definition.

On the other hand, some studies assert the importance of local coordination and have focused on the effects of only local feedback (An and Kiefer, 1995; Cowan and Miller, 1998). An and Kiefer (1995) find partial standardization in which two incompatible technologies can coexist in a market when consumers are embedded in an integer lattice network, the dimension of which is greater than or equal to three. Note that, in An and Kiefer (1995), consumers are assumed to have a tendency to follow a local majority, and so, local feedback is modeled in this way. In Cowan and Miller (1998), consumers are placed on a one-dimensional lattice network (a.k.a. a ring graph) and assumed to conduct best responses with regard to their expected utilities that are affected by local neighbors. The result they obtained is that only partial standardization is possible due to the local feedback that promotes local coordination, unless external forces such as governmental policies are carried out.

Besides the literature cited above, there is a study that has considered the effects of a local and "fixed global" externality (Dalle, 1997). Since the externality is "fixed" and, therefore, does not represent increasing return, this externality is different from the widely accepted definition of the global network externality. Moreover, the two types of externalities affect agents' decision-making additively and independently in Dalle (1997), that is, they are set as linear. In his simulation, heterogeneous agents are embedded in a two-dimensional lattice network and interact with their local neighbors. They are programmed to follow a local majority when they decide which technology to adopt. According to Dalle (1997), the "fixed global" externality is modeled as "a sort of fictitious "global" agent who belongs to each agent's neighborhood and whose influence on the agent's decision is not different from local externalities or interactions." The results he obtained is that total standardization is always observed even with "very heterogeneous" agents as long as the "fixed global" externality is set to assist a certain technology.

Cowan and Cowan (1998) studied the effects of local, regional, and global externalities from the firm's side by examining spatial patterns of coordinated R&D activities formed by the firms. It is interesting to see the formation of the spatial patterns depending on parameters in their simple model in which externalities are all negative and independent of one another. However, in reality, externalities are mutually interconnected to bring positive and negative feedbacks. For example, innovations as the fruits of R&D activities often result in lower prices that promote consumption, which feeds back to bring prices down further. This interrelated feedback amplification plays an important role in the dynamical behaviors of both the supply side and the demand side. Such nonlinear coupling between externalities is one of the main problems to be examined in our present paper.

As was emphasized in the above literature (An and Kiefer, 1995; Cowan and Cowan, 1998; Cowan and Miller, 1998; Dalle, 1997), local coordination matters, indeed, especially for obtaining partial standardization. For example, if a consumer is the only one who uses a certain type of technology, say a PC, within a group to which he belongs, or if there are only a few consumers who use the technology in his group, it can be imagined that his willingness to utilize a PC instead of a Macintosh decreases. This is because, in this extreme case, the technology he is using is not compatible with what his neighbors are using, and, therefore, he cannot coordinate with, say, his colleagues with whom he most likely interacts. People in general act locally on a daily basis even though they may think globally. For instance, we have family, friends, colleagues, and so on, and we interact with them most likely. Not only the number but also the type of people with whom one usually interacts is more or less fixed, even though it is possible to expand one's community. Concerning the effects of local interaction, therefore, is critical due to this restriction from which people cannot be free physically or abstractly. However, at the same time, it is important to take into account the global network externality that characterizes highly technological products. There are two types of externalities, and this implies that we have to be concerned with these two under a single unified framework.

In this present paper, the competitive diffusion of two incompatible technologies, such as PC vs. Macintosh, VHS vs. Betamax and so on, is studied based on an evolutionary game theoretical approach. The framework of the model we used is known as a *spatial game*. In the spatial-game setting, consumers (or agents), who are embedded in a two-dimensional square lattice network, play 3×3 symmetric coordination-like games with their nearest neighbors and aggregate the resulting payoffs. Their aggregated payoffs are treated as utilities that affect decision-making. In the model, consumers are assumed to pursue a *copycat* behavior, under which each consumer imitates the most successful neighbor's strategy in terms of utilities that is given as the aggregated payoff. Note that this decision-making dynamics is different from those utilized by some earlier studies cited above. Through imitation, strategies diffuse on a two-dimensional square lattice network.

In the payoff matrix of the game, payoff elements are determined dynamically. The effects of global feedback are realized in the dynamic payoff matrix of the game, and the framework of the spatial game provides the model with the effects of local feedback. In other words, the consumers can enjoy network externalities by global coordination (global network externality), and at the same time, unlike in the earlier literature, the global network externality is transmitted and enhanced by local coordination (local network externality). Positive effects of the local network externality exist if consumers' utilities obtained by adopting a technology are an increasing function of the number of these of local neighbors adopting a compatible technology. Note that the two types of network externalities are set as multiplicative, that is, as nonlinear. For instance, if one has neighbors with whom one can talk and share information about, say, a Macintosh computer, one may be able to enjoy oneself with a benefit from the global network externality through the local interactions. Moreover, the benefit one receives may be amplified as the number of neighbors with whom one can locally coordinate increases. Introducing the multiplicative effects of global and local network externalities makes our model and its results unique. Not only total but also partial and robust standardization – for example, the phenomenon that Macintosh is still surviving in the market – can occur even with both local and global network externalities and less heterogeneous agents. The scenario of our model is based on an idea that human beings are more or less spatially restricted in a physical and abstract sense, and that their payoffs are affected by their local interactions even though there exists global feedback. A number of studies in evolutionary game theory have shown that the effects of the physical and abstract spatiality are critical for people's behavior (Axelrod, 1984; Pollock, 1989; Nowak and May, 1992, 1993; Herz, 1994; Oliphant, 1994; Nowak, Bohoeffer, and May, 1994a,b, 1996; May, Bohoeffer, and Nowak, 1995; Szabó and Töke, 1997; Tomochi and Kono, 2002).

Details of the model are explained in the next section. In Section 3, results of the simulations based on the rules of the game in Section 2 are shown to clarify the payoff and initial-condition-dependent behavior of the system. The effects of innovation factors that may alter paths toward a lock-in situation (Arthur, 1989) are illustrated in Section 4. In Section 5, a mean-field theory, which is widely used in physical systems, is formulated to approximate the results of the simulations in Sections 3 and 4. Discussions are given in the last section.

2 The model

2.1 Payoffs and utilities

Consumers are placed on a two-dimensional square lattice network. Each consumer has one of the three possible strategies: adopting either a technology A or B, or a strategy C of adopting neither. Note that the technologies A and B are incompatible. Those who follow the strategy C are the potential consumers who will purchase either A or B in the future. In the following, adopting either A, B, or C is expressed by $+1, -1$, or 0, respectively, that is, at time t (≥ 1) a consumer in the i-th cell $(1 \leq i \leq |N|)$ takes a pure strategy $\sigma_i(t)$ that is either $+1, -1$, or 0. The symbol $|N|$ denotes the total number of whole population. It has been confirmed by the simulations that $|N| = 101^2$ is large enough, and, therefore, $|N|$ is set as 101^2 in the following. At each time, a consumer i plays one-shot 3×3 symmetric coordination-like games with his nine immediate local neighbors including himself (the Moore neighborhood), denoted as $n(i)$, under the payoff matrix for a row player given in Table 1. Note that this local interaction under the framework of the spatial game is the source of the local feedback in the model.

