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ORIGINAL ARTICLE

Unification of New Zealand's local vertical datums: iterative gravimetric quasigeoid computations

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Abstract New Zealand uses 13 separate local vertical datums (LVDs) based on geodetic levelling from 12 different tide-gauges. We describe their unification using a regional gravimetric quasigeoid model and GPS-levelling data on each LVD. A novel application of iterative quasigeoid computation is used, where the LVD offsets computed from earlier models are used to apply additional gravity reductions from each LVD to that model. The solution converges after only three iterations yielding LVD offsets ranging from 0.24 to 0.58 m with an average standard deviation of ±0.08 m. The so-computed LVD offsets agree, within expected data errors, with geodetically levelled height differences at common benchmarks between adjacent LVDs. This shows that iterated quasigeoid models have a role in vertical datum unification.

Keywords Vertical datum unification · Iterative quasigeoid computation · Geodetic levelling

1 Introduction, background and motivation

New Zealand (NZ) does not currently have a single vertical datum. Instead, 13 separate local vertical datums (LVDs) based on local mean sea level (MSL) observed at 12 tide-gauges are used (the Dunedin–Bluff 1960 LVD was defined by fixing the heights of two benchmarks from the Dunedin 1958 and Bluff 1955 LVDs instead of using a tide-gauge). Despite some early evidence to the contrary (Humphries 1908), these LVDs were assumed stable and thus capable of being linked to form a single national vertical datum (Hannah 2001).

However, the combination of tectonic motion (e.g. Beavan et al. 2004; Wellman 1979; Walcott 1984), sea surface topography (SST) and sea-level change (Hannah 1990) in NZ means that MSL at each tide-gauge does not lie on the same equipotential surface. Localised height changes are also caused by volcanic activity (Otway et al. 2002), geothermal energy extraction (Bevin et al. 1984) and earthquakes (e.g. Henderson 1933; Lensen and Otway 1971; Beanland et al. 1990; Begg and McSaveney 2005). Thus, the prospect of forming a single vertical datum based solely on the readjustment of the levelling networks based on MSL is becoming more remote with time.

Hannah (2001) proposed a least-squares adjustment of all NZ precise levelling observations to give a single LVD for NZ (or more strictly one LVD for each of the islands). The disadvantage is that during the ∼40 year period that levelling observations have been acquired, many benchmarks have undergone significant vertical deformation. As such, an [unknown] proportion of the adjusted heights will not be representative of current ground positions. Also, because not all of the precise levelling lines are connected into "adjustable" loops, there is a risk that the adjustment would be ill-conditioned. Finally, the problem of subsequently unifying these new LVDs among the NZ islands would remain.

The spatial extent of a LVD is also limited to the location of the precise levelling traverses. In mountainous parts of NZ, LVD coverage is restricted to major highways and urban areas. New levelling observations could be acquired to fill some of the gaps and to identify/quantify the effect of vertical

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deformation, but the high cost makes this impractical on a national scale. There is also no clear demand from users of the existing LVDs for a national adjustment, because it would not provide any practical benefits (on a local scale). Finally, GNSS users require a quasigeoid that is compatible with the LVD, so a quasigeoid model would still need to be computed.

The above limitations make a NZ-wide levelling adjustment unfavourable. Instead, we present an iterative technique for determining a gravimetric quasigeoid model based on the GPS-levelling fit (offsets) among the existing LVDs. It is then necessary to consider these offsets in the reduction of gravity anomalies to a common LVD for subsequent quasigeoid determination through a series of iterations. This novel technique has not been attempted before in practice, although a preliminary mathematical framework is given by Rummel and Teunissen (1988) and Heck and Rummel (1990).

New Zealand uses the normal-orthometric height system for levelled heights (DoSLI 1989). The normal-orthometric height (e.g. Heck 2003; Featherstone and Kuhn 2006) is the distance along the normal plumbline from the quasigeoid to the point of interest (Fig. 1). Its advantage over other types of heights (e.g. orthometric, etc.) is that it does not require gravity observations, which are typically not available along the levelling traverses in NZ. However, the NZ heights are not strictly normal-orthometric because they were derived by the application of a cumulative normal-orthometric correction to the levelled height differences for GRS67 (IAG 1967) using a truncated form of Rapp's (1961) formulas for mean normal gravity along the normal plumbline.

The quasigeoid height (Fig. 1) should be used in conjunction with the normal-orthometric height. The quasigeoid is exactly coincident with the geoid over the oceans and coincides within a few decimetres over most land areas; in NZ, the maximum is ~0.5 m at Aoraki/Mt Cook (Amos and Featherstone 2003). In the current study, we have used some approximations to the Molodensky theory for quasigeoid determination, so the computed surface may not be exactly coincident with the classical quasigeoid. This approximate approach is justified, however, because our computed LVD offsets agree with spirit-levelled height differences among the LVDs.

2 Others' attempts at LVD unification

When a LVD is defined on land, a height or geopotential number is fixed at one or more points. This is normally achieved by making MSL observations over a period of time so that the origin coincides with local MSL. However, phenomena such as long-period tides, SST, sea-level change, land uplift/subsidence and temporal effects on the sea level observations (e.g. Pugh 2004) lead to differences among LVDs. If a direct connection is not possible, e.g. due to a body of water, an alternative method is required. The following summarises approaches proposed by others.

