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Adaptively robust filtering for kinematic geodetic positioning

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Abstract. The Kalman filter has been applied extensively in the area of kinematic geodetic positioning. The reliability of the linear filtering results, however, is reduced when the kinematic model noise is not accurately modeled in filtering or the measurement noises at any measurement epoch are not normally distributed. A new adaptively robust filtering is proposed based on the robust M (maximum-likelihood-type) estimation. It consists in weighting the influence of the updated parameters in accordance with the magnitude of discrepancy between the updated parameters and the robust estimates obtained from the kinematic measurements and in weighting individual measurements at each discrete epoch. The new procedure is different from functional model-error compensation; it changes the covariance matrix or equivalently changes the weight matrix of the predicted parameters to cover the model errors. A general estimator for an adaptively robust filter is developed, which includes the estimators of the classical Kalman filter, adaptive Kalman filter, robust filter, sequential least-squares adjustment and robust sequential adjustment. The procedure can not only resist the influence of outlying kinematic model errors, but also controls the effects of measurement outliers. In addition to the robustness, the feasibility of implementing the new filter is achieved by using the equivalent weights of the measurements and the predicted state parameters. A numerical example is given to demonstrate the ideas involved.

Key words: Adaptive robust filter – Robust estimator – Kinematic positioning – Error compensation

1 Introduction

Applications of the Kalman filter in dynamic or kinematic positioning have sometimes encountered difficulties which have been referred to as divergences. These divergences can often be traced to three factors: (1) insufficient accuracy in modeling the dynamics or kinematics (functional model errors of the state equations); (2) insufficient accuracy in modeling the observations (functional model errors of observation equations); and (3) insufficient accuracy in modeling the distributions or the a priori covariance matrices of the measurements and the updated parameters (stochastic model errors).

The current basic procedure for the quality control of the Kalman filter consists of the following:

(1) Functional model compensation for model errors by introducing uncertain parameters into the state and/or the observation equations. Any model error term can be introduced into the models arbitrarily. We can then augment the state (Jazwinski 1970, p. 308). A similar approach is developed by Schaffrin (1991, pp. 32–34). He partitions the state vector into h groups, each being affected by a common scale error. Then $h \times 1$ vectors of scale parameters are introduced into the models. This kind of approach may, of course, lead to a high-dimensional state vector which, in turn, greatly increases the filter computational load (Jazwinski 1970, p. 305).

(2) Stochastic model compensation by introducing a variance–covariance matrix of the model errors. In taking this approach to prevent divergence, we have to determine what covariance matrix to add. A reasonable covariance matrix may compensate for the model errors. An ineffective covariance matrix, however, adds to the model divergence. For instance, when the model is accurate in some dynamic or kinematic periods, an unsuitable increasing of the covariance matrix of model error will degrade the state estimators. An effective covariance matrix for model errors can only be determined by trial and error.

(3) DIA (detection, identification and adaptation) procedure (Teunissen 1990). This uses a recursive testing procedure to eliminate outliers. In the detection step we look for unspecified model errors. In the identification step we try to find the cause of the model error and its most likely starting time. After a model error has been detected and identified, the bias in the state estimate caused by the model error has to be eliminated as well. This model recovery from errors is called adaptation (Salzmanm 1995).

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The identification of the model is, however, quite difficult, especially when the measurements are not accurate enough to detect the unspecified model errors.

(4) Sequential least-squares (LS) procedure. A quite different procedure that has frequently been used for kinematic positioning does not use the dynamic model information at all but determines discrete positions at the measurement epochs (Cannon et al. 1986). In this case, no assumption on the dynamic model is made, and only the measurements at a discrete epoch are employed to estimate the state parameters. The model error, therefore, does not effect the estimates of new state parameters. Usually, this method is presented as a sequential LS algorithm (Schwarz et al. 1989). The current limitation of this approach is that it wastes the useful information of the state model when the model accurately describes the dynamic process in certain cases.

(5) Adaptive Kalman filtering. An innovation-based adaptive Kalman filtering for an integrated Inertial Navigation System (INS) global positioning system (GPS) is developed by Mohamed and Schwarz (1999), based on the maximum-likelihood criterion by proper choice of the filter weight. Another adaptive Kalman filter algorithm to directly estimate the variance and covariance components for the measurements is studied by Wang et al. (1999). Both of the algorithms need to collect the residuals of the measurements or the update series to calculate the state variance–covariance matrices.