In Table 1, $R_i(\pm 1)$ denotes a consumer i's payoff derived from a technology itself and is given as

$$R_i(\pm 1) = r(\pm 1) \pm \omega\theta_i \; . \tag{1}$$

Table 1. Payoff matrix for a row player. The definitions of $R_i(\pm 1)$ and $S(\pm 1, t)$ are given in Eqs. (1), (2), and (3)

	Strategy A (+1)	Strategy B (-1)	Strategy C (0)
Strategy A (+1)	$R_i(+1) + S(+1, t)$	$R_i(+1)$	$R_i(+1)$
Strategy B (-1)	$R_i(-1)$	$R_i(-1) + S(-1, t)$	$R_i(-1)$
Strategy C (0)	0	0	0

The parameters, $r(\pm 1)$, that represent the performance of the technologies by itself are assumed to be greater than zero. The random variable θ_i is uniformly generated between -0.5 and 0.5, and ω is assumed to be a small positive value compared to $r(\pm 1)$, that is, $0 < w \ll r(\pm 1)$. Introducing such $w\theta_i$ enables $R_i(\pm 1)$ to contain a small amount of fluctuation, reflecting the fact that the benefit obtained from a technology is not exactly the same but is slightly different for each, and, therefore, the model can contain consumers with slightly biased preferences (Cowan and Cowan, 1998; Farrell and Saloner, 1988; Dalle, 1997).

The enhanced payoffs, $S(\pm 1, t)$, are obtained by consuming a technology compatible with one that is used by local neighbors, that is, by coordinating with one's neighbors. These are expressed as an increasing function of global density of the consumers of each technology and defined as

$$S(+1, t) = s(+1)p_A(t-1) \text{ and} \tag{2}$$

$$S(-1, t) = s(-1)p_B(t-1), \tag{3}$$

respectively, so that the benefit from the global network externality is amplified through local coordination. The enhanced payoffs, $S(\pm 1, t)$, reflect the positive effects of the global network externalities, that is, the positive and global feedback is transmitted through local interactions. Here one can also say that the locally obtained payoff is weighted by global feedback. The parameters, $s(\pm 1)$, that control the multiplicative effects of global and local network externalities, are assumed to be greater than or equal to zero. The terms, $p_A(t-1)$ and $p_B(t-1)$, represent the fraction of consumers who are adopting the technologies A and B, respectively, in the whole population at time $t-1$. Note that, for simplicity, $S(\pm 1, t)$ are represented as liner functions of $p_A(t-1)$ and $p_B(t-1)$, respectively, in Eqs. (2) and (3). As one can see, there is time lag on the right hand sides of Eqs. (2) and (3), and it is set as one unit of time for simplicity in this paper. Both $p_A(0)$ and $p_B(0)$ are assumed to be zero, so that the enhanced payoffs become zero at $t = 1$.

Because of the assumption that adopting a technology A or B strictly dominates C, the third row in Table 1 is filled with zeros. This assumption corresponds to the scenario that all consumers sooner or later adopt the technology A or B in the diffusion process.

The payoff function for a consumer i in a game with a consumer j can be denoted as $f_i(\sigma_i(t), \sigma_j(t))$, that is, $f_i(\pm 1, \pm 1) = R_i(\pm 1) + S(\pm 1, t)$, $f_i(+1, -1) = f_i(+1, 0) = R_i(+1)$, $f_i(-1, 0) = f_i(-1, +1) = R_i(-1)$, and $f_i(0, +1) = f_i(0, -1) = f_i(0, 0) = 0$. The utility of a consumer i with the strategy $\sigma_i(t)$ at time t, $u_i(\sigma_i(t))$, is defined as the sum of the resultant payoffs obtained by playing the games with a consumer i's local neighbors including himself:

$$\begin{aligned} u_i(\sigma_i(t)) &= \sum_{j \in n(i)} f_i(\sigma_i(t), \sigma_j(t)) \\ &= |n|(R_i(+1) + S(+1,t)p_i(\sigma_i(t),t))\frac{1}{2}\sigma_i(t)(\sigma_i(t)+1) \\ &+ |n|(R_i(-1) + S(-1,t)p_i(\sigma_i(t),t))\frac{1}{2}\sigma_i(t)(\sigma_i(t)-1) \end{aligned} \tag{4}$$

where $|n|$ stands for the number of local neighbors including himself, which is set as nine. The term, $p_i(\sigma_i(t), t)$, in Eq. (4) is the fraction of the consumers, who have the same strategy as i's strategy among i's nine local neighbors including himself, and is given as

$$\begin{aligned} p_i(\sigma_i(t),t) = \frac{1}{|n|}\sum_{j \in n(i)} \Bigg\{ &\frac{1}{4}\sigma_i(t)(\sigma_i(t)+1)\sigma_j(t)(\sigma_j(t)+1) \\ &+\frac{1}{4}\sigma_i(t)(\sigma_i(t)-1)\sigma_j(t)(\sigma_j(t)-1) \\ &+(\sigma_i(t)+1)(\sigma_i(t)-1)(\sigma_j(t)+1)(\sigma_j(t)-1)\Bigg\} \\ = \frac{1}{2|n|}\sum_{j \in n(i)} \{&\sigma_i(t)\sigma_j(t)(\sigma_i(t)\sigma_j(t)+1)+2(\sigma_i^2(t)-1)(\sigma_j^2(t)-1)\}. \end{aligned} \tag{5}$$

A consumer i's utility in Eq. (4) is an increasing function of $p_A(t-1)$ or $p_B(t-1)$ and $p_i(\sigma_i(t), t)$ depending upon the values of $\sigma_i(t)$ and $\sigma_j(t)$ where $j \in n(i)$, that is, both global and local network externalities are modeled here. Notice that, clearly, both network externalities are integrated through the parameters $s(\pm 1)$ that control nonlinear effects of the two types of externalities. The global-density-dependent payoffs in Eqs. (2) and (3) are taken into the utility in Eq. (4) through the consumers' local interactions with local neighbors.

Since i himself is counted as his own neighbor, it holds that

$$0 < \frac{1}{|n|} \le p_i(\sigma_i(t),t)) \le 1 \quad \forall\, i \text{ and } t\,. \tag{6}$$

The reason a consumer i himself is included in $n(i)$ is that i can expect to acquire the benefit of the global network externality not only through his local neighbors but also by himself. The consumer i is a sort of fictitious neighbor of i himself, and counting i himself as his neighbor guarantees him to enjoy the benefit of economies of scale, and allow him to obtain the minimum amount of the benefit of global network externality, even when none of his neighbors, excluding the fictitious neighbor i himself, adopts a technology compatible with his. For example, especially in these days, one might say that a certain degree of global interaction that is independent of local interactions can be possible by rapidly developing communication technologies such as the internet.

2.2 Updating rule

The updating rule adopted in this paper is the so-called *copycat* rule. Here, each consumer imitates the most successful strategy in his neighborhood in terms of consumers' utilities, that is, the consumer i's strategy at time $t+1$ is defined as follows:

$$\sigma_i(t+1) = \{\sigma_j(t) \mid u_j(t) = \max_{j \in n(i)} u_j(t)\} \,. \tag{7}$$

It is assumed that consumers with the strategy C throw a die to decide which strategy to adopt when there is a tie between maximum utilities of the strategy A and B in the neighborhood. On the other hand, those who already have the strategy A or B are assumed to keep their current strategies when there is a tie. The copycat is adopted in this paper because it is commonly observed that people try to imitate the most successful neighbor's behavior (Axelrod, 1984; Tomochi and Kono, 2002).