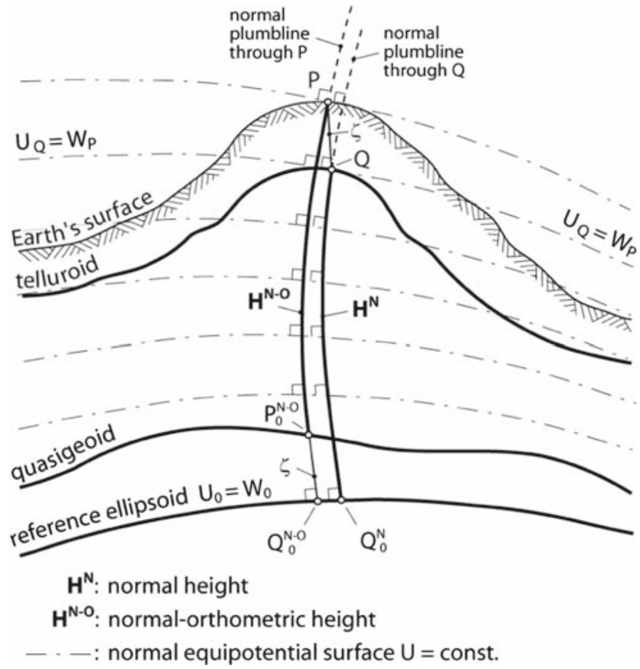

Fig. 1 The normal-orthometric height H^{N-O} is reckoned along the normal gravity plumbline from the point P_0^{N-O} on the quasigeoid to the point P on the Earth's surface. The quasigeoid height ζ is reckoned along the ellipsoidal surface normal from point Q_0^{N-O} on the ellipsoid to point P_0^{N-O} on the quasigeoid. (from Featherstone and Kuhn 2006)

2.1 Geopotential numbers

The global geopotential (W_0) can be used as a reference "level" to relate vertical datums (e.g. Burša et al. 2007). Unification by geopotential numbers uses a global geopotential model (GGM) and GPS-levelling information at each of the LVD origins (tide-gauges) to compute the geopotential for each LVD. This approach has been implemented in several locations (e.g. Grafarend and Ardalan 1997; Burša et al. 2004, 2007). The downside of using a single point for each LVD is that the assumptions must be made that the datum offsets are constant across the LVD and they are not distorted (e.g. due to multiple tide-gauges being fixed). Moreover, GGMs can contain errors that are larger than the likely LVD offsets.

2.2 Gravimetric geoid or quasigeoid

An alternative is to use a regional gravimetric quasi/geoid model and GPS-levelling observations to provide a reference surface to which the LVDs can be related. This technique has been implemented extensively (e.g. Goldan and Seeber 1994; Featherstone 2000; Kumar and Burke 1998;

Nahavandchi and Sjöberg 1998; Rapp 1995; Pan and Sjöberg 1998; Rizos et al. 1991; Rapp and Balasubramania 1992). However, quasi/geoid models contain errors, so this approach will not give exact datum unification. Arabelos and Tscherning (2001) show that satellite gravimetry will improve long-wavelength GGMs, making this and the geopotential numbers approaches more viable.

Laskowski (1983) simulated LVD offsets on a continental scale ranging from −55 to +30 cm (approximating SST) and found that the cumulative error up to degree 180 in the computed geoid was almost 45 cm. His simulation confirmed that the effect of offset LVDs is likely to be seen in the low-frequency geoid (cf. Vaníček and Featherstone 1998). Also, if distortions exist in the LVD, computed offsets change depending on the points used (Featherstone 2000). The problems with this approach in NZ are that (1) a regional quasigeoid model did not exist, and (2) because the data used are reduced to different LVDs, any quasigeoid model will be biased by the effects of the offsets.

3 NZ LVDs

3.1 Tide-gauges and precise levelling networks

Tide-gauges in NZ have been established in harbours and rivers (within a few km of the coast) by local port authorities for prediction and verification of tide tables (Blick et al. 1997), but which are not optimal for LVD definition (e.g. Hipkin 2000; Cross et al. 1987; Merry and Vaníček 1983). Since these were the only data available, Land Information NZ (LINZ) and its predecessor agencies determined MSL at each site, which was then used as the zero height to which a local precise levelling network was referenced and least-squares adjusted to form each LVD. As such, offsets are expected (and observed; see later) among NZ LVDs.

First-order precise levelling ($\pm 2\text{mm}\sqrt{k}$ misclosure tolerance, where k is the two-way distance in km) has been the preferred method for height transfer in NZ. There is currently >16,000 km of two-way first-order levelling that has been observed since the 1960s (cf. Gilliland 1987). These networks were observed in a piece-meal fashion and the large loop around the South Island was only completed in the late 1980s. Approximate normal-orthometric corrections (Sect. 1) were applied to the spirit levelling.

NZ LVDs (Fig. 2) are based on a determination of MSL at different tide-gauges over varying time intervals (normally 3 years) and epochs (1909–1977). The Stewart Island/Rakiura 1977 LVD is not defined by a tide-gauge. Instead, its zero level is based on the MSL value determined from three temporary tide-gauges by averaging the high and low levels of three to five successive (but not simultaneous) tides. It also uses trigonometric heights that could be in error by 0.2–0.3 m, and the MSL could be in error by 0.5 m from the long-term trend. This is a weakly defined LVD.

