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The dynamic effects of general purpose technologies on Schumpeterian growth*

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Abstract. General purpose technologies (GPTs) are drastic innovations characterized by pervasiveness in use and innovational complementarities. The dynamic effects of a GPT are analyzed within a quality-ladders model of scale-invariant Schumpeterian growth. The diffusion path of a GPT across a continuum of industries is governed by S-curve dynamics. The model generates a unique, saddle-path long-run equilibrium. Along the transition path, the measure of industries that adopt the new GPT increases, consumption per capita falls, and the interest rate rises. The growth rate of the stock market depends negatively on the rate of GPT diffusion and the magnitude of the GPT-ridden R&D productivity gains; and positively on the rate of population growth. It also follows a U-shaped path during the diffusion process of the new GPT. Finally, the model generates transitional growth cycles of per capita GNP.

Keywords: General purpose technologies – Schumpeterian growth – Scale effects – R&D races

JEL Classification: E3, O3, O4

1 Introduction

In any given economic "era" there are major technological innovations, such as electricity, the transistor, and the Internet, that have far-reaching and prolonged impact. These drastic innovations induce a series of secondary, incremental innovations. The introduction of the transistor, for example, triggered a sequence of

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secondary innovations, such as the development of the integrated circuit and the microprocessor, which are themselves considered drastic innovations. These main technological innovations are used in a wide range of different sectors, inducing further innovations. For example, microprocessors are now used in many everyday products such as telephones, cars, personal computers, and so forth.

In general, drastic innovations have three key characteristics. The first feature refers to the generality of purpose, i.e., drastic innovations affect a wide range of industries and activities within industries. Consequently, Bresnahan and Trajtenberg (1995) christened these types of drastic innovations "General Purpose Technologies" (GPTs henceforth). Several empirical studies have documented the cross-industry pattern of diffusion for a number of GPTs.[1] In addition, a strand of empirical literature has established that the cross-industry diffusion pattern of GPTs is similar to the diffusion process of product-specific innovations and that it is governed by standard S-curve dynamics.[2] In other words, the internal-influence epidemic model can provide an empirically-relevant framework within which to analyze the dynamic effects of a GPT. During this diffusion process, these drastic innovations could generate growth fluctuations and even business cycles.

Second, the dynamic effects of these GPTs take a long period of time to materialize. For instance, David (1990) argues that it may take several decades before major technological innovations can have a significant impact on macroeconomic activity. Third, these GPTs act as "engines of growth". As a better GPT becomes available, it gets adopted by an increasing number of user industries and fosters complementary advances that raise the industry's productivity growth. As the use of a GPT spreads throughout the economy, its effects become significant at the aggregate level, thus affecting overall productivity growth. In his presidential address to the American Economic Association, Jorgenson (2001) documents the role of information technology in the resurgence of U.S. growth in the late 1990s.[3] There is plenty of evidence that the rise in structural productivity growth in the late 1990s can be traced to the introduction of personal computers and the acceleration in the

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[1] For example, Helpman and Trajtenberg (1998b) provided evidence for the diffusion of the transistor. They state that transistors were first adopted by the hearing aids industry. Later, transistors were used in radios, followed by their adoption by the computer industry. These three industries are known as early adapters. The fourth sector to adopt the transistor was the automobile industry, followed by the telecommunications sector.

[2] Griliches (1957), for example, studied the diffusion of hybrid seed corn in 31 states and 132 crop-reporting areas among farmers. His empirical model generates an S-curve diffusion path. Andersen (1999) confirmed the S-shaped growth path for the diffusion of entrepreneurial activity, using corporate and individual patents granted in the U.S. between 1890 and 1990. Jovanovic and Rousseau (2001) provided more evidence for an S-shaped curve diffusion process by matching the spread of electricity with that of personal computer use by consumers.

[3] At the aggregate level, information technology is identified with the output of computers, communications equipment, and software. These products appear in the GDP as investments by businesses, households, and governments along with net exports to the rest of the world.

price reduction of semiconductors, which constituted the necessary building blocks for the information technology revolution.[4]

The growth effects of GPTs have been analyzed formally by Helpman and Trajtenberg (1998a). In their model, GPTs require complementary inputs before they can be applied profitably in the production process. Complementary inputs developed for previous GPTs are not suited for use with a newly arrived GPT. The sequential arrival of GPTs generates business cycles. A typical cycle consists of two phases, a phase in which firms produce final goods with the old GPT and components are developed for the new GPT, and a second phase in which final goods producers switch to the new GPT and the development of components for that GPT continues. Output declines in the first phase of a cycle as workers switch from production to research to invent new inputs and increases again in the second phase once the new technology is implemented.[5]

Following the Helpman and Trajtenberg (1998a) model, Aghion and Howitt (1998b) explored the macroeconomic effects of GPTs. They derived a simple version of the model from the basic Schumpeterian model of endogenous growth by adding a second stage to the innovation process, a stage of component-building, and they endogenized the arrival times of successive GPTs.[6] Their model results in similar per capita GNP growth cycles due to the adoption of the new GPT.

In this paper, I analyze formally the effects of a GPT within a state-of-the-art model of Schumpeterian growth without scale effects. Schumpeterian (R&D-based) growth is a type of growth that is generated through the endogenous introduction of new goods or processes based on Schumpeter's (1934) process of creative destruction, as opposed to physical or human-capital accumulation.[7]

Earlier models of Schumpeterian growth assumed that the growth rate of technological change depends positively on the level of R&D resources devoted to innovation at each instant in time. As population growth causes the size of the economy (scale) to increase exponentially over time, R&D resources also grow exponentially, as does the long-run growth rate of per capita real output. In other words, long-run Schumpeterian growth in these models exhibits scale effects. Two influential papers by Jones (1995a,b) provided time series evidence for the absence

[4] Another study from OECD documents that U.S. investment in information processing equipment and software increased from 29% in 1987 to 52% in 1999. The diffusion of information and communication equipment accelerated after 1995 as a new wave of information and communication equipment, based on applications such as the World Wide Web and the browser, spread rapidly throughout the economy.

[5] There is a growing literature with this approach. See, for example, Helpman and Rangel (1998), Aghion and Howitt (1998b), and the volume edited by Helpman (1998).

[6] Eriksson and Lindh (2000) explored a variation of the Helpman and Trajtenberg (1998a) model in which technological development occurs partly by discrete replacements of obsolete technologies and the timing of technology shifts is endogenized.

[7] There are two classes of scale invariant Schumpeterian growth models; endogenous and exogenous. Endogenous [exogenous] Schumpeterian growth models are those in which *long-run* growth can [cannot] be affected by permanent policy changes.

of these scale effects. This evidence led theorists to construct Schumpeterian growth models that exclude scale effects.[8]

My approach to modeling the GPTs has the following features. First, the model abstracts from scale effects and generates long-run growth, which is consistent with the time-series evidence presented by Jones (1995a). Second, I take into consideration the above mentioned evidence on long diffusion lags associated with the adoption of a new GPT. I therefore analyze both the transitional dynamics and long-run effects of a new GPT. Third, I assume that a GPT is beneficial to all firms in each industry. Thus, when a GPT is implemented in an industry, it affects the productivity of R&D workers, the size of all future innovations in that industry and its growth rate. Finally, I assume that, although a GPT's rate of diffusion is exogenous, its diffusion path across a continuum of industries is governed by S-curve dynamics.

I incorporate the presence of a GPT into the standard quality-ladder framework of Schumpeterian growth without scale effects that was developed by Dinopoulos and Segerstrom (1999). In the model, there is positive population growth and one factor of production, labor. Final consumption goods are produced by a continuum of structurally identical industries. Labor in each industry can be allocated between two economic activities, manufacturing of high-quality goods and R&D services that are used to discover new products of higher quality.

The arrival of innovations in each industry is governed by a memoryless Poisson process whose intensity depends positively on R&D investments and negatively on the rate of difficulty of conducting R&D. Following Dinopoulos and Segerstrom (1999), I assume that R&D becomes more difficult over time in each industry. Specifically, I assume that the productivity of R&D workers declines as the size of the market (measured by the level of population) increases. This assumption captures the notion that it is more difficult to introduce new products and replace old ones in a larger market.[9]

The main purpose of this paper is to explore the effects of GPTs on Schumpeterian growth. Thus, it is imperative to develop a Schumpeterian growth model

[8] Dinopoulos and Thompson (1999) provided a survey of the empirical evidence on scale and growth, and describe recent attempts to develop models that generate growth without scale effects.

