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Robust estimator for correlated observations based on bifactor equivalent weights

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Abstract. A new robust parameter estimator for the adjustment of correlated observations is developed based on a 'bifactor reduction' model of weight elements. A shrinking factor for weight elements is proposed. The new equivalent weight matrix composed by the bifactor weight elements preserves the symmetry and keeps the original correlation coefficients unchanged. The new parameter estimator with its error influence function is derived. The robustness and efficiency of the new robust estimator is demonstrated with a simulated example and some conclusions are drawn.

Keywords: Robust estimator – Correlated observations – Bifactor reduction model – Equivalent weight

1 Introduction

The approaches to controlling the outlier influence fall into two broad categories: outlier identification and robust parameter estimation. The approach of single or multiple outlier identification is of interest in many practical applications. This interest has given rise to a rapid development of hypothesis testing and corresponding parameter estimation. Most outlier identification methods attempt to separate the data into a 'clean' subset without outliers and a complementary subset that contains all potential outliers. Then the remaining observations are tested relative to the clean subset. A problem with this approach is that the 'clean' subset is rarely known. Hadi and Simonoff (1993) proposed two modified methods to find the initial clean subset in some particular situations. In the geodetic field, outlier identification has also been widely researched (Teunissen 1990). Xu (1989a) even discussed outlier identification in the case of correlated observations. Many other practical methods that have been developed are not mentioned here, because most of them do not deal with correlated observations.

In robust estimation, a good deal of attention has been focused on efficient and highly robust M estimators (see Huber 1981; Hampel et al. 1986). In recent years, robust estimators with high breakdown point have been developed, for example LMS (least median squares) that minimizes the median of the squared residuals rather than their sum and LTS (least trimmed squares) (see Rousseeuw and Leroy 1987). In geodetic applications, robust parameter estimation for independent observations has also received widespread attention (Caspary and Hean 1990; Yang 1993). A number of estimation procedures have been developed (Schaffrin 1989, 1991; Zhou 1989; Koch 1996; Koch and Yang 1998).

Most of the available robust estimation methods appearing in the literature are also based on the presumably independent observations. Although big strides have been made in improving robust estimators and algorithms and in applying them in geodesy, they make the routine geodetic applications problematic. Experience with geodetic practices, however, has shown that correlated observations are very often encountered, especially in preprocessed observations. The main sources for the correlations of preprocessed observations are due to the geometry of observations, physical background and statistical procedures. Thus the correlations between the observations deserve to be carefully taken into account in robust parameter estimation.

By using Cholesky factorization (see Koch 1988, pp. 183–184), we can convert correlated observations into uncorrelated ones using least-square (LS) estimation. This kind of conversion, however, may lead to a transfer of abnormal errors and finally result in a failure in outlier diagnosis and robust estimation. A direct robust approach to deal with the dependent data has received less attention. Gastwirth and Rubin (1975) investigated the effect of serial dependence in the data on the efficiency of some robust location estimators. Portnoy (1977) studied approximately optimal estimators, in the asymptotic minimax sense of Huber (1964, 1981), in dependent

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situations. Genton (1998) discussed the asymptotic behavior of M estimators for dependent Gaussian random variables. None of the available robust estimators for correlated data developed in statistical mathematics is applicable in geodetic adjustment. Two algorithms of robust estimation for correlated observations in geodesy have been proposed by means of a nonlinear iterative ψ function (Xu 1989b) and a so-called IGGIII (Institute of Geodesy and Geophysics) scheme (Yang 1994), which are established based on an M estimation and the principle of equivalent weight (see also Yang 1993, p.104; Zhou et al. 1997, p. 118). The robust estimator based on the IGGIII scheme has the same formation as that based on LS principle. Its calculation and analysis of error influence are simple and quality control is effective. What has been emphasized by the equivalent weight matrix of the IGGIII scheme is that the parameter estimates should be robust. The symmetry of the equivalent weight matrix, however, is ignored. Although this asymmetry does not affect the parameter estimates so much, for only a few outliers are presumed to exist in the observations, it makes the corresponding normal matrix and posterior variance–covariance matrix slightly asymmetric. Thus we cannot make full use of the property of a symmetric matrix to reduce calculation or computer storage when calculating or storing the normal matrix.

