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A solution to EIV model with inequality constraints and its geodetic applications

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Abstract In the field of surveying, mapping and geodesy, there have been a number of works on the error-in-variable (EIV) model with constraints, where equality constraints are imposed on the parameter vector. However, in some cases, these constraints may be inequalities. The EIV model with inequality constraints has not been fully investigated. Therefore, this paper presents an inequality-constrained total least squares (ICTLS) solution for the EIV model with inequality constraints (denoted as ICEIV). Employing the proposed ICTLS method, the ICEIV problem is first converted into an equality-constrained problem by distinguishing the active constraints through exhaustive searching, and it is then resolved employing the method of equality-constrained total least squares (ECTLS). The applicability and feasibility of the proposed method is illustrated in two examples.

Keywords Errors-in-variables model · Inequality constraints · Active constraints · Exhaustive searching · Total least squares

1 Introduction

The classic least squares (LS) method assumes that the coefficient matrix of the Gauss–Markov model is absolutely error free. However, when taking the errors in both the observation vector and the coefficient matrix into account, which is referred to as the error-in-variable (EIV) model, the total least squares (TLS) method is applied. The EIV model is widely used in geodetic data applications, such as straight-line fitting and geodetic coordinate transformation. The reason for using the EIV model in geodetic data processing is that there are various types of random errors due to instrument errors, human errors and sampling errors in almost all measurements, and these errors contribute to the perturbation of the coefficient matrices of straight-line fitting and coordinate transformation models (Davis 1999; Felus 2004; Acar et al. 2006; Akyilmaz 2007; Schaffrin and Wieser 2008; Schaffrin and Felus 2008).

The TLS method was originally introduced by Adcock (1878) for univariate problems, in which errors in both dependent and independent variables are considered independent and the ratio of their variances is assumed to be 1. Kummell (1879) made the method more general with the assumption of an arbitrary variance ratio. Their ideas remained largely unnoticed until 1937, when their work was revived by Koopmans (1937) for linear regression analysis of economic time series, and the ideas were later further extended to clinical chemistry and related fields by Deming (1943). Since then, much effort has been made to develop TLS theory and algorithms (e.g., Gerhold 1969; Hawkins 1973; Gleser 1981; Van Huffel and Vandewalle 1988; Fierro et al. 1997; Schaffrin et al. 2006; Chang and Titley-Peloquin 2009).

In the field of geodesy, significant achievements have been made in applying the TLS method (Felus 2004; Acar et al. 2006; Schaffrin et al. 2006; Akyilmaz 2007; Schaffrin and Felus 2008; Schaffrin and Wieser 2008; Schaffrin and Felus 2009). Felus (2004) presented a TLS-based approach for the trend analysis of a spatial point process. Acar et al. (2006) proposed a generalized TLS method for analyzing geodetic deformation by calculating the relationship between two monitoring networks. Schaffrin et al. (2006) presented

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a TLS adjustment model for straight-line fitting and plane reconstruction from cloud points. Akyilmaz (2007) proposed a TLS approach for distinguishing three-dimensional point displacement in a landslide area using Global Positioning System (GPS) data. Schaffrin and Felus (2008) and Schaffrin and Wieser (2008) presented univariate and multivariate TLS solutions for resolving transformation parameters of affine and similarity models. The results showed the superiority of the TLS method over the traditional LS solution in coordinate transformation. Recently, Schaffrin and Felus (2009) resolved the issues of geodetic resection and surface reconstruction with both linear and quadratic constraints according to the principle of TLS.

In the case that there is a priori information on the parameters in a TLS model, some constraints should be accounted for in the model. In recent decades, much attention has been paid to the constrained TLS problem. Dowling et al. (1992) proposed a constrained subspace linear predictive frequency estimation technique. Using the proposed approach, a numerically stable closed-form expression was derived for the solution of the TLS problem with linear equality constraints. Felus and Schaffrin (2005) and Schaffrin (2006) presented the equality-constrained TLS (ECTLS) method for the EIV model with linear equality constraints (ECEIV). The results showed that the proposed method can be used to easily investigate the statistical properties of the estimated value. Recently, Tong et al. (2011) presented an improved constrained weighted TLS method for resolving the EIV model with constraints by integrating both observation and constraint equations under the assumption that the observation vector and coefficient matrix in the observation equations and the right-hand-side (RHS) vector and the constraining matrix in the constraint equations contain errors.