(6) Robust filter based on min–max robust theory. The deviation of observation error distribution from the Gaussian one may also seriously degrade the performance of the Kalman filtering. Thus, there appears to be considerable motivation for considering filters which are robust enough to perform fairly well in non-Gaussian environments. Facing this problem, Masreliez and Martin (1977) applied the influence function of min–max robust theory to replace the score function of the classical Kalman filter. The basic disadvantages associated with this kind of robust filter are that the estimator requires the unknown contaminating distribution to be symmetried and it cannot work as well as the standard Kalman filter does in Gaussian noise.

(7) Robust filter based on M estimation theory (Huber 1964) and Bayesian statistics. In order to resist the bad influences of both state model errors and measurement outliers, a robust M–M filter is developed (Yang 1991, 1997; Zhou et al. 1997, p. 299) by which the measurement outliers are controlled by robust equivalent weights of the measurements, the model errors are resisted by the equivalent weights of the update parameters according to the divergence of the predicted parameters and the estimated ones. Furthermore, a robust filter for rank-deficient observation models is developed by Koch and Yang (1998), by Bayesian statistics and by applying the robust M estimate.

In the present paper, an adaptively robust filter is developed by combining the adaptive Kalman filter and robust estimation. The main feature of this new filter consists in weighting the effects of the updated parameters in accordance with the magnitude of discrepancy between the dynamic model and the actual measurements.

2 Functional and stochastic compensations for model errors

Let the linear dynamic system be given by

$$X_t = \Phi_{t,t-1}X_{t-1} + \Gamma_t W_t \tag{1}$$

and a time observation series $L_1, L_2, \ldots$ is specified at epoch t by an observation equation

$$L_t = A_t X_t + e_t \tag{2}$$

where $\Phi_{t,t-1}$ and A_t denote the transition and design matrices, respectively, which are assumed known; the vector X_t consists of m unknown parameters. The random error components W_t and e_t have zero expectations and are mutually uncorrelated. The covariance matrices of W_t and e_t will be taken as

$$D(W_t) = E(W_t W_t^T) = \Sigma_{W_t} \tag{3}$$

$$D(L_t) = D(e_t) = \sigma^2 P_t^{-1} = \Sigma_t \tag{4}$$

where P_t denotes a weight matrix of L_t. The classical Kalman filter reads

$$\bar{X}_t = \phi_{t,t-1}\hat{X}_{t-1} \tag{5}$$

$$\hat{X}_t = \bar{X}_t + K(L_t - A_t\bar{X}_t) \tag{6}$$

$$\Sigma_{\hat{X}_t} = [I - KA_t]\Sigma_{\bar{X}_t} \tag{7}$$

where $\hat{X}_t$ and $\hat{X}_{t-1}$ are the estimated state vectors at epoch t and $t-1$ respectively, $\bar{X}_t$ is the predicted state vector at epoch $t-1$; $\Sigma_{\bar{X}_t}$ reads

$$\Sigma_{\bar{X}_t} = \phi_{t,t-1}\Sigma_{\hat{X}_{t-1}}\phi_{t,t-1}^T + \Gamma_t\Sigma_{W_t}\Gamma_t^T \tag{8}$$

K is a Kalman gain matrix, which reads

$$K = \Sigma_{\bar{X}_t}A_t^T(A_t\Sigma_{\bar{X}_t}A_t^T + \Sigma_t)^{-1} \tag{9}$$

To compensate for the model errors, it is possible to parameterize those model errors. We could then augment the state vector with these parameters and estimate them together with the state. We model such a system containing uncertain parameters by (Jazwinski 1970, p. 281)

$$x_t = \phi_{t,t-1}x_{t-1} + H_{t,t-1}u + \Gamma_t W_t \tag{10}$$

$$L_t = A_t x_t + B_t s + e_t \tag{11}$$

The parameters in vector u will be referred to as dynamical parameters, whereas those in the vector s will be called measurement parameters. $H_{t,t-1}$ and B_t are coefficient matrices. The estimator corresponding to the augmented equations can be found in Jazwinski (1970).

This approach can, of course, compensate for the model-error effects to some extent. It may, however, increase the computational load. On the other hand, once we learn the wrong parametric representation or we do not introduce sufficient parameters, the filter can again diverge. Furthermore, if too many parameters are introduced, there may be insufficient data (Jazwinski 1970, p. 305) or it may lead to a rank-deficient model.

