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An approach to GLONASS ambiguity resolution

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Abstract. When processing global navigation satellite system (GLONASS) carrier phases, the standard double-differencing (DD) procedure cannot cancel receiver clock terms in the DD phase measurement equations due to the multiple frequencies of the carrier phases. Consequently, a receiver clock parameter has to be set up in the measurement equations in addition to baseline components and DD ambiguities. The resulting normal matrix unfortunately becomes singular. Methods to deal with this problem have been proposed in the literature. However, these methods rely on the use of pseudo-ranges. As pseudo-ranges are contaminated by multipath and hardware delays, biases in these pseudo-ranges are significant, which may result in unreliable ambiguity resolution. A new approach is addressed that is not sensitive to the biases in the pseudo-ranges. The proposed approach includes such steps as converting the carrier phases to their distances to cancel the receiver clock errors, and searching for the most likely single-differenced (SD) ambiguity. Based on the results from the theoretical investigation, a practical procedure for GLONASS ambiguity resolution is presented. The initial experimental results demonstrate that the proposed approach is useable in cases of GLONASS and combined global positioning system (GPS) and GLONASS positioning.

Key words: GLONASS – GPS – Ambiguity resolution – Satellite geodesy

1 Introduction

The global navigation satellite system (GLONASS) is the Russian equivalent of the global positioning system (GPS). Like the GPS, the GLONASS positioning system consists of 24 satellites and has great potential for precise navigation and geodetic applications, which is evidenced in part by the present international GLONASS experiment (Slater et al. 1998; Willis et al. 1999). It has also been seen that there are many advantages in the integration of the two existing satellite systems. For example, the combination of the two systems offers the increases in accuracy and integrity gained by adding more visible satellites. When the GLONASS system is completely deployed, the integrated GPS and GLONASS constellation will consist of a total of 48 satellites. When using such an integrated satellite constellation, at least 12 satellites will be visible in open areas at any time. The maximum number of satellites in view can reach 20 in the best case. The increase in satellite availability will also make fast static and kinematic positioning much more feasible than it is with just each system alone (e.g. Kleusberg 1990; Ashkenazi et al. 1995; Hein et al. 1997; Langley 1997).

However, due to the fact that the GLONASS satellites transmit their signals at different frequencies, processing the GLONASS carrier-phase measurements is much more complicated than processing only GPS data. In processing the GLONASS carrier phases, one of the critical issues is that the standard double-differencing (DD) procedure cannot cancel receiver clock terms in the DD carrier-phase measurement equations. Consequently, the unknown parameters in the measurement equations include baseline components, DD ambiguities and relative receiver clock terms. The resulting design matrix unfortunately contains a rank deficiency. As a consequence of this, the normal matrix becomes singular (Wang 1998b). In order to remove this singularity, a number of modelling methods have been investigated in the literature. The proposed models can generally be categorized into two types, i.e. *type A*, models excluding the receiver clock parameters, and *type B*, models including the receiver clock parameters. Examples for the models of *type A* are those of e.g. Raby and Daly (1993), Leick et al. (1995), Landau and Vollath (1996) and Pratt et al. (1997); for a detailed review of these models, see Leick (1998) and Wang (1998b). The models of *type B* can be found in e.g. Walsh and Daly (1996), Kozlov and Tkachenko (1997) and Rapoport (1997). An optimal model of *type B* has been identified in Wang (1998a).

In terms of GLONASS carrier-phase ambiguity resolution, a common feature of many existing models is that they rely on the use of pseudo-range measurements, although the pseudo-ranges are used in a different way for each model. Unlike GPS, the effects of inter-channel hardware delays on GLONASS pseudo-ranges are significant (e.g. Pratt et al. 1997; Jonkman et al. 1998; Wang 1998a). For reliable ambiguity resolution, specific efforts must be made to accommodate these biases. Therefore, for high-precision geodetic applications of the GLONASS data, it will also be of great importance to develop suitable algorithms to process carrier-phase data only. Examples in this direction are the works of Rossbach and Hein (1996), Povaliaev (1997) and Habrich (1998).

This paper emphasizes an approach to GLONASS carrier-phase ambiguity resolution which is not sensitive to the biases in pseudo-range data. Mathematical description of the proposed approach and practical strategies for data processing are presented, and tested using real data sets collected on three baselines.

2 Modelling GLONASS phase measurements

Similar to GPS measurements, in the case of short baselines, the so-called *differencing* procedures can considerably reduce some systematic errors existing in the GLONASS measurements, such as atmospheric delay, satellite orbit and clock errors. The resulting mathematical models are simplified. For short baselines, the mathematical model for single-differenced (SD) carrier phases is usually expressed as (e.g. Leick 1995; Teunissen and Kleusberg 1996; Habrich 1998)

$$\varphi_{uv}^{p}(i) = \frac{1}{\lambda_p}\rho_{uv}^{p}(i) + \frac{c}{\lambda_p}t_{uv}(i) + N_{uv}^{p} + \varepsilon_{uv}^{p}(i) \tag{1}$$

where the superscript p identifies the satellite; the subscripts u and v represent the receivers; the index i denotes the epoch at which the data are collected; $\varphi_{uv}^{p}(i)$ is the SD carrier phase expressed in units of cycles; $\rho_{uv}^{p}(i)$ is the SD receiver–satellite range; c is the speed of light; $t_{uv}(i)$ is the relative receiver clock error; N_{uv}^{p} is the integer carrier-phase ambiguity; $\varepsilon_{uv}^{p}(i)$ is the noise error of the SD carrier phase; and λ_p is the wavelength for satellite p. For satellites p and q, the DD carrier phase is further written as

$$\varphi_{uv}^{pq}(i) = \frac{1}{\lambda_p}\rho_{uv}^{p}(i) - \frac{1}{\lambda_q}\rho_{uv}^{q}(i) + \left(\frac{c}{\lambda_p} - \frac{c}{\lambda_q}\right)t_{uv}(i) + N_{uv}^{pq} + \varepsilon_{uv}^{pq}(i) \tag{2}$$