Some inequality constraints may have to be adhered to when there is a priori information on the parameters that should to be taken into account in a geodetic problem. For example, the fitted slope or intercept of a straight line should be within a bound, the estimated variance component should be positive, deformation is in the downstream direction when a dam is subject to water pressure in a flood period, and the height of a GPS antenna is within a limited range when navigating on the surface of water. Several works on the use of inequality-constrained least squares (ICLS) in geodetic applications have been reported. Zhu and Santerre (2002) presented a method for improving GPS ambiguity resolution using a priori height information as inequality constraints in the adjustment model. Peng et al. (2006) proposed an ICLS method for geodetic problems by converting many inequality constraints into one equality constraint. To our knowledge, there is no significant literature on the EIV model with inequality constraints (ICEIV). Thus, this paper might be the first to present a solution to the problem of the EIV model with inequality constraints by converting the ICEIV model into the ECEIV model. In this study, we concentrate on the ICEIV problem, which can be expressed as

$$\begin{cases} \boldsymbol{y} - \boldsymbol{e} = (\boldsymbol{A} - \boldsymbol{E})\boldsymbol{x} \\ \boldsymbol{G}\boldsymbol{x} \le \boldsymbol{w} \end{cases} \tag{1}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of observations, $\boldsymbol{e}$ is an $n \times 1$ vector of observational error (i.e., $\boldsymbol{e} \sim N(0, \sigma_0^2 \boldsymbol{I}_n)$), and σ_0^2 is the variance component. $\boldsymbol{A}$ is an $n \times m$ coefficient matrix of input variables with full column rank (i.e., $rk(\boldsymbol{A}) = m < n$); $\boldsymbol{E}$ is the corresponding $n \times m$ random error matrix of $\boldsymbol{A}$, where the vector of perturbation has zero mean and has a covariance matrix that is equal to the identity up to a scaling factor, namely the jth column of $\boldsymbol{E}$ $(\boldsymbol{E}_{,j}) \sim N(0, \sigma_0^2 \boldsymbol{I}_m)$, with all errors having the same weight; $\boldsymbol{x}$ is an $m \times 1$ deterministic vector of unknowns to be estimated; $\boldsymbol{G}$ is a $k \times m$ matrix of fixed coefficients; and $\boldsymbol{w}$ is a k-dimensional constant vector.

The rest of this paper is organized as follows: Following the introduction, an inequality-constrained total least squares (ICTLS) solution to the ICEIV problem is proposed in Sect. 2. To convert the ICEIV model into the ECEIV model, an exhaustive search (ES) method is presented to distinguish active inequality constraints from inactive inequality constraints. Section 3 illustrates the implementation of the proposed ICTLS method through two examples. The proposed method is compared with traditional methods, and the feasibility of the proposed ICTLS method is demonstrated. Finally, conclusions are presented in Sect. 4.

2 Resolving the ICEIV model

In this section, we put forward an approach to resolve the ICEIV problem by converting the ICEIV model into the ECEIV model, which can be solved employing the ECTLS method. It should be noted that this contribution considers only linear inequalities. Generally, linear inequality constraints in (1) can be divided into two groups at the optimum solution: active and inactive constraints (Ueno et al. 2000). With respect to (1), the active constraints refer to the set of constraints that hold with equality at the optimum solution, and the inactive constraints are those for which a strict inequality holds at the optimum solution (Nocedal and Wright 2006). Therefore, the inequality constraints in (1) can be written as

$$\begin{cases} \boldsymbol{G}_1\hat{\boldsymbol{x}} = \boldsymbol{w}_1 \\ \boldsymbol{G}_2\hat{\boldsymbol{x}} < \boldsymbol{w}_2 \end{cases} \tag{2}$$

where $\mathbf{G}_1\hat{\mathbf{x}} = \mathbf{w}_1$ is the equivalent of active constraint group $\boldsymbol{G}_1\hat{\boldsymbol{x}} \le \boldsymbol{w}_1$, and $\mathbf{G}_2\hat{\mathbf{x}} < \mathbf{w}_2$ is the inactive constraint group.

The primarily idea of the proposed ICTLS solution to resolve the ICEIV problem is (1) to distinguish all active constraints in (2) using an exhaustive searching approach, (2) to convert the ICEIV model into an ECEIV model that includes only active constraints, and (3) to resolve the ECEIV model employing the ECTLS approach.