[9] Several authors have developed microfoundations for this assumption. Young (1998), Dinopoulos and Thompson (1998), and Aghion and Howitt (1998a, chapter 12) have combined tastes for horizontal and vertical product differentiation to generate models in which absolute levels of R&D drive productivity growth at the firm-level, but aggregate R&D in larger economies is diffused over a larger number of product lines or industries. At the steady state, the number of varieties is proportional to the level of population. As population grows, the number of varieties increases and aggregate R&D is diffused over a larger number of product lines or industries, making R&D more difficult.
Dinopoulos and Syropoulos (2000) have provided microfoundations for this specification in a model of Schumpeterian growth, where the discovery of higher quality products is modeled as an R&D contest (as opposed to an R&D race) in which challengers engage in R&D and incumbent firms allocate resources to rent-protecting activities. Rent-protecting activities are defined as costly attempts of incumbent firms to safeguard the monopoly rents from their past innovations. These activities can delay the innovation of better products by reducing the flow of knowledge spillovers from incumbents to potential challengers, and/or increase the costs of copying existing products. Their model postulates that R&D may become more difficult as the size of the economy grows because incumbent firms may allocate more resources to rent-protecting activities.

that is scale effect free and to analyze the behavior of GPTs within this framework. After removing the scale effects property from the model, I can discuss its transitional and long-run properties and implications. I use Mulligan and Sala-i-Martin's (1992) time-elimination method to study the transitional dynamics of the model.

This analysis generates several novel findings. First, there exists a unique globally stable-saddle-path along which the measure of industries that adopt the new GPT increases, per capita consumption expenditure decreases, the market interest rate increases, and the innovation rate of those industries that have adopted the new GPT decreases at a higher rate than that of those that have not adopted the new GPT. Second, the model exhibits transitional growth cycles of per capita GNP.

In previous GPT-driven growth models, GPTs also generate transitional growth cycles of per capita GNP. However, their results are not robust to the introduction of positive population growth. The introduction of positive population growth in Helpman and Trajtenberg (1998a) and Aghion and Howitt's (1998b) models of GPTs, will make these growth cycles shorter and shorter as the size of the economy increases, and in the long-run the GPT-induced cycles disappear. In the present model, the fall in output comes from the reduction in per capita consumption expenditure on final goods and the rise in the per capita R&D investment. As the size of the economy increases, the duration of the per capita GNP cycle remains the same. When all industries have adopted the new GPT and the diffusion process has been completed, the economy experiences a higher per capita income constant growth rate.

In the absence of a new GPT, the economy does not exhibit zero long-run growth as in previous models of GPTs (see Helpman and Trajtenberg, 1998a; Aghion and Howitt, 1998b). That is, the long-run growth rate depends positively on the rate of innovation (which equals per capita R&D) and thus any policy that affects per capita R&D investment has long-run growth effects.[10] In addition, the removal of scale effects allows one to analyze the effects of changes in the rate of growth of population that is absent from earlier models.

I also analyze the effects of a new GPT on the stock market. The growth rate of the stock market depends negatively on the rate of GPT diffusion process and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth. It also follows a U-shaped path during the diffusion process of the new GPT (Proposition 4). During the transition from the old to the new GPT, there are two types of industries in the economy: one that has adopted the new GPT and one that has not yet adopted it. The former type of industry is more innovative in terms of discovering higher quality products than the latter type. In the initial stages of a GPT's diffusion, the aggregate stock value decreases, since most of the industries belong to the latter type. As more industries switch to the new GPT, the aggregate stock value rises. This result is consistent with

[10] The evidence on the empirical validity of endogenous versus exogenous Schumpeterian growth models without scale effects is still limited. However, Zachariadis (2003) found strong support for the Schumpeterian endogenous growth framework without scale effects by using U.S. manufacturing industry data for the period 1963–1988. The manufacturing sector accounted for more than ninety percent of R&D expenditures in the U.S. until the late eighties.

that of previous GPT-driven growth models.[11] In addition, an increase in the GPT diffusion rate increases the economy-wide resources devoted to R&D. Thus, the probability that the incumbent firm will be replaced by a follower firm increases. In other words, when the GPT diffusion process accelerates, the decrease in per capita consumption expenditure is more severe, and per capita R&D investment increases. This last result provides a novel link between the GPT adoption and higher risk for incumbent firms and captures the effects of creative destruction on the stock market valuation of monopoly profits.[12]

However, the mechanism identified in the present model that links the growth rate of the stock market with the GPT differs from that of previous GPT-driven growth models. In Helpman and Trajtenberg's (1998a) model, for example, during the first phase, the components of both the best practice GPT and of the previous one have positive value. When the economy is in the second phase of a typical cycle, only components of the best practice GPT are valuable because at that time it is known that no component of the older technologies will ever be used. Thus, the introduction of a new GPT brings a sharp decline in the real value of the stock market during a substantial part of phase one, but it picks up toward the end of the phase. In the second phase, the stock market rises.

The effect of the GPT diffusion on the aggregate investment during the adoption process is ambiguous (Proposition 5). In the initial stages of the diffusion process, only a limited number of industries adopt the new GPT. These industries are called the early adopters. As more industries adopt the new GPT, aggregate R&D investment increases.

The rest of the paper is organized as follows. Section 2 develops the structure of the model. Section 3 analyzes the long-run properties of the model and Section 4 deals with the transitional dynamics. Section 5 summarizes the model's key findings and suggests possible extensions. The algebraic details and proofs of propositions are relegated to the Appendix.

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[11] Jovanovic and Rousseau (2001) documented empirically how technology has affected the U.S. economy over the past century, using 114 years of U.S. stock market data. Their estimates reveal evidence that entries to the stock market as a percentage of firms listed in each year, were proportionately largest between 1915 and 1929, and that these levels were not again approached until the mid-1980s. About half of American households and most businesses were connected to electricity in 1920, and about one half of the households and most businesses today own or use computers. Both expansions, therefore, coincide with periods during which electricity and information technology saw widespread adoption. During times of rapid technological change, the new entrants of the stock market will grab the most value from previous entrants because the incumbents will find hard to keep up. The downward trend in the starting values of the vintages reflects a slowing down in the growth of the stock market.

[12] The first OPEC shock may also explain a part of the drop in the stock market in the early 1970s, as well as a part of the productivity slowdown. Hobijn and Jovanovic (2001) argued that there are several problems associated with the oil-shock explanation. One problem is that a rise in oil prices should have lowered current profits more than future profits, because of the greater ease of finding substitutes for oil in the long-run, perhaps current output more than future output and, therefore, should have produced a rise in the ratio of market capitalization to GDP, not a fall. This scenario also implies a constant entry in the stock market, something that contradicts their evidence. Another problem that is associated with the oil-price-shock explanation for the stock-market drop is that the energy-intensive sectors did not experience the largest drop in value in 1973–1974. Their evidence supports that the information-technology-intensive sectors experienced the largest drop in 1973–1974.

2 The model

2.1 Industry structure

I consider an economy with a continuum of industries, indexed by $\theta \in [0, 1]$. In each industry θ, firms are distinguished by the quality j of the products they produce. Higher values of j denote higher quality, and j is restricted to taking on integer values. At time $t = 0$, the state-of-the-art quality product in each industry is $j = 0$, that is, some firm in each industry knows how to produce a $j = 0$ quality product and no firm knows how to produce any higher quality product. To learn how to produce higher quality products, firms in each industry engage in R&D races. In general, when the state-of-the-art quality in an industry is j, the next winner of an R&D race becomes the sole producer of a $j + 1$ quality product. Thus, over time, products improve as innovations push each industry up its "quality ladder", as in Grossman and Helpman (1991).

2.2 Diffusion of a new GPT

The diffusion path of a new GPT is modeled as follows: The economy has achieved a steady-state equilibrium, manufacturing final consumption goods with an old GPT. I begin the analysis at time $t = t_0$, when a new GPT arrives unexpectedly. Firms in each industry start adopting the new GPT at an exogenous rate.[13]

I use the epidemic model to describe the diffusion of a new GPT across the continuum of industries.[14] Its form can be described by the following differential equation,

$$\frac{\dot{\omega}}{\omega} = \delta(1 - \omega), \tag{1}$$

where $\dot{\omega} = \partial\omega/\partial t$ denotes the rate of change in the fraction of industries that use the new GPT and $\delta > 0$ is the rate of diffusion. Equation (1) states that the number of new adoptions during the time interval dt, $\dot{\omega}$, is equal to the number of remaining potential adopters, $(1 - \omega)$, multiplied by the probability of adoption, which is the product of the fraction of industries that have already adopted the new GPT, ω, and the parameter δ, which depends upon factors such as the attractiveness of the innovation and the frequency of adoption, both of which are assumed to be exogenous.

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[13] Aghion and Howitt (1998b) model the spread of GPTs using a continuum of sectors. In their model, the innovation process involves three stages. First, the GPT is discovered. Then each sector discovers a "template" on which research can be based. Finally, that sector implements the GPT when its research results in a successful innovation. They have computed paths of the fraction of sectors experimenting with the new GPT and the fraction using the new GPT and found that the time path of the later follows a logistic curve (S-curve).

[14] See Thirtly and Ruttan (1987, pp. 77–89) for various applications of the epidemic model to the diffusion of technology.