Another method of compensating for the model errors involves a stochastic model. Analyzing the estimator of Eqs. (6) through (9) we realize that when the covariance matrix of the model error term is small, the filter gain is therefore small, and subsequent observations have little effect on the state estimate. And when the state model deviates from the actual system model, the estimate and the state will diverge.

It is possible to introduce a suitable covariance matrix to cover the model errors. In taking this approach to avoid divergence, we have to determine $\Gamma_t \Sigma_{W_{t-1}} \Gamma_t^T$ in Eq. (8). Some particular expressions for $\Gamma_t \Sigma_{W_{t-1}} \Gamma_t^T$ are available (see Jazwinski 1970, p. 306). It is evident that none of them work very well. All those techniques which add a covariance matrix to account for the model errors have their special parameters which have to be adjusted in each application by experimentation.

3 Robust Kalman filter

A robust Kalman filter should be applied if data contaminated by outliers are to be processed. This problem was solved by Masreliez and Martin (1977), who applied heavy-tailed Gaussian and non-Gaussian distributions to account for outliers. A more efficient robust Kalman filter based on polynomial interpolation was developed by Tsai and Kurz (1983). Koch and Yang (1998) derived a robust Kalman filter by using Bayesian statistics, by which outliers are looked for not only in the observations but also in the updated parameters.

3.1 Robust Kalman filtering for controlling the observational outliers

If the outliers only in the observations are looked for, then the robust estimator of filter could be

$$\hat{X}_t = \bar{X}_t + \bar{K}_t(L_t - A_t\bar{X}_t) \tag{12}$$

where $\bar{K}_t$ is the Kalman gain matrix based on the equivalent weight matrix of observations, i.e.

$$\bar{K}_t = \Sigma_{\bar{X}_t} A_t^T (A_t \Sigma_{\bar{X}_t} A_t^T + \bar{P}_t^{-1})^{-1} \tag{13}$$

in which $\bar{P}_t$ denotes the equivalent weight matrix of L_t.

In the independent case, $\bar{P}_t$ is a diagonal matrix with elements $\bar{p}_{t_i} (i = 1, 2, \ldots, n_i)$, and $\bar{p}_{t_i}$ can be (Huber 1964; Yang 1993, p. 76)

$$\bar{p}_{t_i} = \begin{cases} p_{t_i} & \left|\frac{V_i}{\sigma_i}\right| = |V_i'| \le c \\ p_{t_i} \frac{c}{|V_i'|} & |V_i'| > c \end{cases} \tag{14}$$

where c is a constant, which is usually chosen as $c = 1.3$–2.0. V_i is the residual of the observation L_{t_i}, V_i' is the standard residual corresponding to V_i, and $p_{t_i} = 1/\sigma_{t_i}$. Of course, other equivalent weight functions can be chosen or constructed according to particular situations (see Yang 1993, pp. 252–255; Zhou 1989). $\bar{p}_{t_i}$ is a descending function with respect to the standard residual, therefore the outlier existing in the observation L_{t_i} is controlled.

In the case of dependent observations, the equivalent weight matrix $\bar{P}_t$ of the observation vector L_t is a non-diagonal matrix because of the dependence of the observations. The dependent equivalent weight matrix was researched by Yang (1994). Here we give an equivalent weight function for the elements of $\bar{P}_t$ as

$$(\bar{p}_t)_{ij} = \begin{cases} (p_t)_{ij} & |V_i'| \le c \text{ and } |V_j'| \le c \\ (p_t)_{ij} \frac{c}{\max\{|V_i'|, |V_j'|\}} & |V_i'| > c \text{ or } |V_j'| > c \end{cases} \tag{15}$$

where $V_i' = V_i/\sigma_i$.