which shows that, unlike GPS, the DD GLONASS carrier phases are sensitive to the receiver clock errors (e.g. Raby and Daly 1993). Because the relative receiver clock term in Eq. (2) does not cancel, a receiver clock parameter has to be set up in addition to the baseline components and the DD ambiguities. Consequently, the normal matrix becomes singular. This means that the DD ambiguity parameters cannot be separated from the receiver clock parameter (Wang 1998b, 1999). In order to cancel the receiver clock errors in DD carrier phases, such strategies as converting original carrier phases into distances, GLONASS mean frequency or GPS L1 frequency have been proposed in the literature (e.g. Leick et al. 1995; Landau 1998). It has been shown that the adjustment results using these strategies are identical (Wang 1998b). Converting the carrier phases into distances and then forming the double differences gives

$$\phi_{uv}^{pq}(i) = \rho_{uv}^{pq}(i) + \bar{N}_{uv}^{pq} + \bar{\varepsilon}_{uv}^{pq}(i) \tag{3}$$

with

$$\bar{N}_{uv}^{pq} = \lambda_q N_{uv}^{pq} + (\lambda_p - \lambda_q)N_{uv}^{p} \tag{4}$$

$$\bar{\varepsilon}_{uv}^{pq}(i) = \lambda_p \varepsilon_{uv}^{p}(i) - \lambda_q \varepsilon_{uv}^{q}(i) \tag{5}$$

In Eq. (3), the SD ambiguity parameters are present. The SD and DD ambiguities are inseparable, however. The lumped parameter $\bar{N}_{uv}^{pq}$ is, by definition, not an integer. Obviously, the key issue here is how to determine the integer SD ambiguity value. One possible option to obtain the SD ambiguity value is to make use of both SD pseudo-ranges and SD carrier phases (e.g. Leick et al. 1995; Landau 1998). For example, at epoch i, the SD ambiguity may be approximated as

$$N_{uv}^{p} = \frac{1}{\lambda_p}\left[R_{uv}^{p}(i) - \lambda_p \varphi_{uv}^{p}(i)\right] \tag{6}$$

where $R_{uv}^{p}(i)$ is the SD pseudo-range. It is easy to see that the correct estimation of SD ambiguities is highly dependent on precise pseudo-ranges. In many practical situations, however, pseudo-ranges may be seriously biased by multipath and hardware delays. For example, a 5-m error in pseudo-range will cause an error of about 26 cycles in the estimated SD ambiguity value. The effect of this error in the DD carrier-phase measurements can reach as much as 0.04 m (about 0.2 cycles) and thus make the ambiguity resolution unfeasible. Therefore, specific strategies for estimating SD ambiguities need to be further investigated.

3 An approach to GLONASS ambiguity resolution

When resolving GPS ambiguities, the integer least-squares (LS) principle is most critical (Teunissen 1993). Within the search space assumed to contain the correct integer ambiguities, all possible ambiguity combinations are fitted to the GPS measurements. The integer ambiguity combination that results in the minimum quadratic form of the LS residuals is considered as the most likely (*best*) solution. In the case of GLONASS ambiguity resolution, however, we have to deal with both the DD ambiguities and the SD ambiguities.

In order to reduce the number of unknown SD ambiguities in the measurement equations, the satellite

p is chosen as a reference satellite in forming all the DD carrier phases. Thus N_{uv}^p is the unique unknown SD ambiguity parameter in the following adjustment. If both SD and DD ambiguities are thought to be integers, the basic principle for fixing DD ambiguities can also be employed to determine this unknown SD ambiguity. The proposed approach to the SD and DD ambiguity resolution is presented in the following.

3.1 *Estimating float DD ambiguities*

Based on Eq. (3), the linearized mathematical model of the DD carrier phases reads

$$Dl_i = DA_i x_c + B_k x_k + f_m x_m + e_i \tag{7}$$

where $i = 1, 2, \ldots, s$ denotes the epoch number and s is the total number of epochs; D is an $(n-1) \times n$ *DD matrix operator* (Teunissen 1997) with n being the number of satellites; l_i is an $n \times 1$ vector of the difference between the SD carrier-phase measurements and their calculated values; x_c is a 3×1 vector of the unknown increments of baseline components; x_k is an $(n-1) \times 1$ vector of the unknown DD ambiguities; x_m is an unknown SD ambiguity parameter; A_i is an $n \times 3$ design matrix capturing the relative satellite–receiver geometry at epoch $i; B_k$ is an $(n-1) \times (n-1)$ diagonal matrix whose diagonal elements are the wavelengths, say λ_i; f_m is an $n \times 1$ vector of the wavelength-difference terms, i.e. $\lambda_i - \lambda_p$, and e_i is an $(n-1) \times 1$ vector of the DD phase noise terms expressed by Eq. (5).

For the adjustment of all s epochs of data, the mathematical model reads

$$l = Ax_c + Bx_k + fx_m + e \tag{8}$$

where

$$A = \left(A_1^T D^T, A_2^T D^T, \ldots, A_s^T D^T\right)^T$$
$$B = (B_k, B_k, \ldots, B_k)^T$$
$$f = (f_m^T, f_m^T, \ldots, f_m^T)^T$$
$$e = (e_1^T, e_2^T, \ldots, e_s^T)^T$$