In order to introduce the effect of both asynchronous updating and social atmosphere by which behaviors of people in a society are more or less affected, the above updating in Eq. (7) is assumed to occur with the updating probability $\mu(t)$ that is given as an increasing function of the fraction of those who adopt either technology:

$$\mu(t) = \nu(p_{AB}(t))^{\xi} \tag{8}$$

where $p_{AB}(t) = p_A(t) + p_B(t) = 1 - p_C(t)$, and ν $(0 < \nu \leq 1)$ and ξ are parameters (see Dalle, 1997; Huberman and Glance, 1993). What this means is that people in general are skeptical about a newly released technology and only a few people initially become so-called "early adopters." As a certain type of technology, for example, OS and VTR, spread and are accepted by more people, the economic activity relating to it becomes lively. However, our main interest in the paper is not to discuss or to find an exact functional relationship between $p_{AB}(t)$ and $\mu(t)$. Therefore, Eq. (8) is introduced as one of the arbitrary increasing functions of the fraction of the people who adopt either A or B.

Additionally, by introducing Eq. (8), it is observed that the growth of $p_{AB}(t)$ coincides with a well-known S-shape curve that is formulated by the logistic equation introduced by Verhulst in 1838 (see Balakrishnan, 1991; Valente, 1995),

$$p_{AB}(t+1) - p_{AB}(t) = \lambda\, p_{AB}(t)\,(1 - p_{AB}(t)) \tag{9}$$

with a certain parameter set of ν, ξ, and λ. Note that the parameters in the payoff matrix do not affect $p_{AB}(t)$ as long as the strategies A and B strictly dominate the strategy C. Figure 1 shows $p_{AB}(t)$ (black circles) that is obtained from the network-based simulation with the initial value $p_{AB}(1) = 20/101^2$ and the parameters $\nu = 1$ and $\xi = 0.5$ in Eq. (8) and $p_{AB}(t)$ (square dots) in Eq. (9) that is numerically solved with the initial value $p_{AB}(1) = 20/101^2$ and the parameter $\lambda = 0.32$. It is observed that two plots, which form an S-shape curve, as shown in Fig. 1 are almost identical. This fact will be utilized later in Section 5 where a mean-field approximation is applied to recover the results of the simulations in Sections 3 and 4.

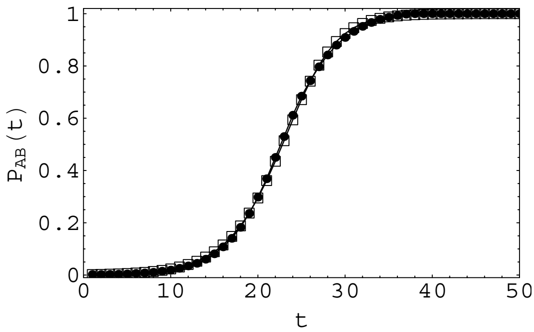

Fig. 1. The black circles are obtained as a result of the cellular-automata-based simulation with the initial value $p_{AB}(1) = 20/101^2$ and the parameters $\nu = 1$ and $\xi = 0.5$. The square dots are attained by numerically solving Eq. (9) with the initial value $p_{AB}(1) = 20/101^2$ and the parameter $\lambda = 0.32$. These two are almost identical, and it suggests that $p_{AB}(t)$ could be described by the logistic equation in Eq. (9). This fact will be utilized later in Section 5 in which a mean-field theory is conducted

3 Numerical results of the model

Simulations are conducted based on the rules of the game explained in the previous section. The results of parameter runs are shown in this section. The consumers with one of the three distinct strategies are uniformly and randomly distributed at time $t = 1$ in the game field of a two-dimensional square lattice network. A periodic boundary condition is used so that the Moore-neighborhood structure can be preserved over the game field. Fifty realizations with fifty different initial random configurations are examined in order to obtain relative frequencies of three possible equilibria, denoted as A*, B*, and P*. The symbol A* denotes the equilibrium where the technology A takes over the whole market, and B* denotes the technology B. In other words, these two equilibria represent total technological standardization. The symbol P* stands for a polymorphic equilibrium, where the technologies A and B coexist; it stands for partial standardization.

The small fluctuation term in payoff, ω, is fixed as 0.001, that is, the level of heterogeneity of consumers is kept very small. It has been confirmed that a small increase in ω first reduces the relative frequency of the realization of P*, since the formation of the equilibrium P* is sensitive to the spatial form of the boundary between the regions of A and B and the boundary is affected by ω. It has been also confirmed that a further increase in ω leads to the growth of the relative frequency of realization of P* up to unity, since it overcomes the effects of both $r(\pm 1)$ and the externalities so that consumers only choose their favorite technologies based on their strong bias. The same kind of "phase transition" has been observed in Cowan and Cowan (1998), Farrell and Saloner (1988), and Dalle (1997).

Extensive parameter runs are performed with regard to the combinations of three conditions,

(ic) an initial ratio of consumers who adopt the technology A, B, or C,
(r) the parameters $r(\pm 1)$, and
(s) the parameters $s(\pm 1)$.

The parameter sets are constructed as combinations of these three conditions. First, the condition (ic) has two categories:

(ic-1) $\{p_A(1), p_B(1), p_C(1)\} = \{10/101^2, 10/101^2, 1 - 20/101^2\}$ and
(ic-2) $\{p_A(1), p_B(1), p_C(1)\} = \{9/101^2, 11/101^2, 1 - 20/101^2\}$.

In the case of (ic-1), there is symmetry in the initial number of those who adopt the technology A and those who adopt the technology B. On the other hand, in the case of (ic-2), the technology B is set to have a very slight superiority in number at the initial point. Secondly, the condition (r) has three categories that are

(r-1) equal: $\{r(+1), r(-1)\} = \{1, 1\}$,
(r-2) unequal (B is better): $\{r(+1), r(-1)\} = \{1, 2\}$, and
(r-3) unequal (A is better): $\{r(+1), r(-1)\} = \{2, 1\}$.

Thirdly, the condition (s) has four categories that are

(s-1) no externality: $\{s(+1), s(-1)\} = \{0, 0\}$,
(s-2) equal externalities: $\{s(+1), s(-1)\} = \{1, 1\}$,
(s-3) unequal externalities (B is better): $\{s(+1), s(-1)\} = \{1, 2\}$, and
(s-4) unequal externalities (A is better): $\{s(+1), s(-1)\} = \{2, 1\}$.

The combinations of the above three conditions give 24 cases. However, only 19 cases, excluding one side of symmetric cases, are examined. Note that not only symmetric but also asymmetric cases can be examined under the parameter sets provided above.

Table 2 shows the relative frequencies of the equilibria A*, B*, and P* for the 19 cases. The robustness of all three equilibria are tested and confirmed in simulations by slightly but continuously perturbing the equilibrium configuration (namely, the ratio between A adopters and B adopters) for two thousand repetitions. The time to reach an equilibrium depends upon the parameters ν and ξ in Eq. (8), and it has been confirmed that $t = 50$ $(= t^*)$ is long enough for the system to reach an equilibrium when $\nu = 1$ and $\xi = 0.5$ are used. The last column in Table 2 will be explained in Section 5.

In cases 1 and 8, there are no effects of global or local network externalities, since both the parameters $s(\pm 1)$ are set as zero which make the terms of externalities in Eq. (4) disappear. In such cases, it is always observed that the system reaches polymorphic equilibria, where clusters of the consumers who adopt A and who adopt B can be seen in the game field. Note that even though there is symmetry in the initial number, the initial configuration of the consumers with each strategy is random. Due to this randomness, the equilibria in case 1 is not exactly as $\{p_A(t^*), p_B(t^*), p_C(t^*)\} = \{0.5, 0.5, 0\}$ but most likely close to the set of these values. On the other hand, the polymorphic equilibria in case 8 move slightly toward B* because of the technology B's slight superiority in number at the initial point.