In fact, in order to control the influence of abnormal observations, we can introduce a suitable contaminated distribution from which a related estimator can be derived based on some score functions, or inflate the covariance elements of the abnormal observations to reduce the effects of the outlying errors on the parameter estimates. In robust statistics, the contaminated error model is usually expressed as (Huber 1981; Hampel et al. 1986)

$$F_{\varepsilon}(\Delta) = (1-\varepsilon)F + \varepsilon\delta L \tag{1}$$

where Δ denotes the observational errors; F_{ε} is a contaminated distributional function and F denotes the empirical distribution of Δ; ε is a small fraction, $0 < \varepsilon < 1$; δL is the point mass at L. Guttman and Lin (1995) proposed a mixed distribution, ie. the error Δ_i of the observation L_i is as follows:

$$\Delta_i \sim \alpha_{i1}N(0,\sigma_{i1}^2) + \alpha_{i2}N(0,\sigma_{i2}^2), \quad \alpha_{i1} + \alpha_{i2} = 1 \tag{2}$$

where α_{i1} and α_{i2} are probabilities corresponding to the two normal distributions $N(0,\sigma_{i1}^2)$ and $N(0,\sigma_{i2}^2)$ respectively; σ_{i1}^2 and σ_{i2}^2 are two variances.

The contaminated distributions above are not suitable to be applied in correlated geodetic data processing. In contaminated distributions, the distributions δL and $N(0,\sigma_{i2}^2)$, the probabilities α_{i1} and α_{i2}, as well as ε, are unknown; the correlations have not been considered in the contaminated model.

2 Bifactor reduction model of weight elements

It should be noted that the variance–covariance matrix and weight matrix as criteria of precision should reliably reflect the accuracy of observations. If an observation is contaminated by an outlier, then its variance ought to be inflated and its weight ought to be reduced. Thus an alternative approach for controlling the influences of the outlying correlated observations is to reduce the weight elements of the outlying observations.

As emphasized above, the correlations usually reflect the intrinsic relations of the observations. Thus the reduction of the weight elements for the outlying observations should keep the intrinsic correlation of the observations unchanged.

Assume that L is an $n \times 1$ observation vector, Δ is an $n \times 1$ error vector, V is an $n \times 1$ residual vector, Σ is a covariance matrix of L, and $P = \Sigma^{-1}$ is the corresponding weight matrix; X is an $m \times 1$ unknown parameter vector, its estimate is $\hat{X}$; A is an $n \times m$ design matrix. The observation equation reads

$$L = AX + \Delta \tag{3}$$

and the corresponding error equation is

$$V = A\hat{X} - L \tag{4}$$

with the ith error equation of the observations

$$v_i = a_i\hat{X} - L_i \tag{5}$$

where v_i and L_i are the ith elements of V and L respectively; a_i is the ith row vector of the design matrix.

Considering the prior weight elements of the observation vector and the robust M estimation principle, we define an M estimator of the unknown parameters under the following condition:

$$\Omega = V^T\bar{P}V = \min \tag{6}$$

where $\bar{P}$ is called an equivalent weight matrix, which adjusts the correlated weight elements to fit the actual accuracy of the corresponding observations.

The estimator follows as

$$A^T\bar{P}V = 0 \tag{7}$$

In order to guarantee the equivalent weight matrix of the observations to be symmetric and the intrinsic correlation of the observations not to be changed, we construct new equivalent weight elements that differ from those of the IGGIII scheme as follows:

$$\bar{p}_{ij} = \gamma_{ij}p_{ij} \tag{8}$$

with

$$\gamma_{ij} = \sqrt{\gamma_{ii}\gamma_{jj}} \tag{9}$$

where γ_{ii} and γ_{jj} are two reduction factors of the weight elements (from which we suggest using the word 'bifactor').

From, the experience of many terrestrial geodetic applications, the reduction factor of the weight elements γ_{ii} could be chosen as

$$\gamma_{ii} = \begin{cases} 1 & |\tilde{v}_i| \le k_0 \\ \frac{k_0}{|\tilde{v}_i|}\left(\frac{k_1 - |\tilde{v}_i|}{k_1 - k_0}\right) & k_0 < |\tilde{v}_i| \le k_1 \\ 0 & |\tilde{v}_i| > k_1 \end{cases} \tag{10}$$

where $\tilde{v}_i$ is a standardized residual; k_0 and k_1 are two constants, usually chosen as 2.0–3.0 and 4.5–8.5 respectively. In theory or in practice, the constants k_0 and k_1 can be determined on the basis of the objective requirements of the problem. There exist many empirical rules to choose the constants, however the first theoretical method was proposed by Xu (1993). He proposed to determine the constants based on the confidence intervals of the parameter estimates.

The curve of γ_{ii} expressed in Eq. (10) is shown in Fig. 1.