The solution to Equation (1) expresses the measure of industries that have adopted the new GPT as a function of time and yields the equation of the sigmoid (S-shaped) logistic curve:

$$\omega = \frac{1}{[1 + e^{-(\gamma+\delta t)}]}, \tag{2}$$

where γ is the constant of integration. Notice that for $t \to \infty$, Equation (2) implies that all industries have adopted the new GPT.[15]

2.3 Households

The economy is populated by a continuum of identical dynastic families that provide labor services in exchange for wages, and save by holding assets of firms engaged in R&D. Each individual member of a household is endowed with one unit of labor, which is inelastically supplied. The number of members in each family grows over time at the exogenous rate $g_N > 0$. I normalize the measure of families in the economy at time 0 to equal unity. Then the population of workers in the economy at time t is $N(t) = e^{g_N t}$.

Each household is modeled as a dynastic family,[16] which maximizes the discounted utility

$$U = \int_0^{\infty} e^{-(\rho - g_N)t} \log\, u(t)dt, \tag{3}$$

where $\rho > 0$ is the constant subjective discount rate. In order for U to be bounded, I assume that the effective discount rate is positive (i.e., $\rho - g_N > 0$). Expression $\log u(t)$ captures the per capita utility at time t, which is defined as follows:

$$\log u(t) \equiv \int_0^1 \log[\sum_j \lambda(\theta)^j q(j,\theta,t)]d\theta \; . \tag{4}$$

In Equation (4), $q(j, \theta, t)$ denotes the quantity consumed of a final product of quality j in industry $\theta \in [0, 1]$ at time t. Parameter $\lambda(\theta)$ measures the size of quality improvements and is equal to

$$\lambda(\theta) = \begin{cases} \lambda_1 & if\ \theta \in [0,\ \omega] \\ \lambda_0 & if\ \theta \in [\omega,\ 1], \end{cases} \tag{5}$$

[15] When $t \to -\infty$, then $\omega = 0$. If one assumes that the new GPT arrives at time $t = 0$, then $\omega > 0$. That is, the new GPT is introduced in the economy by a given fraction of industries ω (i.e., the industry or industries that developed this particular GPT).

[16] Barro and Sala-i-Martin (1995, Ch.2) provide more details on this formulation of the household's behavior within the context of the Ramsey model of growth.

where $\lambda_1 > \lambda_0 > 1$. At each point in time t, each household allocates its income to maximize Equation (4) given the prevailing market prices. Solving this optimal control problem yields a unit elastic demand function for the product in each industry with the lowest quality-adjusted price

$$q(j, \theta, t) = \frac{c(t)N(t)}{p(j, \theta, t)}, \tag{6}$$

where $c(t)$ is per capita consumption expenditure, and $p(j, \theta, t)$ is the market price of the good considered. The quantity demanded of all other goods is zero.

Given this static demand behavior, the intertemporal maximization problem of the representative household is equivalent to

$$\max_{c(t)} \int_0^\infty e^{-(\rho - g_N)t} \log c(t) dt \ , \tag{7}$$

subject to the intertemporal budget constraint $\dot{a}(t) = r(t)a(t) + w(t) - c(t) - g_N a$, where $a(t)$ denotes the per capita financial assets, $w(t)$ is the wage income of the representative household member, and $r(t)$ is the instantaneous rate of return. The solution to this maximization problem obeys the well-known differential equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho \ , \tag{8}$$

According to Equation (8), per capita consumption expenditure increase over time if the instantaneous interest rate exceeded the consumer's subjective discount rate ρ.

2.4 *Product markets*

Every firm in each industry θ uses labor $L(\theta, t)$ as the sole input in its production, according to the following production function

$$Q(\theta, t) = \frac{L(\theta, t)}{\alpha_Q} \ , \tag{9}$$

where α_Q is the unit labor requirement. The monopolist engages in limit pricing, i.e., it charges a price equal to unit cost of manufacturing a product times the quality increment

$$P = \lambda(\theta)\alpha_Q w \ . \tag{10}$$

At each instant in time, the incumbent monopolist produces the state-of-the-art quality product and earns a flow of profits

$$\pi(\theta, t) = \left(\frac{\lambda(\theta) - 1}{\lambda(\theta)} \right) c(t)N(t) \ . \tag{11}$$

2.5 R&D races

Labor is the only input used to do R&D in any industry. Each firm in each industry θ produces R&D services by employing labor $L_R(\theta, t)$ under the constant returns to scale production function[17]

$$R(\theta, t) = \frac{\mu(\theta)}{\alpha_R} L_R(\theta, t) \ , \tag{12}$$

In Equation (12), $\alpha_R/\mu(\theta)$ is the unit-labor requirement in the production of R&D services and $\mu(\theta)$ is equal to

$$\mu(\theta) = \begin{cases} \mu_1 = \mu & \text{if} \quad \theta \in [0, \omega] \\ \\ \mu_0 = 1 & \text{if} \quad \theta \in [\omega, 1], \end{cases} \tag{13}$$

where $\mu > 1$. A firm k that engages in R&D discovers the next higher-quality product with instantaneous probability $I_k dt$, where dt is an infinitesimal interval of time and

$$I_k(\theta, t) = \frac{R_k(\theta, t)}{X(t)} \ . \tag{14}$$

$R_k(\theta, t)$ is firm k's R&D outlays and $X(t)$ captures the difficulty of R&D in a typical industry. I assume that the returns to R&D investments are independently distributed across challengers, across industries, and over time. Therefore, the industry-wide probability of innovation can be obtained from Equation (14) by summing up the levels of R&D across all challengers. That is,

$$I(\theta, t) = \sum_k I_k(\theta, t) = \frac{R(\theta, t)}{X(t)} \ , \tag{15}$$

where and $R(\theta, t)$ denotes total R&D services in industry θ. Variable $I(\theta, t)$ is the effective R&D.[18] The arrival of innovations follows a memoryless Poisson process with intensity I_1 for the industries that have adopted the new GPT, and I_0 for industries that have not adopted the new GPT.

Early models of Schumpeterian growth considered $X(t)$ to be constant over time. This implied that the rates of innovation and the long-run growth increase exponentially as the scale of the economy grows exponentially. This scale-effects property is inconsistent with post-war time-series evidence presented in Jones (1995a).

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[17] The empirical evidence on returns to scale of R&D expenditure is inconclusive. Diminishing returns would make the analysis of the transitional dynamics more complicated. Segerstrom and Zolnierek (1999) among others developed a model in which they allow for diminishing returns to R&D effort at the firm level and industry leaders have R&D cost advantages over follower firms. In their model, when there are diminishing returns to R&D and the government does not intervene, both industry leaders and follower firms invest in R&D.

[18] The variable $I(\theta, t)$ is the intensity of the Poisson process that governs the arrivals of innovations in industry θ.

A recent body of theoretical literature has developed models of Schumpeterian growth without scale effects.[19] Two approaches have offered possible solutions to the scale-effects problem. The first generates exogenous long-run Schumpeterian growth models.[20] The second approach generates models that exhibit endogenous long-run Schumpeterian growth.[21] Here I adopt the second approach and remove the scale-effects property by assuming that the level of R&D difficulty is proportional to the market size measured by the level of population,

$$X(t) = kN(t) \;, \tag{16}$$

where $k > 0$ is a parameter.[22]

Consumer savings are channeled to firms engaging in R&D through the stock market. The assumption of a continuum of industries allows consumers to diversify the industry-specific risk completely and earn the market interest rate. At each instant in time, each challenger issues a flow of securities that promise to pay the flow of monopoly profits defined in (11) if the firm wins the R&D race and zero otherwise. Consider now the stock-market valuation of the incumbent firm in each industry. Let $V(t)$ denote the expected discounted profits of a successful innovator at time t when the monopolist charges a price p for the state-of-the-art quality product. Because each quality leader is targeted by challengers who engage in R&D to discover the next higher-quality product, a shareholder faces a capital loss $V(t)$ if further innovation occurs. The event that the next innovation will arrive occurs with instantaneous probability Idt, whereas the event that no innovation will arrive occurs with instantaneous probability $1 - Idt$. Over a time interval dt, the shareholder of an incumbent's stock receives a dividend $\pi(t)dt$ and the value of the incumbent appreciates by $dV(t) = [\partial V(t) \big/ \partial t]dt = \dot{V}(t)dt$. The absence of profitable arbitrage opportunities requires the expected rate of return on stock issued by a successful innovator to be equal to the riskless rate of return r; that is,

$$\frac{\dot{V}(\theta,t)}{V(\theta,t)}[1 - I(\theta,t)dt]dt + \frac{\pi(\theta,t)}{V(\theta,t)}dt - \frac{[V(\theta,t) - 0]}{V(\theta,t)}I(\theta,t)dt = rdt \;. \tag{17}$$

[19] See Dinopoulos and Thompson (1999) and Dinopoulos and Sener (2003) for an overview of these models.

[20] Jones (1995b), and Segerstrom (1998) have removed scale effects by assuming that R&D becomes more difficult over time because "the most obvious ideas are discovered first." The model that results from their specification is called the *temporary effects of growth* (TEG) model. In these models, the growth rate does not depend on any measure of scale. Increases in the steady-state level of R&D raise technology and income per capita at any point in time, but they do not raise the growth rate.