In case 2, there exist positive and equal effects of both global and local network externalities on the technologies A and B, as well as symmetry in the initial numbers as in the condition (ic-1). In such case, three types of equilibria, A*, B*, and P* are possible, depending upon the initial configuration. Yet, the relative frequencies of

Table 2. Relative frequencies of the equilibria A*, B*, and P*. The symbol A* denotes the equilibrium where the technology A takes over the whole market, and B* denotes the technology B. The symbol P* stands for a polymorphic equilibrium where the technologies A and B coexist. The last column shows equilibria that are approximated by the mean-field theory in Section 5

Initial	Parameters	Parameters	Case	Relative frequencies of equilibria			M.F.
Condition (ic)	(r)	(s)	No.	A*	P*	B*	Approx.
(ic-1)	(r-1)	(s-1) no externalities	1	0	1	0	P*
equal	equal	(s-2) equal externalities	2	0.42	0.14	0.44	P*
		(s-3) unequal (B is better)	3	0	0	1	B*
		(s-4) unequal (A is better)	–	–	–	–	–
	(r-2)	(s-1) no externalities	4	0	0	1	B*
	unequal	(s-2) equal externalities	5	0	0	1	B*
	(B is better)	(s-3) unequal (B is better)	6	0	0	1	B*
		(s-4) unequal (A is better)	7	0	0	1	B*
(ic-2)	(r-1)	(s-1) no externalities	8	0	1	0	P*
unequal	equal	(s-2) equal externalities	9	0.04	0.14	0.82	P*
(B is more)		(s-3) unequal (B is better)	10	0	0	1	B*
		(s-4) unequal (A is better)	11	1	0	0	A*
	(r-2)	(s-1) no externalities	12	0	0	1	B*
	unequal	(s-2) equal externalities	13	0	0	1	B*
	(B is better)	(s-3) unequal (B is better)	14	0	0	1	B*
		(s-4) unequal (A is better)	15	0	0	1	B*
	(r-3)	(s-1) no externalities	16	1	0	0	A*
	unequal	(s-2) equal externalities	17	1	0	0	A*
	(A is better)	(s-3) unequal (B is better)	18	1	0	0	A*
		(s-4) unequal (A is better)	19	1	0	0	A*

these three equilibria are not equally distributed, and P* is less frequent than A* and B*. This is because the initial asymmetric (random) configuration between A and B adopters is magnified due to the existence of the global feedbacks, despite the equal parameters of the system. If the difference of initial scatterness between A and B adopters is negligible, then realization of P* is expected. Moreover, it is expected that the system tends to reach A* if initially the B adopters are closer to each other than the A adopters, since the chance for the initial B adopters to meet those who are potential B adopters (C adopters around the B adopters in this case) in the early stage becomes smaller than the chance for the initial A adopters who are relatively more scattered. This can be very critical for the technology B, since the initial growth rate of the B adopters slows down and the global feedback is weakened, and, as a result, it leads to A*. Let us now denote the average number of shortest steps between A adopters and between B adopters at $t = 1$ as S_A and S_B, respectively, which can be one of the measurements of how much they are spread initially in the game field. For example, S_A tends to be greater than S_B when the initial A adopters form a relatively more dispersed flock than the initial B adopters. In such case, the difference, $S_A - S_B$, becomes positive by the definition of S_A and S_B. Actually, it has been confirmed through the simulations that the system tends to reach P* when $S_A - S_B$ is equal or very close to zero. On the other hand, the system more likely goes toward A* when $S_A - S_B$ is greater than zero, while, B* is observed when $S_A - S_B$ is less than zero. These facts suggest that the initial configuration is the key factor for equilibrium selection in case 2. This result originates from the spatiality of the model that suggests the importance of agent-based modeling. Further explanation is given in Section 5.

In case 9, positive and equal effects of both global and local network externalities on both technologies also exist, but now the initial numbers as well as the initial configuration are asymmetric, as in condition (ic-2). In this case, the system reaches B* more frequently due to B's slight superiority in its initial number that causes the effect of the global feedback to be enlarged. In other words, even if there is a slight difference in initial ratio between consumers who adopt A and those who adopt B, it is eventually amplified. This corresponds to the effects of increasing returns, which is discussed by Arthur (1989). Here, it is worthwhile enough to emphasize that, even though the technology B initially has an slight superiority in number, there still exists a significant possibility for the technology A to coexist with the technology B by forming clusters (see Fig. 2). Moreover, even though the possibility is very small, the technology A still has a chance to conquer the whole market. Note that all the equilibria are robust, as mentioned earlier. Equilibrium selection is again dependent upon an initial configuration, though the level of the dependency is not as high as that of case 2 due to the asymmetry in condition (ic-2). It has been confirmed that the values of $S_A - S_B$ tend to be significantly positive when P* or A* is achieved. If the model does not contain the locality, that is, if $p_i(\sigma_i(t), t)$ in Eq. (4) is invariable for i, then the global feedback drives the technology B's slight superiority in its initial number all the way to B* with probability unity. However, positive frequencies of P* and A* in this case are caused by the existence of multiplicative effects of the global and local feedbacks that originate from local interaction provided by the framework of the spatial game. Also note that, in such a case, motivation of introducing agent-based modeling strongly arises. Detailed explanation on this motivation is given in Section 5.

In cases 3, 4, 5, 6, 10, 12, 13, and 14, the technology B has a strict superiority in its payoff as well as in its initial number of adopters for some cases. As a result, in all those cases, the technology B always takes over the market. The same type of argument applies to cases 11, 16, 17, and 19, but in these cases, the technology A is the one that takes off.

Interestingly enough, in cases 7 and 15, the technology B always dominates the market, even though the technology A has a superiority on its externality parameter $s(+1)$ over $s(-1)$, as seen in the condition (s-4). This is because the technology B has its superiority on the parameter $r(-1)$ over $r(+1)$ in addition to its initial number for case 15, and the level of the superiority in network externality for the technology A could not overwhelm the technology B's other superiority. It is expected that A* would be observed if $s(+1)$ were set higher, and, for example in case 7, it has been observed in the simulations that $s(+1) = 8$ is large enough for the technology A to take over the market with probability one. Case 18 is symmetric in (r) and (s) against case 15, and A* is achieved even though the condition (ic-2) still works against the technology A.

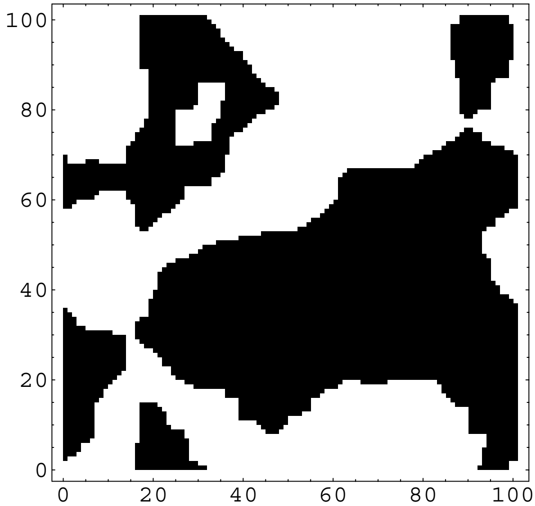

Fig. 2. There are clusters of the technologies A and B adopters in the game field when P* is achieved. Black and White represent the A and B adopters, respectively

