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Short Note: Asymptotic theory for calculating deformations caused by dislocations buried in a spherical earth — gravity change

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Abstract. An asymptotic theory is presented for calculating co-seismic gravity changes, as a complement to a previous article by Sun [J Geod (2003) 77:381–387)], which presents the asymptotic theory for co-seismic geoid changes. Only the important formulas and results are summarized here; all the concepts and conventions are the same as for the companion paper. The theory is given by closed mathematical expressions so that it can be applied as easily as the flat-Earth theory. Moreover, since the asymptotic theory includes a sphericity effect, it is physically more reasonable than the flat-Earth theory.

Key words: Dislocation theory – Asymptotic – Co-seismic gravity change – Spherical Earth

Introduction

The current dislocation theories concerning different Earth models (Okada 1985; Okubo 1991, 1992; Sun and Okubo 1993) have different advantages and disadvantages. For the half-space Earth model, the corresponding theories are mathematically simple and can be easily applied. However, the disadvantage is that the Earth model is physically too simple to reflect the sphericity and stratified structure of the Earth. On the other hand, theories for spherical Earth models are physically better, but are mathematically tedious due to numerical integration and summations. To overcome the disadvantages of the two cases, Sun (2003, in press) presents new theories for calculating asymptotic displacements and geoid changes excited by a point dislocation in a spherical symmetric Earth model as an approximation of the dislocation theories by Sun and Okubo (1993) and Sun et al. (1996).

In this study, gravity change caused by a point dislocation is investigated, and a set of asymptotic formulas for calculating gravity changes caused by four independent sources is presented. The validity of the asymptotic theory at near and regional distances from the source is assessed by numerical comparison with results for flat-Earth and exact spherically symmetric theories. This study is a complement to that of Sun (2003).

Co-seismic gravity changes

A dislocation usually causes a density change and a displacement at any point on the Earth, causing gravity changes. The gravity change on the Earth's surface is composed of three components: a gravity change δ_{g_1} due a potential change caused by a global mass redistribution, a gravity change δ_{g_2} relating to a surface displacement (Bouguer shell), and a gravity change δ_{g_3} coming from the free-air correction. The first component δ_{g_1} can be obtained from the potential change Ψ as (Sun and Okubo 1993)

$$\delta_{g_1}(a,\theta,\varphi) = \sum_{n,m,i,j}(n+1)k_{nm}^{ij}Y_n^m(\theta,\varphi).v_i n_j \frac{g_0 U \mathrm{d}S}{a^3} - 4\pi G\rho u_r(a,\theta,\varphi)$$

where G is Newton's gravitational constant, a is the Earth's radius, and ρ is the surface density. u_r is the vertical displacement on the Earth's surface and U is defined as dislocation movement on the fault plane dS.

The gravity change caused by the Bouguer shell on the Earth's surface is given by $\delta_{g_2}(a,\theta,\varphi) = 4\pi G\rho u_r(a,\theta,\varphi)$. Note that $\delta_{g_2}(a,\theta,\varphi)$ equals the last term in $\delta_{g_1}(a,\theta,\varphi)$ with an opposite sign; they will cancel when summed. The free-air correction to gravity change on the Earth's surface is expressed by $\delta_{g_3}(a,\theta,\varphi) = -\beta u_r(a,\theta,\varphi)$, where β is the linear free-air gravity gradient, which can be spherically approximated by $\beta = 2g_0/a$. Finally, summing up the above three gravity components gives the total co-seismic gravity change on the deformed surface of the Earth as

$$\delta_g(a,\theta,\varphi) = \sum_{n,m,i,j} [(n+1)k_{nm}^{ij} - 2h_{nm}^{ij}] Y_n^m(\theta,\varphi).v_i n_j \frac{g_0 U \mathrm{d}S}{a^3}, \quad (1)$$

Equation (1) indicates that the co-seismic gravity change occurs along with a potential change and vertical displacement. Calculation of gravity changes is composed of two steps: computing dislocation Love numbers and summing them up. The dislocation Love numbers h_{nm}^{ij} and k_{nm}^{ij} are usually obtained by numerical integration (Sun and Okubo 1993), and then the gravity change can be calculated by infinite summations in Eq. (1). Theoretically and technically, these calculations are performable and determinable (Sun and Okubo 1993). However, the numerical work involved is very tedious and greatly constrains its practical application. To overcome this difficulty, this study uses Okubo's reciprocity theorem (Okubo's 1993) and asymptotic theory (Okubo 1988) so that the co-seismic gravity change can be given in a closed mathematical form.

