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GPS signal diffraction modelling: the stochastic SIGMA-Δ model

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Abstract. The SIGMA-Δ model has been developed for stochastic modelling of global positioning system (GPS) signal diffraction errors in high precision GPS surveys. The basic information used in the SIGMA-Δ model is the measured carrier-to-noise power-density ratio ($C/N0$). Using the $C/N0$ data and a template technique, the proper variances are derived for all phase observations. Thus the quality of the measured phase is automatically assessed and if phase observations are suspected to be contaminated by diffraction effects they are weighted down in the least-squares adjustment. The ability of the SIGMA-Δ model to reduce signal diffraction effects is demonstrated on two static GPS surveys as well as on a kinematic high-precision GPS railway survey. In cases of severe signal diffraction the accuracy of the GPS positions is improved by more than 50% compared to standard GPS processing techniques.

Key words. Precise GPS surveying · GPS signal diffraction · Carrier-to-noise power-density ratio · Precision of phase data

1 Introduction

Global positioning systems (GPS) have been used as a reliable method in high-precision surveys in many applications. However, in some results the attainable accuracy is significantly decreased. Most of these poor results have a common error source – GPS signal diffraction (Elosegui et al. 1995). Signal diffraction occurs whenever the direct line-of-sight between the GPS satellite and the antenna is obstructed but the GPS signal is not completely masked.

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We have observed several situations of signal diffraction. Diffuse obstacles such as bushes or trees are common sources for GPS signal diffraction, (e.g. Hartinger and Brunner 1998a). In addition, it is possible that GPS signals ‘bend around’ the edges of a wall, i.e. satellite signals are visible although from the geometrical point of view they should be obstructed. A simulation of this effect was given by Walker and Kubik (1996). In addition, GPS waves can also propagate through certain materials. For example, Kaes and Pauli (1991) estimate a 25–40% energy loss of a 1-GHz wave after propagation through glass with a thickness of 4–6 mm. In all situations of GPS signal diffraction, additional phase delays occur. However, the common GPS processing algorithm interprets the measurement as a signal that has travelled on a straight line from the satellite to the antenna. The consequence is a range error which will significantly affect the attainable accuracy of the GPS result. Thus, for high-precision GPS surveys, e.g. near-real time continuous deformation monitoring with GPS, quality control of the measured phase observation is vital.

The power of a GPS signal is a measure of its quality. One way of expressing the GPS signal power, used by most receiver brands, is the carrier-to-noise power-density ratio ($C/N0$) which is the ratio of the signal carrier to noise power in a 1-Hz bandwidth. However, the raw data files are required to obtain the $C/N0$ values as the current RINEX format does not allow for two-digit information. The $C/N0$ values are measured by-products of the GPS observations, and can be used to calculate the phase variances. Only a few attempts where $C/N0$ measurements are used for modelling GPS phase variances have been published in the literature. Axelrad et al. (1994) and Comp and Axelrad (1996) successfully use the signal-to-noise ratio (SNR) to model the multipath effect. The relationship between the $C/N0$ values and the elevation of a GPS satellite was used to weight GPS observations in order to overcome satellite geometry problems (Hartinger and Brunner 1998b, c). This weighting scheme was called the SIGMA-ε model.

In our investigations of GPS diffraction effects we have observed that diffracted GPS signals are usually associated with low $C/N0$ values. Therefore we developed a new model, named SIGMA-Δ, for the treatment of signal diffraction effects using the measured $C/N0$ values. This is an attempt to generate intelligent GPS processing software which automatically models phase signal diffraction. No interaction between the user and the software is required to prevent the decrease of accuracy. The SIGMA-Δ algorithm is implemented in the GPS software package Grazia. Grazia was developed from 'GPSoft' written by Dr. Joseph Czompo at the University of Calgary.

In Sect. 2, the background information necessary for the SIGMA-Δ model development is presented. After a brief discussion of the $C/N0$ measurements, the SIGMA-ε model is explained. In Sect. 3 the antenna $C/N0$ template is introduced as the core of the SIGMA-Δ model. The estimation of phase variances using the SIGMA-Δ model is discussed in detail in Sect. 3. In Sect. 4 the effectiveness of the SIGMA-Δ model is investigated using three examples of GPS surveys in which GPS signal diffraction occurred. GPS results are shown without any phase weighting, with the SIGMA-ε and with the SIGMA-Δ model applied.