It is easy to prove that

$$(A, B, f)\begin{pmatrix} 0 \\ B_k^{-1} f_m \\ -1 \end{pmatrix} = 0 \tag{9}$$

which indicates that a linear dependent combination exists in the column vectors of the design matrices (e.g. Teunissen and Kleusberg 1996). The resulting normal matrix is therefore singular. This theoretically confirms the above statement, that the SD and DD ambiguities are inseparable. In order to remove the singularity in the normal matrix, the SD ambiguity parameter is assigned an approximate value (m_0) and Eq. (9) is rewritten as

$$\bar{l}(m_0) = Ax_c + Bx_k + e \tag{10}$$

with $\bar{l}(m_0) = l - fm_0$. In Eq. (10), the unknown SD ambiguity disappears and, thus, the unknown DD ambiguity parameters can be estimated together with the baseline components. In order to do this, a stochastic model (covariance matrix) for DD carrier phases is also required. By assuming that the unscaled SD carrier phases are statistically independent and have the same variance σ_0^2, the covariance matrix for the DD carrier phases at epoch i is derived as

$$C_i = \sigma_0^2 \begin{bmatrix} \lambda_1^2 + \lambda_p^2 & \lambda_p^2 & \ldots & \lambda_p^2 \\ \lambda_p^2 & \lambda_2^2 + \lambda_p^2 & \ldots & \lambda_p^2 \\ \ldots & \ldots & \ldots & \ldots \\ \lambda_p^2 & \lambda_p^2 & \ldots & \lambda_n^2 + \lambda_p^2 \end{bmatrix} = \sigma_0^2 P_i^{-1} \tag{11}$$

Moreover, with the assumptions that the time correlation between epochs is absent and the covariance matrices for each epoch are identical, the whole covariance matrix in the adjustment is then written as

$$\mathrm{Cov}(e) = C = \mathrm{diag}(C_i) = \sigma_0^2 \mathrm{diag}(P_i^{-1}) = \sigma_0^2 P^{-1} \tag{12}$$

where P is the weight matrix and P_i are its block diagonal matrices. Based on the mathematical and stochastic models given by Eqs. (10) and (12), respectively, the LS estimators of the unknowns x_c and x_k are derived as

$$\hat{x}_c(m_0) = Q_{\hat{x}_c} A^T P \bar{l}(m_0) + Q_{\hat{x}_c \hat{x}_k} B^T P \bar{l}(m_0) \tag{13}$$

$$\hat{x}_k(m_0) = Q_{\hat{x}_k \hat{x}_c} A^T P \bar{l}(m_0) + Q_{\hat{x}_k} B^T P \bar{l}(m_0) \tag{14}$$

with the matrices $Q_{\hat{x}_c}, Q_{\hat{x}_k}, Q_{\hat{x}_c\hat{x}_k}$ and $Q_{\hat{x}_k\hat{x}_c}$ being determined by

$$\begin{bmatrix} Q_{\hat{x}_c} & Q_{\hat{x}_c\hat{x}_k} \\ Q_{\hat{x}_k\hat{x}_c} & Q_{\hat{x}_k} \end{bmatrix} = \begin{bmatrix} A^T PA & A^T PB \\ B^T PA & B^T PB \end{bmatrix}^{-1} \tag{15}$$

Hence, the LS residuals read

$$\hat{e}(m_0) = \bar{l}(m_0) - A\hat{x}_c(m_0) - B\hat{x}_k(m_0) \tag{16}$$

and the estimated variance factor is

$$\hat{s}^2(m_0) = \frac{\Omega(m_0)}{r} \tag{17}$$

where $\Omega(m_0) = \hat{e}^T(m_0) P \hat{e}(m_0)$ is the quadratic form of the residuals and $r = (s-1)(n-1) - 3$ is the model redundancy.

Intuitively, it may be expected that if the SD ambiguity is fixed to its correct integer value, the mathematical model expressed by Eq. (10) will have a good performance in the following DD ambiguity-float solution. The closer the approximate SD ambiguity m_0 to its correct value, the smaller the resulting quadratic form of residuals. However, the following theorem shows that this is untrue.

Theorem 1. (DD ambiguity-float solutions with the SD ambiguity fixed to various values) Suppose that fixing the SD ambiguity parameter x_m in Eq. (10) to any two integer values m_i and m_j leads to two sets of statistics for the DD ambiguity-float solutions, namely, $\hat{x}_c(m_i)$, $\hat{x}_k(m_i)$, $\hat{e}(m_i)$, $\Omega(m_i)$, and $\hat{x}_c(m_j)$, $\hat{x}_k(m_j)$, $\hat{e}(m_j)$, $\Omega(m_j)$. It is then concluded that

1. $\hat{x}_k(m_i) = \hat{x}_k(m_j) + B_k^{-1} f_m \cdot (m_j - m_i)$ (18)
2. $\hat{x}_c(m_i) = \hat{x}_c(m_j)$ (19)
3. $\hat{e}(m_i) = \hat{e}(m_j)$ (20)
4. $\Omega(m_i) = \Omega(m_j)$ (21)

Proof. See Appendix.

Equation (18) shows that, when fixing the SD ambiguity to different values, the DD ambiguity parameters can be easily calculated without need to reprocess the whole data set. Equation (19) indicates that the baseline components estimated from DD ambiguity-float solutions are not influenced by the fixed SD ambiguity values. These two equations can be used to simplify the process of searching for the best SD ambiguity value.

Equations (20) and (21) indicate that, in DD ambiguity-float solutions, the DD phase residuals and the quadratic form of the residuals are independent of the fixed SD ambiguity value. These results may be explained by using the concept of estimable quantity in the framework of a rank defect LS adjustment (e.g. Teunissen 1985, 1996; Koch 1988). In this situation, the fixed SD ambiguity value is considered as a datum constraint and the residuals are estimable quantities, which are invariant to the changes in the fixed SD ambiguity value (Teunissen 1985, 1996; Koch 1988). Therefore, with the statistics of the DD ambiguity-float solutions, it is impossible to search for the most likely SD ambiguity value. Ambiguity validation criteria need to be further discussed.

3.2 Ambiguity validation criteria

Given the SD ambiguity fixed to an approximate value m_i, the real-valued DD ambiguity parameters are estimated, and the so-called ambiguity search process is then performed using a search criterion based on the minimization of the quadratic form of the LS residuals. During the search process, the compatibility of all the potential integer ambiguity combinations with the associated measurements is statistically tested. If no integer ambiguity combination passes this *acceptance test* (e.g. Tiberius and de Jonge 1995; Walsh et al. 1995) under the given confidence level, the correct integer ambiguities cannot be identified with the available data.