Of course, other reduction factors can be chosen, for instance Huber's ψ function (Huber 1964) or Hampel's ψ function (see Hampel et al. 1986, p. 150). A reduction function similar to Huber's ψ function can be chosen as

$$\gamma_{ii} = \begin{cases} 1 & |\tilde{v}_i| \le c \\ \frac{c}{|\tilde{v}_i|} & |\tilde{v}_i| > c \end{cases} \tag{11}$$

where c is a constant which is chosen to be 1.0–1.5.

By using the two reduction factors for all the elements of the weight matrix of the observations, we obtain the equivalent weight matrix as

$$\bar{P} = \begin{bmatrix} \gamma_{11}p_{11} & \gamma_{12}p_{12} & \cdots & \gamma_{1n}p_{1n} \\ \gamma_{21}p_{21} & \gamma_{22}p_{22} & \cdots & \gamma_{2n}p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{n1}p_{n1} & \gamma_{n2}p_{n2} & \cdots & \gamma_{nn}p_{nn} \end{bmatrix} \tag{12}$$

Considering Eqs. (7), (10), and (12), as well as the error equation [Eq. (4)], we obtain the new robust estimator as

$$\hat{X} = \left(A^T\bar{P}A\right)^{-1}A^T\bar{P}L \tag{13}$$

The posterior covariance matrix reads (Yang 1997; Wisniewski 1999)

$$\Sigma_{\hat{X}} = \left(A^T\bar{P}A\right)^{-1}\hat{\sigma}_0^2 \tag{14}$$

with

$$\hat{\sigma}_0^2 = \frac{V^T\bar{P}V}{n-m} \tag{15}$$

The robust estimation for correlated observations, based on the bifactor reduction model of the weight elements, is simply called the RECO scheme.

The empirical influence function of the RECO scheme is

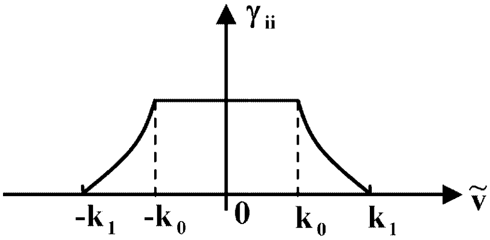

Fig. 1. Curve of the reduction factor of weight elements

$$IF\left(\Delta_j;\hat{X}\right) = \left(A^T\bar{P}A\right)^{-1}\sum_{i=1}^{n} a_i^T p_{ij}\gamma_{ij}\Delta_j \tag{16}$$

where a_i is the ith row vector of the error equation; Δ_i is the error of L_i. The influence function Eq. (16), shows that the impacts of the outlying observations on the parameter estimates, $\hat{X}$, depend on the bifactor γ_{ij}. Because γ_{ij} depends on $\sqrt{\gamma_{ii}}$ and $\sqrt{\gamma_{jj}}$, both of which are descending functions, the impacts of outlying observations on robust parameter estimates are weakened by the reduction factors. It is the bifactor that makes the new robust estimator for correlated observations differ from that of the IGGIII scheme or that of the LS method.

3 Characteristics of the new robust estimator

The characteristics of the robust estimator based on the bifactor reduction model of the weight elements are presented as follows.

1. γ_{ii} and γ_{jj} expressed in Eqs. (10) and (11) are continuous descending factors, which descend when the absolute values of the observational errors increase, and lead to the decrease of the absolute values of $\bar{p}_{ij}$ and $\bar{p}_{ji}$ as well as the values of $\bar{p}_{ii}$ and $\bar{p}_{jj}$. Thus the errors of L_i and L_j are controlled by the equivalent weight elements.
2. γ_{ii} in Eq. (10) is a three-part function. When L_i is normal, $\gamma_{ii} = 1$; when L_i is outlying, $\gamma_{ii} = 0$, i.e. L_i is eliminated; when L_i falls in the interval between k_0 and k_1, γ_{ii} decreases continuously. γ_{jj} is similar to γ_{ii} . Thus both γ_{ii} and γ_{jj} play a role of adjusting the observation weights continuously and reasonably.
3. The equivalent weight matrix determined by Eq. (12) is symmetric and keeps the original correlation coefficients of the observations unchanged. It makes not only the calculation of the robust estimator easy, but also the expression of the posterior covariance matrix relatively simple.

4 Computation and comparison

A practical global positioning system (GPS) baseline network, which is composed of eight stations with point 1 fixed as reference and 27 baseline observations divided into six sessions (see Fig. 2), is used as a reference for the subsequent computations. The baselines are correlated since they are derived from the site coordinates.