2 Background on the SIGMA-Δ model

2.1 Carrier-to-noise power-density measurement $C/N0$

GPS signals are transmitted from satellites with different powers, at +19.7dBW (P-code L_2), +23.8 dBW (P-code L_1) and +26.8 dBW (C/A-code L_1). The propagation effects between the satellite and the antenna attenuate the GPS signals which are finally received with a power near the noise level. Usually the power is expressed as the ratio of the power level of the signal carrier to the noise level in a 1 Hz bandwidth, termed the $C/N0$. The lowest possible power at which a signal can be distinguished from noise is −204 dBW-Hz. At reception the expected minimum C/A-code level is −160 dBW-Hz. A signal with such a power has a carrier-to-noise power-density ratio $C/N0$ of 44 dBW-Hz. Antenna design and receiver processing techniques have a significant influence on the $C/N0$ value. A more detailed discussion is given by Ward (1996) and Langley (1997).

Geodetic GPS antennas are designed to operate with a few dB gain at the zenith and a negative gain at very low elevations, i.e. the $C/N0$ will vary with the elevation of the arriving signal. Ward (1996) derived a formula which expresses the phase variance σ_i^2 in mm^2 as a function of the measured $C/N0$ values:

$$\sigma_i^2 = C_i \cdot 10^{-(C/N0_{\text{measured}})/10} \qquad (1)$$

where the subscript i indicates the L_i signal (L_1 or L_2) and the effect of the oscillator stability on the phase variances is considered negligible. The factor C_i consists of the carrier loop noise bandwidth and a conversion term from cycle^2 to mm^2 which includes the L_i wavelength, whereas the dimension of the $C/N0$ is in dB-Hz. Equation (1) can be used to estimate variances for the original phase observations at one station to a satellite, i.e. undifferenced.

2.2 SIGMA-ε model

Equation (1) forms the basis of a purely random noise model which we have termed SIGMA-ε for the calculation of the variance matrix Σ_ε of the phase measurements (Hartinger and Brunner 1998b, c). The measured $C/N0$ values can be used to calculate the variance of a double-difference (DD) phase observation using Eq. (1) and the law of the propagation of variances leading to DD–$C/N0$ variances. Figure 1 shows the relationship of σ_{DD}^2 calculated from a time series of DD phase residuals using 5 min observation intervals to calculate the variances and, alternatively, the DD–$C/N0$ variances for the same observation interval. Figure 1 suggests for C_1 a value of $1.61 \cdot 10^{+4}$ (mm^2). C_1 depends on the bandwidth of the tracking loop used by the receiver tracking channel. However, in all our investigations we have used a constant value for C_1 due to the lack of bandwidth information.

The advantage of the SIGMA-ε weight model, as compared with not weighting the phase observations or calculating phase variances as a function of elevation, was discussed by Hartinger and Brunner (1998b, c). In this paper the GPS data of all three examples will be processed using the SIGMA-ε model with the above quoted value of C_1.

3 Development of the SIGMA-Δ model

3.1 $C/N0$ template

Next we investigated the distinction between a 'clear' signal and a signal which is affected by diffraction effects. We found out that such signals generally have

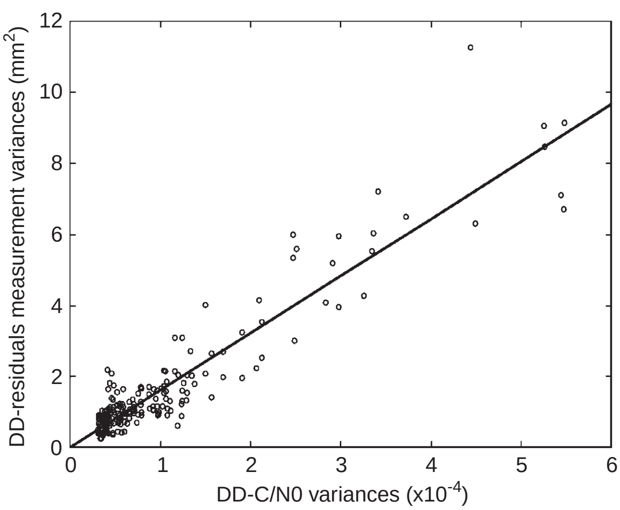

Fig. 1. DD phase variances for a 5 min interval versus DD–$C/N0$ variances without C_1. The slope of the regression line is C_1

lower $C/N0$ values than 'clear' signals at the same azimuth and elevation. Thus we constructed $C/N0$ versus elevation plots, termed $C/N0$ templates. The envelope of the highest $C/N0$ values represents the best signal quality to be achieved at a certain GPS site.