When one or more integer ambiguity combinations are accepted, the integer ambiguity combinations that result in the minimum and second minimum quadratic forms of the LS residuals will be considered as the most likely (*best*) and *second-best* solutions, respectively. The next and most critical step for ambiguity resolution is to apply a so-called *discrimination test* (Tiberius and de Jonge 1995; Walsh et al. 1995) to ensure that the most likely integer ambiguity combination, denoted as K_1, is statistically better than the second-best combination, denoted as K_2. In this study, the integer ambiguities obtained from the ambiguity search process are treated as nonstochastic quantities. Investigations into the stochasticity and distribution of the integer ambiguities can be found in e.g. Teunissen (1998).

Suppose that fixing the DD ambiguities to K_1 and K_2 produces the quadratic forms of the residuals $\bar{\Omega}(K_1, m_i)$ and $\bar{\Omega}(K_2, m_i)$, respectively. With the SD ambiguity being fixed to m_i, the acceptance of the best DD ambiguity combination can be evaluated by the following statistic:

$$T = \frac{\Omega(m_i)}{\bar{\Omega}(K_1, m_i)} \tag{22}$$

with $\bar{\Omega}(K_1, m_i) = \Omega(m_i) + R(m_i)$ and $R(m_i) = [K_1 - \hat{x}_k(m_i)]^T \; Q_{\hat{x}_k}^{-1}[K_1 - \hat{x}_k(m_i)]$. If the measurement errors are assumed to be normally distributed, the quadratic forms $\Omega(m_i)$ and $R(m_i)$ are independent (Koch 1988, p. 301) and, furthermore, each of them has a chi-square distribution, which is a particular form of the gamma distribution (Johnson and Kotz 1970, p. 167). Consequently, the statistic T has a *beta* distribution (e.g. Koch 1988, p. 133). For the discrimination test, a classic statistic is defined by (e.g. Frei and Beutler 1990)

$$F = \frac{\bar{\Omega}(K_2, m_i)}{\bar{\Omega}(K_1, m_i)} \tag{23}$$

A disadvantage of this statistic F is that its distribution is, if not impossible, very difficult to identify. A more rigorous statistic for ambiguity discrimination testing, called W ratio, can be defined by (Wang et al. 1998a)

$$W = \frac{d}{\hat{s}_1(m_i)\sqrt{Q_d}} \tag{24}$$

with

$$d = \bar{\Omega}(K_2, m_i) - \bar{\Omega}(K_1, m_i)$$

$$Q_d = 4 \cdot (K_1 - K_2)^T Q_{\hat{x}_k}^{-1}(K_1 - K_2)$$

and

$$\hat{s}_1(m_i) = \sqrt{\frac{\bar{\Omega}(K_1, m_i) - \omega_1}{s \cdot (n-1) - 4}} \tag{25}$$

where $\omega_1 = (Q_d - 4d)^2/16Q_d$. The statistic W has a student's t distribution (Wang et al. 1998a).

The above acceptance and discrimination test statistics are, as expected, dependent on the fixed SD ambiguity value. The closer the SD ambiguity is to its correct value, the larger the T statistic. Similar to the case of the DD ambiguity search, therefore, the SD ambiguity values that result in the maximum value of the T statistic are considered as the most likely (*best*) SD ambiguity values, denoted as m_1. With the SD ambiguity fixed to its most likely value, the correct DD ambiguities should be more easily recovered.

When the DD ambiguities are fixed to their correct values, there are two methods to deal with the SD ambiguity parameter in the final baseline solutions, namely, *method A*, fixing the SD ambiguity to its most likely value, or *method B*, treating the SD ambiguity as a real-valued parameter. In the case of using *method B*, the DD ambiguities may be considered as constant parameters in Eq. (8), and then one can obtain the following measurement equation:

$$\tilde{l}(K_1) = Ax_c + fx_m + e \tag{26}$$

where $\tilde{l}(K_1) = l - BK_1$ with K_1 being the validated best DD ambiguity combination. Based on the mathematical and stochastic models defined in Eqs. (26) and (12), the SD ambiguity-float solution can be obtained. The treatment of the SD ambiguity in the final baseline solutions will be further discussed in Sect. 4.4 using a real data set.

3.3 *A practical procedure for resolving GLONASS ambiguities*

An important feature of this approach to GLONASS ambiguity resolution is that the SD ambiguity value is involved. In order to start the process of ambiguity resolution, the initial SD ambiguity value must be determined. With the help of Eq. (6), an approximate SD ambiguity value m_0 can be estimated from pseudoranges. By assuming that possible biases in SD pseudoranges are less than 20 m, a search window for the correct SD ambiguity is then constructed as $(m_0 - 100, m_0 + 100)$.

On the other hand, it is easy to see from Eq. (10) that before the correct SD ambiguity value is identified, systematic model errors caused by the approximate SD ambiguity value will always be present. Denoting ∇m as the integer error in the fixed SD ambiguity value, the systematic error in the DD carrier phase for satellite pair p and q can be derived as

$$\nabla l^{pq} = (\lambda_p - \lambda_q) \cdot \nabla m \tag{27}$$

which indicates that the size of the systematic error is reference-satellite dependent. However, numerical experiments show that choosing different reference satellites gives almost identical results. The reason for this is that the GLONASS satellite frequencies are very close. In order to explain this point clearly, the matrix P_i in Eq. (11) is approximated as

$$P_i^{-1} \cong \lambda_0^2 \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 2 \end{bmatrix} = \lambda_0^2 DD^T \tag{28}$$

where λ_0 may be one of the wavelengths $\lambda_1, \lambda_2, \dots, \lambda_n$. The errors made in the elements of the above matrix are always less than 0.8%. With Eq. (28), the influence imposed by the SD ambiguity error ∇m on the ambiguity-fixed solution is derived as