2.1 ECTLS approach

Assuming that $\boldsymbol{G}_1$ is an active constraining matrix, the ICEIV model presented in (1) can be converted into an ECEIV model:

$$\begin{cases} \boldsymbol{y} - \boldsymbol{e} = (\boldsymbol{A} - \boldsymbol{E})\boldsymbol{x} \\ \boldsymbol{G}_1\boldsymbol{x} = \boldsymbol{w}_1 \end{cases} \tag{3}$$

where $\boldsymbol{G}_1$ is an $r \times m$ matrix, r is the number of active constraints, and $\boldsymbol{w}_1$ is the corresponding $r \times 1$ RHS vector of the constraint equations. The meanings of the other symbols are the same as in (1).

Therefore, the model presented in (3) can be resolved by taking the ECTLS approach proposed by Schaffrin (2006). It is an Euler–Lagrange approach, and the target function is written as

$$\begin{aligned} \Phi &= \boldsymbol{e}^T\boldsymbol{e} + vec\,(\boldsymbol{E})^T\,vec\,(\boldsymbol{E}) + 2\boldsymbol{\lambda}^T\,(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x} + \boldsymbol{E}\boldsymbol{x} - \boldsymbol{e}) \\ &\quad -2\boldsymbol{\mu}^T(\boldsymbol{w}_1 - \boldsymbol{G}_1\boldsymbol{x}) \end{aligned} \tag{4}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the $n \times 1$and $r \times 1$ vectors of Lagrange multipliers respectively, and the "*vec*" operator stacks one column of a matrix under another, moving from left to right.

Letting $\boldsymbol{c} := \boldsymbol{A}^T\boldsymbol{y}, \boldsymbol{N} := \boldsymbol{A}^T\boldsymbol{A}, \bar{\boldsymbol{\mu}} := \hat{\boldsymbol{\mu}}(1+\boldsymbol{x}^T\boldsymbol{x})$ and $\hat{\varphi} := \left(\boldsymbol{y} - \boldsymbol{A}\,\hat{\boldsymbol{x}}\right)^T\left(\boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}\right)(1+\hat{\boldsymbol{x}}^T\hat{\boldsymbol{x}})^{-1}$, then the Euler–Lagrange necessary condition is achieved, and the nonlinear normal equations is derived as

$$\begin{bmatrix} \boldsymbol{N} & \boldsymbol{c} & \boldsymbol{G}_1^T \\ \boldsymbol{c}^T & \boldsymbol{y}^T\boldsymbol{y} & \boldsymbol{w}_1^T \\ \boldsymbol{G}_1 & \boldsymbol{w}_1 & \hat{\varphi}\boldsymbol{I}_r \end{bmatrix} \cdot \begin{bmatrix} \hat{\boldsymbol{x}} \\ -1 \\ \bar{\boldsymbol{\mu}} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{x}} \\ -1 \\ \bar{\boldsymbol{\mu}} \end{bmatrix} .\hat{\varphi} \tag{5}$$

The ECTLS algorithm is detailed as follows (Schaffrin 2006):

1. Calculate initial values of $\hat{\boldsymbol{x}}$ and $\bar{\boldsymbol{\mu}}$ according to

$$\begin{bmatrix} \hat{\boldsymbol{x}}^{(1)} \\ \bar{\boldsymbol{\mu}}^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{N} & \boldsymbol{G}_1^T \\ \boldsymbol{G}_1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{w}_1 \end{bmatrix} \tag{6a}$$

2. Calculate $\hat{\boldsymbol{\mu}}$ and new $\hat{\boldsymbol{x}}$ and $\bar{\boldsymbol{\mu}}$ according to

$$\hat{\boldsymbol{\mu}}^{(i)} := \bar{\boldsymbol{\mu}}^{(i)}(1 + \hat{\boldsymbol{x}}^{(i)^T}\hat{\boldsymbol{x}}^{(i)})^{-1} \tag{6b}$$

$$\hat{\varphi}^{(i)} := (\boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}^{(i)})^T(\boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}^{(i)})(1 + \hat{\boldsymbol{x}}^{(i)^T}\hat{\boldsymbol{x}}^{(i)})^{-1} \tag{6c}$$

$$\begin{bmatrix} \hat{\boldsymbol{x}}^{(i+1)} \\ \bar{\boldsymbol{\mu}}^{(i)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{N} & \boldsymbol{G}_1^T \\ \boldsymbol{G}_1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{c} + \hat{\boldsymbol{x}}^{(i)}\hat{\varphi}^{(i)} \\ \boldsymbol{w}_1 \end{bmatrix} \tag{6d}$$

