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ORIGINAL ARTICLE

The topographic bias by analytical continuation in physical geodesy

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Abstract This study emphasizes that the harmonic downward continuation of an external representation of the Earth's gravity potential to sea level through the topographic masses implies a topographic bias. It is shown that the bias is only dependent on the topographic density along the geocentric radius at the computation point. The bias corresponds to the combined topographic geoid effect, i.e., the sum of the direct and indirect topographic effects. For a laterally variable topographic density function, the combined geoid effect is proportional to terms of powers two and three of the topographic height, while all higher order terms vanish. The result is useful in geoid determination by analytical continuation, e.g., from an Earth gravity model, Stokes's formula or a combination thereof.

Keywords Analytical continuation · Downward continuation · Stokes's formula · Topographic effects

1 Introduction

Physical geodesy primarily deals with the external gravity field of the Earth, implying that the gravitational potential and acceleration are harmonic functions down to the surface of the Earth (if we disregard the mass of the atmosphere). However, a main goal of physical geodesy is to determine the geoid from gravity-related observables on and/or outside the Earth's surface. As the geoid is roughly related with the undisturbed sea level and its continuation inside the continents, the harmonic or analytical downward continuation of the external harmonic observables of gravity to the geoid will be biased in continental areas.

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Analytical continuation could be used as a tool to determine the quasi-geoid (Bjerhammar 1962; Heiskanen and Moritz 1967, Sect. 8.10), but if the same procedure is used to estimate the geoid inside the topography, the estimator will experience the topographic bias. By analytical continuation, we mean a mathematical process (calculation) that extends the operation valid on and above the Earth's surface to points within or below the topographic masses. The harmonic downward continuation thus treats the potential of the Earth as if it were harmonic also inside the topographic masses, which obviously is in error and leads to the topographic bias. This means that there is no topographic bias for the downward continuation to any point located at the Earth's surface or even in a narrow drill hole.

We will not address the possible error committed by the specific method used for the analytical continuation. (Examples of such methods are Taylor expansion, solid spherical harmonic expansion and solving Poisson's integral equation.) In other words, we will assume that the downward continuation of the potential by a harmonic function is achieved without error, or at least without significant error. This assumption is justified, e.g., by the approximation theorems of Runge (Krarup 1969) and Keldysh–Lavrentieff (Landkof 1972, p. 341; Bjerhammar 1974). The fictitious potential field so obtained, being harmonic also within the topographic masses, experiences the above topographic bias. In practice, however, there will usually be an additional error related with the method chosen for downward continuation. This error/bias is thus not part of the present study.

Sjöberg (1977) showed that the topographic bias at sea level of an external type series of solid spherical harmonics of the geopotential can be expressed as a volume integral over the topographic masses, which can be approximated by a power series of topographic height. This study included also numerical estimates of errors of the low-degree spherical harmonic coefficients (complete to degree and order 16) of the external potential series when applied at sea level; also see Martinec (1998). Later, Sjöberg (1996a,b, 2000, 2001) and Ågren (2004) showed that this bias can also be expressed as the negative sum of the downward-continued direct topographic effect on the geoid and the (primary) indirect topographic effect.

However, when using this type of spherical harmonic representation of a power series of topographic elevation, the solution becomes an unstable asymptotic series, whose convergence or divergence with an increasing degree of the spherical harmonic expansion is an open question (see Ågren 2004). Although the standard procedure to determine the geoid by Stokes's formula is to remove the effect of the topography from surface gravity prior to Stokes's integration and to apply the indirect effect afterwards, the topographic bias seen as the negative sum of the direct and indirect effects is closely related with such methods. Finally, the method of applying Stokes's formula with additive corrections of Sjöberg (2003a) relies directly on the topographic bias as being the negative of the combined topographic effect.

The goal of the present study is to derive the topographic bias with clarity. This will be described in two steps: first, we derive the biases for the Bouguer shell and a symmetric cap of finite height on the sphere. Then, we will present the general solution. Similar to our general solution, Wang (1990) divided the topographic bias into the contributions from a Bouguer shell and a residual terrain model. However, the downward continuation of the residual topographic potential by a Taylor series, as used by Wang (1990), is not very fruitful, as the truncated series also contains a truncation error, which cannot be separated from the possible topographic bias (cf. the approach by spherical harmonics above). The solution presented here will not suffer from such a problem.

