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Period variations of the Chandler wobble

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Abstract. Variations in the period of the Chandler wobble have been discussed since its discovery by Chandler in 1892. Various authors engaged in the investigation of polar motion time series suggest both a variable and an invariable period. It cannot be resolved by the analysis of time series whether the Chandler period is variable. By studying the influence of mass redistributions on the Chandler period it has been found that it is in fact variable, but the magnitude of such variation is much smaller than that found by polar motion time series analysis. For the currently available time series of polar motion, it is sufficient to assume an invariable Chandler period.

Keywords: Polar motion – Chandler period – Geophysical processes – Mass redistributions

1 Introduction

The temporal variation of the Chandler period has been discussed since its discovery by Chandler in 1892. Chandler (1892) suggested a variable period increasing by ca. 2 months within a century. Newcomb (1892), who had explained the difference between the Eulerian (306 days) and Chandler period (435 days), replied that, perturbations aside, any variation of the period is in such direct conflict with the laws of dynamics that it can be considered impossible. These contradictory points of view have been discussed by several authors engaged in the analysis of polar motion time series. A number of these (e.g. Okubo 1982; Kuehne and Wilson 1996; Vicente and Wilson 1997) accepted and provided evidence for a invariable Chandler period, while others (e.g. Melchior 1954, 1957; Sekiguchi 1972, 1976; Carter 1981) suggested a temporally variable period.

Recently, the increased accuracy of polar motion data sets has inspired new efforts to study the characteristics of polar motion and some authors (Liu et al. 2000; Schuh et al. 2001; Hoepfner 2002, submitted) have found a temporal variation of the Chandler period.

These different and partly contradictory results show that analyses of polar motion time series are not suitable to decide whether the Chandler period changes with time. Subsequently, the Chandler period will not be treated as a quality of a polar motion time series, but as an eigenvalue of the differential equations of polar motion.

2 The Chandler frequency and mass redistributions of the Earth

The polar motion of an Earth model passively coupled to a fluid core is described by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} - i\sigma_{\mathrm{CH}} m = -i\sigma_{\mathrm{CH}}\psi \tag{1}$$

in complex notation, where the circular frequency of the Chandler wobble is given by

$$\sigma_{\mathrm{CH}} = \frac{C - \frac{A+B}{2}}{A_M}\omega_0 \left(1 - \frac{D_{11} - iE_{11}}{C - \frac{A+B}{2}}\right) \tag{2}$$

In Eq. (1) the pole coordinates are

$$m = m_1 + im_2 \tag{3}$$

where $i = \sqrt{-1}$ and the excitation function ψ is given by

$$\psi = \frac{A}{A_M}\left(\frac{c}{C-A} + \frac{h}{(C-A)\omega_0} - \frac{i}{\omega_0}\left(\frac{c}{C-A} + \frac{h}{(C-A)\omega_0}\right)\right) \tag{4}$$

In Eq. (2) A, B, C, and A_M are the principal moments of inertia of the Earth and the Earth's mantle respectively. D_{11} and E_{11} are parameters depending upon the rheology of the Earth (see Moritz and Mueller 1989). The principal moments of inertia depend upon the distribution of the Earth's masses, and if this distribution varies over time a corresponding modulation of the

Chandler frequency will also take place. Since the mass redistribution caused by geophysical processes mainly influences the excitation function by variations of the products of inertia

$$c_{13} = \int\limits_M x_1 x_3 \,\mathrm{d}m; \quad c_{23} = \int\limits_M x_2 x_3 \,\mathrm{d}m \tag{4a}$$

[in Eq. (4) combined in the complex quantity c], a similar variation of the principal moments of inertia written as

$$A = \int\limits_M (x_1^2 + x_3^2)\,\mathrm{d}m; \quad B = \int\limits_M (x_2^2 + x_3^2)\,\mathrm{d}m;$$
$$C = \int\limits_M (x_1^2 + x_2^2)\,\mathrm{d}m \tag{5}$$

can be supposed. This contradicts the statement of Newcomb and apparently supports the assertions of authors claiming a variable Chandler period. However, the question remains whether the variations found in analyses of polar motion time series are physically reasonable.

In addition to a temporal variation in the principal moments of inertia, the Chandler frequency could vary because of changes in the coefficients D_{11} and E_{11}. Subsequently, the influence of only mass redistributions will be considered, since it can be assumed that globally distributed variations in rheology do not take place over shorter time scales.