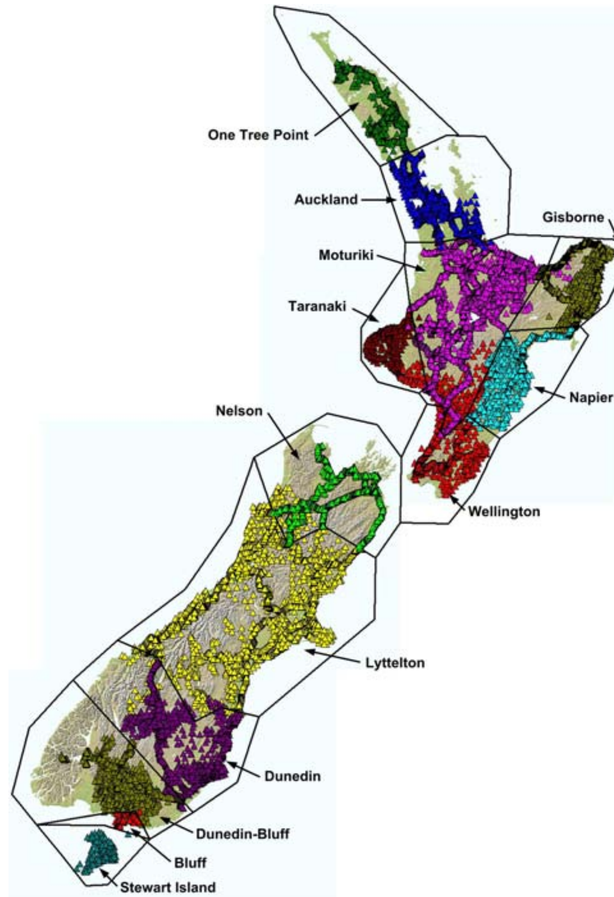

Fig. 2 NZ LVD extents. Triangles show the location of benchmarks with normal-orthometric heights (all orders of levelling). *Solid lines* show the presumed spatial extents of the LVDs

3.2 Offsets among NZ LVDs

Since sea-level observed at tide-gauges varies on annual, inter-annual and inter-decadal cycles (e.g. Pugh 2004), the epoch used will affect the computed MSL (Bell et al. 2000). Analysis of sea-level observations by LINZ (Rowe 2006, personal communication) shows that variations in observed MSL can differ from the long-term average by >10 cm over a 3-year period. Given that most of NZ LVDs were defined by only around 3 years of observations, it is very likely that they are based on a MSL that is not representative of the long-term average. For example, if MSL for the Wellington tide-gauge was computed from data indicated by either of the two horizontal lines in Fig. 3 rather than the full record, it could be offset from the long term average by >50 mm. As such, part of the offsets can be attributed to the epoch for the shorter duration MSL observations.

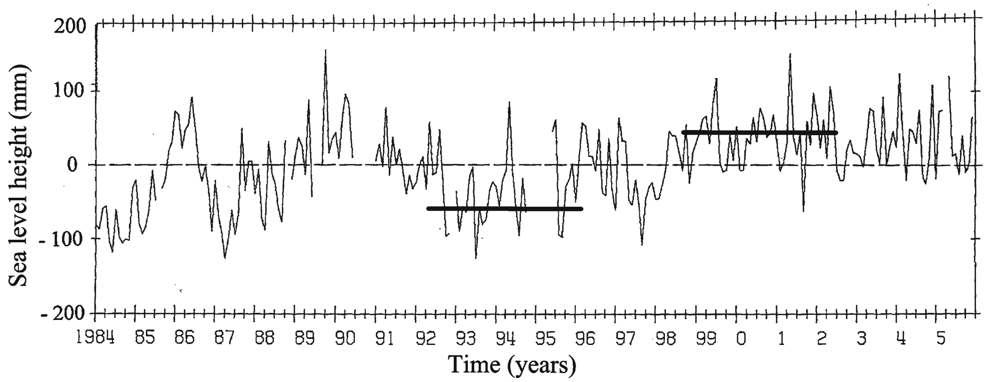

Fig. 3 Monthly sea-level observations (mm) for the Wellington tide-gauge from LINZ records, 1984–2006

Table 1 LVD offsets determined from spirit-levelled height differences at junction points (m)

Mark	LVD1	LVD2	Offset
ABHL	One Tree Point 1964	Auckland 1946	+0.206
AGD8	Auckland 1946	Moturiki 1953	−0.069
ABTE	Auckland 1946	Moturiki 1953	−0.075
ABV5	Auckland 1946	Moturiki 1953	−0.067
ABX2	Gisborne 1926	Moturiki 1953	−0.075
AD2J	Napier 1962	Gisborne 1926	+0.166
AEVR	Napier 1962	Moturiki 1953	+0.099
AE54	Napier 1962	Taranaki 1970	+0.046
AE54	Taranaki 1970	Wellington 1953	+0.191
AE54	Napier 1962	Wellington 1953	+0.237
AHBB	Taranaki 1970	Moturiki 1953	−0.455
B48K	Taranaki 1970	Moturiki 1953	−0.014
AEXF	Taranaki 1970	Moturiki 1953	−0.019
AEXF	Taranaki 1970	Wellington 1953	+0.102
AEXF	Moturiki 1953	Wellington 1953	+0.121
AEJ5	Nelson 1955	Lyttelton 1937	+0.014
AP5E	Nelson 1955	Lyttelton 1937	+0.039
ADHE	Nelson 1955	Lyttelton 1937	−0.086
ADCK	Nelson 1955	Lyttelton 1937	−0.076
B4A2	Lyttelton 1937	Dunedin 1958	−0.054
AE7N	Lyttelton 1937	Dunedin 1958	−0.087
ADP2	Dunedin-Bluff 1960	Dunedin 1958	−0.019
AB9T	Dunedin-Bluff 1960	Bluff 1955	−0.001