3.2 Robust filtering for controlling outliers of observations and updated parameters

An estimator of the robust filtering for controlling the outliers of the observation and the updated parameters is given by Koch and Yang (1998)

$$\hat{X}_t = \bar{X}_t + \tilde{K}_t(L_t - A_t\bar{X}_t) \tag{16}$$

$$\tilde{K}_t = \tilde{\Sigma}_{\bar{X}_t} A_t^T (A_t \tilde{\Sigma}_{\bar{X}_t} A_t^T + \sigma^2 \bar{P}_t^{-1})^{-1} \tag{17}$$

$$\tilde{\Sigma}_{\bar{X}_t} = \sigma^2 (G_t \bar{P}_{\bar{X}_t} G^T)^{-1} \tag{18}$$

$$G_t G_t^T = \sigma^2 \Sigma_{\bar{X}_t}^{-1} \tag{19}$$

where $\bar{P}_{\bar{X}_t}$ denotes the equivalent weight matrix of the updated parameter vector $\bar{X}_t$. The determination of the element of $\bar{P}_{\bar{X}_t}$ is similar to that of $\bar{P}_t$.

An alternative expression to Eq. (12) is

$$\hat{X}_t = (A_t^T \bar{P}_t A_t + P_{\bar{X}_t})^{-1} (A_t^T \bar{P}_t L_t + P_{\bar{X}_t} \bar{X}_t) \tag{20}$$

which is called the M–LS filter (Zhou et al. 1997, pp. 295–296), similar to the M–LS Bayesian estimator (Yang 1991). Here $P_{\bar{X}_t}$ denotes the original weight matrix of $\bar{X}_t$. An alternative expression to Eq. (16) is

$$\hat{X}_t = (A_t^T \bar{P}_t A_t + \bar{P}_{\bar{X}_t})^{-1} (A_t^T \bar{P}_t L_t + \bar{P}_{\bar{X}_t} \bar{X}_t) \tag{21}$$

which is called the M–M filter (Yang 1997; Zhou et al. 1997, p. 299).

The robust M–M filter above may cover the dynamic model errors and the measurement outliers in theory. However, the main problem is that when both the dynamic model and the measurements are distorted by outliers, the filter system cannot distinguish them, so the system cannot determine the equivalent weights of both the measurements and updated parameters.

4 Adaptively robust filter

All the methods described depend on the knowledge of the dynamic model errors, with which the functional or stochastic models for compensation for the model errors and the equivalent weights for the robust filter are constructed. In practical applications, it is very difficult to predict the error distribution or the error type of the updated parameters or the dynamic model errors, thus it is very difficult to construct functional and stochastic models. Furthermore, when a moving vehicle is accelerated from zero or decelerated to a stop, the acceleration profile is discontinuous. If this discontinuity falls between two measurement epochs, the dynamics cannot be accurately modeled or predicted by state equations; in this case the predicted information from the dynamic model should be treated with caution. Thus the filter procedure should weaken the effects of the updated parameters. In addition, if the updated parameter vector is contaminated by model error, then it is usually distorted in its entirety. Thus we do not need to consider the error influence of the individual elements of the updated parameter vector like the robust M–M filter does. An adaptive filter is suitable in this case to balance the dynamic model information and the measurements.

4.1 General estimator of adaptively robust filtering

An adaptively robust filter is constructed based on the estimator of Eq. (20) as

$$\hat{X}_t = (A_t^T \bar{P}_t A_t + \alpha P_{\bar{X}_t})^{-1}(A_t^T \bar{P}_t L_t + \alpha P_{\bar{X}_t}\bar{X}_t) \tag{22}$$

$$\Sigma_{\hat{X}_t} = (A_t^T \bar{P}_t A_t + \alpha P_{\bar{X}_t})^{-1}\sigma_0^2 \tag{23}$$

where σ_0^2 is a scale factor, α is an adaptive factor which can be chosen as

$$\alpha = \begin{cases} 1 & |\Delta\tilde{X}_t| \le c_0 \\ \frac{c_0}{|\Delta\tilde{X}_t|}\left(\frac{c_1 - |\Delta\tilde{X}_t|}{c_1 - c_0}\right)^2 & c_0 < |\Delta\tilde{X}_t| \le c_1 \\ 0 & |\Delta\tilde{X}_t| > c_1 \end{cases} \tag{24}$$

where c_0 and c_1 are constants which are found to have the values $c_0 = 1.0$–1.5, $c_1 = 3.0$–4.5

$$\Delta\tilde{X}_t = \left\|\tilde{X}_t - \tilde{\bar{X}}_t\right\| \Big/ \sqrt{\mathrm{tr}\{\Sigma_{\tilde{\bar{X}}_t}\}} \tag{25}$$

and $\tilde{X}_t$ is a robust estimate of the state vector (state position) which is only evaluated by new measurements at epoch t and the raw velocity observations are not included in it. $\tilde{\bar{X}}_t$ is a predicted position from Eq. (5) in which the a priori velocity components are not included. In our opinion, the change of the position expressed by Eq. (25) can also reflect the stability of the velocity.