$$\begin{aligned} \nabla \hat{x}_c &= (A^T PA)^{-1} A^T P \bar{f} \cdot \nabla m \\ &= \left(\sum_{i=1}^{s} A_i^T D^T P_i D A_i \right)^{-1} \cdot \sum_{i=1}^{s} A_i^T D^T P_i D f_m \cdot \nabla m \\ &= \left(\sum_{i=1}^{s} A_i^T D^T (DD^T)^{-1} D A_i \right)^{-1} \\ &\quad \cdot \sum_{i=1}^{s} A_i^T D^T (DD^T)^{-1} D f_m \cdot \nabla m \end{aligned} \tag{29}$$

in which $D^T(DD^T)^{-1}D = E_n - \frac{1}{n} e_n e_n^T$ is an orthogonal projector matrix. It is apparent that this matrix is independent of the structure of the DD matrix D and thus the choice of the reference satellite (Teunissen 1997). Although using different reference satellites does produce similar results, one may argue that for a good approximation of the SD ambiguity value, it is still better to choose the highest satellite as the reference satellite. Actually, if the search window is large enough to contain the correct SD ambiguity, the ambiguity resolution and position solutions before ambiguity resolution are independent of the approximate SD ambiguity values.

Based on the above analysis, a procedure for GLONASS ambiguity resolution is given as follows:

1. Choose a reference satellite.
2. Compute an approximate SD ambiguity for the reference satellite.
3. Set up a search window for the SD ambiguity.
4. Identify the most likely SD ambiguity using the statistic T.
5. Validate the most likely (best) DD ambiguity combination using a testing procedure.

It should be noted that although the above discussion focuses on the processing of GLONASS data only, the basic equations and procedure are also valid when combing GLONASS and GPS carrier phases. For the combined GPS and GLONASS data processing, it is proposed that the GPS–GPS and GLONASS–GLONASS DD carrier phases are formulated (e.g. Wang 1998a, b). The reason for this is that the GPS–GLONASS DD ambiguities may be sensitive to the biases caused by incompatibilities between the GPS and GLONASS systems. It has also been commented in the literature (e.g. Walsh and Daly 1998; Zarraoa et al. 1998) that the GLONASS and GPS carrier phases may have various noise levels, which should be taken into accounted in the stochastic model. For a realistic stochastic model for the carrier phases, a rigorous statistical method should be used (e.g. Wang et al. 1998b), but this will not be discussed further here.

4 Experiments

Three experiment data sets were collected in Perth, Australia, using two Ashtech GG24 GPS/GLONASS receivers. All the data sets are free of cycle slips. The details of the data sets are presented in Table 1. The zero

Table 1. Details of the experiment data sets

Baseline names	Zero baseline	1.2-km baseline	0.3-km baseline
Baseline length (m)	0	1216	285
Cut-off angle (degrees)	15	15	15
GLONASS satellites	5	4	4
GPS satellites	0	5	6
Data interval (s)	10	10	10
Data span (min)	5	5	10
Survey date	22 July 1997	16 February 1998	9 February 1998

baseline and 1.2-km baseline data sets are analysed in detail to show the performance of the proposed approach to GLONASS ambiguity resolution, whereas the 0.3-km baseline data set (with a longer data span) is processed in a batch mode to compare two different methods of dealing with the SD ambiguity in final baseline solutions.

4.1 Ambiguity search and acceptance test results

Following the procedure proposed above, the SD and DD ambiguity search was performed for the zero and 1.2-km baseline data sets. The DD ambiguity search was conducted using the LAMBDA method (Teunissen 1993). The resulting statistics for ambiguity validation tests are presented in Figs. 1–6. It is shown in Fig. 1 that, in the case of the zero baseline, the most likely SD ambiguity is -6, with the maximum value of statistic T being 0.964. If the confidence level is chosen as 99%, the critical value for statistic T is set up to 0.384. Therefore, the best DD ambiguity combination can pass the acceptance test. Figure 2 indicates that in the case of the 1.2-km baseline, the peak value of the statistic T reaches 0.620, with which the most likely SD ambiguity is identified as 0. With the same confidence level, the critical value for statistic T is 0.411, and thus, the best DD ambiguity combination can also be statistically accepted. Both Figs. 1 and 2 show, as expected, that with the fixed SD ambiguity value approaching its correct value, the value of statistic T increases.

4.2 DD ambiguity discrimination test results

The DD ambiguity discrimination test statistics F and W are shown in Figs. 3–6, indicating, as expected, similar trends as in Figs. 1 and 2 for the statistic T. Overall, the closer the SD ambiguity to the most likely value, the bigger the values of the discrimination test statistics F and W. With the SD ambiguity fixed to its most likely value, the values of both statistics, F and W, for both data sets are larger than 2.0. This indicates that the two best DD ambiguity combinations can be distinguished very well. With the statistic W, the confidence level of

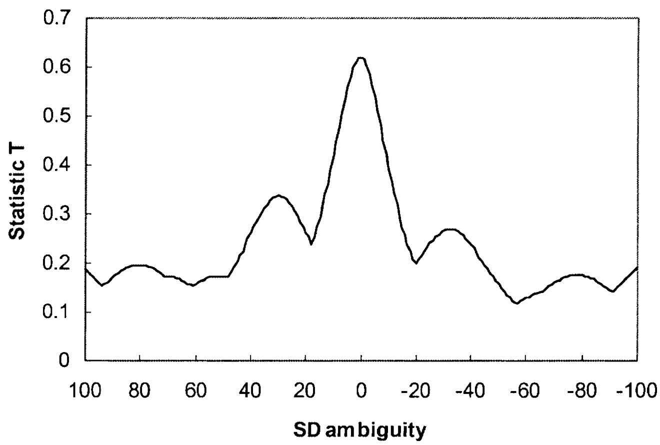

Fig. 2. Statistic T for the acceptance test of the best DD ambiguity set (1.2-km baseline)