Asymptotic gravity changes

Inserting the asymptotic Love numbers (Sun and Okubo 1993; Sun in press) into Eq. (1), one obtains analytical expressions of the co-seismic gravity changes for the four types of dislocation sources, with some simple but cumbersome mathematical work, as

$$\begin{aligned}\delta_g^{12}(a,\theta,\varphi) = 2\sin 2\varphi \Bigg\{ & \left(y_{130}^{12} - 2y_{230}^{12}\right)\left[\frac{2c}{w^3} - \frac{1}{w^5}\left(2c - \varepsilon\left(3 + c^2 - 2\varepsilon c\right)\right)\right] \\ & + \left(y_{130}^{12} + y_{131}^{12} - 2y_{231}^{12}\right)\frac{1}{\varepsilon}\left[\frac{2}{s^2} - 1 - \frac{1-\varepsilon c}{w^3} + \frac{2c(\varepsilon - c)}{s^2 w}\right] \\ & + \left(y_{131}^{12} + y_{132}^{12} - 2y_{232}^{12}\right)\frac{1}{\varepsilon}\left[1 - \frac{1}{w} - \frac{2c}{\varepsilon s^2}(1 - \varepsilon c - w)\right] \\ & + \left(y_{131}^{12} + 2y_{132}^{12} - 2y_{232}^{12}\right)\Bigg[-\frac{1}{\varepsilon}\left(1 - \frac{1}{w} + \frac{\varepsilon(\varepsilon - c)}{w^3} + \frac{2c}{s^2}\left(c + \frac{\varepsilon - c}{w}\right)\right) \\ & + \frac{\varepsilon s^2}{2w^3}\left(1 + \frac{2w^3 + 2w^2 + \varepsilon c}{1 - \varepsilon c + w} + \frac{\varepsilon c w (1+w)^2}{(1 - \varepsilon c + w)^2}\right) \\ & + \frac{c}{\varepsilon^2 s^2} + \frac{1}{2\varepsilon^2 s^2 w}\left(\varepsilon\left(1 + c^2\right) - 2c + \frac{\varepsilon s^2}{w^2}(\varepsilon c - 1)\right)\Bigg]\Bigg\}.\frac{g_o U \mathrm{d}S}{a^3} \quad (2)\end{aligned}$$

$$\begin{aligned}\delta_g^{32}(a,\theta,\varphi) = 2\sin\varphi \Bigg[& \left(y_{140}^{23} - 2y_{240}^{23}\right)\frac{s\left(1 + \varepsilon c - 2\varepsilon^2\right)}{\varepsilon w^5} \\ & + \left(y_{140}^{23} + y_{141}^{23} - 2y_{241}^{23}\right)\frac{s}{\varepsilon w^3} \\ & + y_{141}^{23}\frac{1+w}{\varepsilon w(1 - \varepsilon c + w)}\Bigg].\frac{g_o U \mathrm{d}S}{a^3} \quad (3)\end{aligned}$$

$$\begin{aligned}\delta_g^{22,0}(a,\theta,\varphi) = \Bigg\{ & \left[\left(y_{120}^{22} + y_{130}^{22}\right) - 2\left(y_{220}^{22} + y_{230}^{22}\right)\right]\frac{1}{w^5}\left[c(1-\varepsilon^2) + \varepsilon\left(c^2 - 2 + \varepsilon^2\right)\right] \\ & + \left[\left(y_{110}^{22} + y_{120}^{22} + 2y_{130}^{22} + y_{131}^{22}\right) - 2\left(y_{210}^{22} + y_{221}^{22} + y_{230}^{22} + y_{231}^{22}\right)\right]\frac{1}{w^3}(c - \varepsilon) \\ & + \left[\left(y_{110}^{22} + y_{111}^{22} + 2y_{131}^{22} + y_{132}^{22}\right) - 2\left(y_{211}^{22} + y_{222}^{22} + y_{231}^{22} + y_{232}^{22}\right)\right]\frac{1}{\varepsilon w} \\ & + \left[\left(y_{111}^{22} + y_{131}^{22} + y_{132}^{22}\right) - 2\left(y_{212}^{22} + y_{232}^{22}\right)\right]\frac{1}{\varepsilon}(\ln 2 - \ln(1 - \varepsilon c + w)) \\ & + y_{132}^{22}\left[-\frac{1}{2\varepsilon^2}(\ln(w + \varepsilon - c) - \ln(1-c) - \varepsilon) + \frac{1}{2\varepsilon}(1 - w) - \frac{c}{2}\ln(1 - \varepsilon c + w) + \frac{c}{2}(\ln 2 - 1)\right]\Bigg\}.\frac{g_o U \mathrm{d}S}{a^3} \quad (4)\end{aligned}$$