Such a template can be obtained in the field by recording phase and $C/N0$ data at the GPS site and with the specific antennas to be used in the GPS surveys. The final result is a plot of the $C/N0$ values as a function of the elevation angles. In order to distinguish between 'good' and 'bad' phase observations, the $C/N0$ template for a certain antenna type is defined by the highest $C/N0$ value at a certain elevation. The basic assumption for this template definition is an azimuthally constant antenna pattern. Otherwise azimuth variations must also be modelled, which was not done in the present investigations. Sometimes the $C/N0$ values start to oscillate as a result of the superposition of the direct and an indirect GPS signal. The present definition of the template avoids this issue. However, we are investigating other template definitions in order to treat oscillating $C/N0$ values.

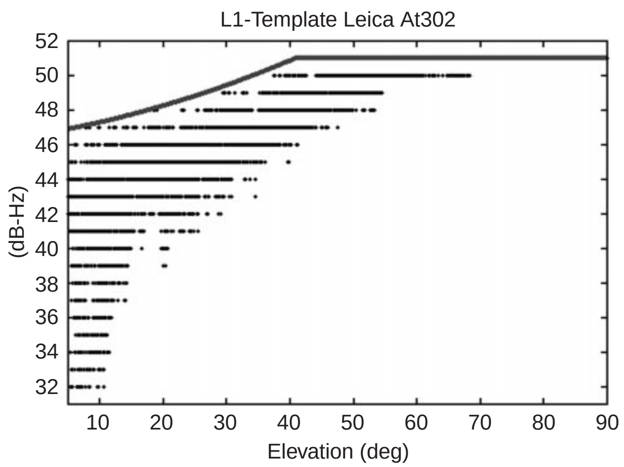

Fig. 2. Leica AT302 GPS antenna – $C/N0$ and template for 5–90° (L_1)

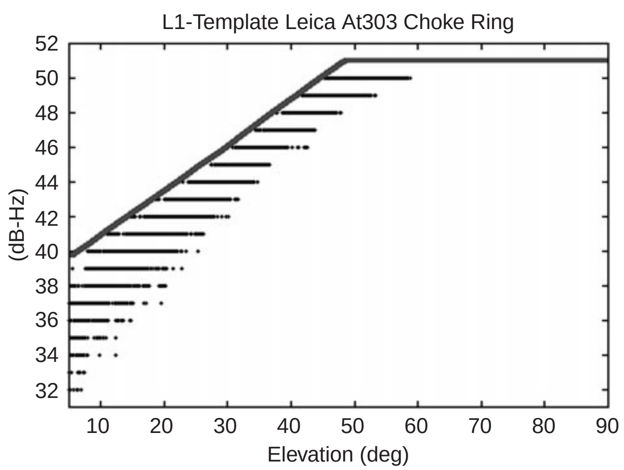

Fig. 3. Leica AT303 choke-ring antenna – $C/N0$ and template for 5–90° (L_1)

Figures 2 and 3 show the $C/N0$ characteristics of a Leica geodetic antenna (AT302) and a choke-ring antenna (AT303), each connected to a Leica SR399 sensor. The signal power of this sensor ranges from +32 to + 51 dB-Hz. Both antennas were used at the same position, with the same satellite geometry and at a site with hardly any satellite masking. The template of the Leica choke-ring antenna shows a steeper drop compared to the geodetic antenna of the $C/N0$ values at lower elevations.

As an alternative to the use of a template, an easier approach would be to compare directly the $C/N0$ values between the reference and the rover station; this approach has two drawbacks: it assumes an undisturbed reference station and requires the same antenna type at both stations.

3.2 SIGMA-Δ model

The dispersion of the phase observations will represent the stochastic behaviour of the so far unmodelled phase diffraction delay in the presence of signal diffraction. In Fig. 4 the variance–$C/N0$ relationship is plotted according to Eq. (1). From a measured $C/N0$ value, the appropriate σ_i^2 can be calculated. Let us introduce Δ as the difference between the measured $C/N0$ value and the template value at the appropriate satellite elevation

$$\Delta = C/N0_{\text{template}} - C/N0_{\text{measured}} \tag{2}$$

Now Δ can be used as an indicator for the diffraction noise, which yields a larger variance than σ_i^2. Therefore we can calculate the variance of the phase observations, σ_Δ^2, using Eq. (1) but with a $C/N0$ value which is reduced by $\alpha\Delta$

$$\sigma_\Delta^2 = C_i \cdot 10^{-(C/N0_{\text{measured}} - \alpha \cdot \Delta)/10} \tag{3}$$

The factor α is included in Eq. (3) to allow for the empirical scaling of the effect of Δ on the variance σ_Δ^2. This model is schematically shown in Fig. 4. Using Eq. (3) the variance matrix of the phase observations, Σ_Δ, can now be calculated, where Δ controls the noise level due to suspected additional diffraction effects. Σ_Δ refers now to the DDs of phase observations.