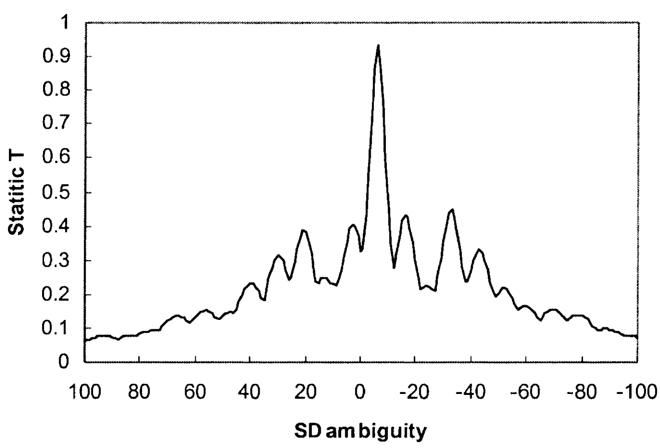

Fig. 1. Statistic T for the acceptance test of the best DD ambiguity set (zero baseline)

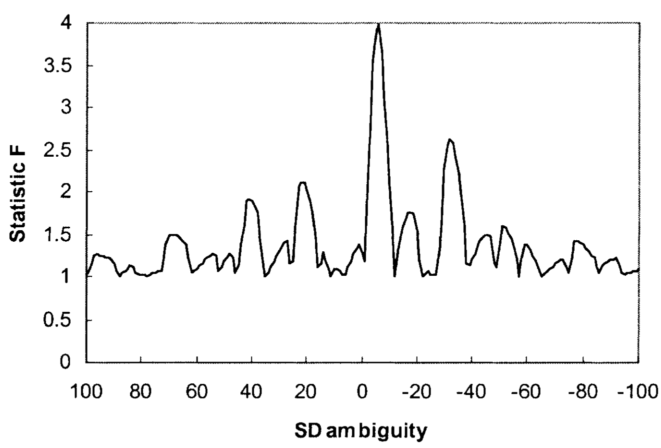

Fig. 3. Statistic F for the discrimination test between the best and second-best sets of DD ambiguity (zero baseline)

the DD ambiguity discrimination test in each data set is extremely close to 100%.

From Figs. 1 and 2, it is easy to see that a range of possible values for the SD ambiguity can pass the ambiguity acceptance test. This situation is somewhat similar to that of DD ambiguity resolution. However, the important thing here is that any accepted incorrect (or approximate) SD ambiguity values must cause *systematic* model errors and may unfortunately result in incorrect DD ambiguity resolution. This was the case for the zero baseline. For example, when the SD ambiguity was fixed to an incorrect value of -33, where the correct SD ambiguity value was -6 cycles (a bias of -27 cycles), an incorrect DD ambiguity combination was subsequently identified as the best DD ambiguity combination, and this incorrect ambiguity combination also passed both acceptance and discrimination tests, in which $T = 0.45$, $f = 2.62$ and $W = 6.26$ (with a confidence level close to 100%). For reliable DD ambiguity resolution, therefore, it is critical to fix the SD ambiguity to the most likely value.

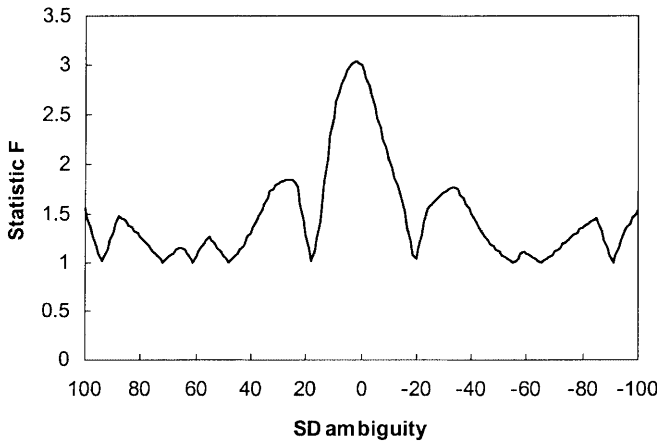

Fig. 4. Statistic F for the discrimination test between the best and second-best sets of DD ambiguity (1.2-km baseline)

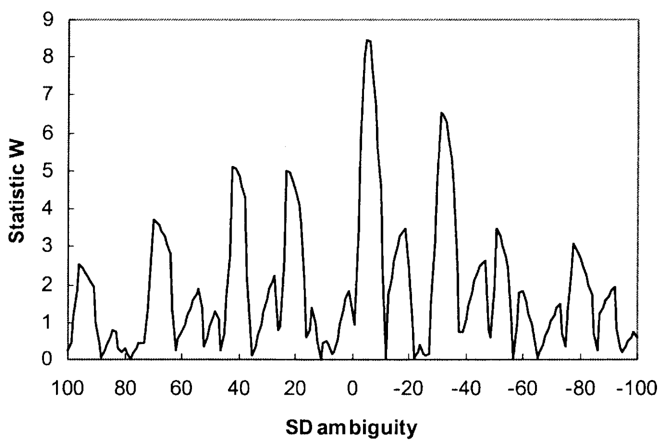

Fig. 5. Statistic W for the discrimination test between the best and second-best sets of DD ambiguity (zero baseline)

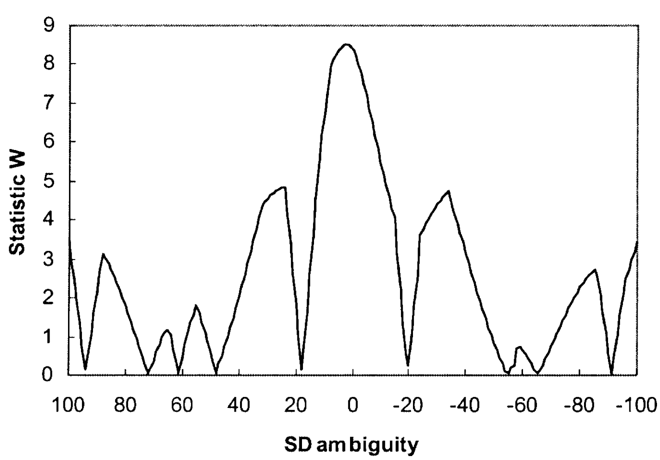

Fig. 6. Statistic W for the discrimination test between the best and second-best sets of DD ambiguity (1.2-km baseline)

4.3 Baseline errors caused by incorrect SD ambiguity values

In order to evaluate the effects of incorrect (or approximate) SD ambiguities on the final baseline solutions, the DD ambiguities were first fixed to their correct values (verified with the known baseline length). Then, with the SD ambiguity being fixed to all possible values in the search space, the differences between the solved baseline lengths and the known value were produced for each data set, and are shown in Figs. 7 and 8.