Where two or more LVDs abut or overlap, it is possible to directly estimate the offsets. However, this is affected by the distance and route of the levelling traverse to get to the junction point, any deformation that has occurred while the levelling was being carried out (although this deformation will be "spread-out" by the least-squares adjustment), and observation and reduction errors. As such, when LVDs join at multiple places, the observed offsets will differ. For instance, the Taranaki–Moturiki offset at AHBB in Table 1 is abnormally large. This is probably due to benchmark movement between the observation times, but it was not possible to confirm or disprove this from the levelling records at LINZ.

3.3 GPS-levelling data

The current horizontal geodetic datum is NZ Geodetic Datum 2000 (NZGD2000; LINZ 2007). It is a 3D geocentric datum that uses GRS80 (Moritz 1980) and is aligned to ITRF96 epoch 2000.0 (Boucher et al. 1998). NZGD2000 uses a horizontal deformation and velocity model to "correct"

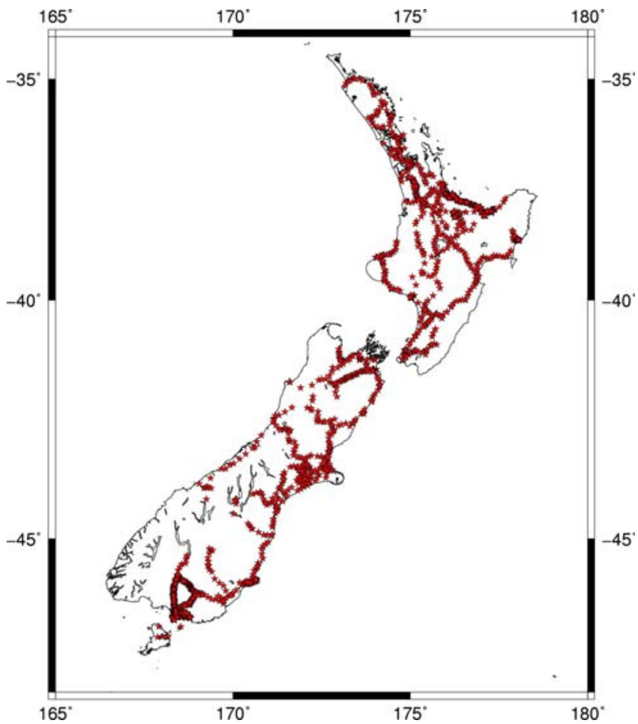

Fig. 4 1,422 NZ GPS-levelling points (Mercator projection)

observations for the effects of deformation from the time of acquisition to the datum's reference epoch (01 January 2000). No vertical deformation or velocity model is used in NZGD2000.

A total of 1,422 points in NZ have both NZGD2000/GRS80 ellipsoidal and LVD normal-orthometric heights (first- and second-order levelling). The spatial distribution of the GPS-levelling points is not uniform, and there are significant gaps in the South Island (Fig. 4). This is where the topography is particularly rugged so that the precise levelling traverses are restricted to roads.

The levelled normal-orthometric heights were then divided among the 13 LVDs. No GPS-levelling points exist on the Chatham Islands and the five points on Stewart Island have less accurate heights (see earlier). The absolute accuracy of the ellipsoidal heights and the normal-orthometric heights is estimated to be on average 10 cm (OSG 2003) and the combined accuracy 14 cm. This error estimate assumes independence and does not account for the offsets among the LVDs.

4 NZ quasigeoid input data

4.1 Land gravity

Terrestrial gravity data in NZ is held by GNS Science (http://www.gns.cri.nz). The database (2007) consists of 40,737 observations covering the NZ and Chatham Islands. They were primarily collected for the production of gravity anomaly maps in the 1960s and 1970s (Reilly 1972). Reilly (*ibid.*) estimates the accuracy of the gravity observations to be ∼0.1–0.5 mGal. Their horizontal positions were transformed to NZGD2000 (Amos and Featherstone 2003) and their heights are assumed to be in terms of the 13 LVDs, although this could not be confirmed in all cases. The gravity observations were referenced to the Potsdam (NZ) datum. The accepted conversion of 15.27 mGal (Hunt and Ferry 1975) was applied to convert these to IGSN71 (Morelli et al. 1974).

Molodensky free-air gravity anomalies (at the Earth's surface) were first computed by subtracting the value of normal gravity at the geocentric observation latitude, then adding the second-order free-air correction (Hackney and Featherstone 2003) and an atmospheric correction for the observation's height (on the LVD). The difference between the linear and second-order free-air gravity anomalies reaches 1.149 mGal at the summit of Aoraki/Mount Cook (3,754 m). The atmospheric correction is 0.550 mGal at the summit of Aoraki/Mount Cook.

4.2 Ship-track gravity

Marine gravity observations in the vicinity of NZ have been collected over the past 45 years by various agencies at different times for different purposes. The databases (2007) comprise 1,300,266 gravity anomalies bounded by 160°E $\leq \lambda \leq$ 190°E and 25°S $\leq \phi \leq$ 60°S and auxiliary information. Woodward (2001, personal communication) estimates the overall accuracy of the marine data to be approximately 1 mGal.