Equation (22) is a general estimator of an adaptively robust filter. In the case of $\alpha \neq 0$, Eq. (22) is changed into, by using the matrix identities (Koch 1988, p. 40)

$$\hat{X}_t = \bar{X}_t + \Sigma_{\bar{X}_t} A_t^T (A_t \Sigma_{\bar{X}_t} A_t^T + \alpha \Sigma_t)^{-1}(L_t - A_t \bar{X}_t) \tag{26}$$

4.2 Special estimators

The adaptive factor α changes between 0 and 1, which balances the contribution of the new measurements and the updated parameters to the new estimates of state parameters.

Case 1. If $\alpha = 0$ and $\bar{P}_t = P_t$, then

$$\hat{X}_t = (A_t^T P_t A_t)^{-1} A_t^T P_t L_t \tag{27}$$

which is an LS estimator using only the new measurements at epoch t. This estimator is suitable when the measurements are not contaminated by outliers and the updated parameters are biased so much that the $\Delta\tilde{X}_t$ in Eq. (24) is larger than c_1 (rejecting point), and the information of updated parameters is completely ignored.

Case 2. If $\alpha = 1$ and $\bar{P}_t = P_t$, then

$$\hat{X}_t = (A_t^T P_t A_t + P_{\bar{X}_t})^{-1}(A_t^T P_t L_t + P_{\bar{X}_t}\bar{X}_t) \tag{28}$$

which is a general estimator of the classical Kalman filter. Equation (28) is equivalent to Eq. (6).

Case 3. If α is determined by Eq. (24) and $\bar{P}_t = P_t$, then

$$\hat{X}_t = (A_t^T P_t A_t + \alpha P_{\bar{X}_t})^{-1}(A_t^T P_t L_t + \alpha P_{\bar{X}_t}\bar{X}_t) \tag{29}$$

which is an adaptive LS estimator of the Kalman filter. It balances the contribution of the updated parameters and the measurements. The only difference between Eq. (22) and Eq. (29) is the weight matrix of L_t; the former uses the equivalent weights and the latter uses the original weights of L_t.

Case 4. If $\alpha = 0$, then we obtain

$$\hat{X}_t = (A_t^T \bar{P}_t A_t)^{-1} A_t^T \bar{P}_t L_t \tag{30}$$

which is a robust estimator using only the new measurements at epoch t.

Case 5. If $\alpha = 1$, then

$$\hat{X}_t = (A_t^T \bar{P}_t A_t + P_{\bar{X}_t})^{-1}(A_t^T \bar{P}_t L_t + P_{\bar{X}_t}\bar{X}_t) \tag{31}$$

which is an M–LS filter estimator (Yang 1997).

5 Test computation and analysis

A data set was collected on 20 June 1996, using Trimble 4000SSE by a flight. The available measurements are C/A code, P2-code pseudoranges, L1 and L2 carrier phases and Doppler measurements with 1-s data rate.

The rover receiver was mounted in an aircraft, and the reference receiver was fixed at a site about 1 km from the initial aircraft location. After about 10 minutes of static tracking the aircraft took off, and the flight time was about 90 minutes. The flight trajectories and velocities are shown in Fig. 1.

The double-differenced C/A-code and P2-code measurements are employed in the test performance. An outlier of 50 m was given to the C/A-code measurements of Satellite 2 every other 500 epochs in order to test the performance of the robust algorithm. The constant-velocity model of the Kalman filter was employed. The reasons for choosing the constant velocity model are that so far we have not found any suitable acceleration model to fit the test flight, and it is also very difficult for us to construct a new reliable model. Furthermore, the small time differences between the observations may weaken the effects of the acceleration on the dynamic model. Therefore we apply a transition matrix as

$$\phi_{t,t-1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

with initial variances of $0.2\,\mathrm{m}^2$ for positions, $9 \times 10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-2}$ for velocities, and $1\,\mathrm{m}^2$ for code measurements, and with a spectral density of $0.2\,\mathrm{m}^2\,\mathrm{s}^{-3}$ for velocities. The dynamic model covariance matrix in Eq. (8) is chosen as (Schwarz et al. 1989)

$$\Gamma_t \Sigma_{W_t} \Gamma_t^T = \begin{bmatrix} 1/3 Q_2 \Delta t^3 & 1/2 Q_2 \Delta t^2 \\ 1/2 Q_2 \Delta t^2 & Q_2 \Delta t \end{bmatrix}$$

where Q_2 denotes the spectral density for velocities and Δt denotes a sampling time interval.