We investigated the value of the factor α using the data set of the experiment described in Sect. 4.1. In Fig. 5 the difference δ between the terrestrial 'true' and the GPS-derived baseline length is shown as a function of α in order to investigate the sensitivity of the selection of the empirical value of α. The GPS session length was selected so that it mainly contained the diffraction effect. Thus the bias δ is caused by the diffraction effect. From Fig. 5 it appears that $\alpha = 2$ is a reasonable numerical value. The results discussed in Sect. 4 were obtained using $\alpha = 2$. However, we are continuing to investigate the selection of the most appropriate value of α and its possible dependence on other influences.

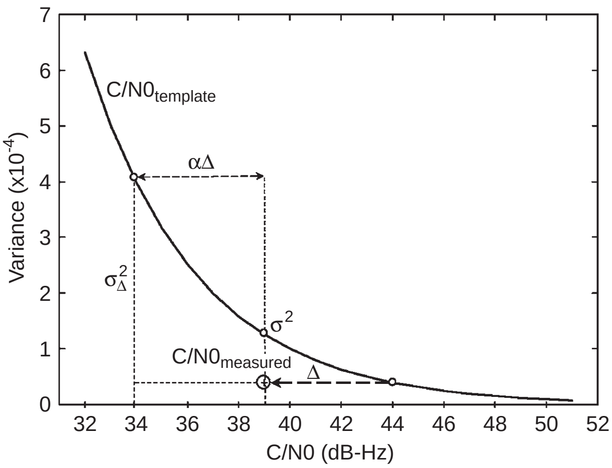

Fig. 4. Schematic diagram of the variance computation to account for the possible delay noise

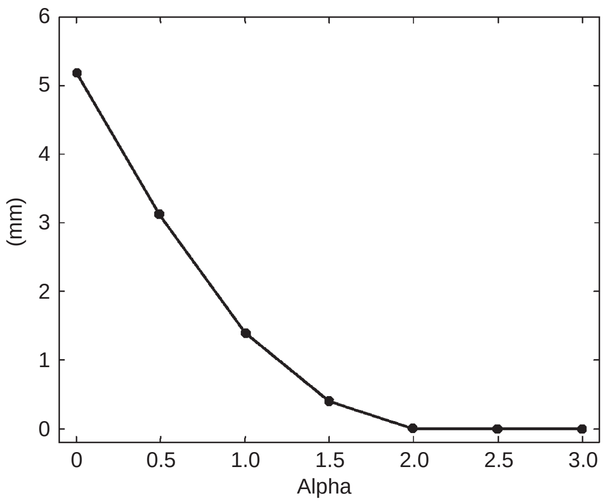

Fig. 5. Relationship between the baseline length bias δ and α using experimental data

4 Experimental results using the SIGMA-Δ model

4.1 Static GPS survey close to a building

In order to study the GPS diffraction effects on a static baseline, the rover station was situated at a distance of 6 m from a concrete building about 8 m high (see Fig. 6). The reference station was set up at a distance of 114 m without any obstructions above an elevation of 5°. Two Leica AT303 choke-ring antennas and SR399 sensors were used. The length of the session was 1 h 45 min, at a sampling rate of 3 sec. The L_1 results will be discussed in the following.

The concrete building caused a significant bias in the phase measurements of PRN 01. Figure 7 shows the positions of PRN 01 as observed by the rover antenna. The rover antenna was already able to track the satellite signals of PRN 01 during the first 30 min of the session, even though a propagation along the line-of-sight was not possible. This is a typical case of signal diffraction where a satellite signal is received although the direct line-of-sight is obstructed.

The $C/N0$ values of PRN 01 indicate this diffraction effect (see Fig. 8). During the first 30 minutes the template suggests a $C/N0$ value for PRN 01 of around 50 dB-Hz; however, the actual $C/N0$ measurement is about 13 dB-Hz less. When the signals of PRN 01 were no longer physically obstructed, the measured $C/N0$ values needed another 30 minutes until they approached the corresponding template values. In addition to the $C/N0$ values the DD residuals also display the diffraction effect (see Fig. 8). The DD residuals were calculated from the phase observations of PRN 01, holding the coordinates of the reference and rover station at fixed values. These residuals display the diffraction phase errors of the signals of PRN 01 (second row of Fig. 8). The maximum phase error is about 60 mm, which is 1/3 of an L_1 cycle. At 7.5 h, when direct line-of-sight propagation is again possible, and thus no diffracted

Fig. 6. The rover station beside the concrete building

signals reach the antenna, the phase errors revert back to the usual range.