These observations were previously stored in different formats, in terms of different (horizontal and gravity) datums, and no attempt had been made to ensure consistency among individual cruises, let alone the datasets. To remedy gravimeter-drift-induced offsets and tilts in marine gravimetry (Wessel and Watts 1988), a crossover adjustment of ∼900,000 line-km of observations surrounding NZ was carried out by *Intrepid Geophysics* under contract to LINZ (Brett 2004; Amos et al. 2005).

4.3 Satellite altimeter-derived gravity

To achieve better gravity data coverage over the NZ computation area, the ship-track observations were combined with gravity anomalies derived from satellite altimetry. However, altimetry-derived gravity anomalies are known to be less accurate close to the coast (e.g. Hipkin 2000; Andersen and Knudsen 2000; Deng and Featherstone 2006). For instance, Amos et al. (2005) compared four altimetry-derived gravity anomaly grids, and found 100 mGal discrepancies around NZ and the Chatham Islands. These were attributed to a combination of the problems with coastal altimetry and the very steep gravity gradients at the boundary of the Australian and Pacific plates. Based on comparisons with the crossover-adjusted ship-track data, no single grid was significantly better. Consequently, we made an arbitrary decision to use KMS02 (Andersen et al. 2005).

The altimeter-derived gravity anomalies are probably of better quality than the poorly constrained sparse ship-track data far from shore (cf. Kirby and Forsberg 1998), conversely near the coast. To reduce the expected error in KMS02 near the coast, the crossover-adjusted ship-tracks were used to "correct" it (cf. Strykowski and Forsberg 1998). This was achieved using the least-squares collocation interpolation routines in GRAVSOFT (Tscherning et al. 1992) to "drape" the altimetry anomalies onto the crossover-adjusted ship track data within 400 km of the coast (Amos et al. 2005). This reduced the standard deviation of the fit to independent data from 9.9 to 3.2 mGal.

4.4 Digital elevation data

Although no "official" digital elevation model (DEM) is published in NZ, a number of companies sell DEMs that are derived from the official LINZ vector data (used for 1:50,000 topographic mapping). For this study, a 1.8 arc-second (0.0005 degree or ∼56 m) resolution DEM was purchased from *GeographX*. It has an estimated precision of ±22 m horizontally and ±10 m vertically (Smith 2001, personal communication). All heights in the DEM are related to the "zero" contour line which approximates the level of mean high water springs. The heights are not explicitly referenced to any of the 13 LVDs, but the accuracy of the DEM heights is less than the LVD offsets (given later).

5 Initial model computation

The above data were used, together with a "cut and paste" combination of GGM02S (Tapley et al. 2005) and EGM96 (Lemoine et al. 1998) at degree 100, to compute a preliminary model. This combination does not account for the different error characteristics of each GGM, but because the error coefficients are not used, it is still valid. The first solution is only a preliminary model because the gravity anomalies used refer to the 13 disparate LVDs, and—as such—the result is biased because the input data have been reduced to different LVDs. Topographical corrections, as approximations to the Molodensky G1 term, were computed from the 56-m DEM using prism integration (Nagy 1966a,b), taking ∼2.5 months on a Sun E4500 server (8 × 400 MHz processors, 8 GB RAM).

Prism integration was used to avoid numerical instabilities in Moritz's (1968) algorithm (cf. Martinec et al. 1996). These terrain corrections were also used in the gridding/interpolation of gravity anomalies so as to smooth the highly variable NZ gravity field (cf. Janák and Vaníček 2005). Mean gravity anomalies on the Earth's surface were computed from the interpolated anomalies using the "reconstruction" and averaging technique of Featherstone and Kirby (2000).

The initial model was computed via an adapted remove-compute-restore (RCR) approach with the GGM02S/EGM96 GGM to degree 100/360 and the Featherstone et al. (1998) deterministically modified Stokes kernel (cf. Featherstone et al. 2001; Amos and Featherstone 2004; Amos 2007). Stochastically modified kernels were not considered because reliable estimates of the error variances of the NZ gravity data are not currently known. In addition to kernel modifications reducing truncation errors, they also have preferential filtering properties that can reduce the effect of errors in the gravity observations and GGMs (Vaníček and Featherstone 1998).

Firstly, the degree-100/360 GGM02S/EGM96 gravity anomaly contribution was removed from the gridded gravity anomalies. Residual quasigeoid undulations were computed from these residual gravity anomalies using the 1D-FFT (Haagmans et al. 1993) with the modified kernel over a spherical cap (Featherstone and Sideris 1998). The restore stage added the GGM quasigeoid contribution to the residual quasigeoid. Note that this is not the classical Molodensky approach to quasigeoid computation. Therefore, we believe that our approximate approach taken to quasigeoid computation delivers values that are more compatible with levelled heights in NZ.

Five different deterministic kernel modifications (Meissl 1971; Wong and Gore 1969; Vaníček and Kleusberg 1987; Heck and Grüninger 1987; Featherstone et al. 1998) were trialled for different cap radii (ψ_0) and degree (L) of modification (where applicable), as well as the unmodified spherical Stokes kernel (Amos 2007). These results were compared with the GPS-levelling data on a LVD-by-LVD basis to try to optimise these parameters. While there was little difference among the results (cf. Featherstone et al. 2004; Ellmann 2005), the $L = 40$ Featherstone et al. (1998) kernel for a $\psi_0 = 1.5°$ cap was chosen because the offsets among LVDs were relatively insensitive for this kernel. This insensitivity relates to the cap/modification combinations that exhibited low standard deviations for the computed LVD offsets.