So far, in this paper, switching costs that consumers pay when they switch from the technology A to B or B to A are assumed to be negligible and set as zero for simplicity. If sufficient switching costs, which can be treated as new parameters, are considered in the model, then the consumers' incentive to switch technologies decreases, and, therefore, P* is conjectured to be encouraged. Actually, it has been confirmed that this conjecture is correct by simulations with switching costs. Figure 3 shows the relative frequency of P* for case 2 ((ic-1), (r-1), and (s-2)) with a equal switching cost, c. It can be observed that the relative frequency of P* increases linearly as the value of c increases when it holds $c_1\ (= 1.4) < c < c_2\ (= 3.2)$, while the relative frequency of P* seems independent of c for $c \leq c_1$ and $c \geq c_2$. This result is intuitive because, when switching costs are too small, they are conceived as negligible, while the realization of total standardization for which switching is indispensable most likely becomes hopeless when the costs are too large. When we look at switching from, for example, B to A, the value of c_1 is interpreted as a maximum cost below which consumers are willing to pay to switch from B to A. From Eq. (4), c_1 is analytically calculated as $u_{max}(+1) - u_{max}(-1) = 9(\frac{8}{9}0.5 - \frac{4}{9}0.5) = 2$, which can be said close to c_1 since it is an averaged value over fifty trials. On the other hand, the value of c_2 is interpreted as a minimum cost above which consumers are unable or unwilling to pay to switch from B to A and calculated as $u_{max}(+1) - u_{min}(-1) = 9(\frac{8}{9}0.5 - \frac{1}{9}0.5) = 3.5$, which can be said close to c_2 due to the same reason mentioned above. The analytical approach when $c_1 < c < c_2$ will be discussed in Section 5. Until then, c is set as zero again for simplicity.

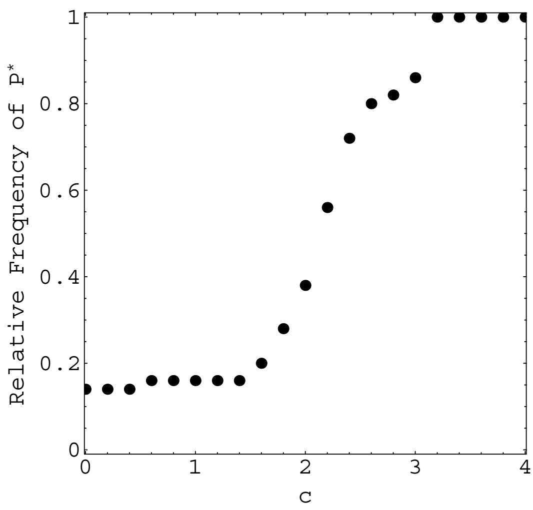

Fig. 3. Relative frequency of P* of the case 2 ((ic-1), (r-1), and (s-2)) as a function of the equal switching cost, c. It can be observed that the relative frequency of P* increases linearly as the value of c increases, while it seems independent of c when $c \leq c_1$ $(= 1.4)$ and $c \geq c_2$ $(= 3.2)$

4 Effects of an innovation factor

Now an innovation factor is introduced into the model. Applying the innovation factor to the externality might sound odd, but it signifies an innovation that improves the compatibility of a technology. Due to the innovation, consumers or users can enjoy more benefit of the externality, since as a result they are more connected. Note that, for example, the idea of trying to line up VHS on shelves of as many rental video shops also counts as an innovation on externality.

Table 3 shows relative frequencies of the three equilibria, A*, B*, and P* when the effect of an innovation factor is applied to case 5 shown in the previous section. The last column in Table 3 will be explained in Section 5. For cases 5(a), 5(b), and 5(c) in Table 3, all the parameters as well as initial conditions are the same as case 5 up to $t = 24$, and after $t = 25$ the parameter $s(+1)$, now denoted as $s(+1, t \geq 25)$, is increased to 5(a) $s(+1, t \geq 25) = 2$, 5(b) $s(+1, t \geq 25) = 4$, and 5(c) $s(+1, t \geq 25) = 8$, that is, an innovation is introduced to technology A's side at $t = 25$. Note that once an innovation factor has been introduced, it remains. One can see that the innovation factor in 5(c) is large enough for the technology A to almost always retake the market, while 5(a) and 5(b) are both too small. Figure 4a represents the trajectories of $\{p_A(t), p_B(t)\}$ projected on the $p_A(t)$-$p_B(t)$ plane with (triangle dots) and without (black circles) the innovation factor $s(+1, t \geq 25) = 8$. It is observed that the technology A regains its market share after $t = 25$ and eventually takes over the whole market. Here, note that introducing an innovation factor could sometimes make the system arrive at a polymorphic equilibrium (case not shown).

In case 5(d) in Table 3, the same size of innovation factor as case 5(c) is introduced, but at time $t = 35$, that is, $s(+1, t \geq 35) = 8$. In this case, technology

Table 3. Relative frequencies of the three equilibria, A*, B*, and P* when an innovation factor is introduced in case 5 shown in Table 2. The last column shows equilibria that are approximated by the mean-field theory in Section 5

Initial Condition (ic)	Parameters (r)	Parameters (s)	Case No.	Relative frequencies of equilibria A*	P*	B*	M.F. Approx.
(ic-1)	(r-2)	(s-2)	5(a) $s(+1, t \geq 25) = 2$	0	0	1	B*
equal	unequal	initially equal	5(b) $s(+1, t \geq 25) = 4$	0.02	0	0.98	B*
	(B is better)	externalities	5(c) $s(+1, t \geq 25) = 8$	0.98	0	0.02	A*
			5(d) $s(+1, t \geq 35) = 8$	0.02	0	0.98	B*

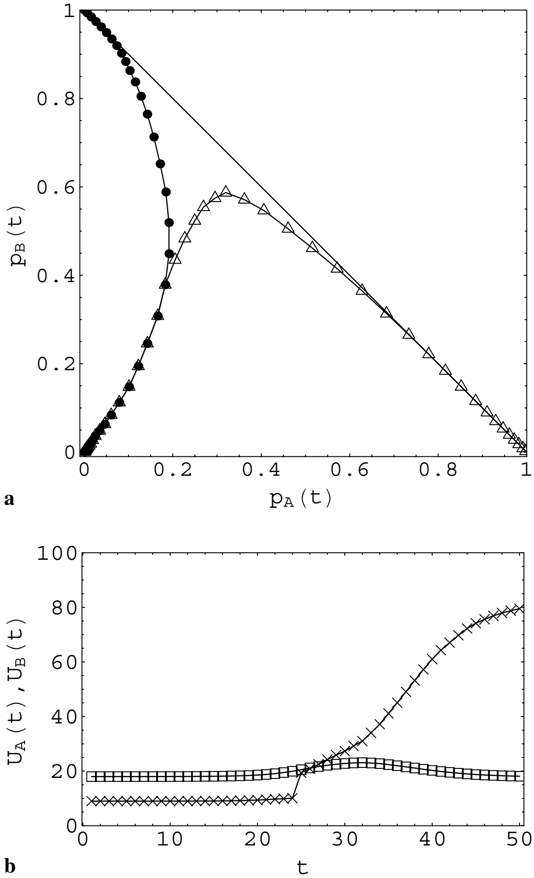

Fig. 4a,b. The results from the simulations in case 5 with the innovation factor. **a** The trajectories of $\{p_A(t), p_B(t)\}$ on the $p_A(t)$-$p_B(t)$ plane with (triangle dots) and without (black circles) the innovation factor $s(+1, t \geq 25) = 8$, respectively. **b** The overall mean utilities corresponding to the case with the innovation factor in Fig. 4a. The x-marks and square dots are for $U_A(t)$ and $U_B(t)$, respectively

B still almost always takes over the market, that is, the time $t = 35$ is too late for technology A with the innovation factor given as eight, or the innovation factor $s(+1, t \geq 35) = 8$ is too small for introducing it at $t = 35$ to retake the market. These results suggest that both the timing and the size of the innovation factor matter for altering paths toward a lock-in situation.