Next, the diffraction effects on the coordinate results will be investigated. The height component result is displayed in the first row of Fig. 9. It shows the epoch-to-epoch coordinate variation of this component minus the actual height without any phase weighting applied. Although seven satellites were visible during the first 30 min and only the data of PRN 01 was diffracted, the height component has a maximum error of 50 mm. At 7.5 h PRN 01 is not obstructed and the GPS coordinates are no longer corrupted. If we use the SIGMA-ε model this effect is reduced by about 50%, but the GPS height is still biased by about 25 mm. This shows that weighting the phase observations with Σ_ε does not model adequately the strong diffraction effect. If, however, the diffraction effect is modelled by the Σ_Δ variance matrix, accounting for the stochastic delay, this error is removed; see the third row in Fig. 9.

Averaging the kinematic GPS results over the whole session yields the static results for this 1 h 45 min session. For the above example, the height component of the static solution without any phase weighting is biased by 5.6 mm. This bias was determined by a precise terrestrial survey. The application of the SIGMA-ε model reduces this bias to 2.3 mm. Finally, the use of the SIGMA-Δ model reduces the bias to 0.1 mm, which is below the noise level of current GPS receivers. It should be pointed out that, due to the diffraction effect at the beginning of the session, the time needed to fix all ambiguities correctly is significantly increased if no phase weighting is applied. With the SIGMA-Δ model all ambiguities are fixed much faster to their correct integer values.

The diffraction effect on the east component is not as severe, but still significant. As in the case of the height component, the impact of the diffraction effect is minimised by de-weighting 'bad' phase observations. Due to the relative geometrical situation between the antenna, the building and the satellites, the north coordinates are not biased.

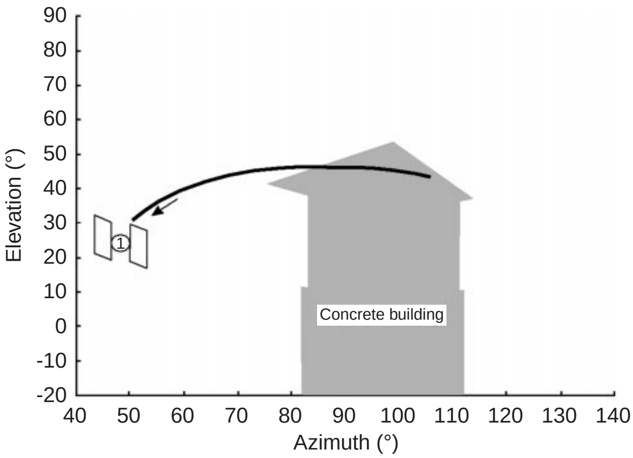

Fig. 7. Positions of PRN 01 seen from the rover antenna

4.2 Static GPS survey near trees

A geodetic network consisting of six stations was surveyed using precise terrestrial and GPS surveying methods. The maximum baseline length was 1.1 km with a maximum height difference of 350 m. The aim of this investigation was to study the attainable accuracy of GPS for engineering surveying applications. This research project was carried out by the Geodetic Institute of the Technical University of Munich. Leica SR399

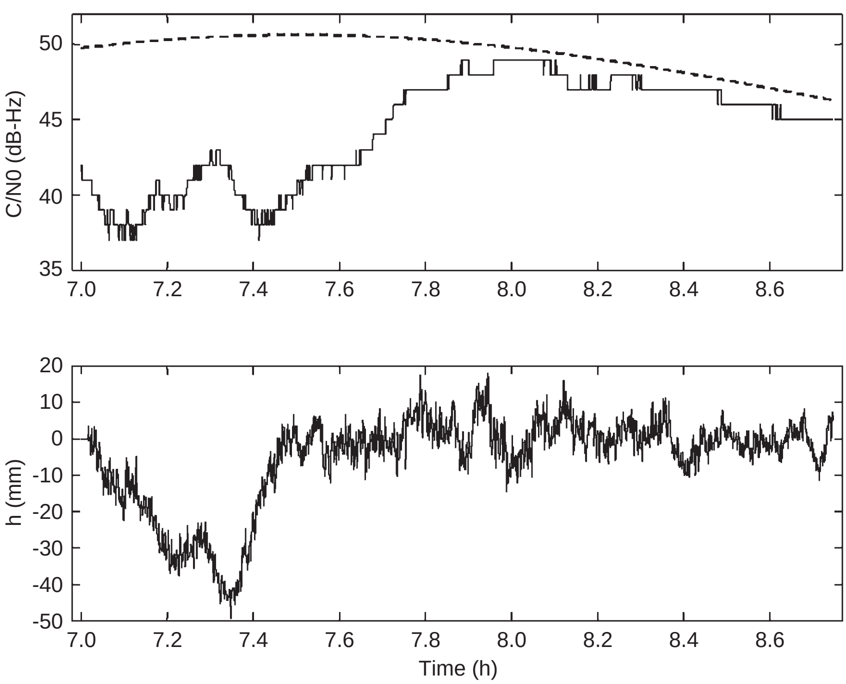

Fig. 8. Diffraction effect in the $C/N0$ of PRN 01 and in the DD residuals. In the first row the template $C/N0$ (*dashed*) and measured $C/N0$s of PRN 01 (*solid*) are shown. In the second row the DD residuals of PRN 01 with the reference satellite PRN 15 are visualised