In the following discussions, it is necessary to take into account the relationship between the components of the inertia tensor and the second degree Stokes coefficients, which read for fully normalized spherical harmonics (see e.g. Hopfner 1933, Lambeck 1980)

$$\begin{aligned} C_{20} &= \frac{1}{\sqrt{5}}\left(\frac{\frac{A+B}{2} - C}{Ma^2}\right), \\ C_{21} &= -\sqrt{\frac{3}{5}}\frac{c_{13}}{Ma^2}, \quad S_{21} = -\sqrt{\frac{3}{5}}\frac{c_{23}}{Ma^2} \\ C_{22} &= \frac{1}{2}\sqrt{\frac{3}{5}}\frac{B-A}{Ma^2}, \quad S_{22} = -\sqrt{\frac{3}{5}}\frac{c_{12}}{Ma^2} \end{aligned} \tag{6}$$

Generally, the coefficients A, B, and C in Eq. (6) are not principal moments of inertia because they are related to the axes of the reference system used for determining the Stokes coefficients. Only when the coefficients C_{21}, S_{21}, and S_{22} are zero will the axes of the defined coordinate system coincide with the axes related to the principal moments of inertia. In the case of the gravity field, the x_3 axis deviates little from the axis related to the principal moment of inertia C. Therefore, C obtained from Eq. (6) can be considered the principal moment of inertia. The axes in the equatorial plane are rotated at a larger rate with respect to the axes of the principal moments of inertia, but because rotations in the equatorial plane do not change the sum $A+B$, ΔC_{20} does not vary due to these rotations. These conditions allow the following relationships between variations in ΔC_{20} and the real part of the Chandler frequency to be written

$$\begin{aligned} \Delta\sigma_{\mathrm{CH}} &= \frac{\Delta C - \frac{\Delta A + \Delta B}{2}}{A_M}\left(\frac{1 - D_{11}}{C - A}\right)\omega_0 \\ &= -\sqrt{5}\frac{Ma^2}{A_M}\Delta C_{20}\left(1 - \frac{D_{11}}{C-A}\right)\omega_0 \\ &= \omega_0 \Delta f_{\mathrm{CH}} \end{aligned} \tag{7}$$

Equation (7) allows the modulation of the Chandler frequency caused by geophysical processes to be studied. A convenient way could be the use of the temporal variations in ΔC_{20} given by gravity field representations as measured by modern satellite missions (e.g. CHAMP and GRACE). If information dealing with geophysical processes allows the determination of the accompanying mass redistribution, the temporal variation in the second-degree Stokes coefficient is obtained according to

$$\Delta C_{20} = \frac{1}{4\sqrt{5}Ma^2}\int\limits_V r^2 \Delta\rho(r, \varphi, \lambda, t)(1 - \cos^2\varphi)\,\mathrm{d}V \tag{8}$$

where the mass redistribution is described by the temporal variation of the local densities ρ.

3 The influence of different geophysical processes on the Chandler period

The influence of secular seasonal and tidal mass redistributions on the length of the Chandler period will now be discussed. Secular variations of mass redistributions are caused by different geophysical processes, e.g. land uplift due to glacial rebound, and water mass redistributions between the continents and oceans. These processes cause a secular increase in the C_{20} component . In Marchenko and Schwintzer (2003), the time rates of change are given for different gravity field models. The most accurate value is

$$\frac{\mathrm{d}C_{20}}{\mathrm{d}t} 10^{11}\ \mathrm{year}^{-1} = 1.1655 \pm 0.04$$

It can be assumed that this value represents the secular variation of C_{20} because it only changes by 10^{-14} between 1986 and 1997. Following Eq.(7) the secular variation of the Chandler frequency is given by

$$\frac{\mathrm{d}\sigma_{\mathrm{CH}}}{\mathrm{d}t} = 6.293 \cdot 10^{-11}\omega_0\ \mathrm{year}^{-1}$$

where the corresponding change of the period is

$$\frac{\mathrm{d}T_{\mathrm{CH}}}{\mathrm{d}t} = -\frac{1}{\omega_0}T_{\mathrm{CH}}^2\frac{\mathrm{d}\sigma_{\mathrm{CH}}}{\mathrm{d}t} = -1.1907 \cdot 10^{-5}\ \mathrm{days\ year}^{-1}$$

indicating a decrease of 1 day over 10^5 years.

Seasonal variations of C_{20} are mainly caused by the dynamics of geophysical surface fluids. In Table 1 (Jochmann et al. 2001; Reigber et al. 2003) the parameters describing the annual variations of ΔC_{20} due to

Table 1. $\Delta C_{20} 10^{11}$

	a_1	a_2	a	$\gamma\,(\circ)$
Atmosphere	3.43	2.87	4.48	39.9
Ocean dynamics	4.51	0.47	4.54	6.0
Sea level	0.60	0.80	1.00	53.1
Continental water storage	2.20	1.06	2.44	25.7
Sum	10.74	5.20	11.93	25.8

Table 2. $\mathrm{d}f_{\mathrm{CH}} 10^9$ and $\mathrm{d}T_{\mathrm{CH}}$ due to the tidal influence of the Moon and the Sun