2 The topographic bias for the Bouguer shell and the symmetric cap

Let us first consider a Bouguer shell with constant mass-density ρ, of internal radius R and external radius $r_s = R + H$ = constant with respect to the geocenter. $H = r_s - R$ is the topographic height. Then, the potential of the shell at any point P of geocentric radius r_P can be written as

$$\begin{aligned} V_P^B &= 2\pi\mu \int\limits_{r=R}^{r_s}\int\limits_{\psi=0}^{\pi}\int\limits_{\alpha=0}^{2\pi} \frac{\mathrm{d}\alpha \sin\psi \mathrm{d}\psi}{l_P} r^2\,\mathrm{d}r \\ &= \frac{2\pi\mu}{r_P}\int\limits_{R}^{r_s}(r + r_P - |r - r_P|)r\,\mathrm{d}r \\ &= 2\pi\mu \begin{cases} \frac{2}{3r_P}\left(r_s^3 - R^3\right) & \text{if } r_P \geq r_s \\ r_s^2 - \frac{2R^3}{3r_P} - \frac{r_P^2}{3} & \text{if } R \leq r_P < r_s \end{cases}, \end{aligned} \tag{1}$$

where $l_P = \sqrt{r_P^2 + r^2 - 2r_P r\cos\psi}$, (r, ψ, α)= (radius, geocentric angle, azimuth) are spherical coordinates, and $\mu = G\rho$, with G being the gravitational constant.

If we continue the external potential (i.e., the solution for $r_P \geq r_s$) to sea level with radius $r_P = R$, we obtain

$$\left(V_P^B\right)^* = C/R, \tag{2a}$$

where

$$C = 4\pi\mu\left(r_s^3 - R^3\right)/3, \tag{2b}$$

and it differs from the internal (correct) potential at sea level

$$V_g^B = 2\pi\mu(r_s^2 - R^2) \tag{3}$$

by the bias

$$V_{\text{bias}}^B = \left(V_P^B\right)^* - V_g^B = 2\pi\mu\left(H^2 + \frac{2}{3}\frac{H^3}{R^2}\right). \tag{4}$$

It can be seen from Eq. (1) that the bias is caused by the term $|r_P - r|$ under the integral, and this term causes discontinuities at the Earth's surface of the radial derivatives of the topographic potential.

It should be noted that the downward-continued potential of the Bouguer shell in Eq. (2a) can be correctly determined by a Taylor expansion of the external potential at the point P with $r_P \geq r_s$. This is so, because the series becomes

$$\left(V_P^B\right)^* = \sum_{k=0}^{\infty}\frac{(-H_P)^k}{k!}\left(\frac{\partial^k V_P^B}{\partial r_P^k}\right), \tag{5}$$

where $H_P = r_P - R$, and by inserting V_P^B for $r_P \geq r_s$ from Eq. (1), we arrive at

$$\left(V_P^B\right)^* = \frac{C}{r_P}\sum_{k=0}^{\infty}\left(\frac{H_P}{r_P}\right)^k = \frac{C}{r_P - H_P} = \frac{C}{R}, \tag{6}$$

which agrees with Eq. (2a).

We now turn to the case with a homogeneous cap of height $H = r_s - R$ = constant, of constant mass-density and geocentric angle ψ_0 on top of the sphere of radius

R, yielding the potential V_P^c at any point P of radius r_P along its axis:

$$V_P^c = 2\pi\mu \int_0^{\psi_0}\int_R^{r_s} \frac{r^2 \mathrm{d}r \sin\psi \,\mathrm{d}\psi}{l_P} = \frac{2\pi\mu}{r_P}\int_R^{r_s} (l_{P0} - |r_P - r|) r \,\mathrm{d}r, \tag{7}$$

where $l_{P0} = \sqrt{r_P^2 + r^2 - 2r_P r \cos\psi_0}$. This potential has the following solutions in the exterior space and at the sphere of radius R:

$$V_P^c = \frac{2\pi\mu}{r_P}\left(I(r_P) - r_P\frac{r_s^2 - R^2}{2} + \frac{r_s^3 - R^3}{3}\right) \text{ if } r_P \geq r_s, \tag{8a}$$

and

$$V_g^c = 2\pi\mu\left(\frac{I(R)}{R} - \frac{H^2}{2} - \frac{H^3}{3R}\right) \text{ if } r_P = R, \tag{8b}$$

where

$$\begin{aligned} I(r_P) &= \int_R^{r_s} l_{P0} r \,\mathrm{d}r \\ &= \left[\frac{l_{P0}^3}{3} + r_P t_0 \left\{\frac{r - r_P t_0}{2} l_{P0} + \frac{r_P^2}{2}\right.\right. \\ &\quad \left.\left.\times\left(1 - t_0^2\right)\ln 2(r - r_P t_0 + l_{P0})\right\}\right]_{r=R}^{r=r_P} \end{aligned} \tag{8c}$$