Figures 7 and 8 demonstrate that, as expected, the best baseline solution is achieved when the SD ambiguity is fixed to the most likely value. In the case of the 1.2-km baseline (GPS + GLONASS data), as shown in Fig. 8, an error of 5 cycles in the SD ambiguity leads to a baseline change of about 1.0 mm (the standard deviation of the baseline length is 2.3 mm). In some cases, however, small errors in the fixed SD ambiguity values may result in large baseline errors. For example, in the case of the zero baseline (GLONASS data only), as shown in Fig. 7, an error of 5 cycles in the SD ambiguity

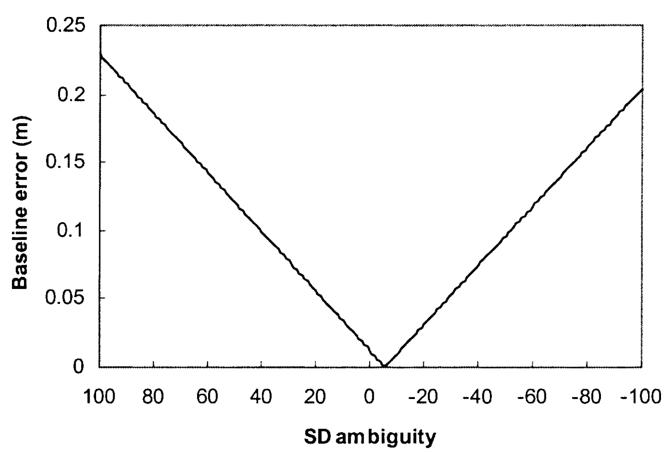

Fig. 7. Baseline errors vs SD ambiguity (zero baseline)

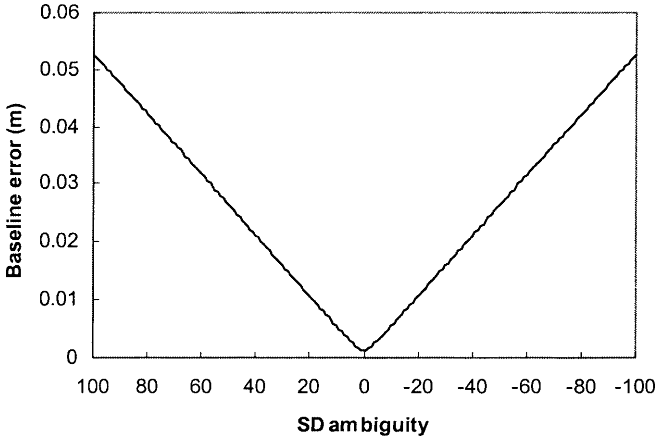

Fig. 8. Baseline errors vs SD ambiguity (1.2-km baseline) (the zero line is given by the ground truth value of the baseline length

value can bias the baseline length by as much as 10.0 mm (the standard deviation of the baseline length is 0.8 mm). This again indicates that the approximate SD ambiguity value computed by pseudo-ranges may not be accurate enough for precise baseline solutions.

4.4 Treatment of the SD ambiguity in final baseline solutions

In order to compare the two methods of treating the SD ambiguity parameter in final baseline solutions, the 0.3-km baseline data set was processed in a batch mode with segments of 2, 5 and 10 minutes, respectively. A total of eight solutions were obtained, and the results are listed in Table 2.

In Table 2, we see that methods A and B produce almost identical baseline components. We should also note that the standard deviations of the baseline components for methods A and B are also very close (almost identical). One possible reason for this, as mentioned above, is that the coefficients of the SD ambiguity parameter in Eq. (26) are much smaller than those of other parameters. This leads to a geometry which is not sensitive to the SD ambiguity parameter. However, in order to check the SD ambiguity search process, it is wise to treat the SD ambiguity as a real-valued parameter in the final baseline solution. The integer nearest to this estimated SD ambiguity parameter should be the same as the most likely SD ambiguity value produced by the SD ambiguity research process.

From Table 2, it can be seen that the most likely (best) SD ambiguity value varies slightly for different segments in the same data set. A possible reason for this is that some systematic errors in carrier-phase measurements change slightly over time, influencing the selection of the SD ambiguity value. It should be noted, however, that the most likely SD ambiguity value is statistically the best one fitting the measurements, and thus this value might reduce some of the systematic errors existing in carrier-phase measurements. When processing the GLONASS carrier phases, therefore, the use of the most likely SD ambiguity value will improve the reliability of the DD ambiguity resolution.

5 Concluding remarks

Although converting the GLONASS carrier phases to distances before forming the DD measurements can remove the receiver clock terms from the DD measurement equations, both SD and DD ambiguity parameters simultaneously appear in the DD measurement equations. This results, however, in a singularity in the normal matrix. In theory, without additional information, it is impossible to resolve the SD and DD ambiguities at the same time. This kind of information may be obtained from pseudo-ranges, which can be used to calculate an approximate SD ambiguity value. With the SD ambiguity fixed to this approximate value, the DD ambiguity resolution can be performed. The SD ambiguity value determined by the pseudo-ranges, however, may not be accurate enough for reliable DD ambiguity resolution, as pseudo-ranges may be significantly contaminated by multipath and hardware delays.