Figure 4b illustrates the overall mean utilities corresponding to Fig. 4a, where technology A successfully retakes the market. The overall mean utilities, denoted as $U_A(t)$ and $U_B(t)$, are defined as

$$\begin{aligned} U_A(t) &= \frac{1}{|N_A(t)|} \sum_{i \in N_A(t)} u_i(+1, t) \\ &= \frac{1}{|N_A(t)|} \sum_{i \in N_A(t)} |n| \{R_i(+1) + S(+1, t) p_i(+1, t)\} \\ &= |n| \{r(+1) + s(+1) p_A(t-1) \frac{\sum_{i \in N_A(t)} p_i(+1, t)}{|N_A(t)|}\} \quad \text{and} \end{aligned} \tag{10}$$

$$U_B(t) = |n| \{r(-1) + s(-1) p_B(t-1) \frac{\sum_{i \in N_B(t)} p_i(-1, t)}{|N_B(t)|}\} \tag{11}$$

where $u_i(\pm 1, t) = u_i(\sigma_i(t) = \pm 1)$ in Eq. (4), $N_A(t)$ and $N_B(t)$ are the sets of consumers who consume the technology A and B in the whole population N, respectively, and $|N_A(t)|$ and $|N_B(t)|$ are the sizes of the sets $N_A(t)$ and $N_B(t)$, respectively. The x-marks and square dots represent $U_A(t)$ and $U_B(t)$, respectively, in Fig. 4b. As in Fig. 4b, when the disadvantaged technology A comes from behind to retake the market due to the innovation factor, there exists a crossover between $U_A(t)$ and $U_B(t)$. In the case where the system reaches P* instead, there is no significant difference between $U_A(t)$ and $U_B(t)$. From these, it is observed that evaluating the overall mean utilities in Eqs. (10) and (11) suggests an equilibrium at which the system most likely arrives.

5 Mean-field theory

In this section, macroscopic equations for the densities of the agents with different strategies are derived under the mean-field approximation in which local densities are replaced by the global density and shown to replicate some of the simulation results in the previous sections. Microscopically, the local densities of the agents are different from position to position. However, the deviations of the local densities from the global density may be assumed randomly distributed if the configuration of initial agents is chosen by throwing a die. The mean-field approximation is expected to work as a zero-th order approximation, if the disagreements between the local and global densities do not play crucial roles. This depends upon the parameters and initial configurations, and the approximation works well for most of the cases in our model. However, as will be discussed later in this section, if the effects of equal amount of externalities are taken into account for both the products A and B, the discrepancies between the local and global densities are amplified and prevent the approximation from being effective.

The following rate equations hold for the fraction of A, B and C adopters:

$$p_A(t+1) - p_A(t) = \alpha(t)\, p_B(t) - \beta(t)\, p_A(t) + \gamma(t)\, p_C(t), \tag{12}$$
$$p_B(t+1) - p_B(t) = -\alpha(t)\, p_B(t) + \beta(t)\, p_A(t) + \delta(t)\, p_C(t), \text{ and} \tag{13}$$
$$p_C(t+1) - p_C(t) = -\epsilon(t)\, p_C(t), \tag{14}$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\delta(t)$, and $\epsilon(t)$ represent transition probabilities (see Appendix for the derivation of the rate equations). Now, if we let $\sigma^M_{k\in n(j)}(t)$ symbolize the $\sigma_j(t+1)$ that satisfies the right hand side of Eq. (7), then $\alpha(t)$ is obtained as

$$\begin{aligned}
\alpha(t) &= Pr(\sigma^M_{k\in n(j\in n(i))}(t) = +1,\ \forall i \in B) \\
&= Pr(\sigma^M_{k\in n(j\in n(i))}(t) = +1,\ \forall i \in B_{in} \cup B_{ed}) \\
&= Pr(\sigma^M_{k\in n(j\in n(i))}(t) = +1,\ \forall i \in B_{in}) \\
&\quad + Pr(\sigma^M_{k\in n(j\in n(i))}(t) = +1,\ \forall i \in B_{ed}) \\
&= 0 + Pr(\sigma^M_{k\in n(j\in n(i))}(t) = +1,\ \forall i \in B_{ed}) \\
&= Pr(B_{ed} \longrightarrow A_{ed}) \\
&\simeq \begin{cases} \frac{a}{2}\sqrt{p_A(t)p_B(t)}\{1 + \mathrm{sign}[U_{A_{ed}}(t) - U_{B_{in}}(t)]\} & \text{if } \mathrm{p_A}(1) \neq 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{15}$$

where sign[0] is assumed to be -1. The symbols A_{ed} (B_{ed} and C_{ed}) and A_{in} (B_{in} and C_{in}) stand for the set of technology A (B and C) adopters who are located at the edges of the clusters of A (B and C) adopters and the set of technology A (B and C) adopters who are inside the clusters of A (B and C) adopters, respectively. The value of $\alpha(t)$ becomes positive when $U_{A_{ed}}(t) > U_{B_{in}}(t)$. In other words, only when an agent judges that being at the edge of a cluster of the alternative product is strictly better than being inside a cluster of the current product, the agent switches. This shows his attachment to the cluster to which he presently belongs. The size of the probability, which reflects the ratio of encounter between A and B adopters, is approximated as the product of the length of circumferences of the clusters of A and B adopters that are measured by the square root of $p_A(t)$ and $p_B(t)$, respectively, with a parameter a ($0 \leq a \leq 1$). The same logic works for $\beta(t)$ that is obtained as

$$\begin{aligned}
\beta(t) &= Pr(A_{ed} \longrightarrow B_{ed}) \\
&\simeq \begin{cases} \frac{b}{2}\sqrt{p_A(t)p_B(t)}\{1 + \mathrm{sign}[U_{B_{ed}}(t) - U_{A_{in}}(t)]\} & \text{if } \mathrm{p_B}(1) \neq 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{16}$$

with a parameter b ($0 \leq b \leq 1$). From Eq. (10) and (11), the utilities, $U_{A_{in}}(t), U_{A_{ed}}(t), U_{B_{in}}(t)$, and $U_{B_{ed}}(t)$ in Eqs. (15) and (16), are defined as

$$U_{A_{in}}(t) = |n|\{r(+1) + s(+1)\, p_A(t-1)\}, \tag{17}$$

$$U_{A_{ed}}(t) = |n|\{r(+1) + s(+1)\, p_A(t-1)\, \ell_a\}, \tag{18}$$
$$U_{B_{in}}(t) = |n|\{r(-1) + s(-1)\, p_B(t-1)\}, \text{ and} \tag{19}$$
$$U_{B_{ed}}(t) = |n|\{r(-1) + s(-1)\, p_B(t-1)\, \ell_b\} \tag{20}$$

since we have

$$\frac{1}{|N_{A_{in}}(t)|} \sum_{i \in N_{A_{in}}(t)} p_i(+1, t) = 1, \tag{21}$$
$$\frac{1}{|N_{B_{in}}(t)|} \sum_{i \in N_{B_{in}}(t)} p_i(-1, t) = 1, \tag{22}$$
$$\frac{1}{|N_{A_{ed}}(t)|} \sum_{i \in N_{A_{ed}}(t)} p_i(+1, t) = \ell_a, \text{ and} \tag{23}$$
$$\frac{1}{|N_{B_{ed}}(t)|} \sum_{i \in N_{B_{ed}}(t)} p_i(-1, t) = \ell_b \tag{24}$$