Again, similar to the case with the Bouguer shell, it is only the term $|r_P - r|$ under the integral of Eq. (7) that causes the bias in the downward continuation of the external representation of the gravitational potential. This implies that the downward-continued external type solution of Eq. (8a) becomes for $r_P = R$:

$$\left(V_P^c\right)^* = 2\pi\mu\left(\frac{I(R)}{R} + \frac{H^2}{2} + \frac{H^3}{3R}\right), \tag{9}$$

and the topographic bias becomes

$$V_{\text{bias}}^c = 2\pi\mu\left(H^2 + \frac{2H^3}{3R}\right), \tag{10}$$

which is exactly the same as for the Bouguer shell Eq. (4).

To help in understanding why the shell and cap biases are the same, consider that V_P^c can be written as

$$V_P^c = V_P^B - \left(V_P^B - V_P^c\right) \tag{11}$$

As the potential $V_P^B - V_P^c$ is generated by a Bouguer shell with a conic hole of geocentric angle ψ_0 with respect to the radial axis through point P, it follows that this potential has no topographic bias, i.e., the external potential at P can be downward-continued unbiasedly to the level $r_P = R$. Hence, it follows that $V_{\text{bias}}^c = V_{\text{bias}}^B$, and it also follows that the bias is independent of the choice of the geocentric angle $\psi_0 > 0$ of the spherical cap used.

3 The topographic bias in the general case

In the general case, we assume that the laterally variable topographic density (times gravitational constant) is $\mu = \mu(\Omega, r)$, where $\Omega = (\theta, \lambda)$ and r, θ, λ are the spherical coordinates (radius, co-latitude and longitude). Then, the topographic potential at any point P can be expressed as the Newton integral

$$V_P^t = \iint_\sigma \int_R^{r_s} \frac{\mu r^2 \mathrm{d}\, r \,\mathrm{d}\sigma}{\sqrt{r_P^2 + r^2 - 2r_P r \cos\psi}}, \tag{12}$$

where $r_s = R + H$, $H = H(\Omega)$ varies laterally (as does μ), and σ is the unit sphere. Based on Eq. (12), Proposition 1 follows.

Proposition 1 *Let P_0 denote a point at sea level (with radius R) along the geocentric radius through an arbitrary point P. Then the topographic bias at P is given by*

$$V_{\text{bias}}^t = 4\pi \int_R^{r_Q} \mu_P(r)\left(\frac{r^2}{R} - r\right) \mathrm{d}r, \tag{13}$$

where Q is the point at radius $r_Q = r_s(\Omega_P)$ along the geocentric radius through P, and $\mu_P(r) = \mu(\Omega_P, r)$.

Proof Let us decompose V_P^t in the form

$$V_P^t = \bar{V}_P^B + \delta V_P^t, \tag{14a}$$

where

$$\begin{aligned} \bar{V}_P^B &= \iint_\sigma \int_R^{r_Q} \frac{\mu_P(r) r^2 \,\mathrm{d}\, r \,\mathrm{d}\sigma}{\sqrt{r_P^2 + r^2 - 2r_P r \cos\psi}} \\ &= 2\pi \int_R^{r_Q} \mu_P(r) r^2 \int_0^\pi \frac{\sin\psi \,\mathrm{d}\psi}{\sqrt{r_P^2 + r^2 - 2r_P r \cos\psi}} \mathrm{d}r \end{aligned} \tag{14b}$$

is the contribution from a Bouguer shell with a radial symmetric density $\mu_P(r)$, and δV_P^t is the residual topographic potential generated by the remaining topographic masses. Equation (14b) can be further simplified to

$$\bar{V}_P^B = 2\pi \int_R^{r_Q} \frac{\mu_P(r)}{r_P}(r_P + r - |r_P - r|) r \,\mathrm{d}r, \tag{15}$$

from which the external case solution becomes

$$\bar{V}_P^B = \frac{4\pi}{r_P} \int_R^{r_Q} \mu_P(r) r^2 \, \mathrm{d}r, \quad r_P \geq r_Q, \tag{16}$$

and for $r_P \leq R$(internal case), the solution becomes

$$\bar{V}_P^B = 4\pi \int_R^{r_Q} \mu_P(r) r \, \mathrm{d}r, r_P \leq R. \tag{17}$$