[21] Young (1998), Aghion and Howitt (1998a, chapter 12), Dinopoulos and Thompson (1998), Peretto (1998), and Peretto and Smulders (1998) removed the scale effects property by essentially the same mechanism as the one developed by exogenous Schumpeterian growth models. They introduced the concept of localized intertemporal R&D spillovers. Dinopoulos and Syropoulos (2000) proposed a novel mechanism based on the notion of innovation-blocking activities that removes the scale-effects property and generates endogenous long-run Schumpeterian growth. This model offers a novel explanation to the observation that the difficulty of conducting R&D has been increasing over time.

[22] Informational, organizational, marketing, and transportation costs can readily account for this difficulty. Arroyo, et al. (1995) have proposed this specification under the name of the *permanent effects of growth* (PEG) model, and have provided time-series evidence for its empirical relevance.

Taking limits in Equation (17) as $dt \to 0$ and rearranging terms appropriately gives the following expression for the value of monopoly profits

$$V(\theta,t) = \frac{\pi(\theta,t)}{r(t) + I(\theta,t) - \frac{\dot{V}(\theta,t)}{V(\theta,t)}}. \tag{18}$$

Consider now the maximization problem of a typical challenger k. This firm chooses the level of R&D investment $R_k(\theta,t)$ to maximize the expected discounted profits

$$V(\theta,t)\frac{R_k(\theta,t)}{X(t)}dt - w\frac{\alpha_R}{\mu(\theta)}R_k(\theta,t)dt \ , \tag{19}$$

where $I_k dt = [R_k(\theta,t)/X(t)]dt$ is the instantaneous probability it will discover the next higher-quality product and $w\alpha_R R_k(\theta,t)/\mu(\theta)$ is the R&D cost of challenger k.

Free entry into each R&D race drives the expected discounted profits of each challenger down to zero and yields the following equilibrium condition:

$$V(\theta,t) = \frac{w\alpha_R k N(t)}{\mu(\theta)} \ . \tag{20}$$

2.6 Labor market

All workers are employed by firms in either production or R&D activities. Taking into account that each industry leader charges the same price p, and that consumers only buy goods from industry leaders in equilibrium, it follows from (9) that total employment of labor in production is $\int_0^1 Q(\theta,t)d\theta$. Solving (12) for each industry leader's R&D employment $L_R(\theta,t)$ and then integrating across industries, total R&D employment by industry leaders is $\int_0^1 [R(\theta,t)\alpha_R/\mu(\theta)]d\theta$. Thus, the full employment of labor condition for the economy at time t is

$$N(t) = \int_0^1 Q(\theta,t)\alpha_Q d\theta + \int_0^1 \frac{\alpha_R R(\theta,t)}{\mu(\theta)}d\theta \ . \tag{21}$$

Equation (21) completes the description of the model.

3 Long-run equilibrium

The dynamic behavior of the economy is governed by two equations that determine the evolution of the per capita consumption expenditure, c, and the number of industries that adopt the new GPT, ω. To facilitate the interpretation and understanding of my results, I begin by deriving expressions for long-run per capita real output and long-run growth. Following the standard practice of Schumpeterian growth models, one can obtain the following deterministic expression for sub-utility $u(t)$, which is appropriately weighted consumption index and corresponds to real per capita income[23]

$$\log u(t) = \log c - \log \alpha_Q + I(\theta,t)t \log \lambda(\theta) - \log \lambda(\theta) \ . \tag{22}$$

The economy's long-run Schumpeterian growth rate is defined as the rate of growth of sub-utility $u(t)$, $g_u = \dot{u}(t)/u(t)$. By differentiating Equation (22) with respect to time, I obtain:

$$g_u = \frac{\dot{u}(t)}{u(t)} = I(\theta,t) \log \lambda(\theta) \ , \tag{23}$$

which is a standard expression for long-run growth in quality-ladders growth models. Because the size of each innovation becomes larger (i.e., $\lambda_1 > \lambda_0$) after all industries have adopted the new GPT (i.e., the diffusion process has been completed), long-run growth, g_u, can be affected not only through changes in the rate of innovation, but also through the diffusion of the new GPT.

At this point it is useful to choose labor as the numeraire of the model and set

$$w \equiv 1 \ . \tag{24}$$

Using Equations (6), (10), (12), (15), (16), and (21), and taking into account (24), I obtain the *resource condition*

$$1 = c\left(\frac{\omega}{\lambda_1} + \frac{(1-\omega)}{\lambda_0}\right) + k\alpha_R\left(\frac{\omega}{\mu}I_1 + (1-\omega)I_0\right) \ , \tag{25}$$

which defines a negative linear relationship between per capita consumption expenditure, c, and the effective R&D, I. The above resource condition holds at each instant in time because by assumption factor markets clear instantaneously.

I now derive the differential equation that determines the growth rate of per capita consumption expenditure, $\dot{c}/c$, as a function of its level and the rate of innovation. Equation (20) holds at each instant in time, and yields $\dot{V}(\theta,t)\Big/V(\theta,t) = \dot{X}(t)\Big/X(t) = g_N$. In other words, the values of expected discounted profits, $V(t)$, and the level of R&D difficulty, $X(t)$, grow at the constant rate of population growth, g_N. Using Equations (18) and (20), I obtain

$$c = \frac{\lambda(\theta)\alpha_R k}{(\lambda(\theta)-1)\mu(\theta)}][\rho + I(\theta,t) - g_N] \ , \tag{26}$$

which defines a positive linear relationship between per capita consumption expenditure, c, and the effective R&D, I. It also implies the familiar condition that $r = \rho$, which means that the market interest rate must be equal to the subjective discount rate in the steady-state equilibrium. This property is shared by all Schumpeterian models in which growth is generated by the introduction of final consumption (as opposed to intermediate production) goods.

[23] See Dinopoulos (1994) for an overview on Schumpeterian growth theory.

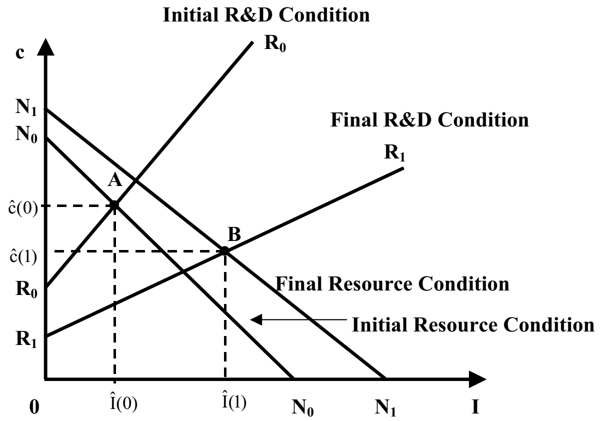

Fig. 1. Steady-state equilibria: Point A: No industry has adopted the new GPT. Point B: All industries have adopted the new GPT

Let a hat "^" over variables denote their market value in steady-state equilibrium. The resource condition (25) and the equilibrium R&D condition (26) determine simultaneously the long-run equilibrium values of per capita consumption expenditure, $\hat{c}$, and the rates of innovation, $\hat{I}_0$ and $\hat{I}_1$. Figure 1 illustrates the two steady-state equilibria: the initial steady-state (point A) in which no industry has adopted the new GPT (i.e., $\omega = 0$) and the final steady-state (point B) in which all industries have adopted the new GPT (i.e., $\omega = 1$). When $\omega = 0$ the balanced-growth resource condition is

$$1 = \frac{c}{\lambda_0} + k\alpha_R I_0 \ , \tag{27}$$

and the balanced-growth R&D condition is given by Equation (26) (when $\omega = 0$). The vertical axis measures consumption expenditure per capita, c, and the horizontal axis measures the rate of innovation, I. The resource condition is reflected by the negatively-sloped line N_0N_0 and the R&D equilibrium condition is represented by the positively-sloped line R_0R_0. Their unique intersection at point A determines the long-run values $\hat{c}(0)$ and $\hat{I}_0(0)$, where $\hat{c}(0)$ denotes the per capita consumption expenditure evaluated at $\omega = 0$ and $\hat{I}_0(0)$ denotes the innovation rate for industries that have adopted the new GPT evaluated at $\omega = 0$. Therefore, I arrive at:

Proposition 1. *For a given $\omega \in [0, 1]$, where ω is the measure of industries with a new GPT, there exists a unique steady-state equilibrium such that the long-run Schumpeterian growth, $\hat{g}_u$, is endogenous and does not exhibit scale effects: it depends positively on policies that affect the size of innovations, λ, the labor productivity in R&D services, $\mu(\theta)/\alpha_R$, and the rate of population growth, g_N; it depends negatively on the consumer's subjective discount rate, ρ. At each steady-state equilibrium, consumption expenditure per capita, $\hat{c}$, is constant, the interest rate, $\hat{r}(t)$, is equal to the constant subjective discount rate, ρ, and the aggregate stock value, $\bar{V}$, increases at the same rate as the constant rate of population growth, g_N.*

Proof. See Appendix.