where $|N_{A_{in}}(t)|$, $|N_{B_{in}}(t)|$, $|N_{A_{ed}}(t)|$, and $|N_{B_{ed}}(t)|$ are the sizes of the sets $N_{A_{in}}(t)$, $N_{B_{in}}(t)$, $N_{A_{ed}}(t)$, and $N_{B_{ed}}(t)$, respectively. Note that the parameters, ℓ_a and ℓ_b, satisfy $1/9 < \ell_a, \ell_b < 1$. The transition probabilities, $\gamma(t)$, $\delta(t)$, and $\epsilon(t)$, are approximated by applying the logistic equation in Eq. (9) and obtained as

$$\begin{aligned} \gamma(t) &= Pr(C_{ed} \longrightarrow A_{ed}) \\ &\simeq \begin{cases} k(1 - p_C(t)) & \text{if } \mathrm{p_A}(1) \neq 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{25}$$

$$\begin{aligned} \delta(t) &= Pr(C_{ed} \longrightarrow B_{ed}) \\ &\simeq \begin{cases} k'(1 - p_C(t)) & \text{if } \mathrm{p_B}(1) \neq 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{26}$$

$$\begin{aligned} \epsilon(t) &= Pr(C_{ed} \longrightarrow A_{ed} \text{ or } B_{ed}) \\ &= \begin{cases} \gamma(t) + \delta(t) & \text{if } \mathrm{p_A}(1) \neq 0 \text{ and } \mathrm{p_B}(1) \neq 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{27}$$

with parameters, k and k' ($0 < k, k' < 1$), which control the magnitude of the transition probabilities.

The last columns in Tables 2 and 3 show equilibria that are obtained by numerically solving Eqs. (12) to (14) and Eqs. (15) to (27). Note that these equilibria are consistent with the stationary solutions obtained by analytically solving Eqs. (12) to (14) with Eqs. (15) to (27), which show stability of the equilibria. The parameters a ($= b$), ℓ_a ($= \ell_b$), and k ($= k'$) are chosen as 0.5, 6/9, and 0.16 ($= \lambda/2$), respectively. The parameter k ($= k'$) is set as $\lambda/2$, since $p_A(t) + p_B(t)$ ($= 1 - p_C(t)$) can be described by the logistic equation in Eq. (9), the parameter of which is λ. It has been confirmed that the robustness of all the three equilibria (A*, B*, and P*) against mutation that is tested in the simulations is preserved in the mean-field theory. One can see that the mean-field theory with the above parameters successfully approximates the equilibria the system most likely reaches. However, note that the mean-field theory reaches its limit of approximation in cases 2 and 9 in Table 2 where equilibrium selection is highly sensitive to the effects of the initial configuration that are enlarged by the feedbacks under the parameter sets.

In Fig. 5a and b, the trajectory of $\{p_A(t), p_B(t)\}$ and overall mean utilities, $\hat{U}_A(t)$ ($\simeq U_{A_{in}}(t)$) and $\hat{U}_B(t)$ ($\simeq U_{B_{in}}(t)$), which are approximated by the above mean-field theory, are shown. These figures correspond to the case in Fig. 4a and b, respectively, which are obtained as a result of a simulation. It is observed that features of Fig. 4a and b are successfully recovered in Fig. 5a and b.

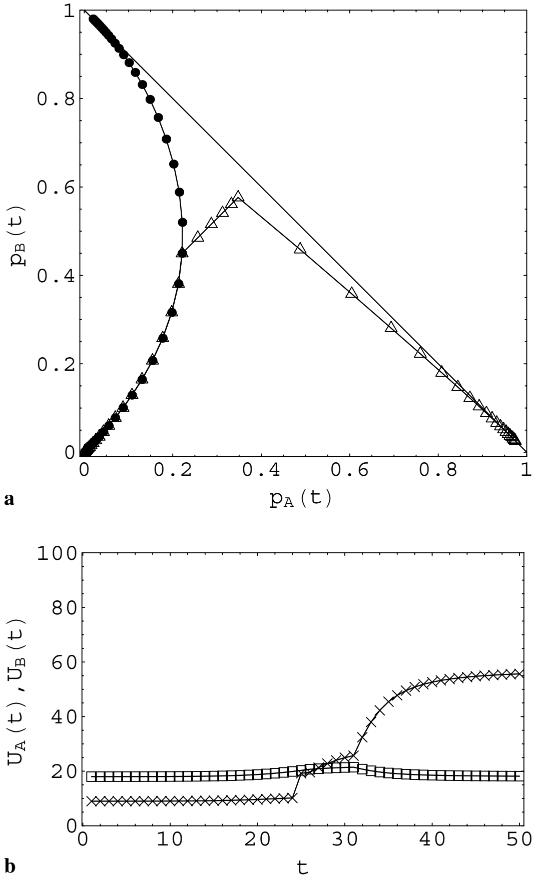

Fig. 5a,b. The results from the mean-field approximation that corresponds to the case in Fig. 4. The parameters a $(= b)$, ℓ_a $(= \ell_b)$, and k $(= k')$ are chosen as 0.5, 6/9, and 0.16 $(= \lambda/2)$, respectively. **a** The trajectories of $\{p_A(t), p_B(t)\}$ to A* and B* on the $p_A(t)$-$p_B(t)$ plane with and without the innovation factor $s(+1, t \geq 25) = 8$, respectively. **b** The overall mean utilities. The ex marks and square dots are for $U_A(t)$ and $U_B(t)$, respectively

So far, in this section, switching costs have been set as equal to zero. The following utilizes the mean-field approximation to case 2 with the equal switching cost, c (see Fig. 3). When the equal switching cost is introduced, the terms inside the sign functions in Eqs. (15) and (16) become

$$U_{A_{ed}}(t) - c - U_{B_{in}}(t) = |n| \left\{ p_A(t-1)\ell_a - p_B(t-1) - \frac{c}{|n|} \right\} \text{ and} \quad (28)$$

$$U_{B_{ed}}(t) - c - U_{A_{in}}(t) = |n| \left\{ p_B(t-1)\ell_b - p_A(t-1) - \frac{c}{|n|} \right\}, \quad (29)$$

respectively. Two unstable fixed points, P_1 and P_2, in Fig. 6 are calculated as intersections of $p_A(t) + p_B(t) = 1$ ($p_C(t) = 0$) and equations that are obtained by setting Eqs. (28) and (29) as equal to zero. Stable fixed points (A*, B*, and P*) and the initial point are also depicted in Fig. 6. The segment P_1P_2 in Fig. 6 corresponds to the region for P* to be stable. Clearly, the length of P_1P_2, which is calculated as $\frac{\sqrt{2}}{(1+\ell_a)(1+\ell_b)|n|}((2+\ell_a+\ell_b)c + (1-\ell_a\ell_b)|n|)$, is an increasing function of c, which corresponds to the results of the simulations for $c_1 < c < c_2$ shown in Fig. 3.