	$\mathrm{d}f_{\mathrm{CH}} 10^9$ day^{-1}	$\mathrm{d}f_{\mathrm{CH}}$ [sec]
$r^M_{\max}$	9.31	152
$r^M_{\min}$	14.01	229
$r^S_{\max}$	3.37	55
$r^S_{\min}$	3.72	61

atmosphere and ocean dynamics and continental water storage are given for the notation

$$\Delta C_{20} = a_1 \cos \sigma t + a_2 \sin \sigma t = a \cos(\sigma t - \gamma) \tag{9}$$

These seasonal mass redistributions create an annual modulation of the Chandler frequency with an amplitude of

$$\Delta f_{\mathrm{CH}} = 6.44 \times 10^{-10} \text{ day}^{-1}$$

corresponding to a variation in the period of 10.53 sec.

The influence of tidal forces on the principal moments of inertia has been comprehensively studied in Burså (1983) and Vondrák (1984). For this discussion, we refer to Rikitake et al. (1986), where the changes in the moments of inertia due to variations of the pole distances [in Eqs. (10), (11), and (12) replaced by the declination δ] and longitudes of the attracting celestial bodies are given. From these relations and Eq. (2) the modulation of the Chandler frequency follows as

$$\begin{aligned}\Delta\sigma_{\mathrm{CH}} &= \frac{1}{4}\frac{kM^* a^5}{r^{*^3} A_M}(3\cos 2\delta(t) - 1)\left(1 - \frac{D_{11}}{C-A}\right)\omega_0 \\ &= \omega_0 \Delta f_{\mathrm{CH}}\end{aligned} \tag{10}$$

for the tides of the solid Earth and

$$\begin{aligned}\Delta\sigma_{\mathrm{CH}} &= \frac{1}{5}\frac{\pi k'(1+k-h)GM^*\rho_w a^6}{gr^{*^3} A_M}(3\cos 2\delta(t) - 1) \\ &\quad \times \left(1 - \frac{D_{11}}{C-A}\right)\omega_0 = \omega_0 \Delta f_{\mathrm{CH}}\end{aligned} \tag{11}$$

for the ocean tides.

In Eqs. (10) and (11) M^* and r^* are the masses and distances of the tide-generating celestial bodies. a is the Earth's radius. k, h, and k' are the Love numbers for deformation and load respectively, G is the gravitational constant and ρ_w the density of sea water. To calculate the influence of Eqs. (10) and (11) upon the Chandler frequency, only the variable part of $\Delta\sigma_{\mathrm{CH}}(t)$ needs to be taken into account. Therefore, the difference $d\sigma_{\mathrm{CH}} = \Delta\sigma_{\mathrm{CH}}(\delta = \max) - \Delta\sigma_{\mathrm{CH}}(\delta = 0)$ will be considered. From Eqs. (10) and (11)

$$d\sigma_{\mathrm{CH}} = 3G_z(1 - \cos 2\delta_{\max})\left(1 - \frac{D_{11}}{A_M}\right)\omega_0 = \omega_0\,\mathrm{d}f_{\mathrm{CH}} \tag{12}$$

is obtained, where

$$G_z = \frac{1}{4}\frac{kM^* a^5}{r^{*^3} A_M} + \frac{1}{5}\frac{\pi k'(1+k-h)GM^*\rho_w a^6}{gr^{*^3} A_M}$$

The tidal deformations of the Earth depend largely on the attraction towards the Moon and the Sun. Introducing in Eq. (12) the maximal declinations for the Moon ($\delta_{\max} = 28.5°$) and the Sun ($\delta_{\max} = 23.5°$), the results displayed in Table 2, for the maximal and minima distances of both celestial bodies, are obtained.

Assessing the values in Table 2, we note that they are amplitudes of periodic variations. Therefore, their influence on the results of Eq. (1) is much smaller, as indicated by the already small values in Table 2.

Polar motion creates a mass redistribution similar to that caused by tidal forces. This pole tide effect produces changes in sea level due to varying centrifugal forces. If the pole tide is considered as an equilibrium tide, it contributes only to the amount of D_{11} in Eq. (2) (see Munk and McDonald 1960; Lambeck 1980). In the case of a time delay between polar motion and the reaction of the oceans, a non-equilibrium theory must be applied that results in the following expression of the Chandler frequency

$$\sigma_0 = \sigma_{\mathrm{CH}}\left(1 - \frac{1}{2}Q_w^{-2} + \frac{1}{2}Q_w^{-1} i\right) \tag{13}$$