In the first step, we take advantage of the site positions from GAMIT software to calculate the baseline vector and its covariance matrix Σ, but only the covariance matrix is used as reference; an observational vector L calculated from the estimated coordinates is used as true observational vector. A random error vector Δ which follows standard normal distribution is simulated in the second step by Monte Carlo method. In order to make the simulated errors correlated in accordance with the original covariance matrix, we decompose Σ into FF^T by using Cholesky decomposition (see

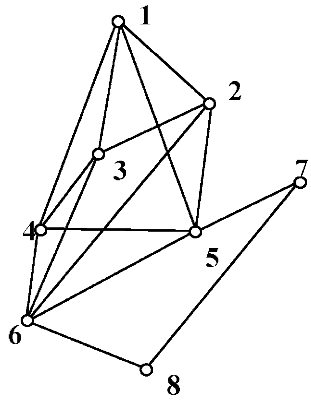

Fig. 2. GPS network

Koch 1988, p. 36) and transform Δ into $\Delta' = F\Delta$ in the third step. Then Δ', as the true error vector shown in columns 4, 5, and 6 in Table 1, is added to the baseline vector L to obtain the new observation vector $L' = L + \Delta'$ with its covariance matrix Σ. Seven simulated outliers are added to $L'(Y_1, X_8, X_{12}, Z_{15}, X_{16}, X_{19}, Z_{26})$, which are also shown in columns 4, 5, and 6 of Table 1, as bold values.

The true errors and the covariance matrix of the simulated correlated baselines described above are known, so that the results calculated from the observations can easily be analyzed and compared for all the computation schemes. Three schemes of adjustment for the GPS network are performed.

Scheme 1: LS adjustment based on the simulated observation vector L' without any additional outliers;
Scheme 2: LS adjustment with seven additional outliers;
Scheme 3: RECO with seven additional outliers.

The shrink function of Eq. (11) is applied as an example. The residuals are shown in columns 7, 8, and 9 (LS), and columns 10, 11, and 12 (RECO) in Table 1. The estimated coordinate differences are shown in Table 2. From the calculation results, we find the following.

1. Some of the outliers have not been reflected in the corresponding residuals in the LS adjustment, in particular the absolute of the residual of the outlying observation Y_1 is masked. There are 19 outlying residuals which correspond to good observations (see columns 7, 8, and 9 with italic values in Table 1). The residuals resulting from the RECO have successfully shown the simulated outliers (see the last three columns in Table 1 with bold values). Similar facts were indicated by Xu (1989b).
2. The sums of the squares of the differences between the residuals and the true errors, $\sum(\Delta_i - V_i)^2$, are shown in the last row in Table 1, illustrating that the residuals of the RECO method fit the true errors much better than those of the LS method.