Hence, the topographic bias at sea level with $r_P = R$ becomes

$$V_{\mathrm{bias}}^t = \left(\bar{V}_P^B\right)^* + \left(\delta V_P^t\right)^* - \bar{V}_{r_P=R}^B - \delta V_{r_P=R}^t, \tag{18}$$

and, as there are no topographic masses along the radial axis at P after the removal of the Bouguer shell, it must hold that

$$\left(\delta V_P^t\right)^* = \delta V_{r_P=R}^t. \tag{19}$$

Hence, Eqs. (16), (17) and (19) inserted into Eq. (18) yield Eq. (13) as postulated in the proposition. □

Corollary 1 *If the topographic density along radius vector at P is $\mu_P(r) = \mu_P$ =constant, then the topographic bias becomes*

$$V_{\mathrm{bias}}^t = 2\pi\mu_P \left(H^2 + \frac{2H^3}{3R}\right), \tag{20}$$

where H is the topographic height at the computation point. This result follows directly from the proposition.

Corollary 2 *The topographic bias at any point P with $R \leq r_P < r_Q$ becomes*

$$(V_{r_P}^t)_{\mathrm{bias}} = 4\pi \int_{r_P}^{r_Q} \mu_P(r) \left(\frac{r^2}{r_P} - r\right) \mathrm{d}r. \tag{21}$$

In particular, for $\mu_P(r) = \mu_P$ = constant, it follows that

$$\left(V_P^t\right)_{\mathrm{bias}} = 2\pi\mu_P \left[(\Delta H_P)^2 + \frac{2(\Delta H_P)^3}{3r_P}\right], \tag{22a}$$

where

$$\Delta H_P = \begin{cases} r_Q - r_P & \text{if } r_Q > r_P \\ 0 & \text{otherwise} \end{cases}. \tag{22b}$$

The corollary follows directly from the proposition.

Note The important Eq. (19) can be seen as a result of that, after the removal of the Bouguer shell, there is a hole of infinitesimal width in the topography from the surface down to sea level. Along this hole δV_P^t obeys Laplace's equation, and Eq. (19) follows.

4 Implications in physical geodesy

As will be shown next, the topographic bias is of basic importance in physical geodesy. Two applications will be presented: geoid determination by an Earth gravity model, and geoid determination by Stokes's formula. In both cases, the external type of harmonic representation of the geopotential or gravity anomaly needs to be downward continued to the geoid.

4.1 Application to an Earth gravity model

In applying the external type representation of the geopotential to the continental geoid, the correction for the topographic bias is the negative of Eq. (13). In the case of a spherical harmonic Earth gravity model, the harmonic representation of the external gravity field is truncated at some upper degree and order $n_{\max}$. Consequently, the geoid correction δN_P^t for the topographic bias in such a satellite-derived spherical harmonic model of the geopotential will also be limited to the degree of truncation, i.e.,

$$\delta N_P^t = -\frac{2\pi}{\gamma} \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left[\left(\mu H^2\right)_{nm} + \frac{2\left(\mu H^3\right)_{nm}}{3R}\right] Y_{nm}(P), \tag{23}$$

where Y_{nm} is the fully normalized spherical harmonic function of degree n and order m with coefficients

$$\left(\mu H^\nu\right)_{nm} = \frac{1}{4\pi} \iint_\sigma \mu H^\nu Y_{nm} \, \mathrm{d}\sigma, \tag{24}$$

and γ is normal gravity on the reference ellipsoid. Again, we have assumed that the topographic density (μ) is only laterally variable.

Sjöberg (1977) computed the coefficients normalized according to $R\mu_0 H_{nm}^2/(4\pi GM)$, where R is the mean Earth radius, $\mu_0 = G\rho_0, \rho_0 = 2{,}670\,\mathrm{kg/m^3}$ is a constant topographic density and GM is the geocentric gravitational constant, to $n_{\max} = 16$. Martinec (1998, Sect. 7.4) computed the coefficients of Y_{nm} of Eq. (23) for the constant density ρ_0 to degree and order 20. If the series goes to infinity, the topographic geoid correction becomes

$$\delta N_P^t = -\frac{2\pi\mu_P}{\gamma} \left(H^2 + \frac{2H^3}{3R}\right), \tag{25}$$

which agrees with *Corollary* 1. The magnitude of δN_P^t increases rapidly with topographic elevation. For Mt. Everest it is of the order of -9 m.