3. Repeat Step 2 until $\|\hat{\boldsymbol{x}}^{(j)} - \hat{\boldsymbol{x}}^{(j-1)}\| < \boldsymbol{\varepsilon}$ (for a given $\boldsymbol{\varepsilon}$) The variance component σ_0^2 can be estimated from the value of $\hat{\varphi}$

$$\hat{\sigma}_0^2 = \hat{\varphi}/(n - m + r) \tag{6e}$$

where n is the number of observations, m is the number of unknown parameters, and r is the number of equality constraints.
The first-order variance–covariance matrix of $\hat{\boldsymbol{x}}$ is (Schaffrin 2006)

$$D(\hat{\boldsymbol{x}}) \approx \hat{\sigma}_0^2\left(\boldsymbol{N} - \hat{\varphi}\boldsymbol{I}_m\right)^{-1}\boldsymbol{N}(\boldsymbol{N} - \hat{\varphi}\boldsymbol{I}_m)^{-1} \tag{6f}$$

2.2 Distinguishing the active constraints

As stated above, the solution to (1) equals that with the objective function only having active constraints, which is presented in (3). Therefore, if active inequality constraints in (1) are distinguished, they can be treated as equality constraints.

With respect to the inequality-constrained LS problem, there are generally two main classes of algorithms: the active set method (Gill et al. 1981) and interior point method (Baldick 2006). In this section, considering that ICTLS method is more complicated than the ICLS problem, we introduce an exhaustive search (ES) method to distinguish active constraints.

The principal idea of the ES method is described as follows: We first select a number of constraints, and by assuming that these selected constraints are active, an ECEIV model is constructed accordingly. If these selected constraints are distinguished to be active, then the ECTLS estimate will satisfy all the constraints. However, when the selected number of inequality constraints are $j(0 < j \leq m)$, we need to try all possible combinations of the selected inequality constraints to find feasible solutions. The main steps of finding the optimal solution are detailed as follows:

1. Let $kk = 0$, which means that the active constraint combination set (S) is empty, and the inequality-constraint combination set (C) includes all possible combinations of the inequality constraints(the number of combinations is $N(N = \sum_{t=1}^{n} C_k^t$, k is the number of constraints). Let $i = 1$.
2. If $i > N$ (which means that all combinations have been checked), go to Step 5. Otherwise, select the ith combination from C.
3. Assume that the selected combination is a combination of active constraints; since they are now equivalent to equality constraints, add them to the EIV model. Thus, we obtain the estimation of $\boldsymbol{x}$ by using ECTLS as discussed in Sect. 2.1.
4. $i = i + 1$, check whether the estimation satisfies all constraints. If it does, $kk = kk + 1$ and add the chosen set

Table 1 Measured coordinates of 16 points

Point ID	1	2	3	4	5	6	7	8	9
x	−2.17	−1.72	−1.12	−1.03	−1.02	−0.85	−0.63	−0.79	−0.12
y	−0.02	−0.10	1.09	0.55	0.52	0.57	0.76	−0.56	0.76
Point ID	10	11	12	13	14	15	16	17	
x	−0.10	0.44	0.52	0.75	1.01	1.17	1.22	1.83	
y	−0.79	0.86	0.53	0.13	−0.85	−1.56	−0.62	−2.35	

Table 2 Four scenarios of inequality constraints on the intercept and slope of the fitted straight line

Scenario	I	II	III	IV
Constraints	$-1 \le a \le 0$	$-0.3 \le b \le 0$	$0 \le a \le 0.1$	$0 \le a \le 0.1$, $-0.3 \le b \le 0$
Number of active constraints (r')	0	1	1	2

Table 3 Distinguished active constraints and estimated parameters of the fitted straight line in the four scenarios

Scenario	I	II	III	IV
Distinguished active constraints	None	$b \ge -0.3$	$a \ge 0$	$b \ge -0.3$, $a \ge 0$
Number of the distinguished active constraints (r)	0	1	1	2
Slope (b)	−0.726	−0.300	−0.698	−0.300
Intercept (a)	−0.302	−0.240	0.000	0.000

of active inequality constraints to S, then go to Step 2. If not, goto Step 2.

5. If $kk == 0$, which means that all constraints are inactive, then the TLS estimation of $\boldsymbol{x}$ is exactly what we need, get the TLS estimation as in Golub and VanLoan (1980). If $kk == 1$, namely S includes only one element which is the only feasible solution to problem (1), then this element is the ICTLS solution we are searching for. If $kk > 1$, we search for a solution with the minimum variance component within S.