The 1422 GPS-levelling points (Fig. 4) were then used to estimate the initial offsets for each LVD from this initial model, where the GPS-levelling points were divided into their respective LVDs (Fig. 2). These points are not evenly distributed among the 13 LVDs because the levelling routes are located along highways. The normal-orthometric (H^{N-O}), quasigeoid (ζ) and GPS ellipsoidal (h) heights of a point are related by $h = \zeta + H^{N-O}$. Therefore, assuming the absence of other systematic error sources, the offsets (on LVD "a") were computed according to:

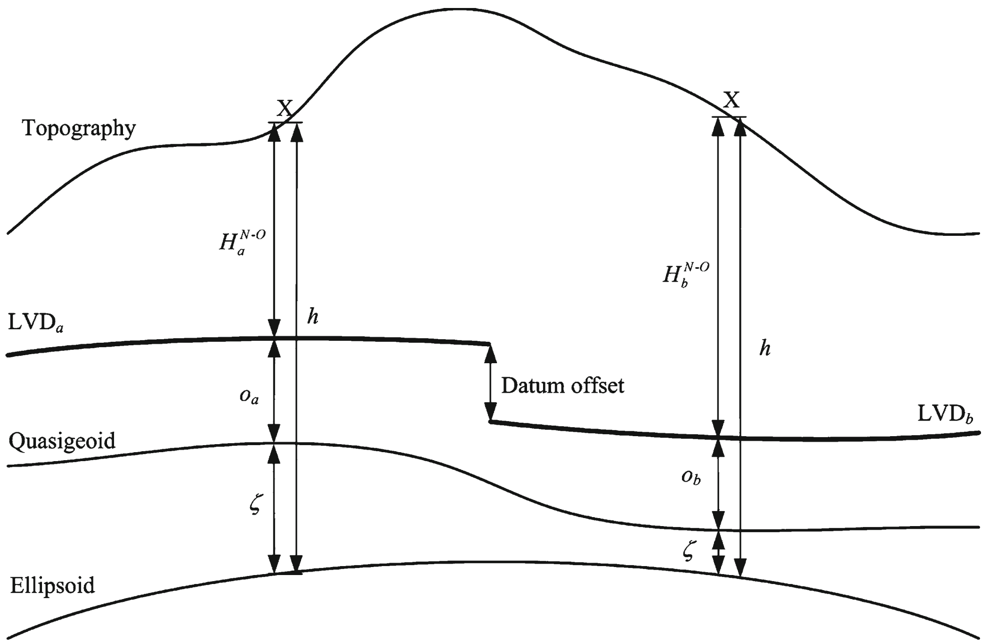

Fig. 5 Schematic of LVD offsets and their effect on the initial quasigeoid

$$H_a^{N-O} - h + \zeta = o_a \tag{1}$$

After removing the mean offset for each LVD, the standard deviation (STD) for all 1,422 points is reduced from 0.124 m (if ignoring the offsets) to 0.078 m (max +0.360 m, min −0.315). It was not possible to evaluate the offsets for the Chatham Islands because there is currently no ellipsoidal height information at the small number levelling points on the island. As such, it is assumed to be coincident with the quasigeoid models in all computations (i.e. zero offset).

6 The iterative quasigeoid computation scheme

Where heights are not on the same LVD (or the LVD used is distorted), the heights and any quantities derived from them (e.g. gravity anomalies) will be inconsistent. When inconsistent gravity anomalies are converted to a quasi/geoid, distortion will occur (Fig. 5). Laskowski (1983) proposed a datum offset correction ($\delta\Delta g$) to correct gravity observations for the effect of offset LVDs and thus convert them to a consistent reference system prior to computation.

$\delta\Delta g$ has the form of the (first-order) free-air gravity correction and units of mGal applied over the LVD offset o.

$$\delta\Delta g = \Delta g^* - \Delta g = \frac{\partial\gamma}{\partial h} o \cong 0.3086\, o \tag{2}$$

where

$$\Delta g^* = g_{obs} + \frac{\partial\gamma}{\partial h}(h_D + o) - \gamma \tag{3}$$

$$\Delta g = g_{obs} + \frac{\partial\gamma}{\partial h} h_D - \gamma \tag{4}$$

and g_{obs} is the observed value of gravity, $\partial\gamma/\partial h$ the linear vertical gradient of normal gravity, h_D is the height of the gravity observation on LVD D, and γ is normal gravity on the reference ellipsoid. It is not necessary to use a second-order free air correction in Eq. (2) because of the small height differences involved ($o \leq 2$m).

A limitation of Laskowski (1983) approach is that it needs the magnitude of the offsets to be known before Eq. (2) can be applied and the quasigeoid computed. In many situations, the offset will not be known beforehand, e.g. across water bodies. The iterative scheme proposed and used here, on the other hand, utilises the initial model (Sect. 4) and GPS-levelling on each LVD to estimate the offsets (Table 2) and then uses Eq. (2) to 'correct' the gravity anomaly values for the effect of the offset LVDs. This attempts to make them consistent, thus lessening the distortion that will occur if offset LVDs are used only. This procedure is iterated until the so-computed LVD offsets converge.