$$\begin{aligned}\delta_g^{33}(a,\theta,\varphi) = \Bigg\{ & \left(y_{120}^{33} - 2y_{220}^{33}\right)\frac{1}{\varepsilon w^5}\left[c\left(1 - \varepsilon^2\right) + \varepsilon\left(c^2 - 2 + \varepsilon^2\right)\right] \\ & + \left(y_{120}^{33} - 2y_{221}^{33}\right)\frac{1}{\varepsilon w^3}(c - \varepsilon) \\ & + \left(-2y_{222}^{33}\right)\frac{1}{\varepsilon^2 w}\Bigg\}.\frac{g_o U \mathrm{d}S}{a^3} \quad (5)\end{aligned}$$

where $w = (1 - 2\varepsilon\cos\theta + \varepsilon^2)^{1/2}, c = \cos\theta, s = \sin\theta, \varepsilon = r_s/a$, and the symbols y_{kmn}^{ij} are constants determined by an Earth model (See Appendix). Note that by using the asymptotic technique, co-seismic gravity changes can be calculated much more easily because integrations and summations are no longer needed. Numerical results in Sect. 4 show that asymptotic gravity changes describe the co-seismic deformation field well. Equations (2)–(5)

show the main results of this research. They can be used to calculate gravity changes excited by four types of sources buried in a spherically symmetric Earth model. Once an Earth model and source parameters are provided, the gravity change can be calculated analytically. This is the merit of asymptotic solutions. The above results are given for a point source on a polar axis. In combination, these components allow calculation of a gravity change caused by an arbitrary seismic source. Co-seismic gravity change for a limited fault can be obtained easily by integration of the point dislocations over the fault plane.