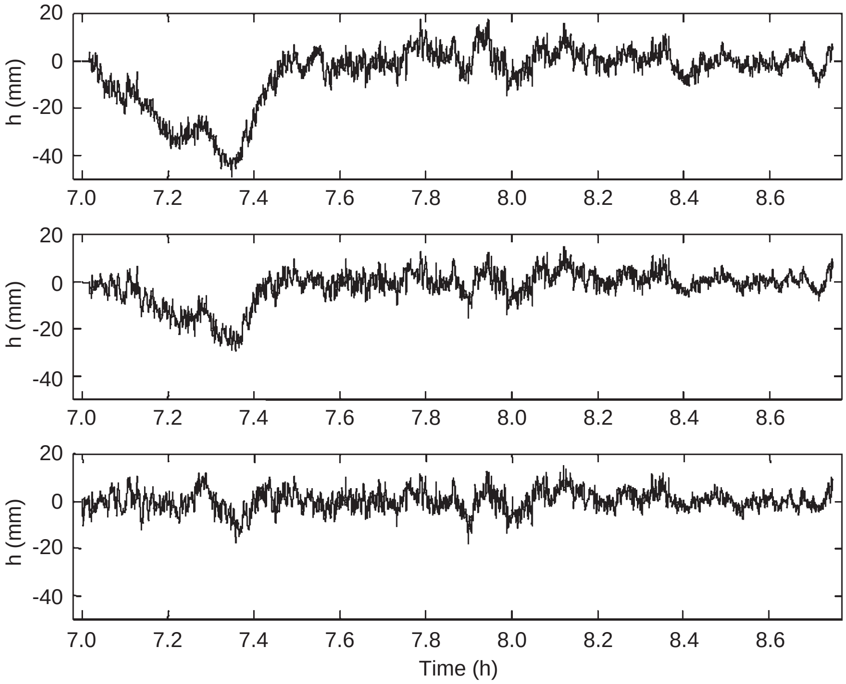

Fig. 9. Height components for three different processing schemes. *First row* No phase weighting; *second row* the SIGMA-ε model; *third row* the SIGMA-Δ model

Fig. 10. The GPS station beside the coniferous tree at point BRE3

GPS receivers with internal antennas were used at all points. Data were collected for 24 h with a sampling rate interval of 15 s.

Signal diffraction effects may also be caused by diffuse obstacles such as bushes or trees. In the following example, a GPS antenna was mounted on an already existing pillar (BRE3) near a coniferous tree which obstructed the satellite signals from the north-east to the south direction up to an elevation of 70° (see Fig. 10). We will consider the baseline BRE3 to BRE5, which is about 601.26 m long. At station BRE5 there is no satellite masking. The terrestrial survey result for the baseline coordinates is the average of several precise measurements.

Table 1. Baseline length results between the points BRE5 and BRE3

Software	Terrestrial minus GPS result [mm]
Leica SKI V2.2	24
Bernese V4.0	22
Grazia without SIGMA weighting	23
Grazia with SIGMA-ε	18
Grazia with SIGMA-Δ	7

First, the 24 h of continuous GPS data were processed by the commercial GPS processing software SKI which uses an elevation-dependent mathematical phase-weighting model. The GPS baseline between the reference point BRE5 and the rover point BRE3 is biased by 24 mm compared to the terrestrial result (see Table 1). Next, the GPS net was processed with the Bernese software V4.0. This software uses no elevation-dependent weighting scheme for phase observations. The baseline between BRE5 and BRE3 calculated with the Bernese software is biased by 22 mm compared to the terrestrial result. Using Grazia the bias is 23 mm if no phase weighting is applied. By processing the 24 h of data with the random $C/N0$ weighting model (SIGMA-ε), the difference between the terrestrial and the GPS-estimated baseline length is reduced to 18 mm. However, if the possible diffraction delay noise is modelled by Σ_Δ, the bias is reduced to 7 mm. The gain in precision of the SIGMA-Δ model is about 71%. So far, the reason for the remaining difference of 7 mm has not been clearly identified. However, it should be remembered that the SIGMA-Δ model minimises through de-weighting only those signal diffraction effects which are different at both stations.