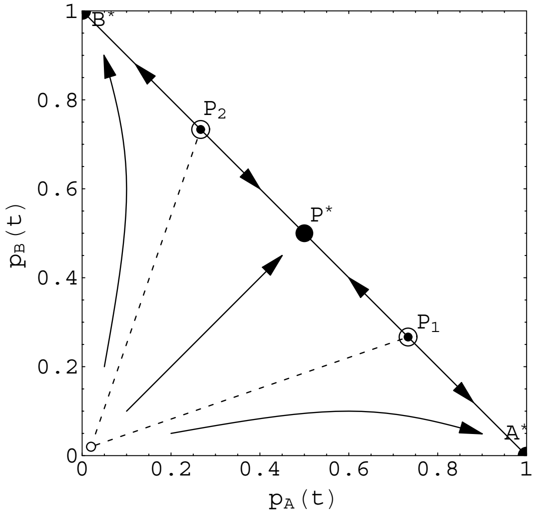

Fig. 6. Two unstable fixed points, P_1 and P_2, are calculated as the intersections of $p_A(t) + p_B(t) = 1$ ($p_C(t) = 0$) and equations that are obtained by setting Eqs. (28) and (29) as equal to zero. Stable fixed points (A*, B*, and P*) and initial point are also depicted in this figure. The segment P_1P_2 corresponds to the region for P* to be stable

6 Discussion

Competitive diffusion of two incompatible technologies, such as PC vs. Macintosh, VHS vs. Betamax and so on, is studied under the framework of a spatial game where consumers are distributed on a two-dimensional square lattice network and play 3×3 symmetric coordination-like games with their nearest neighbors. The consumers can enjoy network externalities by global coordination (global network externality), and, at the same time, the global network externality is transmitted and enhanced by local coordination (local network externality). The two types of externalities are set as multiplicative, that is, the multiplicative effects of global and local network externalities are introduced.

Both simulations and mean-field approximation show that the system always reaches robust partial standardization for the parameter sets under which there are no effects of global or local network externalities. On the other hand, when there are effects of both global and local network externalities, not only total but also robust partial standardization is observed, even with less heterogeneous agents, depending upon the parameters and initial configuration. The model, with not only global but also local network externalities, appears to be able to produce a variety of possible equilibria. Additionally, from the study on the model with an innovation factor, it is shown that both the timing and the size of the innovation factor matter for altering paths toward a lock-in situation.

Finally, introducing a random connection between consumers into the model is now underway, which may provide outcomes that are more realistic.

Appendix

Here we introduce the local densities of i's neighbors who are adopting either the strategy A, B, or C at time t that are given as $\ell_i(+1,t)$, $\ell_i(-1,t)$, and $\ell_i(0,t)$, respectively, as follows:

$$\ell_i(\sigma,t) = \frac{1}{2|n|}\sum_{j\in n(i)} \{\sigma\sigma_j(t)(\sigma\sigma_j(t)+1) + 2(\sigma^2-1)(\sigma_j^2(t)-1)\} \tag{30}$$

where $\sigma = +1, -1$, or 0. Now, if we let $\sigma^M_{k\in n(j)}(t)$ symbolize the $\sigma_j(t+1)$ that satisfies the right hand side of Eq. (7), then from Eq. (30) the time evolutions of the local densities are given as

$$\begin{aligned}
&\ell_i(+1,t+1) - \ell_i(+1,t)\\
&= \frac{1}{2|n|}\sum_{j\in n(i)} \left\{(\sigma^M_{k\in n(j)}(t))^2 - \sigma_j^2(t) + (\sigma^M_{k\in n(j)}(t) - \sigma_j(t))\right\}\\
&= \frac{1}{2|n|}\left\{\sum_{j\in n(i)} \sigma_j(t)(\sigma_j(t)-1)\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)+1)}{2}\right.\\
&\quad - \sum_{j\in n(i)} \sigma_j(t)(\sigma_j(t)+1)\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)-1)}{2}\\
&\quad \left. + \sum_{j\in n(i)} 2(1+\sigma_j(t))(1-\sigma_j(t))\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)+1)}{2}\right\}, \qquad (31)\\
&\ell_i(-1,t+1) - \ell_i(-1,t)\\
&= \frac{1}{2|n|}\sum_{j\in n(i)} \left\{(\sigma^M_{k\in n(j)}(t))^2 - \sigma_j^2(t) - (\sigma^M_{k\in n(j)}(t) - \sigma_j(t))\right\}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2|n|}\Bigg\{ -\sum_{j\in n(i)} \sigma_j(t)(\sigma_j(t)-1)\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)+1)}{2} \\
&\quad + \sum_{j\in n(i)} \sigma_j(t)(\sigma_j(t)+1)\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)-1)}{2} \\
&\quad + \sum_{j\in n(i)} 2(1+\sigma_j(t))(1-\sigma_j(t))\frac{\sigma^M_{k\in n(j)}(t)(\sigma^M_{k\in n(j)}(t)-1)}{2}\Bigg\}, \qquad (32)
\end{aligned}
$$

and

$$
\begin{aligned}
&\ell_i(0,t+1)-\ell_i(0,t) \\
&= \frac{1}{2|n|}\sum_{j\in n(i)}(-2)\left\{(\sigma^M_{k\in n(j)}(t))^2-\sigma_j^2(t)\right\} \\
&= \frac{1}{2|n|}\Bigg\{-\sum_{j\in n(i)} 2(1+\sigma_j(t))(1-\sigma_j(t))(\sigma^M_{k\in n(j)}(t))^2\Bigg\}. \qquad (33)
\end{aligned}
$$

Here the global densities of the strategy A, B, and C consumers are introduced as

$$p_A(t) = \frac{1}{|N|}\sum_{i\in N}\ell_i(+1,t), \qquad (34)$$

$$p_B(t) = \frac{1}{|N|}\sum_{i\in N}\ell_i(-1,t), \ \text{and} \qquad (35)$$

$$p_C(t) = \frac{1}{|N|}\sum_{i\in N}\ell_i(0,t), \qquad (36)$$

respectively. Certainly, it holds

$$p_A(t)+p_B(t)+p_C(t)=1. \qquad (37)$$

Now the local densities $\ell_i(+1,t)$, $\ell_i(-1,t)$, and $\ell_i(0,t)$ are replaced by the global density $p_A(t)$, $p_B(t)$, and $p_C(t)$, respectively, and the following equations are obtained:

$$
\begin{aligned}
&\frac{1}{|N|}\sum_{i\in N}[\,\ell_i(\pm1,t+1)-\ell_i(\pm1,t)\,] \qquad (38) \\
&= \pm Pr(\sigma^M_{k\in n(j\in n(i))}(t)=+1,\ \forall i\in B)\ \frac{1}{|N|}\sum_{i\in N}\frac{1}{2|n|}\sum_{j\in n(i)}\sigma_j(t)(\sigma_j(t)-1) \\
&\quad \mp Pr(\sigma^M_{k\in n(j\in n(i))}(t)=-1,\ \forall i\in A)\ \frac{1}{|N|}\sum_{i\in N}\frac{1}{2|n|}\sum_{j\in n(i)}\sigma_j(t)(\sigma_j(t)+1) \\
&\quad + Pr(\sigma^M_{k\in n(j\in n(i))}(t)=\pm1,\ \forall i\in C)\ \frac{1}{|N|}\sum_{i\in N}\frac{1}{2|n|}\sum_{j\in n(i)}2(1+\sigma_j(t))(1-\sigma_j(t))
\end{aligned}
$$

and

$$\frac{1}{|N|}\sum_{i\in N}[\,\ell_i(0,t+1)-\ell_i(0,t)\,] = Pr(\sigma^M_{k\in n(j\in n(i))}(t)$$
$$= +1 \cup -1,\ \forall i \in C)\ \frac{1}{|N|}\sum_{i\in N}\frac{1}{2|n|}\sum_{j\in n(i)} 2(1+\sigma_j(t))(1-\sigma_j(t)) \quad (39)$$

that lead to Eqs. (12), (13), and (14).

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