The removal of scale effects from the long-run growth rate, g_u, depends on the assumption that the level of R&D difficulty is proportional to market size. At the steady-state equilibrium, the level of R&D difficulty, $X(t)$, increases exponentially at the rate of population growth g_N, i.e., $\dot{X}(t)\big/X(t) = g_N$, as can be seen from Equation (16). The absence of a new GPT does not result in zero long-run growth rate, as in the Helpman and Trajtenberg (1998a) and Aghion and Howitt (1998b) models. That is, the long-run growth rate depends positively on per capita R&D and, thus, any policy that affects this variable has long-run growth effects. The following proposition describes the long-run properties of the economy:

Proposition 2. *If ω is governed by S-curve dynamics, there are only two steady-state equilibria: the initial steady-state equilibrium arises before the adoption of the new GPT, where $\omega = 0$, and the final steady-state equilibrium is reached after the diffusion process of the new GPT has been completed, where $\omega = 1$. At the final steady-state equilibrium: aggregate investment is higher, $\hat{I}(1) > \hat{I}(0)$, long-run growth rate is higher, $\hat{g}_u(1) > \hat{g}_u(0)$, per capita consumption expenditure is lower, $\hat{c}(1) < \hat{c}(0)$, per capita stock market valuation of the incumbent in each industry is lower, $\hat{V}(1)\big/N < \hat{V}(0)\big/N$, relative to the initial steady-state equilibrium. In both steady states the market interest rate is equal to the subjective discount rate, $\hat{r} = \rho$.*

Proof. See Appendix.

These comparative steady-state properties can be illustrated with the help of Figure 1. Before the introduction of the new GPT, the economy is in a steady state (point A), where $\omega = 0$, with per capita consumption expenditure $\hat{c}(0)$, and with innovation rate $\hat{I}_0$. An increase in the measure of industries that adopt the new GPT makes the R&D condition in Figure 1 shift downward from R_0R_0 (where $\omega = 0$) to R_1R_1 (where $\omega = 1$) and the resource condition shift upward from N_0N_0 to N_1N_1, resulting in higher long-run rate of innovation and in lower long-run consumption expenditure per capita. In other words, when all industries have adopted the new GPT, the long-run Schumpeterian growth rate increases. The new steady state is at point B, where $\omega = 1$, with per capita consumption expenditure $\hat{c}(1)$, and innovation rate $\hat{I}_1$.

4 Transitional dynamics

I analyze the transitional dynamics of the model by adapting the time-elimination method described by Mulligan and Sala-i-Martin (1992).[24] The time-elimination method enables one to construct a system of two differential equations that govern the evolution of c and ω. Since Equation (26) holds at each instant in time (when the subjective discount rate, ρ, is replaced by the interest rate, r), I can solve for the rates of innovation for the two types of industries, I_0 and I_1. After substituting

[24] See also Mulligan and Sala-i-Martin (1991) for more details on this method.

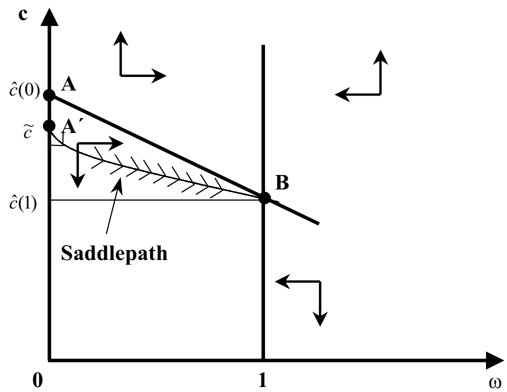

Fig. 2. Stability of the balanced-growth equilibrium

these rates into the resource condition (25), which holds at each instant in time, I can solve for the market interest rate along any path and obtain

$$r = \frac{\mu(c-1)}{k\alpha_R[\mu - \omega(\mu-1)]} + g_N \ . \tag{28}$$

Substituting (28) into (8) yields the following differential equation:

$$\frac{\dot{c}}{c} = r - \rho = \frac{\mu(c-1)}{k\alpha_R[\mu - \omega(\mu-1)]} + g_N - \rho \ . \tag{29}$$

Equations (29) and (1) determine the evolution of the two endogenous variables of the model, per capita consumption expenditure, c, and the number of industries that have adopted the new GPT, ω.

Since the right-hand side of Equation (29) is decreasing in ω, $\dot{c} = 0$ defines the downward-sloping curve in Figure 2. Starting from any point on this curve, an increase in ω leads to $\dot{c} > 0$ and a decrease in ω leads to $\dot{c} < 0$. The right-hand side of Equation (1) is independent of c, and therefore the $\dot{\omega} = 0$ locus is a vertical line. Starting from any point on this line, decrease in ω leads to $\dot{\omega} > 0$. The area to the left of the vertical line (i.e., locus $\dot{\omega} = 0$) identifies a region in which the potential number of adopters is greater than one. Therefore, this region is not feasible. There exists a downward-sloping saddle path going through the unique balanced-growth equilibrium point B. Thus, I arrive at:

Proposition 3. *Assume that $\delta > (g_N - \rho)$. Then, there exists a unique negative-sloping globally stable-saddle-path going through the final unique balanced-growth equilibrium point B. Along the saddle path, the measure of industries that adopt the new GPT, ω, increases, the per capita consumption expenditure, c, decreases, the market interest rate, r, increases, the innovation rate of the industries that have adopted the new GPT, I_1, decreases at a higher rate than that of those that have not adopted the new GPT, I_0. In addition, there exist transitional growth cycles of per capita GNP.*

Proof. See Appendix.

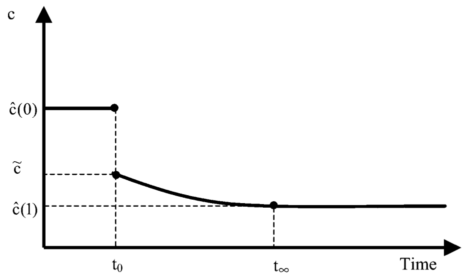

Fig. 3. Time path of the per captia consumption expenditure after a GPT arrives in the economy

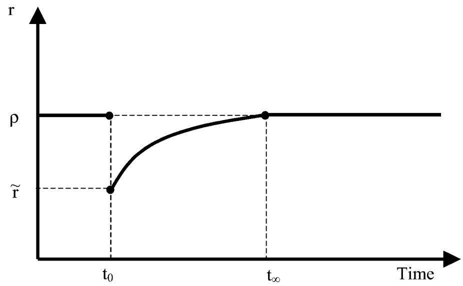

Fig. 4. Time path of the market interest rate after a GPT arrives in the economy

The analysis is predicated on the assumption of perfect foresight.[25] When the new GPT arrives, per capita consumption expenditure, c, jumps down instantaneously to $\tilde{c}$ (point A' in Fig. 2). This per capita consumption expenditure jump lowers the interest rate to $\tilde{r}$ (Fig. 4) since there are more savings available. The downward jumps on the per capita consumption expenditure and on the interest rate imply an upward jump in the innovation rates of both types of industries; those that have adopted the new GPT and those that have not adopted the new GPT ($\tilde{I}_1$ and $\tilde{I}_0$ in Fig. 5).

Figure 1 illustrates that the R&D line R_0R_0 will shift downwards and the resource line N_0N_0 will shift upwards with the arrival of the new GPT resulting in lower per capita consumption expenditure. Going back to Figure 2, the instantaneous decrease in c is reflected by a movement from point A to point A'. The decrease in per capita consumption expenditure leads to a decrease in the market interest rate r (from Eq. (28), which always hold). When the market interest rate r is lower than the subjective rate ρ, per capita consumption expenditure decreases even further, until the market interest rate approaches the subjective discount rate at the new steady state (point B in Figs. 1 and 2). During the transition dynamics (i.e., as the equilibrium moves from point A' to point B in Fig. 2), the interest

[25] There also exists a degenerate equilibrium at which the adoption of the new GPT is not completed. Suppose that when a new GPT arrives, every potential consumer expects that no one will decrease his consumption expenditure in order to finance innovation. As a result, it does not pay to decrease consumption expenditure of a single consumer, because the new GPT will never be fully adopted. In this event, the pessimistic expectations are self-fulfilling, and no new GPTs are fully adopted. I do not discuss these types of equilibria in what follows.

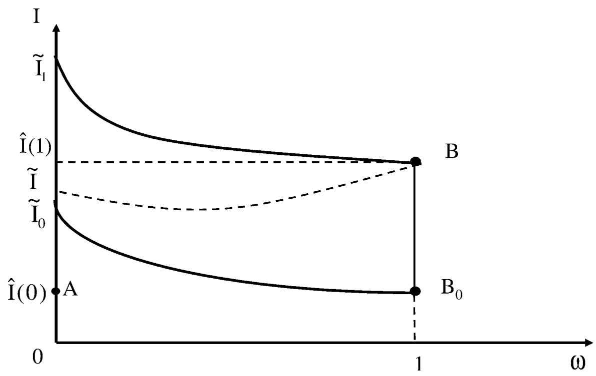

Fig. 5. Evolution of the aggregate investment during the diffusion path: the initial steady-state equilibrium is at point A. There is an upward jump in the aggregate investment with the introduction of the new GPT. One possible path of the aggregate investment, along the diffusion path, is depicted in the figure by the dotted curve $\tilde{I}B$

rate increases leading to more savings and a decrease in per capita consumption expenditure. At point B in Figure 2, all industries have adopted the new GPT.