4.2 Application to Stokes's formula

Stokes's formula for geoid determination, extended with possible zero- and first-degree harmonic terms of the disturbing potential (T_0 and T_1), can be written (e.g., Sjöberg 2000)

$$N = \frac{T_0}{\gamma} + \frac{T_1}{\gamma} + \frac{R}{4\pi\gamma} \iint_\sigma S(\psi)\Delta g^* \,\mathrm{d}\sigma + \delta N^t_{\mathrm{comb}}, \quad (26)$$

where $\delta N^t_{\mathrm{comb}}$ (the combined topographic effect) is the sum of the direct topographic effect and the (primary) indirect effect on the geoid. The integral of Eq. (26) is the original Stokes formula with the surface gravity anomaly Δg downward-continued to sea level (denoted Δg^*). This integral implies a biased representation of the geoid height due to the analytical continuation of the gravity anomaly through the topography, and the bias is compensated by the combined topographic effect.

Using Bruns's formula, we can thus postulate that the combined topographic effect on the geoid height becomes

$$\delta N^t_{\mathrm{comb}} = -\frac{V^t_{\mathrm{bias}}}{\gamma}. \quad (27)$$

Equation (27) can be verified as follows. As the direct topographic effect is the negative of Eq. (26) with $T_0 = (V^t_P)_0 = \bar{V}^B_P + \left(\delta V^t_P\right)_0$, $T_1 = (V^t_P)_1 = (\delta V^t_P)_1$, where the subscripts denote degrees of harmonics, and $(\Delta g)^* = \left(\Delta g^t\right)^*$, this implies that Stokes' integral for the direct effect becomes

$$\begin{aligned}\delta N^t_{\mathrm{dir}} &= -\frac{\left(V^t_P\right)^*_0 + \left(V^t_P\right)^*_1}{\gamma} - \frac{R}{4\pi\gamma}\iint_\sigma S(\psi)\left(\Delta g^t\right)^* \mathrm{d}\sigma \\ &= -\frac{\left(V^t_P\right)^*}{\gamma} = -\frac{\left(\bar{V}^B_P\right)^* + \left(\delta V^t_P\right)^*}{\gamma}, \end{aligned} \quad (28)$$

where the integral contributes with all harmonics from degree 2 to infinity.

Here we have used the fact that $r\Delta g^t$ (based on the boundary condition of physical geodesy; Heiskanen and Moritz (1967), Sect. 2.14) is harmonic exterior to the topography and also when downward-continued (by an harmonic operation) to sea level, and $(\Delta g^t)^*$ therefore satisfies Stokes's formula (in contrast to the real gravity anomaly inside the topographic masses).

Considering Eq. (19), we obtain

$$\delta N^t_{\mathrm{dir}} = -\frac{\left(\bar{V}^B_P\right)^* + \delta V^t_{r_P=R}}{\gamma}, \quad (29)$$

and by adding the direct effect and indirect effect given by

$$\delta N_I = \frac{V^t_{r_p=R}}{\gamma} = \frac{\bar{V}^B_{r_P=R} + \delta V^t_{r_P=R}}{\gamma}, \quad (30)$$

we finally arrive at:

$$\delta N^t_{\mathrm{comb}} = \delta N^t_{\mathrm{dir}} + \delta N^t_I = \frac{-\left(\bar{V}^B_P\right)^* + \bar{V}^B_{r_P=R}}{\gamma} = -\frac{V^t_{\mathrm{bias}}}{\gamma}, \quad (31)$$

and we have thus proved Eq. (27)

In the past, Sjöberg (1977, 1996a,b, 2000, 2001) and Ågren (2004) presented the combined topographic geoid effect as a power series of topographic height with the first two terms of this series represented by Eq. (20). The present study proves that only these terms contribute to the effect, while all higher power terms vanish. Importantly, only local topographic data is needed to compute the effect. In other words, as suggested in Sjöberg (2000, 2001), all other topographic contributions to the direct and indirect topographic effects cancel in the combined effect. This cancellation does not completely occur when one considers only the masses outside the local cap, as shown in Smith (2002). Nor was this the case in Ågren (2004), whose spherical harmonic approach turned out to be very unstable at higher degrees.

It should be emphasized that the application of Eq. (26), i.e., Stokes's formula with additive corrections (Sjöberg 2003a), is not the standard method of using Stokes's formula. The direct downward continuation of the unreduced gravity anomaly to Δg^* is usually an unstable procedure, but Sjöberg (2003b) avoids this reduction by using a method for directly computing its effect as an additive correction to the preliminary geoid height.

5 Concluding remarks

We have derived an exact formula for the topographic bias or combined topographic effect in gravimetric geoid determination. This bias is purely local, and the formula is remarkably simple for a constant or only laterally variable topographic density model, as it is then proportional only to terms of second and third power of topographic height of the computation point (Corollary 1).

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