3 Case study

In this section, two examples are taken to test the proposed ICTLS solution to the ICEIV model. The first example is a simulated straight-line fitting problem under the assumption of knowing in advance the activeness of each constraint. The purpose of this example is to check if the ES method can distinguish the active constraints. The second example is the analysis of the dataset of Peng et al. (2006) using the ICEIV model. In this example, the number of the inequality constraints is 11, and the target of this example is to examine whether the ES method is feasible when the number of inequality constraints increases to 11.

3.1 Straight-line fitting

In this example, a total of 16 pairs of coordinates were measured (Table 1).

From these measured coordinates, the intercept (a) and slope (b) of the fitted straight line can be estimated using TLS without constraints as −0.302 and −0.726, respectively. From the coefficients of the fitted straight line estimated using the TLS method, we designed four scenarios of inequality constraints on the intercept and slope in the same straight-line fitting problem. In addition, the activeness of each constraint was known in advance. Table 2 shows the designed four scenarios of inequality constraints on the intercept and slope of the fitted straight line. From the table, we see that, in scenario I, the constraint is $-1 \le a \le 0$, and the estimation of the intercept based on TLS satisfies the interval constraints. As a result, the designed number of active constraints (r') should be zero. In scenario II, the added constraints is $-0.3 \le b \le 0$, in which $b \le 0$ is inactive while $b \ge -0.3$ is active. Thus, r' should be one. Scenario III was designed similarly.

Table 3 presents the result for the fitted straight line in the four scenarios using our proposed ICTLS method. The table gives the distinguished active constraints, the number of distinguished active constraints, and the estimated parameters of the fitted straight line.

Table 3 shows that both the number of distinguished active constraints and the corresponding active constraints obtained using our proposed ICTLS method are consistent with the designed values. In scenario I, there are no distinguished active constraints, and the result of the estimated coefficients of the fitted straight line is the same as that using the TLS solution without any constraint. In scenario II, there is one distinguished active constraint, which is the designed number of active constraints. The distinguished active constraint

Table 4 Data from Peng et al. (2006)

A				***y***
0.9501	0.7620	0.6153	0.4057	0.0578
0.2311	0.4564	0.7919	0.9354	0.3528
0.6068	0.0185	0.9218	0.9169	0.8131
0.4859	0.8214	0.7382	0.4102	0.0098
0.8912	0.4447	0.1762	0.8936	0.1388
G				***w***
0.2027	0.2721	0.7467	0.4659	0.5251
0.1987	0.1988	0.4450	0.4186	0.2026
0.6037	0.0152	0.9318	0.8462	0.6721
$-0.1 \leq x_i \leq 2.0, \quad i = 1, 2, 3, 4$				

is $b \geq -0.3$, which is consistent with our expectation, and the estimate of the unknown *slope* (b) satisfies the inequality constraints $-0.3 \leq b \leq 0$. Similar results are obtained in scenario III. In scenario IV, we cannot be sure from only the TLS estimation and inequality constraints that there are two active constraints in advance. The reason is that the activeness of the constraints on *slope* (b) may affect the activeness of the constraints on *intercept* (a) or vice versa. However, we can refer to the ICTLS estimation for scenarios II and III in Table 3, and we know that there are two active constraints. Therefore, in scenario IV, the number of the distinguished active constraints equals the designed number, the distinguished active constraint is consistent with our expectation, and the estimates of both *intercept* (a) and *slope* (b) satisfy the inequality constraints. Therefore, it is concluded that in the straight-line fitting example, the ES method distinguishes correctly the active constraints and the ICTLS estimates can satisfy all constraints.

3.2 Example of data from Peng et al. (2006)

In this example, we used the dataset from Peng et al. (2006), presented as Table 4. In the table, $\boldsymbol{A}$ is the elements of the coefficient matrix, and $\boldsymbol{y}$ is the elements of the observation vector. At the same time, there are three inequality constraints as shown in matrix $\boldsymbol{G}$ and $\boldsymbol{w}$, and eight constraints on the domain of x_i.

Peng et al. (2006) used the ICLS method to estimate the unknown variable:

$$\begin{cases} \boldsymbol{y} - \boldsymbol{e} = \boldsymbol{A}\boldsymbol{x} \\ \boldsymbol{G}\boldsymbol{x} \leq \boldsymbol{w} \end{cases} \tag{9}$$

However, our goal is to determine the unknown variable $\boldsymbol{x}$ using the proposed ICTLS method (10) for model (1).