The highly precise results from double-differenced carrier measurements are used only as "true values" for comparing with the results from the code measurements, in which the ambiguities are resolved on the fly using the LAMBDA method (Teunissen et al. 1997). The following six schemes are performed.

Scheme 1: LS estimation, i.e. $\alpha = 0$ and $\bar{P}_t = P_t$.
Scheme 2: classical Kalman filtering, i.e. $\alpha = 1$ and $\bar{P}_t = P_t$.
Scheme 3: adaptive Kalman filtering, i.e. α is determined by Eq. (24) and $\bar{P}_t = P_t$.
Scheme 4: robust estimation, i.e. $\alpha = 0$ and the equivalent weight element of $\bar{P}_t$ is determined by Eq. (15).
Scheme 5: robust Kalman filtering, i.e. $\alpha = 1$.
Scheme 6: adaptively robust Kalman filtering.

The position differences for the X component between the results from the six computation schemes and the "true values" are shown in Figs. 2–7. The position differences for Y and Z components are similar to those for the X component, and are omitted here.

Figure 1a and b shows that the flight states have two notable sudden changes, one is close to epoch 1000 when the plane takes off and the other one is between epoch 3000 and 4000 when the flight turns around. From the test computation and comparisons, the following facts can be stated.

(1) The two unstable states of the flight are obviously reflected in the results of the classical Kalman filtering (Scheme 2, Fig. 3) and the robust filtering (Scheme 5, Fig. 6). The dynamic errors, however, have little influence on the results of the LS adjustment (Scheme 1, Fig. 2) and the robust estimation (Scheme 4, Fig. 5), since the a priori dynamic model information is not considered in these two estimation procedures. The adaptive filters do resist the influences of the dynamic model errors (Scheme 3, Fig. 4 and Scheme 6, Fig. 7).

(2) Comparing the robust estimators, Schemes 4, 5 and 6, to the non-robust algorithms, Schemes 1, 2 and 3, we recognize that the robust estimators (see Figs. 5b, 6b and 7b) have effectively resisted the influence of the outliers and the errors from the satellites of lower elevation angle on the estimates of the state parameters.

(3) Among the above algorithms, the results from the adaptively robust Kalman filter are the best. It cannot only resist the impact of outliers, but also measure the dynamic errors in time. Then, the adaptive factor α can

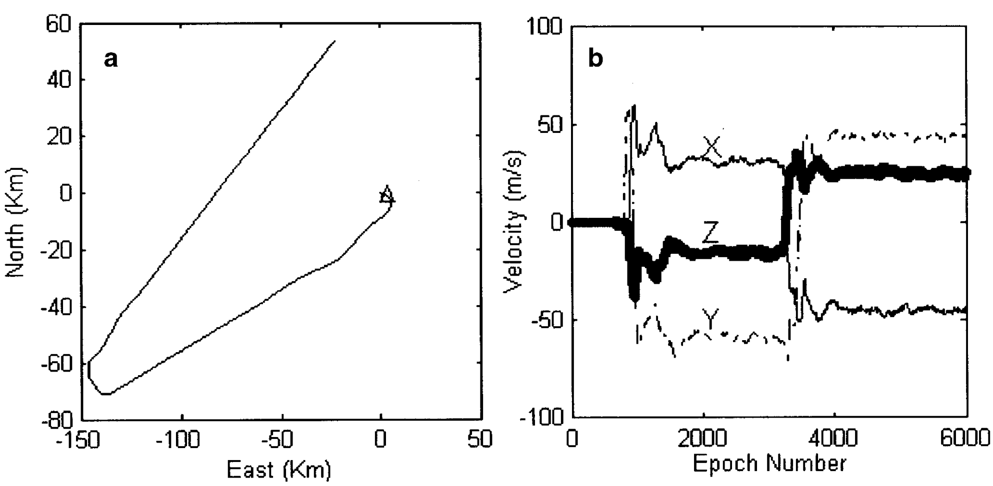

Fig. 1a, b. Flight trajectories and velocities relative to fixed receiver. **a** Position relative to fixed receiver; **b** velocities of X, Y and Z components