Table 1. Residual of comparison between LS and RECO

Observation no.	Site no.	Simulated errors			Residuals by LS			Residuals by RECO		
		X	Y	Z	X	Y	Z	X	Y	Z
1	2 1	−0.026	**−0.958**	−0.005	−0.072	−0.063	0.502	−0.009	**−0.978**	−0.014
2	3 2	0.001	−0.004	0.007	−0.123	−0.380	−0.345	0.012	−0.028	−0.033
3	4 3	−0.002	0.021	0.009	0.135	0.045	0.041	0.013	0.032	−0.024
4	5 4	0.001	0.008	−0.002	−0.015	−0.360	−0.222	−0.007	0.043	0.017
5	3 6	0.017	0.053	−0.006	0.053	−0.255	*−0.534*	−0.019	0.045	0.070
6	4 3	−0.012	−0.052	−0.002	0.124	−0.029	0.030	0.002	−0.042	−0.035
7	5 4	−0.000	0.016	−0.001	−0.016	−0.353	−0.221	−0.008	0.050	0.018
8	7 5	**−3.011**	0.038	0.004	*−2.151*	0.278	−0.020	**−3.012**	0.025	0.010
9	2 6	−0.005	0.012	0.003	0.156	0.079	−0.173	−0.051	0.027	0.119
10	3 2	0.008	0.036	−0.003	−0.117	−0.340	−0.355	0.018	0.012	−0.044
11	4 3	−0.016	−0.053	−0.000	0.120	−0.030	0.031	−0.001	−0.043	−0.033
12	5 4	**−0.991**	0.007	0.000	*−1.006*	−0.361	−0.220	**−0.998**	0.042	0.019
13	7 5	−0.109	0.038	0.020	*0.751*	0.278	−0.004	−0.110	0.025	0.025
14	8 7	0.003	0.008	−0.001	*−1.129*	−0.055	*0.785*	0.083	−0.005	−0.016
15	6 2	0.010	0.085	**1.512**	−0.151	0.017	*1.688*	0.056	0.069	**1.395**
16	6 4	**0.879**	−0.016	0.010	*0.707*	0.268	*0.506*	**0.900**	−0.018	−0.034
17	6 5	0.007	0.058	0.005	−0.150	0.711	0.720	0.035	0.022	−0.058
18	6 7	−0.011	0.028	0.003	*−1.028*	0.441	0.743	0.019	0.005	−0.065
19	6 8	**−0.997**	0.007	−0.002	*−0.882*	0.482	−0.049	**−1.047**	−0.003	−0.055
20	2 6	0.064	0.052	−0.093	0.225	0.120	−0.269	0.018	0.068	0.024
21	5 2	−0.025	−0.110	−0.002	−0.029	*−0.831*	*−0.542*	−0.007	−0.090	−0.056
22	7 5	−0.004	0.012	0.002	*0.856*	0.252	−0.022	−0.006	−0.002	0.007
23	8 7	−0.014	0.034	0.005	*−1.147*	−0.028	*0.791*	0.066	0.021	−0.010
24	1 2	−0.106	0.000	0.023	−0.061	*−0.895*	−0.484	−0.124	0.020	0.032
25	1 3	−0.109	0.031	0.010	0.061	−0.488	−0.146	−0.137	0.074	0.059
26	1 4	−0.125	−0.034	**−2.103**	−0.091	*−0.577*	*−2.290*	−0.167	−0.001	**−2.021**
27	1 5	−0.022	0.077	0.010	0.028	−0.098	0.043	−0.057	0.075	0.073
$\sum_{i=1}^{18}(\Delta_i - V_i)^2$			0			14.870			0.127	

Table 2. True errors of the simulated coordinates

Site no.	LS without outliers			LS with outliers			RECO with outliers		
	Δ_X	Δ_Y	Δ_Z	Δ_X	Δ_Y	Δ_Z	Δ_X	Δ_Y	Δ_Z
2	0.010	−0.017	0.001	−0.046	0.895	0.507	0.017	−0.019	−0.009
3	0.019	−0.021	0.017	−0.170	0.520	0.155	0.028	−0.043	−0.049
4	0.031	−0.005	0.006	−0.034	0.543	0.187	0.043	−0.033	−0.082
5	0.027	0.027	0.018	−0.050	0.175	−0.033	0.035	0.001	−0.063
6	0.040	0.039	0.016	−0.206	0.828	0.683	0.063	−0.035	−0.125
7	0.035	0.005	0.015	0.810	0.415	−0.057	0.033	−0.012	−0.058
	0.074	0.033	0.074	−0.322	0.352	0.730	0.113	−0.025	−0.072
Sums of the error square		0.021			4.536			0.065	

3. In the adjustments, the true coordinates of the network sites are chosen as the approximations of the unknown parameters, thus the corrections of the coordinates after the adjustments reflect the true errors of the coordinates (see Table 2). The results also illustrate that the corrections of the coordinates obtained from the RECO method are more reasonable and reliable than those from LS method, especially when some additional outliers are added to the observations (compare the coordinate corrections of scheme 2 and scheme 3 shown in Table 2). The sums of the error squares confirm the above conclusion.

5 Concluding remarks

The occurrence of violations of the famous 'IID' (independently identical distributed) assumption in geodetic observations is acknowledged by most geodesists. The error effects of correlated observations in robust estimation and in error diagnosis are quite different from those of LS estimation. We cannot equivalently transform the dependent observations into independent ones in robust estimation, for the error value may be converted and masked, as indicated also by Xu (1989b). It is necessary to take the correlation of the observations into account in robust estimation. So far, we have developed robust estimators for correlated observations by means of the equivalent weight matrix of the IGGIII scheme and the bifactor reduction model of the weight elements of the observations respectively. We strongly propose that the robust estimator based on the bifactor reduction model of the weight elements be applied in correlated geodetic data processing. This kind of robust estimator takes the calculation procedures of the common LS estimation and keeps the equivalent weight matrix symmetrical and the original correlation coefficients of the observations unchanged. Its robustness and efficiency are guaranteed by the reduction factor of the weight elements. A lot of actual and Monte Carlo calculations have shown its effectiveness.

It should be mentioned that the results of the robust estimation for the correlated observations are very preliminary; further theoretical and practical studies are needed – in particular the criteria of the reduction factor of the weight elements should be determined reasonably and theoretically.

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