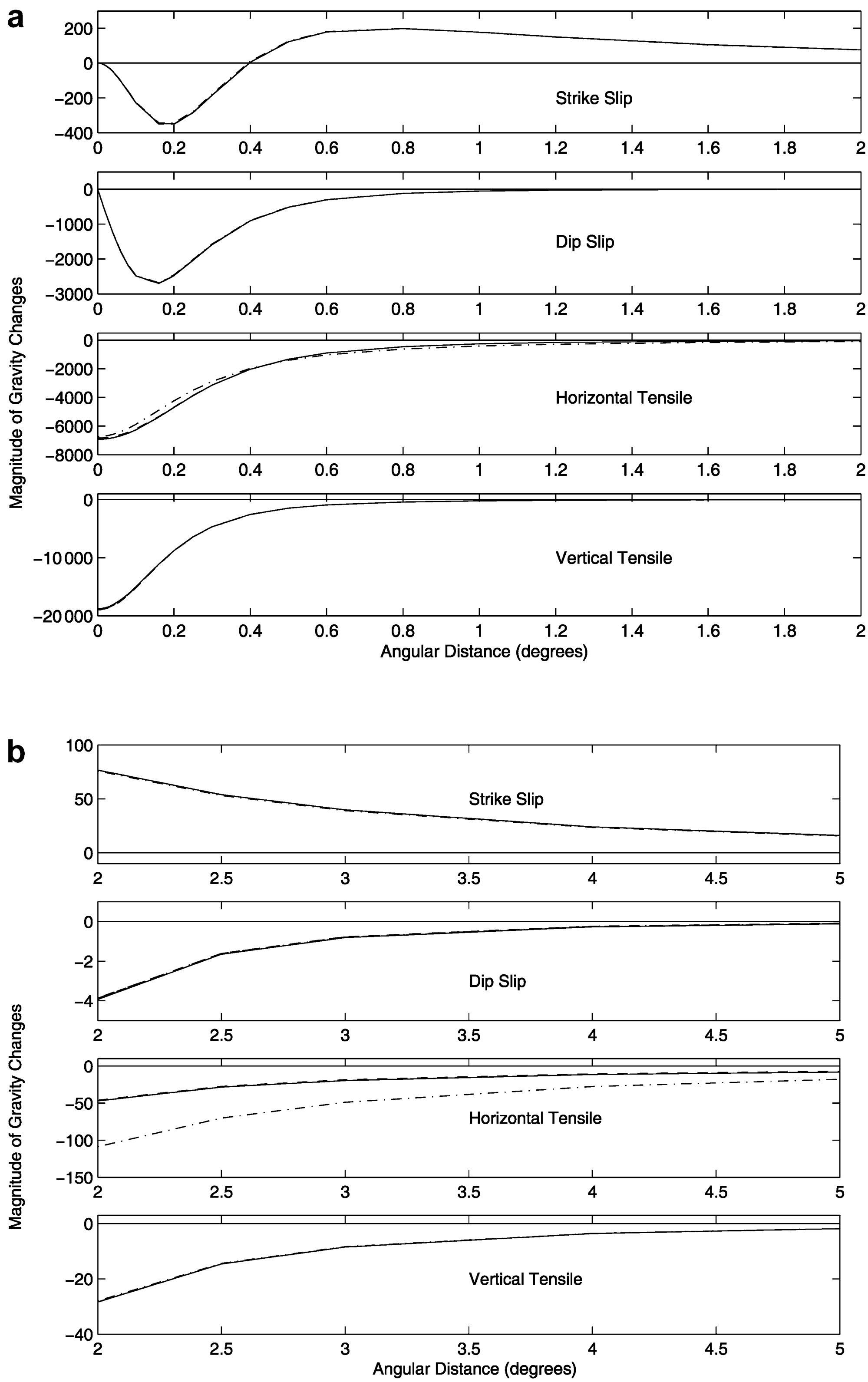

Fig. 1. Comparison of co-seismic gravity changes within angular distance of **a** 0–2 degrees and **b** 2–5 degrees, caused by four types of dislocation sources at a depth of 32 km in a homogeneous Earth model, calculated by using three dislocation theories: a flat-Earth (*dash–dot line*); a spherically symmetric Earth (*dashed line*); and the asymptotic theory for a spherical Earth (*solid line*)

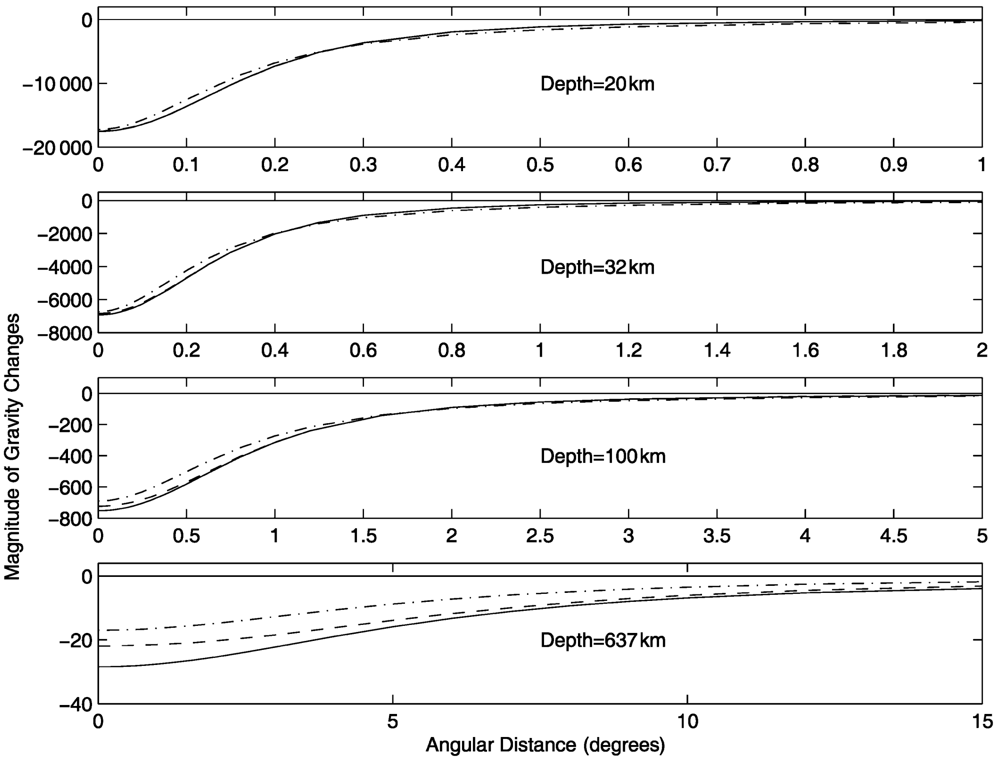

Fig. 2. Co-seismic gravity changes caused by a horizontal source on a vertical fault, for four source depths, 20, 32 100 and 637 km, calculated by using the three dislocation theories for flat Earth and an homogeneous sphere.