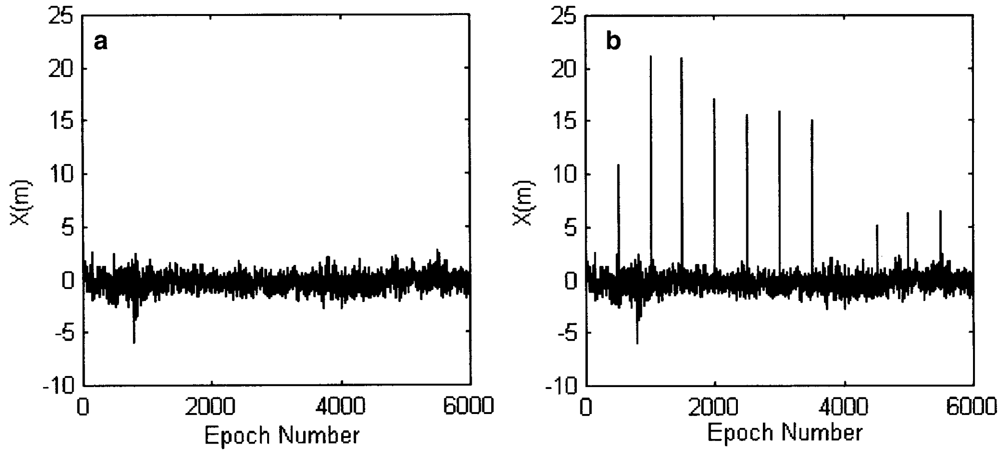

Fig. 2a, b. LS adjustment. **a** Without outlier; **b** with outliers

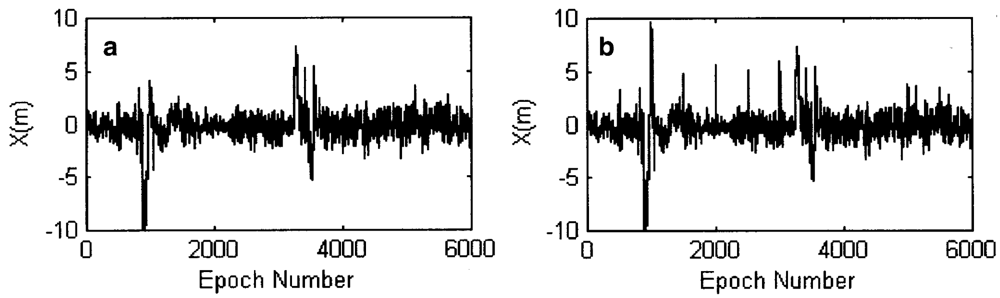

Fig. 3a, b. Classical Kalman filter. **a** Without outlier; **b** with outliers

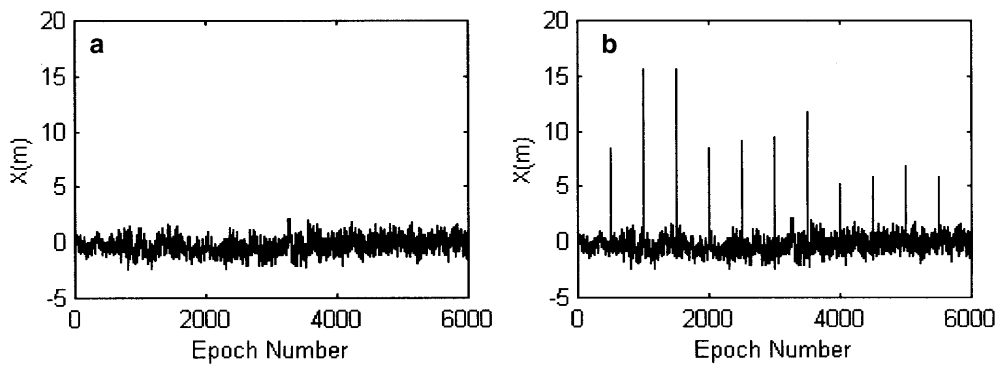

Fig. 4a, b. Adaptive LS Kalman filter; **a** without outlier; **b** with outliers

be appropriately determined to employ a priori information reasonably, so that the estimator avoids the divergences or large biases which often occur in the classical Kalman filter because the dynamics are not tracked in time.