Along the transition path, the aggregate investment may increase or decrease. One possible path of the aggregate investment is shown in Figure 5 by the dotted curve. There is an upward jump in the innovation rate of industries that have not adopted the new GPT (from point B_0 to point B in Fig. 5).

Figures 3 and 4 show the time paths of per capita consumption expenditure and the market interest rate (where t_0 indicates the time when the new GPT arrives in the economy and t_∞ indicates the time when all industries in the economy have adopted the new GPT). Figure 6 shows the effect of a GPT on the Schumpeterian growth rate. The adoption of the new GPT entails cyclical growth patterns.[26] The growth rate decreases in the initial stages of the adoption of the new GPT. There exist transitional growth cycles.

4.1 Stock market behavior

The fact that the adoption of a new GPT affects positively the productivity of R&D together with free entry into each R&D race is a key factor in explaining the behavior of the stock market. The probability of discovering the next higher quality product in each industry increases with the adoption of the new GPT and so does the probability that the incumbent in each industry will be replaced by a follower firm (i.e., the hazard rate). This link between the GPT adoption and higher risk for incumbent firms captures the effects of creative destruction on the stock market valuation of monopoly profits. In other words, during the diffusion of a GPT, per capita consumption declines, the market interest rate rises, and the hazard

[26] Earlier contributions on this issue include the macroeconomic model of Cheng and Dinopoulos (1996) in which Schumpeterian waves obtain as a unique non-steady-state equilibrium solution and the current flow of monopoly profits follows a cyclical evolution.

rate increases. These changes lower the per capita expected discounted profits of the successful innovator and drive down its per capita stock market valuation.[27] Furthermore, the larger the productivity gains associated with the new GPT, the larger the slump of the stock market. For example, it may be that the productivity gains generated by the introduction of the new GPT are large not because the new GPT is technologically very advanced at that initial stage, but because the previous GPTs are particularly inadequate for the needs of these sectors.[28] However, the size of the slump in the stock market is more severe, when the new GPT is diffused at a higher rate.

The aggregate stock value is given by the following equation:

$$\bar{V} = \left[\omega \frac{V_1}{N(t)} + (1-\omega)\frac{V_0}{N(t)}\right] N(t) \ , \tag{30}$$

where V_1 and V_0 are given by Equation (20) after taking account Equation (24). Thus, the growth rate of the aggregate stock value is given by:

$$g_V = \frac{\dot{\bar{V}}}{\bar{V}} = g_N - \frac{(\mu-1)\omega\delta(1-\omega)}{[\omega + (1-\omega)\mu]} \ . \tag{31}$$

At the initial steady-state, where $\omega = 0$ and at the final steady-state, where $\omega = 1$, the growth rate of the aggregate stock value is equal to the rate of the population growth.

That is,

$$\hat{g}_V = g_N \ . \tag{32}$$

The effects of a GPT on the stock market valuation of monopoly profits are summarized in the following proposition:

Proposition 4. *The growth rate of the stock market, g_v, depends negatively on the rate of GPT diffusion process, δ, and the magnitude of the GPT-ridden R&D productivity gains, μ, and positively on the rate of population growth, g_N. It also follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT.*

Proof. See Appendix.

These comparative properties, which differentiate the model from several others in its class, can be illustrated with the help of Figure 7, which shows the growth

[27] Hobijn and Jovanovic (2001) argue that U.S. stock market decline in the early 1970s was due to the arrival of information technology and the fact that the stock-market incumbents were not ready to implement it. They state "Instead, new firms would bring in the new technology after the mid-1980s. Investors foresaw this in the early 1970s and stock prices fell right away." The U.S. stock market value relative to GDP plummeted to 0.4 in 1973, just after Intel developed the microprocessor in late 1971. The decrease of the stock market value relative to GDP did not recover until the mid-1980s, and then rose sharply. Leading OECD countries also experienced similar movements in their stock markets, following a U-shaped path.

[28] This was clearly the case for early computers, where even though valves had been getting smaller for over a decade prior to the arrival of the transistor, the transistor was still an order of magnitude smaller.

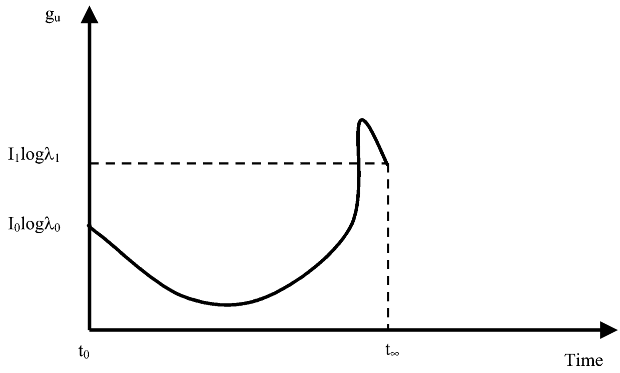

Fig. 6. The effects of a GPT on the Schumpeterian growth rate: when all industries have adopted the new GPT, the economy experiences higher steady-state Schumpeterian growth. There also exist transitional growth cycles of per capita GNP

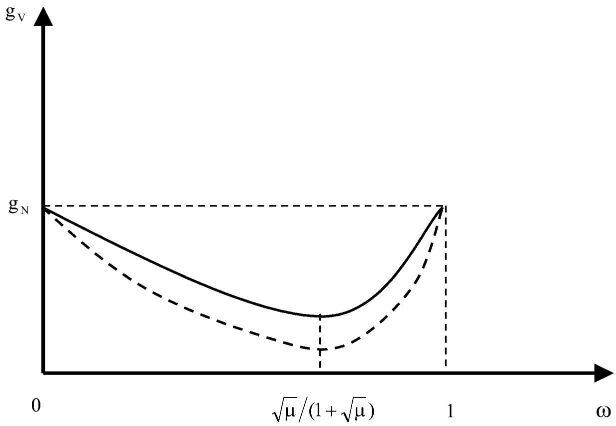

Fig. 7. The effects of a GPT on the stock market: The growth rate of the stock market depends on the rate of GPT diffusion process, the magnitude of the GPT-ridden R&D productivity gains, and the rate of population growth. It also follows a U-shapepd path relative to the population growth

rate of stock market as a function of the measure of industries that have adopted the new GPT. The initial adoption of the new GPT decreases the growth rate of the stock market below the rate of the population growth. In the later stages of the adoption of the new GPT, the growth rate of the stock market increases. When the diffusion process of the new GPT has been completed, the growth rate of the stock market is equal to the rate of the population growth. That is, it follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT.

This last result can be seen from the second term of the right hand side in Equation (31).[29] The free entry condition in each R&D race (Eq. 20) implies that the per capita stock value in any industry θ, $V_\theta/N(t)$, is constant over time. It jumps down instantaneously with the adoption of the new GPT, and it remains constant thereafter. The aggregate stock value, which increases exponentially at the rate of the population growth, jumps down with the arrival of the new GPT, and then increases again at the population growth rate. The slump in the aggregate stock value is due to the realization of the R&D productivity gains associated with the new GPT. The higher these R&D productivity gains are, the higher is the jump in the per capita industry and aggregate stock value at the time of the adoption of the new GPT.[30]

An increase in the GPT diffusion rate, δ, increases the economywide resources devoted to R&D. Thus, the probability that the incumbent firm will be replaced by a follower firm increases. This can be seen from Equation (18), which gives the value of monopoly profits. In other words, when the GPT diffusion process accelerates, the decrease in per capita consumption expenditure is more severe, and the per capita R&D investment increases. In this case, the U-shaped path of the growth rate of the stock market sags (this is shown by the dotted-shaped curve in Fig. 7). That is, the slope of the curve representing the growth of the stock market gets steeper at the initial stages of the diffusion process of the new GPT and gets flatter at the final stages of the process.[31]

An increase in the productivity gains generated by the new GPT, μ, lowers the cost of discovering the next higher quality product. This, in turn, will affect negatively the stock market valuation of the incumbent firm (see Eq. (20)).

An increase in the rate of population growth, g_N, shifts the U-shaped curve in Figure 7 upwards and increases the growth rate of the stock market.

4.2 Aggregate investment

Proposition 5. *The effect of the GPT diffusion on the aggregate investment during the adoption process is ambiguous.*

Proof. See Appendix.

[29] The numerator in Equation (31), which is positive and reflects the slope of a truncated S-curve, is equal to $\dot{\omega}$ times a positive fraction that depends on the magnitude of the GPT-ridden R&D productivity gains and on the number of the industries that have adopted the new GPT.

[30] This can be seen from Equation (31), where the first term on the right-hand side gets smaller when each industry adopts the new GPT relative to the second term of the right-hand side of the same equation.