Numerical test

A numerical test is carried out to verify the asymptotic theory, considering a homogenous sphere with identical parameters to the 1066 A top layer (Gilbert and Dziewonski 1975). The constants y_{kmn}^{ij} (See Appendix) are computed for the homogenous sphere. Assuming a point source at a depth of 32 km in the homogenous sphere, co-seismic gravity changes (Fig. 1) are calculated using numerical summations (Sun and Okubo, 1993) and the above asymptotic formulations. This is repeated for four types of point sources: (1). vertical strike-slip (when $\sin 2\varphi = 1$ and $\cos 2\varphi = 1$); (2). vertical dip-slip (when $\sin \varphi = 1$ and $\cos \varphi = 1$); (3). horizontal opening along a vertical fault; and (4). vertical opening along a horizontal fault. Note that the calculated gravity changes are normalized by the factor $g_o U dS/a^3$. Figure 1 gives results for two epicentral distance segments: 0–2° and 2–5°; it can be seen that the asymptotic results approximate well the true (integrated) results. The discrepancy between them for most of the components cannot be identified, since they overlap, except for the difference between the flat-Earth and other two models for the horizontal-tensile dislocation source.

For comparison, gravity changes calculated by using the theory of a flat Earth (Okubo 1991) are also plotted in Fig. 1 (dash–dot line). The comparison shows that the asymptotic results approximate the exact results better than the flat-Earth results, since the discrepancy between them becomes larger as the epicentral distance increases; it even appears in the near field for the horizontal opening on a vertical fault (Fig. 1). This is expected because of the neglection of sphericity in the flat-Earth model.

To investigate the effect of sphericity, co-seismic gravity changes of the three theories for different source depths (20, 32, 100 and 637 km) are plotted in Fig. 2. It is seen that the asymptotic results agree closely with the integrated ones, but the flat Earth always shows large discrepancies. Since the asymptotic gravity changes include a sphericity effect, the asymptotic theory for a homogeneous sphere is theoretically and empirically more reasonable than a flat-Earth model. Furthermore, the asymptotic theory is mathematically simple, so that its practical application becomes as simple as the theory for a flat-Earth. Note that a relatively large discrepancy appears for the case of 637 km; this is understandable because the contribution of low-degree dislocation Love numbers becomes greater as the depth increases. However, the asymptotic results remain better that those of a flat Earth.

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Appendix

Expressions for y_{ijk}^{lm} used in Eqs. (2)–(5)

The symbols y_{ijk}^{lm} used in this paper are obtained from Okubo's asymptotic theory (Okubo 1988), and can be determined by an Earth model. This discussion is limited to a homogeneous sphere without losing generality. The superscripts and subscripts *ijkmn* of the constants y_{kmn}^{ij} have the following meanings: *ij* indicate the four independent dislocation solutions; *k* stands for the three kinds (tide, press and shear) of asymptotic solutions; *m* denotes the six members of each asymptotic solution; and *n* gives the coefficients of power *n* ranked from high to low.

$$y_{230}^{12} = -\frac{\mu d}{16\pi\rho\beta^2 r_s}(-2+\delta)$$

$$y_{231}^{12} = -\frac{a\mu}{16\pi\rho\beta^2 r_s}\left[2M + \delta\left(2\frac{\alpha t}{\beta} - 3\Lambda\right)\right]$$

$$y_{232}^{12} = -\frac{a\mu}{16\pi\rho\beta^2 r_s}\left(1 + \frac{1}{N}(2T-6)\right)$$

$$y_{130}^{12} = \frac{Ga\mu d}{4g_0\beta^2 r_s}\left(1 - \frac{\delta P}{2}\right)$$

$$y_{131}^{12} = \frac{Ga\mu d}{4g_0\beta^2 r_s}\left(1 - \frac{\alpha t}{\beta}\right)$$