Table 5 Proposed ICTLS estimation and the ICLS estimation

Parameters	$\hat{\boldsymbol{x}}_{\text{ICTLS}}$	$\hat{\boldsymbol{x}}_{\text{ICLS1}}$	$\hat{\boldsymbol{x}}_{\text{ICLS2}}$[a]	$\hat{\boldsymbol{x}}_{\text{ICLS3}}$[a]
x_1	−0.1000	−0.1000	−0.1000	−0.1000
x_2	−0.1000	−0.1000	−0.1000	−0.1000
x_3	0.1685	0.2152	0.2152	0.2137
x_4	0.3998	0.3502	0.3502	0.3518

[a] Solution reported by Peng et al. (2006)

In Table 4, there are 11 inequality constraints (k), and the constraint equations for $-0.1 \leq \boldsymbol{x}_1 \leq 2.0$ can be written as

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \boldsymbol{x} \leq \begin{pmatrix} 2.0 \\ -0.1 \end{pmatrix} \tag{10}$$

Similar equations can also be written for x_2, x_3 and x_4.

Employing the ES method, three inequality constraints were distinguished as active constraints:

$$\boldsymbol{G}_1 = \begin{bmatrix} 0.1987 & 0.1988 & 0.4450 & 0.4186 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

The ICTLS estimation, denotes as $\hat{\boldsymbol{x}}_{\text{ICTLS}}$ is given in Table 5. Substituting $\hat{\boldsymbol{x}}_{\text{ICTLS}}$ into $\boldsymbol{G}\hat{\boldsymbol{x}} - \boldsymbol{w}$, we obtained the vector $(-0.2606\ 0\ -0.2386\ -2.1000\ 0\ -2.1000\ 0\ -1.8314\ -0.2685\ -1.6002\ -0.3998)^T$. The inequality domain constraints are obviously satisfied. Table 5 compares the proposed ICTLS estimation and the ICLS estimation. In the table, $\hat{\boldsymbol{x}}_{\text{ICLS1}}$ is the results obtained using the Lemke algorithm (Murty 1988) according to the principle of LS, and the Lemke algorithm distinguished the same three active constraints that were distinguished using the ICTLS. Additionally, the ICLS estimations of Peng et al. (2006) employing a simplex algorithm and aggregate function method are listed as $\hat{\boldsymbol{x}}_{\text{ICLS2}}$ and $\hat{\boldsymbol{x}}_{\text{ICLS3}}$, respectively.

Table 5 shows that the estimations of x_1 and x_2 obtained using the four methods (ICTLS, ICLS1, ICLS2 and ICLS3) are the same. This could be due to the active domain constraints on the two parameters. The table also shows that the eight constraints on the domain are all satisfied, and the three ICLS estimations are very close or even identical (see $\hat{\boldsymbol{x}}_{\text{ICLS1}}$ and $\hat{\boldsymbol{x}}_{\text{ICLS2}}$in Table 5). Finally, it is seen that the differences between ICTLS and ICLS estimations for x_3 and x_4 are notable. When substituting $\hat{\boldsymbol{x}}_{\text{ICLS1}}$,$\hat{\boldsymbol{x}}_{\text{ICLS2}}$ and $\hat{\boldsymbol{x}}_{\text{ICLS3}}$ into $\boldsymbol{G}\hat{\boldsymbol{x}} - \boldsymbol{w}$, we obtain vectors with all the elements non-positive, which tells us that all four estimations satisfy the inequality constraints defined by the matrix $\boldsymbol{G}$ and vector $\boldsymbol{w}$.

4 Conclusion

In this contribution, we proposed an ICEIV model that meets the demand of handing the EIV model with inequality constraints. We presented an ICTLS solution for the ICEIV model. By distinguishing active constraints employing the ES approach, the ICEIV model was converted to an ECEIV model, which can be dealt with by employing ECTLS.

The method was implemented in two examples. The first example was a simulated straight-line fitting problem under the assumption of knowing in advance the activeness of each constraint. The result showed that the ES method can distinguish correctly the active constraints and the ICTLS estimates can satisfy all the constraints. The second example was an ICEIV problem with the number of the inequality constraints increased to 11, and the elapsed time was endurable. The feasibility of the proposed method was thus demonstrated in the three examples.

We did not discuss the statistic properties of the estimation which will be our further interest.

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