[31] Hobijn and Jovanovic (2001) provide evidence that the drop in the stock market in the early 1970s was due to the arrival of information technology. Their evidence supports that the information-technology-intensive sectors experienced the largest drop in 1973–1974, reducing the role of the first OPEC shock in explaining the decrease in the stock market. This result is also consistent with the empirical evidence provided by Jovanovic and Rousseau (2001) on how the U.S. economy is affected by new technologies. They show by using 114 years of U.S. stock market data that the growth of the stock market slows down due to the fact that the new entrants of the stock market will find hard to keep up.

The initial steady-state equilibrium is at point A in Figure 5. There is an upward jump in the aggregate investment with the introduction of the new GPT (from $\hat{I}$ to $\tilde{I}$). Along the diffusion path, both innovation rates (I_0 and I_1) decrease until the economy reaches the final steady-state equilibrium point B, where the aggregate investment is higher relative to the initial steady-state equilibrium point A. There is an upward jump in the innovation rate of the industries that have not adopted the new GPT at the final steady state (from point B_0 to B). One possible picture of how the aggregate investment behaves along the diffusion of the new GPT is shown in Figure 5. Along the transition path, the aggregate investment decreases and then increases. In the initial stages of the diffusion process, only a limited number of industries adopt the new GPT (see Eq. (1)). These industries are called the early adopters. As more industries adopt the new GPT, the aggregate investment increases.

5 Concluding remarks

Previous models that have analyzed GPTs exhibit the scale effects property. The present paper analyzed the effects of a GPT on short-run and long-run Schumpeterian growth without scale effects. The absence of growth scale effects and the modeling of the diffusion process through S-curve dynamics generate several novel and interesting results.

First, the long-run growth rate of the economy depends positively on the magnitude of quality innovations. Any policy that affects this magnitude has long-run growth effects. However, the absence of the arrival of a new GPT in the economy does not reduce the long-run growth rate to zero, as in the previous GPTs-based growth models. All the previous R&D-based models that analyze the effects of GPTs exhibit scale effects.

The assumption that the diffusion of the new GPT follows an S-curve generates two steady-state equilibria: one is the initial steady-state before the adoption of the new GPT begins and the other is the final steady-state after the diffusion process of the new GPT has been completed. At the final steady-state relative to the initial steady-state the long-run growth rate is higher, the aggregate investment is higher, the per capita consumption expenditure is lower, and the market interest rate is equal to the subjective discount rate.

The growth rate of the stock market depends negatively on the rate of GPT diffusion process and the magnitude of the GPT-ridden R&D productivity gains, and positively on the rate of population growth. It also follows a U-shaped path relative to the population growth rate during the diffusion process of the new GPT. This is consistent with the empirical evidence provided by Jovanovic and Rousseau (2001), who empirically document that, during times of rapid technological change, the growth of the stock market slows, since the new entrants will grab the most value from previous entrants (because the incumbents will find hard to keep up). Hobijn and Jovanovic (2001) also provide evidence that the drop in the stock market in the early 1970s was due to the arrival of information technology. Their evidence supports the notion that the information-technology-intensive sectors experienced the largest drop in 1973–1974, reducing the role of the first OPEC shock in explaining the decrease in the stock market.

One could also develop a dynamic general equilibrium model to study the effects of a GPT diffusion on a global economy that exhibits endogenous Schumpeterian growth. As in this model, the adoption of a GPT by a particular industry can generate an increase in the productivity of R&D workers, and the magnitude of all future innovations and its diffusion across industries can be governed by S-curve dynamics. The diffusion of the GPT within an industry from one country to the other can occur with a time lag. Under this framework, it would be interesting to analyze the long-run and transitional dynamic effects of a new GPT on trade patterns, product cycles and (transitional) divergence in per capita growth rates between the two countries. This is a fruitful direction for future research.

Appendix

A.1. Proposition 1

Equations (25) and (27) define a unique steady-state equilibrium.

The long-run Schumpeterian growth rate is endogenous and does not exhibit scale effects. This follows from Equation (23).

Solving the expression for I in (25) (by substituting Eq. (25) into Eq. (26) in the main text), substituting it into (23) and differentiating the resulting expression with respect to the appropriate parameter.

At each steady-state, per capita consumption expenditure is constant. That is $\hat{\dot{c}} = 0$. Then Equation (8) implies that the interest rate, $\hat{r}$, is equal to the subjective discount rate, ρ.

The aggregate stock value is given by: $\bar{V} = [\omega \frac{V_1}{N(t)} + (1-\omega)\frac{V_0}{N(t)}]N(t)$, where V_1 and V_0 are given by Equation (20) after taking account Equation (24). After substitution of these values into the aggregate stock value and taking logs and derivatives with respect to time, I obtain the growth rate of the aggregate stock value:

$$g_V = \frac{\dot{\bar{V}}}{\bar{V}} = g_N - \frac{(\mu-1)\omega\delta(1-\omega)}{[\omega + (1-\omega)\mu]} \, . \tag{A.1}$$

At the initial steady-state, where $\omega = 0$ and at the final steady-state, where $\omega = 1$, the growth rate of the aggregate stock value is equal to the rate of the population growth. That is $\hat{g}_V = g_N$.

This completes the proof of Proposition 1.

A.2. Proposition 2

Equations (1) and (29) define two loci: one where $\omega = 0$ and one where $\omega = 1$.

Evaluating the aggregate investment (which is given by $I = I_0(1-\omega) + I_1\omega$) at the two steady-states implies that it is higher at the final steady state relative to the initial steady state.

Substituting Equations (6) and (10) into Equation (4) in the main text (after taking account Eq. (24)), I obtain $\log u(t) = \log c(t) - \log \alpha_Q + \int_0^1 \log \lambda(\theta)^{j(\theta)} d\theta - \int_0^1 \log \lambda(\theta) d\theta$. I invoke the properties of the Poisson distribution to argue that the expected number of improvements is $I_1 t$ (Feller, 1968, p.159) and obtain:

$$\log u(t) = \log c(t) - \log \alpha_Q + \omega(I_1 t - 1)\log \lambda_1 + (1-\omega)(I_0 t - 1)\log \lambda_0 \ . \tag{A.2}$$

The economy's long-run Schumpeterian growth is defined as the rate of growth of sub-utility $u(t)$, $g_u = \dot{u}(t)/u(t)$. By taking logs and differentiating Equation (A.2) with respect to time and substituting Equation (29) from the main text, I obtain:

$$\begin{aligned} g_u = {} & \frac{\mu(c-1)}{k\alpha_R[\mu - \omega(\mu-1)]} + g_N - \rho + \dot{\omega}[(I_1 t - 1)\log \lambda_1 - (I_0 t - 1)\log \lambda_0] \\ & + \omega(I_1 + t\dot{I}_1)\log \lambda_1 + (1-\omega)(I_0 + t\dot{I}_0)\log \lambda_0 \ . \end{aligned} \tag{A.3}$$

Evaluating Equation (A.3) at the two steady-states implies that the long-run Schumpeterian growth rate is higher at the final steady-state than at the initial steady-state.

By setting Equation (29) equal to zero, I obtain the per capita consumption expenditure as a function of the measure of industries that have adopted the new GPT, $c(\omega) = \frac{(\rho - g_N)k\alpha_R[\mu - \omega(\mu-1)]}{\mu} + 1$. Evaluating the per capita consumption expenditure at the two steady-states implies that $\hat{c}(1) < \hat{c}(0)$.

At each steady-state, per capita consumption expenditure is constant. That is $\dot{\hat{c}} = 0$. Then Equation (8) implies that the interest rate, $\hat{r}$, is equal to the subjective discount rate, ρ.

Equations (20) and (13) in the main text imply that $\frac{V(1)}{N} = \frac{V_1}{N} < \frac{V(0)}{N} = \frac{V_0}{N}$.

This completes the proof of Proposition 2.

A.3. Proposition 3

In order to prove that there exists locally a negative sloping saddle path, I use a polynomial of order one to linearize the nonlinear differential Equations (1) and (29) around their steady-state values ($\hat{c} = \frac{(\rho - g_N)k\alpha_R}{\mu} + 1, \hat{\omega} = 1$). Equations (31) and (1) in the main text can be written as $\dot{c} = a_1 c + b_1 \omega$ and $\dot{\omega} = b_2 \omega$, respectively (where $a_1 = \frac{[(\rho - g_N)k\alpha_R + \mu]}{k\alpha_R}$, $b_1 = \frac{[(\rho - g_N)k\alpha_R + \mu][(\rho - g_N)k\alpha_R(\mu - 1)]}{\mu k \alpha_R}$ and $b_2 = -\delta$).