$$y_{132}^{12} = \frac{Ga^2\mu}{8g_0\beta^2 r_s}(4-T)$$

$$y_{240}^{23} = -\frac{d}{8\pi a}(-2+\delta)$$

$$y_{241}^{23} = -\frac{d}{8\pi a}\left(-1 - 2M + \frac{2\alpha t}{\beta}(1-M)\right)$$

$$y_{140}^{23} = \frac{1}{2g_o}G\rho d\left[1 - \frac{\delta P}{2}\right]$$

$$y_{141}^{23} = -\frac{G\rho d N\alpha t}{2g_0\beta}$$

$$y_{210}^{22} = -\frac{3k\mu d}{4\pi\rho\beta^2\sigma r_s}(-2+\delta)$$

$$y_{211}^{22} = -\frac{3ak\mu}{4\pi\rho\beta^2\sigma r_s}\left(-\frac{2}{N} + \frac{2\alpha t\delta}{\beta}(2-\Lambda) + \delta\Lambda\right)$$

$$y_{212}^{22} = -\frac{3ak\mu}{4\pi\rho\beta^2\sigma r_s}\left[\frac{2\alpha t}{\beta N}(2-\Lambda) + 2M^2\left(\frac{\alpha t}{\beta} - 1\right)\right]$$

$$y_{220}^{22} = -\frac{\lambda d}{4\pi\sigma r_s}(-2+\delta)$$

$$y_{221}^{22} = -\frac{\lambda a}{4\pi\sigma r_s}[-2 + \delta(5-2T)]$$

$$y_{222}^{22} = \frac{\lambda a}{4\pi\sigma r_s}$$

$$y_{230}^{22} = \frac{3k\mu d}{8\pi\rho\beta^2\sigma r_s}(-2+\delta)$$

$$y_{231}^{22} = \frac{3aK\mu}{8\pi\rho\beta^2\sigma r_s}\left[2M + \delta\left(\frac{2\alpha t}{\beta} - 3\Lambda\right)\right]$$

$$y_{232}^{22} = \frac{3aK\mu}{8\pi\rho\beta^2\sigma r_s}\left[1 + \frac{1}{N}(2T-6)\right]$$

$$y_{110}^{22} = \frac{3aGK\mu d}{2g_0\beta^2\sigma r_s}(2 - \delta P)$$

$$y_{111}^{22} = \frac{3a^2GK\mu}{2g_0\beta^2\sigma r_s}\left[2 + \delta\left(2P - 4 - \frac{2\alpha t}{\beta}\right)\right]$$

$$y_{112}^{22} = \frac{3a^2GK\mu}{2g_0\beta^2\sigma r_s}\left(-2 + T - \frac{2\alpha t}{\beta}\right)$$

$$y_{120}^{22} = \frac{G\lambda\rho d}{g_0\sigma}\left(1 - \frac{\delta P}{2}\right)$$

$$y_{130}^{22} = -\frac{3GaK\mu d}{2g_0\beta^2\sigma r_s}\left(1 - \frac{\delta P}{2}\right)$$

$$y_{131}^{22} = -\frac{3GaK\mu d}{2g_0\beta^2\sigma r_s}\left(1 - \frac{\alpha t}{\beta}\right)$$

$$y_{132}^{22} = -\frac{3Ga^2K\mu}{4g_0\beta^2\sigma r_s}(4-T)$$

$$y_{220}^{33} = -\frac{\delta}{4\pi}(-2+\delta)$$

$$y_{221}^{33} = -\frac{1}{4\pi}[-2 + \delta(5-2T)]$$

$$y_{222}^{33} = \frac{1}{4\pi}$$

$$y_{120}^{33} = \frac{G\rho d}{g_0}\left(1 - \frac{\delta P}{2}\right)$$

where a is the radius of the Earth, α and β are the P- and S-wave velocities, respectively, λ and μ are Lame's constants and d is the source depth. $\delta = d/a$, $\sigma = \lambda + 2\mu$, $K = \lambda + 2\mu/3$, $N = 1 - \beta^2/\alpha^2$, $M = \beta^2/(\alpha^2 - \beta^2)$, $\Lambda = (\alpha^2 + \beta^2)/(\alpha^2 - \beta^2)$, $\gamma = 4\pi G\rho/3$, $t = g_0 a/2\alpha\beta$, $T = (\alpha + \beta t)\alpha/(\alpha^2 - \beta^2)$ and $P = 1 + \beta t/\alpha$.

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