There is a slight complication in that the extent over which the LVD applies had to be estimated (Fig. 2). If LVD metadata had been included in the gravity database, this would have simplified matters. Instead, we estimated the lateral extents of the LVDs, which becomes problematic when the LVDs overlap. Therefore, the LVD boundaries were determined by visual inspection of maps that showed the locations of all geodetic marks with normal-orthometric heights on each LVD (this includes low-order heights), which allowed boundaries

Table 2 Descriptive statistics of the comparison of the final quasigeoid with GPS-levelling points on the 13 LVDs (metres)

LVD	Points	Max	Min	Average (offset)	Standard deviation
One Tree Point 1964	51	−0.145	−0.411	−0.242	0.063
Auckland 1946	137	−0.309	−0.651	−0.491	0.068
Moturiki 1953	258	−0.165	−0.517	−0.314	0.058
Gisborne 1926	61	−0.424	−0.690	−0.578	0.087
Taranaki 1970	70	−0.318	−0.592	−0.451	0.067
Napier 1962	54	−0.117	−0.472	−0.301	0.067
Wellington 1953	78	−0.415	−0.608	−0.504	0.039
Nelson 1955	111	−0.077	−0.430	−0.258	0.073
Lyttelton 1937	251	+0.012	−0.612	−0.349	0.092
Dunedin 1958	73	−0.147	−0.722	−0.485	0.163
Dunedin–Bluff 1960	181	−0.022	−0.572	−0.256	0.076
Bluff 1955	92	−0.200	−0.463	−0.376	0.051
Stewart Island 1977	5	−0.236	−0.589	−0.400	0.116
All Data	1422	+0.012	−0.722	−0.364	0.124
All Data, Zero Datum Average	1422	+0.361	−0.316	0.000	0.079

to be drawn to approximate the extent of each LVD (Fig. 2). The gravity data was then split among LVDs using *MapInfo* v 6.5 software.

The first step is to compute a preliminary/initial model using gravity anomalies reduced to their respective LVDs (Sect. 4). This model and GPS-levelling observations are used to make a first estimate of the offsets (i.e. mean of the GPS-levelling-quasigeoid differences) for each LVD (Eq. 1). These offsets were then used in Eq. (2) to determine $\delta\Delta g$ for each LVD. The original gravity anomalies were then "corrected" by adding the applicable $\delta\Delta g$ for each LVD. The effect of using $\delta\Delta g$ to unify two datums with the iterative scheme is shown in Fig. 6: where the two LVDs meet, a step (smoothed by the Stokes filtering) occurs in the computed quasigeoid as a result of the offset.

These "corrected" gravity anomalies were then used to evaluate a second model (ζ_2) (shown as a dashed line in Fig. 6). The step in the second model at the datum boundary has been smoothed further in comparison to the preliminary model. This is because the offset bias is being better modelled by the LVD offset "correction" applied above. The original GPS-levelling data is then used again with the second model to re-evaluate the datum offsets.

The "again-corrected" gravity anomalies (g_{2a}, g_{2b}) were then used to compute a third model (ζ_3), shown as a solid line in Fig. 6. This model is even smoother than the second model across the LVD boundary. Again, the GPS-levelling data were used to evaluate the LVD offsets, o_{3a} and o_{3b}. This process was repeated until the offsets computed in successive iterations are constant. Alternatively, the "corrections" to the gravity anomalies alone can be subjected to Stokesian integration, but the final result will not change.

7 Final results and discussion

Our iterative quasigeoid computation approach to LVD unification it has been implemented over NZ. The NZ gravity observations were assumed to have been reduced to the LVD in which they are located (Fig. 2). Because the spatial extents of the 13 LVDs are not explicitly defined and the LVD used to reduce the gravity observations had not always been recorded, it was not possible to categorically ascertain whether the reduction to the LVDs has occurred or not. However, with the lack of evidence to the contrary, it was necessary to make this assumption.

This iterative procedure converged after only three iterations in NZ. The final LVD offsets and their standard deviations (Table 2) were compared with the levelled differences at junction points (Table 3). Ten of the 13 levelled offsets agree statistically with the so-computed offsets. When taking into account the crudely estimated precision of the GPS-levelling data of ∼14 cm, all results in Table 3 are consistent, showing that the iterative approach can be used to unify LVDs.

The converged/final NZ gravimetric quasigeoid model represents a surface that has been "corrected" for the biases otherwise introduced as a result of the gravity anomalies being computed in terms of offset LVDs. For this reason, the converged quasigeoid solution can then be used as a transformation surface from GRS80 to each of the LVDs when combined with the respective LVD offset. For example, an ellipsoidal height can be transformed to a normal-orthometric height on LVD "a" using:

$$H_a^{N-O} = h - \zeta + o_a \quad (5)$$

where o_a is the offset for LVD "a".