(Lambeck 1980, 1989), where Q_w is the quality factor and σ_{CH} the Chandler frequency corrected for the influence of the equilibrium pole tide. According to Eq. (13) the non-equilibrium pole tide influences the period and damping of the Chandler wobble. This may explain positive correlations between the variations of the amplitude and the period of the Chandler wobble determined by several authors (e.g. Melchior 1957; Carter 1981). Lambeck (1980) suggested that the larger the amplitude, the greater the pole tide currents and the greater the dissipation and lag, assuming that non-linear bottom friction is a dominant mechanism. However, it has not been determined whether the pole tide should be treated as an equilibrium or non-equilibrium tide. Some authors suggest only a small departure from the equilibrium pole tide (Schweydar 1916; Dickman 1985; Carton and Wahr 1986; O'Conner and Starr 1986; Lambeck 1989), which justifies the application of the equilibrium theory. This conclusion was disproved by Molodenskiy (1985), who studied the problem for a more detailed land–sea–distribution. These more realistic ocean geometries led to the conclusion that the departures from the equilibrium tide become significant.

Although the problem of whether the pole tide is an equilibrium or non-equilibrium tide is still debated, a possible influence on the Chandler period can be estimated according to Eq. (13). If a very small value

$Q_w = 50$ is assumed, and provided that it is the maximal variation due to the pole tide, a modulation of $\Delta f_{CH} \approx -4.6 \cdot 10^{-7}$ day^{-1} would be produced, corresponding to an increase of the period of 0.087 days (or two hours). However, even such an unlikely large pole tide effect would produce a small variation in the Chandler period.

4 Discussion and conclusions

If the variations of the Chandler period created by mass redistributions resulting from different geophysical processes are compared with those obtained from analyses of polar motion data, it is found that large variations of several days (e.g. Carter 1981, 10 days within 3 years) are very unlikely. To confirm this conclusion, a mass redistribution between an equatorial zone (bounded by the latitudes $\varphi = -30°$ and $\varphi = 30°$) and both polar caps is simulated. According to Eq. (8) and taking into account mass conservation for a periodic mass movement (circular frequency σ and amplitude Δh), the corresponding modulation of the Chandler frequency

$$\Delta f_{CH} = \frac{3}{2}\frac{a^4\pi}{A_M}\rho(1+k')\left(1-\frac{D_{11}}{C-A}\right)\Delta h \cos\sigma t \qquad (14)$$

is obtained. Equation (14) reads for the mean density of the Earth ($\rho = 5.514 \cdot 10^3\ kg\ m^{-3}$)

$$\Delta f_{CH} = 2.603 \cdot 10^{-7}\Delta h_0 \cos\sigma t \qquad (15)$$

A period variation of 1 day is equivalent to a frequency modulation of $\Delta f_{CH} = 5.275 \cdot 10^{-6}$ day^{-1}. The necessary amplitude of the corresponding mass movement amounts to $\Delta h = 20.2$ m, which shows that the above-mentioned large period variations are physically not reasonable.

This conclusion requires a discussion of the causes for the apparently large period variations found from the analyses of polar motion time series. Mathematical methods used to determine periodic constituents of time series (Fourier analysis, spectral analysis, wavelet analysis) are based on the calculation of the expression

$$a_k \cos(2\pi kt + \alpha) \qquad (16)$$

which can be exactly determined only if the physical process consists of a series of harmonic functions, i.e. if the amplitudes a_k and phases α do not vary over time. This is generally not the case and a periodic function with temporally varying parameters will be split into a number of periodic functions by Fourier or spectral analyses. The properties of the periodic constituents obtained by these analyses depend on the variations of the amplitude and phase of the physically given function. In limited intervals of the time series, a periodic term can be considered harmonic if its parameter variations are sufficiently small. Calculations over different time intervals will possibly result in different values for the amplitude, the phase, and the period, and imitate in this way an apparent period variation.

The Chandler wobble is in fact not harmonic, but its period can be considered as invariable following the above discussion. An apparent period variation can be easily explained if the effect of a phase variation

$$\alpha_0 + \alpha_n t^n$$

is considered. This expression can appear as phase or frequency modulation in Eq. (16)

$$a_k \cos(2\pi kt + \alpha_0 + \alpha_n t^n) = a_k \cos\left(2\pi\left(k + \frac{\alpha_n}{2\pi}t^{n-1}\right)t + \alpha_0\right) \qquad (17)$$

According to Eq. (17), it is impossible to differentiate between phase and period variations with the available analysis methods. Even if the function is apparently harmonic, the frequency cannot be exactly determined because a linear phase variation ($n = 1$) can possibly cause a frequency delay. It can therefore be decided only by physical arguments whether the frequency of a periodic term is variable.

The frequency of the Chandler wobble is in fact variable, but the order of magnitude of these variations allows it to be treated as invariable for the length of the time series presently available. Period variations presently identified must be considered as phase variations caused by the influence of poorly known excitations of the Chandler wobble.

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