In this paper, the SD ambiguity value is determined using other information, which is mainly based on two basic facts, namely (1) that the possible range or space of the SD ambiguity is sufficiently known, and (2) that the closer the SD ambiguity parameter is to its correct value, the better the performance of the DD ambiguity resolution, or the larger the statistic T. By searching through all the possible integer SD ambiguity values, the best

Table 2. The results for the 0.3-km baseline data set

Batch no.	Data span (min)	Best SD ambiguity	DD ambiguity resolution statistics			Final solution: method A			Final solution: method B		
			T	F	W	Corrections[a] (mm)			Corrections[a] (mm)		
1	2	−12	0.87	12.9	16.1	5.1	4.2	4.1	5.2	4.2	4.1
2	2	−13	0.86	9.6	13.5	5.1	2.6	1.0	5.0	2.6	1.0
3	2	−14	0.61	8.8	12.8	9.1	7.6	0.1	9.0	7.6	0.1
4	2	−14	0.63	11.5	15.1	6.8	2.3	3.6	6.7	2.4	3.6
5	2	−13	0.43	4.6	8.3	7.1	5.8	2.1	7.1	5.8	2.1
6	5	−12	0.86	10.9	23.4	6.2	4.7	1.5	6.2	4.7	1.5
7	5	−14	0.65	20.9	33.6	7.1	4.3	2.7	7.0	4.4	2.7
8	10	−13	0.81	23.9	51.9	6.5	4.4	2.1	6.5	4.4	2.1

[a] Estimated corrections to the approximate baseline components

(most likely) SD ambiguity value can be identified, which is associated with the maximum value of the statistic T. When the SD ambiguity is fixed to its most likely value, the correct DD ambiguities can be more easily recovered. Therefore, the proposed GLONASS ambiguity resolution approach is actually based on a two-level search process, in which all the SD and DD ambiguity combinations are compared using the statistic T. A mathematical relationship between any two DD ambiguity-float solutions based on different SD ambiguity values has been established, which makes the two-level search process more efficient.

The initial experimental results indicate that the proposed approach is feasible for resolving the carrier-phase ambiguities in the cases of GLONASS and combined GPS/GLONASS positioning. However, it should be pointed out that, like any other new approaches, the proposed approach needs to be extensively tested under varying circumstances. This approach is based on models for short baselines. Its possible application for long baselines remains a topic for further research.

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Appendix

Proof of theorem 1. (DD ambiguity-float solutions with the SD ambiguity fixed to various values)

From Eq (15), we can obtain

$$Q_{\hat{x}_c} = G^{-1} + G^{-1}A^TPBQ_{\hat{x}_k}B^TPAG^{-1} \tag{A1}$$

$$Q_{\hat{x}_c}Q_{\hat{x}_k} = Q^T_{\hat{x}_c\hat{x}_k} = -G^{-1}A^TPBQ_{\hat{x}_k} \tag{A2}$$

$$\begin{aligned} Q_{\hat{x}_k} &= [B^TPB - B^TPAG^{-1}A^TPB]^{-1} \\ &= [s \cdot B_kP_iB_k - B_kP_iHG^{-1}H^TP_iB_k]^{-1} \\ &= [B_kP_i\Lambda P_iB_k]^{-1} \end{aligned} \tag{A3}$$

where

$$G = A^TPA \tag{A4}$$

$$H = \sum_{i=1}^{s} DA_i \tag{A5}$$

$$\Lambda = s \cdot P_i^{-1} - HG^{-1}H^T \tag{A6}$$

Case (a):

$$\begin{aligned} &\hat{x}_k(m_i) - \hat{x}_k(m_j) \\ &= Q_{\hat{x}_k\hat{x}_c}A^TP[\bar{l}(m_i) - \bar{l}(m_j)] + Q_{\hat{x}_k}B^TP[\bar{l}(m_i) - \bar{l}(m_j)] \\ &= Q_{\hat{x}_k}[B^TP - B^TPAG^{-1}A^TP]f \cdot (m_j - m_i) \\ &= Q_{\hat{x}_k}[B_kP_i\Lambda P_i]f_m \cdot (m_j - m_i) \\ &= B_k^{-1}P_i^{-1}\Lambda^{-1}P_i^{-1}B_k^{-1}B_kP_i\Lambda P_if_m \cdot (m_j - m_i) \\ &= B_k^{-1}f_m \cdot (m_j - m_i) \end{aligned} \tag{A7}$$

Case (b):

$$\begin{aligned} &\hat{x}_c(m_i) - \hat{x}_c(m_j) \\ &= Q_{\hat{x}_c}A^TP[\bar{l}(m_i) - \bar{l}(m_j)] + Q_{\hat{x}_c\hat{x}_k}B^TP[\bar{l}(m_i) - \bar{l}(m_j)] \\ &= G^{-1}A^TPf \cdot (m_j - m_i) - G^{-1}A^TPBQ_{\hat{x}_k} \\ &\quad \times [B^TP - B^TPAG^{-1}A^TP]f.(m_j - m_i) \\ &= G^{-1}H^TP_if_m \cdot (m_j - m_i) - G^{-1}H^TP_iB_kQ_{\hat{x}_k} \\ &\quad \times [B_kP_i\Lambda P_i]f_m \cdot (m_j - m_i) \\ &= G^{-1}H^TP_if_m \cdot (m_j - m_i) - G^{-1}H^TP_if_m \cdot (m_j - m_i) \\ &= 0 \end{aligned} \tag{A8}$$

Case (c):

$$\begin{aligned} &\hat{e}(m_i) - \hat{e}(m_j) \\ &= \bar{l}(m_i) - A\hat{x}_c(m_i) - B\hat{x}_k(m_i) - [\bar{l}(m_j) - A\hat{x}_c(m_j) - B\hat{x}_k(m_j)] \\ &= f \cdot (m_j - m_i) - B[\hat{x}_k(m_i) - \hat{x}_k(m_j)] \\ &= f \cdot (m_j - m_i) - B \cdot B_k^{-1}f_m \cdot (m_j - m_i) \\ &= f \cdot (m_j - m_i) - f \cdot (m_j - m_i) \\ &= 0 \end{aligned} \tag{A9}$$

Based on the result of (c), case (d) is obviously true. □

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