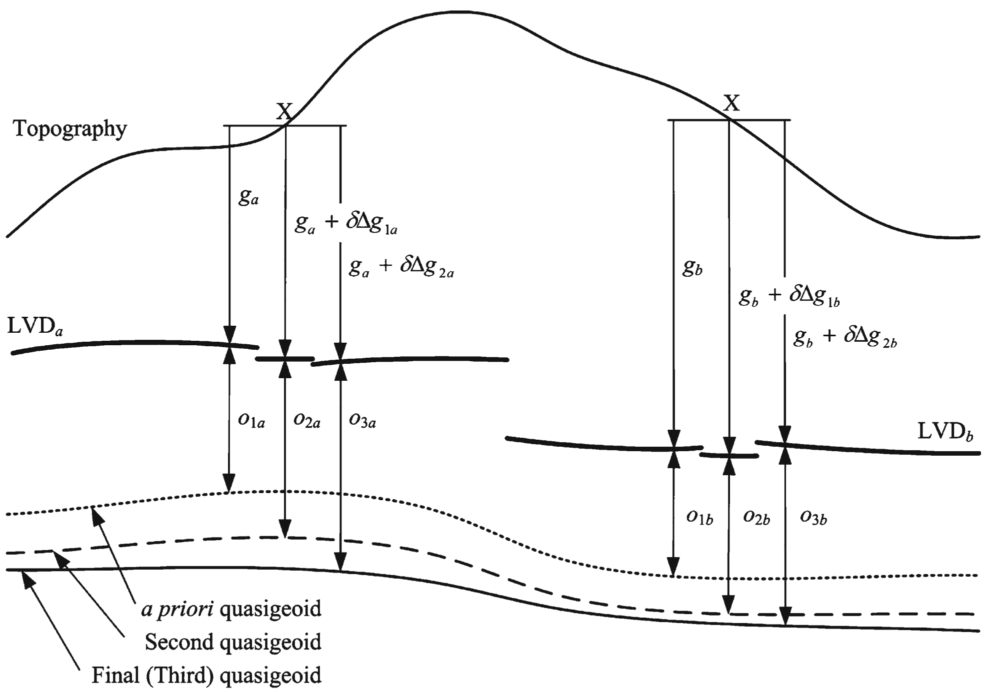

Fig. 6 Iterative quasigeoid datum unification scheme

Table 3 Summary of comparison between final quasigeoid and observed precise levelling offsets (95% CI, Student *t* distribution, metres)

From	To	Final Quasigeoid		Levelling		Offsets agree?
		Offset	95% CI	Offset	95% CI	
Auckland	One Tree Point	−0.245	±0.021	−0.206	±0.139	Yes
Auckland	Moturiki	−0.177	±0.013	−0.070	±0.139	Yes
Gisborne	Moturiki	−0.264	±0.023	−0.075	±0.139	No
Gisborne	Napier	−0.270	±0.028	−0.166	±0.139	Yes
Moturiki	Napier	−0.007	±0.019	−0.099	±0.139	Yes
Taranaki	Napier	−0.150	±0.024	−0.046	±0.139	Yes
Taranaki	Wellington	0.053	±0.018	0.147	±0.139	Yes
Taranaki	Moturiki	−0.143	±0.017	−0.162	±0.139	Yes
Napier	Wellington	0.203	±0.020	0.237	±0.139	Yes
Nelson	Lyttelton	0.093	±0.018	−0.027	±0.139	Yes
Lyttelton	Dunedin	0.137	±0.039	−0.071	±0.139	No
Dunedin-Bluff	Dunedin	0.230	±0.039	−0.019	±0.139	No
Dunedin-Bluff	Bluff	0.118	±0.015	−0.001	±0.139	Yes

A relatively large STD (0.163 m) was found for the Dunedin 1958 LVD (Table 2). This is because the initial model and GPS-levelling residuals get systematically larger northwest from the Dunedin tide-gauge (cf. Fig. 1). This could be due to a tilt in the LVD, rather than the constant offset that has been assumed for all other NZ LVDs. However, the limited number and geographical extent of GPS-levelling points (cf. Fig. 3) meant that it was not possible to verify this. Future studies (with additional GPS-levelling data) that investigate the use of inclined planes may help to isolate the cause.

All 13 offsets in Table 2 are significantly different to zero at the 95% confidence interval (CI). Of the 16 abutting LVDs, the offsets at 14 were significantly different and only the Napier–Moturiki and Bluff–Stewart Island were not. An additional validation can be obtained by comparing the so-estimated LVD offsets with the observed differences at junction points (Table 3). Ten of the 13 observed offsets agreed with the computed values (a combined STD of 0.071 m was conservatively estimated for the levelled LVD offsets).

The Lyttelton–Dunedin and Dunedin–Dunedin–Bluff LVD differences are likely to be caused by the high STD of the Dunedin 1958 offset (Table 2), resulting from a potential tilt in this LVD. The Gisborne–Moturiki difference might be caused by the poor spatial coverage of the GPS-levelling points used to evaluate the offset (cf. Fig. 4). The majority of the Gisborne 1926 LVD, notably a large levelling loop around East Cape (37°41′S, 178°32′E), has no GPS observations on it.

8 Conclusion

We have presented a new concept of iterative quasigeoid computation for the unification of LVDs. An initial model is computed from (distorted) gravity anomalies computed on each LVD. GPS-levelling on each LVD is then used to compute preliminary offsets, which are used to apply additional reduction to the gravity anomalies to refer them to a more consistent reference surface. These are then Stokes-integrated and new offsets computed iteratively. For NZ, this approach converges after only three iterations and yields offsets ranging from 0.24 to 0.58 m (Table 2). The average offset is 0.36 m with a standard deviation of ±0.08 m (when the each LVD offset is removed). Importantly, the so-computed offsets agree with levelled offsets (Table 3), within expected data errors, showing that such iterated quasi/geoid models do have a role in vertical datum unification. In this sense, it can be said that the NZ LVDs are better unified.

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