4.3 Kinematic GPS survey near a bush

Having shown the influence and the modelling of signal diffraction for two static surveys, the following example shows the use of the SIGMA-Δ model for the reduction of diffraction effects on kinematic surveys. GPS surveys were performed on a 145 m long, straight part of a rarely used railway line. The straight geometry of the railway tracks was purposely chosen in order to maintain parallel orientation of all antennas. The goal of the experiments was to assess the achievable accuracy of GPS surveys for the determination of the railway track geometry.

Ashtech Z-12 receivers and Ashtech choke-ring antennas as well as Leica SR399 sensors and MicroPulse choke-ring antennas were used. In order to obtain a high density of measurement points, the survey trolley (see Fig. 11) was moved with an average speed of 0.25 m/s and the tracking interval of the receivers was 1. The cut-off angle of the GPS observations was chosen to be 15°. In addition, a terrestrial survey was carried out to obtain an independent result of the track position for assessing the accuracy of the GPS results. The experiments and the results were described previously by Hartinger and Brunner (1998a). Here it was desired to concentrate on the use of the SIGMA-Δ model.

At positions between 91.5 and 95.5 m the signals of PRN 03 had to propagate through a bush to reach the GPS antenna (see Fig. 11). This diffraction effect is indicated by the changes in the $C/N0$ values, see Fig. 12. The $C/N0$ values of PRN 03 are expected to be around 48 dB-Hz. However, the diffraction effect caused by the bush reduces the $C/N0$ values of this satellite to much smaller values of about 36 dB-Hz. Although six

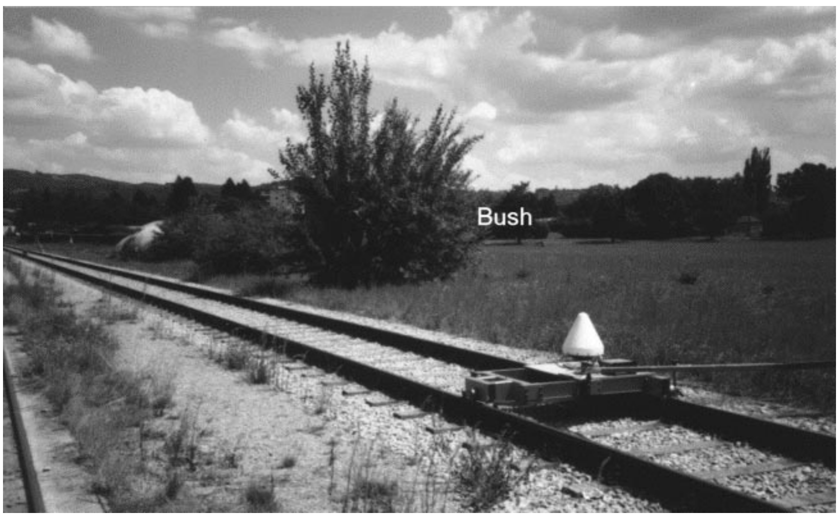

Fig. 11. The surveying trolley before it passes the 'diffracting' bush

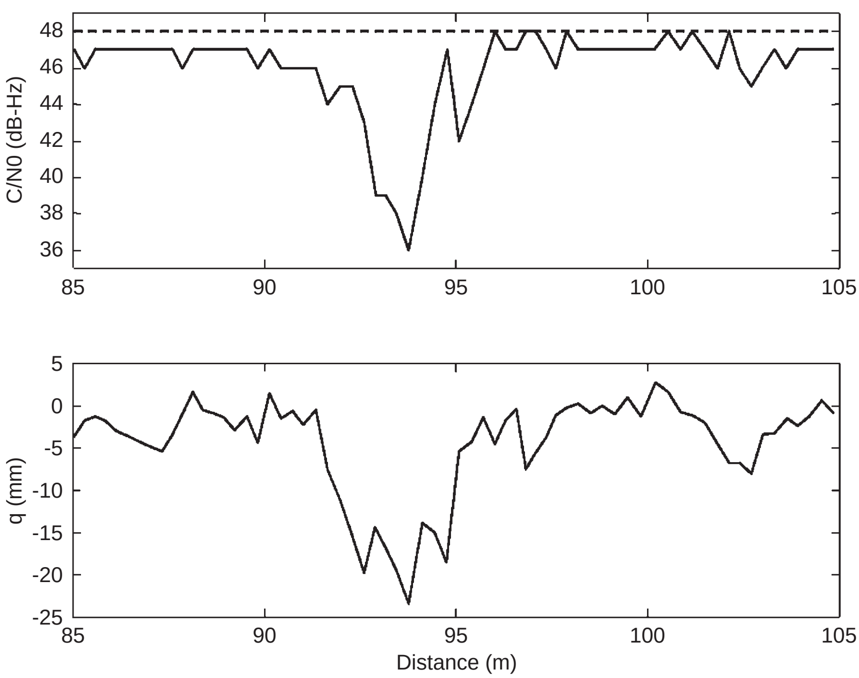

Fig. 12. Bush diffraction effects in the $C/N0$ of PRN 03 and in the results. The *first row* shows the $C/N0_{\text{template}}$ (*dashed*) and the $C/N0_{\text{measured}}$ of PRN 03 (*solid*). In the *second row* the difference between the terrestrial survey and GPS result for a position between 85 and 105 m is visualised