(4) As a by-product, we find that from the computation, when the measurements are heavily contaminated by outliers, the results of the adaptive LS filter are poorer than those of the classical Kalman filter. The outliers distort the current estimates of the state parameters at the current epoch; consequently they make the adaptive factor α believe that a dynamic state error occurs, and result in a bad determination of α. The a priori information is degraded inappropriately and the noise of the estimates of state parameters increases; compare Figs. 4b and 3b.

6 Concluding remarks

From our theoretical derivation, analysis, and actual computation and comparison, the following conclusions can be drawn.

(1) The classical Kalman filter can fully employ the model information to improve the precision and reliability of kinematic positioning by smoothing the measurement noises based on support of the accurate state equation. It cannot, however, control the dynamic model biases.
(2) The LS adjustment can resist the dynamic model errors, but it cannot fully employ the reliable information from the dynamic model, even though the model may be accurate in most cases.

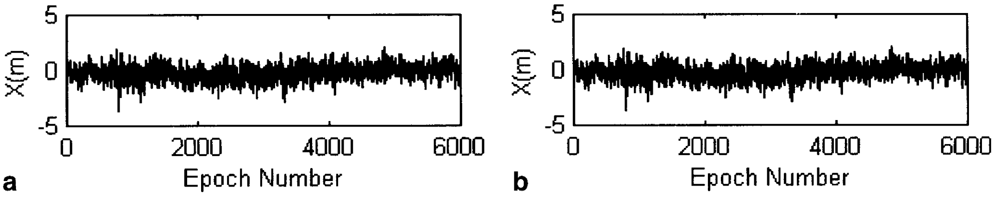

Fig. 5a, b. Robust adjustment. **a** Without outlier; **b** with outliers

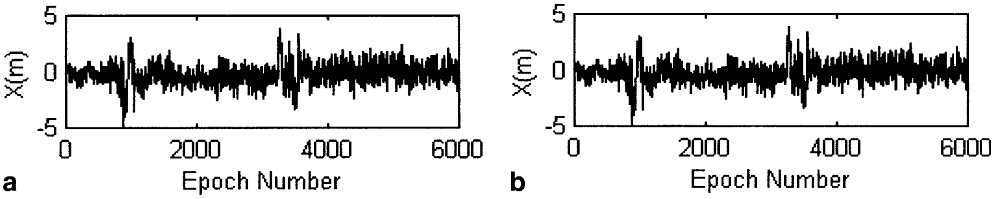

Fig. 6a, b. Robust Kalman filter. **a** Without outlier; **b** with outliers

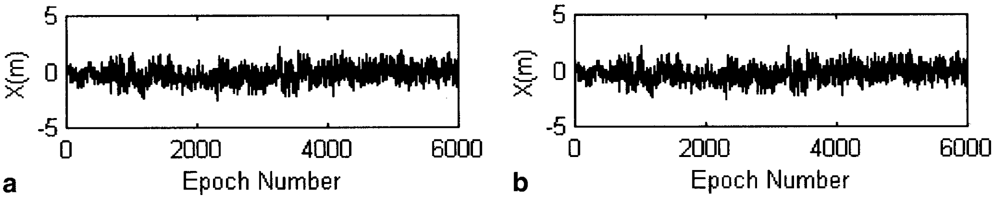

Fig. 7a, b. Adaptively robust Kalman filter; **a** Without outlier; **b** with outliers

(3) The adaptively robust Kalman filter can balance the contribution of updated parameters and the new measurements, but it needs the support of reliable measurements.
(4) None of the estimators based on LS can resist the measurement outliers.
(5) The adaptively robust Kalman filter proposed in this paper can not only balance the contribution between the updated parameters and measurements in accordance with the magnitudes of their discrepancy, but also resist the influences of measurement outliers. It can be combined with any other error compensation methods if a suitable stochastic covariance matrix for dynamic model errors or some reliable functional models are available. The general estimator of the new adaptively robust Kalman filter includes the estimators of LS adjustment, robust adjustment, Kalman filter, robust Kalman filter and adaptive LS filter. All of the special estimators can be achieved by an effectively adaptive factor α, and an equivalent weight function.

The adaptively robust filtering developed in this paper is very preliminary; further theoretical and practical research and analyses are needed.

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