Suppose $c = Ae^{rt}$, $\omega = Be^{rt}$ are the particular solutions to this homogeneous system $(\dot{c}, \dot{\omega})$. I can substitute these proposed solutions into this system and I can write it in matrix notation as follows:

$$\begin{bmatrix} a_1 - r & b_1 \\ 0 & b_2 - r \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{A.4}$$

In order that the solution to (A.4) be other than $A = B = 0$, the coefficient matrix in (A.4) must be singular. Thus, I obtain a quadratic equation in $r : r^2 - r(a_1 + b_2) + a_1 b_2 = 0$. Computing the roots of this quadratic equation, I obtain:

$$r_1 = \frac{(a_1 + b_2) + \sqrt{(a_1 + b_2)^2 - 4a_1 b_2}}{2}$$
$$r_2 = \frac{(a_1 + b_2) - \sqrt{(a_1 + b_2)^2 - 4a_1 b_2}}{2} \, . \tag{A.5}$$

Since $a_1 > 0$ and $b_2 < 0$, the roots are real. Because the argument of the square root function exceeds $(a_1 + b_2)^2$, the smaller of the roots is negative. The larger root is positive. Hence, the roots are real and of opposite sign; the stationary point is a saddlepoint.

In order to prove that there exists globally a negative sloping saddle path, I use the time elimination method. The slope of the policy function can be obtained by taking the ratio of the two differential equations that govern the dynamic behavior of the economy:

$$\frac{\partial c}{\partial \omega} = c'(\omega) = \frac{\dot{c}}{\dot{\omega}} = \frac{\frac{[c^2(\omega)\mu - c(\omega)\mu]}{k\alpha_R[\mu - \omega(\mu - 1)]} + c(\omega)(g_N - \rho)}{\delta\omega - \delta\omega^2} \, . \tag{A.6}$$

Time does not appear in the above equation. To solve this equation numerically, there must be one boundary condition; that is, one point, (c, ω), that lies on the stable arm. Although the initial pair, $[c(0), \omega(0)]$, is unknown, the policy function goes through the steady state $(\hat{c}, \hat{\omega})$.

The slope of the policy function at the steady state is $c'(\hat{\omega}) = \frac{\dot{\hat{c}}}{\dot{\hat{\omega}}} = \frac{0}{0}$, which is indeterminate. Applying the L'Hôpital's rule to this slope and evaluating it at the steady state values, I obtain:

$$c'(\hat{\omega}) = \frac{[1 - c(\hat{\omega})](\mu - 1)c(\hat{\omega})}{\{\mu[2c(\hat{\omega}) - 1] + (\delta + g_N - \rho)k\alpha_R\}} \, . \tag{A.7}$$

From the phase diagram in Figure 2, I can sign the following expressions: $[2c(\hat{\omega}) - 1] > 0$ and $[1 - c(\hat{\omega})] < 0$. If $\delta > (g_N - \rho)$, then the expression in (A.7) is negative. Thus, if the rate of diffusion is high enough (higher than the effective discount rate), the slope of the stable arm is negative.

Along the saddle path, the measure of industries that adopt the new GPT evolves according to Equation (1). As ω increases, Equation (A.7) implies that per capita consumption expenditure decreases. Along the saddle path, $\dot{c}/c < 0$, which implies that the market interest rate, r, is lower than the subjective interest rate, ρ (from Eq. (8) in the main text). Once the diffusion process is about to complete and the economy approaches the new steady state, the market interest rate increases to become equal with the constant subjective interest rate at the new steady state.

In order to analyze the behavior of the two innovation rates along the diffusion process, I differentiate equations them with respect to ω. By taking the difference between these innovation rates, I obtain:

$$\frac{\partial I_1}{\partial \omega} - \frac{\partial I_0}{\partial \omega} = \left(\frac{\mu(\lambda_1 - 1)}{k\alpha_R \lambda_1} - \frac{(\lambda_0 - 1)}{k\alpha_R \lambda_0}\right) \frac{\partial c}{\partial \omega} < 0 \,. \tag{A.8}$$

That is, along the diffusion process, the innovation rate for the industries that have adopted the new GPT decreases more than that of the industries that have not adopted the new GPT.

In terms of the growth rate, differentiating Equation (A.3) with respect to time and taking the limit when the economy approaches the two steady-states, I obtain: $\lim\limits_{\omega \to 0}[\frac{\partial g_u}{\partial t}] = \frac{\dot{c}}{k\alpha_R} + \ddot{I}_0 t \log \lambda_0 + 2\dot{I}_0 \log \lambda_0 < 0$ and $\lim\limits_{\omega \to 1}[\frac{\partial g_u}{\partial t}] = \frac{\mu \dot{c}}{k\alpha_R} + \ddot{I}_1 t \log \lambda_1 + 2\dot{I}_1 \log \lambda_1 < 0$. Since the signs of these equations are negative, the growth rate decreases both in the initial stage and towards the final stage of the diffusion process. Since the quantity of the first equation is smaller than that of the second equation in absolute value, the growth rate decreases more towards the final stage than in the initial stage.

A.4. Proposition 4

The growth rate of the stock market is given by Equation (A.1). From Equation (A.1), it is obvious that the growth rate of the stock market depends on the rate of GPT diffusion process, δ, the magnitude of the GPT-ridden R&D productivity gains, μ, and the rate of population growth, g_N.

By differentiating Equation (A.1) with respect to ω, I obtain: $\frac{\partial g_V}{\partial \omega} = \frac{\delta(1-\mu)(k\alpha_R)^2[(1-\omega)^2\mu - \omega^2]}{[\omega k\alpha_R + (1-\omega)k\alpha_R \mu]^2}$. The sing of this expression depends on the sign of the expression $[(1-\omega)^2\mu - \omega^2]$. When $\omega < \sqrt{\mu}/(1+\sqrt{\mu})$, the growth rate of the aggregate stock value decreases. When $\omega > \sqrt{\mu}/(1+\sqrt{\mu})$, the growth rate of the aggregate stock value increases. That is, along the diffusion process, the growth rate of the aggregate stock value follows a U-shaped path.

Differentiating Equation (A.1) with respect to the diffusion rate, δ, I obtain:

$$\frac{\partial g_V}{\partial \delta} = \frac{\omega(1-\omega)(1-\mu)}{[1+(1-\omega)\mu]} < 0 \,. \tag{A.9}$$

Equation (A.9) implies that when the diffusion rate increases, the growth rate of the aggregate stock market decreases.

Differentiating expression $\frac{\partial g_V}{\partial \omega}$ with respect to the diffusion rate, δ, I obtain the expression $\frac{\partial(\partial g_V/\partial \omega)}{\partial \delta} = \frac{(1-\mu)[(1-\omega)^2\mu - \omega^2]}{[\omega + (1-\omega)\mu]^2}$, which sign depends on the sign of the expression $[(1-\omega)^2\mu - \omega^2]$. When the rate of the diffusion process of the new GPT increases, the slope of the growth of the stock market increases (i.e., gets steeper) for $\omega < \sqrt{\mu}/(1+\sqrt{\mu})$ and decreases (i.e., gets for $\omega > \sqrt{\mu}/(1+\sqrt{\mu})$.

By differentiating Equation (A.1) with respect to μ, I obtain:

$$\frac{\partial g_V}{\partial \mu} = -\frac{\omega\delta(1-\omega)}{[\omega + (1-\omega)\mu]^2} < 0 \,. \tag{A.10}$$

The sign of Equation (A.10) implies that the growth rate of the stock market decreases when the productivity gains from the new GPT are larger.

By differentiating Equation (A.1) with respect to g_N, I obtain:

$$\frac{\partial g_v}{\partial g_N} = 1 > 0 \,. \tag{A.11}$$

The sign of Equation (A.11) implies that the growth rate of the stock market increases at the rate of population growth.

A.5. Proposition 5

Differentiating the equation that expresses the aggregate investment with respect to the measure of industries that adopt the new GPT, I obtain:

$$\frac{\partial I}{\partial \omega} = \frac{\partial I_0}{\partial \omega}(1-\omega) + \frac{\partial I_1}{\partial \omega}\omega + (I_1 - I_0) \,. \tag{A.12}$$

The first two terms in Equation (A.12) are negative. The third term is positive. Thus, the aggregate investment will decrease or increase during the diffusion process depending on the magnitude of these signs.

The first two terms are the slopes of the solid curves depicted in Figure 7. These slopes depend on the rate of diffusion of the new GPT. By contrast, the third term of Equation (A.12) does not depend on the rate of diffusion of the new GPT. Thus, when the rate of diffusion of the new GPT increases and more industries switch to the new GPT faster, the negative effect dominates the positive effect and the aggregate investment decreases.

$$\lim_{\omega \to 1} I_1(\omega) = \frac{\mu(\lambda_1 - 1)}{k\alpha_R \lambda_1} c(\omega) + g_N - \rho \,. \tag{A.13}$$

$$\lim_{\omega \to 1} I_0(\omega) = \frac{(\lambda_0 - 1)}{k\alpha_R \lambda_0} c(\omega) + g_N - \rho \,. \tag{A.14}$$

Equations (A.13) and (A.14) imply that when the measure of industries that adopt the new GPT approaches the final steady-state, the innovation rates of the industries that have adopted the new GPT and the innovation rate of industries that have not adopted the new GPT are not the same. That implies that the innovation rate of the industries that have not adopted the new GPT jumps upward (i.e., when the last industry switches to the new GPT, I_0 becomes I_1).

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