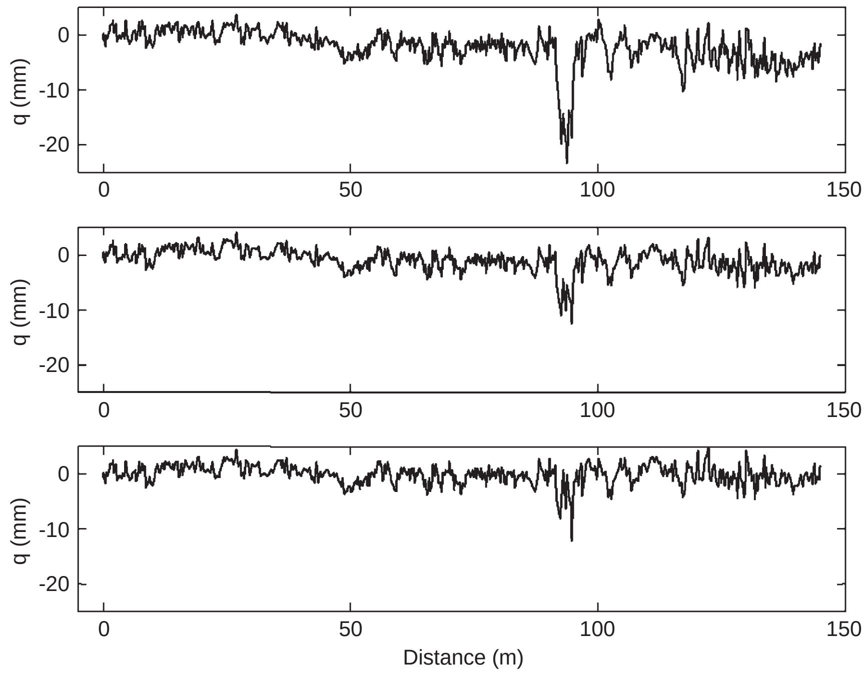

Fig. 13. Difference between the terrestrial survey and GPS for different processing strategies. The *first row* shows the results with no phase weighting, the *second row* with the SIGMA-ε model and *third row* with the SIGMA-Δ model

satellites were visible at this period, the diffracted signal of PRN 03 introduces a 23 mm error in the position results if all GPS phase observations are considered to have the same variance in the least-square adjustment (see Fig 12).

By processing the data of the whole session with Σ_ε, the 23 mm error is reduced to 12 mm (see Fig. 13). The GPS results are improved by about 50%. Modelling the diffraction noise by the variance matrix Σ_Δ reduces the diffraction effect in the results even more. However the maximum error in the GPS results is still 12 mm. Figure 12 shows that the SIGMA-Δ model cannot completely remove this error. At these epochs it appears that the $C/N0$ values of the rover station already approach the appropriate template values, although the phase residuals are still contaminated.

5 Conclusion

The SIGMA-Δ model is a first step in the development of an automatic GPS software which has the ability to overcome signal diffraction effects without any user interaction. The user may not know anything about the antenna environment and does not have to investigate which signals are diffracted – the SIGMA-Δ model does it automatically. In contrast to the method of manually eliminating the whole data of a certain satellite, the number of good observations is kept as large as possible. The SIGMA-Δ model works at the zero difference level of phase data and on an epoch-to-epoch basis. The core of the SIGMA-Δ model is the computation of the phase noise in the Σ_Δ variance matrix using the measured $C/N0$ data and a $C/N0$ template. Due to the antenna template technique, it can also be used for different antenna types at the rover and reference stations.

We have shown the effectiveness of the SIGMA-Δ model in two static and one kinematic GPS surveys. The errors due to GPS signal diffraction were reduced by about 50–85%. The SIGMA-Δ algorithm was compared with commercial software packages which assume equal weights for the phase observations or model phase observations with elevation-dependent mathematical functions. The Grazia software with the SIGMA-Δ model implemented shows a superior performance in the case of signal diffraction.

We are continuing the investigation of the SIGMA-Δ model and possible extensions of the present model.

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