J Manag Control (2018) 29:37–61
https://doi.org/10.1007/s00187-018-0262-4

ORIGINAL PAPER

A matrix approach to valuation and performance measurement based on accounting information considering different financing policies

Dirk Beyer[1]

Published online: 26 February 2018

Abstract Two of the most important issues related to value-based management are a company's value and the contribution made to it in a certain period. Variations in residual income have been discussed and used for such kinds of valuation purposes and performance measurement for many years to link the value of a company to traditional accounting data. Considering certain financing policies with changing levels of debt, the technical problem of circularity has to be solved concerning the readjustment of the cost of capital. As a practical way to handle that issue, a matrix-based approach is presented in this article. The result of this technique is a vector of the current and future expected amounts of a firm's goodwill. This vector provides a useful base simultaneously for valuation purposes and for measuring the contribution made to the corporate value in a particular period. Thus, the method presented tackles these two main issues of value-based management at once solely by focusing on traditional accounting data.

Keywords Residual income · Corporate valuation · Value-based management · Performance measurement

JEL Classification G 320

✉ Dirk Beyer
dbeyer@hs-harz.de

[1] Professur für ABWL/Betriebliches Rechnungswesen und Controlling, Fachbereich Wirtschaftswissenschaften, Hochschule Harz, Friedrichstraße 57-59, 38855 Wernigerode, Germany

1 Introduction

During the last decades, many tools for management accounting purposes in the field of value-based management have been discussed in academic research and corporate practice. This topic generally focuses on a company's responsibility to create value for its shareholders, beside other obligations, such as corporate reporting, corporate governance or corporate social responsibility (Bhimani and Soonawalla 2005; Ruangviset et al. 2014). Value-based management implies the need to define appropriate metrics to enable an evaluation of organisational and managerial performance with reference to planned and realised actions in certain periods (Hahn and Kuhn 2012; Ittner and Larcker 2001; Malmi and Ikäheimo 2003). Such measures are intended to help the management to act in the interest of the shareholders (Schultze and Weiler 2010). Two fundamental requirements can be derived from these general goals, which are of a methodical and behavioural nature:

1. *Value-based measures of performance should always be in line with the contemporary methods of corporate valuation*. Value in this sense, claimed to be the fundamental guideline for management, should be considered in a theoretically accurate way. The so-called discounted cash flow approach (DCF), with its specific variations, nowadays provides the most important benchmark for this aspect. These valuation techniques put considerable effort into defining appropriate discount rates in accordance with the determined cash flows and other surrounding assumptions, especially concerning the general financing pattern of the firm. It is necessary to ensure that any value-based indicator of economic success is compatible with these leading concepts of valuation; otherwise, the decisions of the managers are likely to be incongruent with the goal of the shareholders (Reichelstein 1997).
2. *Measures of performance should fit the managerial pattern and the systems that are used in practice for planning and control*. Management accounting still relies on traditional accounting terms, namely income statements and balance sheets, rather than on cash flows. Bound to fiscal periods, variance analyses of planned and realised costs and revenues have been well-established tools in the field of management control for many years (e.g. Bhimani et al. 2012). Much time and effort is spent on the budgeting procedures for the planning and controlling of these traditional accounting terms. Despite ongoing criticism (e.g. Banham 2012; Hope and Fraser 2003; Messer 2017), companies adhere to this practice with a tendency to implement rolling planning instruments (Heupel and Schmitz 2015; Libby and Lindsay 2010; Rickards and Ritsert 2012), often supported by new IT solutions (Amato 2013; Leon et al. 2012). If value-based measurement instruments try to gauge the value created in a certain period, it should easily be integrated into this common frame of management accounting systems.

This article targets both aspects, presenting a specific modification of the residual income approach, which is traditionally seen as providing a 'link' between accounting and valuation (O'Hanlon and Peasnell 2002; Ohlson 2002; Schueler and Krotter 2008). Residual income charges the cost of equity to the net income of a company. It is often interpreted as an indication of the creation of value. Discounted by the cost of equity, the residual income can also be used for corporate valuation (e.g. Pinto et al. 2015). A company's assumed policy of debt financing has a deep impact on these results if interest payments are deducted from its taxable income. Two fundamental variants of such financing policies, which are often used for valuation purposes, imply on the one hand passive management of debt with predetermined autonomous amounts of debt and on the other an active form of debt management bound to corporate values (Dierkes and Schaefer 2016). If budgetary planning provides information about fixed scheduled or expected future amounts of debt according to these policies, periodical readjustment of the cost of equity becomes necessary, because the relation between debt and value is likely to vary over time. Since these readjustments rely on values, the inherent problem of circularity occurs. This means that the costs of capital needed to calculate residual incomes and corporate values depend on these values themselves. The mathematical equations for valuation then adopt an implicit nature, whereby the variable sought cannot be isolated. In this case, the residual income approach cannot be used directly without procedures to solve this problem of circularity. This aspect, however, is rarely tackled in the literature. Some previous research in this field has not considered debt financing at all (O'Hanlon and Peasnell 2002). Other studies have simply postulated a constant cost of capital (O'Byrne 2016; Schultze and Weiler 2010) or assumed constant value-based target leverage to ensure this (Schueler and Krotter 2008). Explicit consideration of debt financing can be found in the study by Schueler et al. (2008), focusing only on an autonomous financing policy. However, the readjusted cost of capital for residual incomes was based on the outcomes from other valuation techniques and hence only reproduced the already-existing results.

The method presented here takes a further step and contributes to the existing literature in the following way: it incorporates an implicit readjustment of the cost of capital to changing debt levels under two basic financing policies into the residual income approach for both valuation and performance measurement purposes. The first of these financing policies is so-called autonomous financing, in which debt is known in fixed scheduled amounts. The second policy implies proportional debt-to-value-related financing with an estimated amount of debt but an unknown and possibly changing relation to the corporate value (Ruback 2002). In both cases, the procedure considers information from common financial planning of income statements and balance sheets, particularly including quantifications of the forecasted fixed or expected volumes of debt. The matrix-based solution presented here solves the inherent problem of circularity mentioned above and enables the direct and independent use of the residual income approach instead of either avoiding this issue by making the simplifying assumption of constant cost of capital or relying on other valuation methods respectively using iterative procedures. The method produces simultaneously a vector of the current and the prospectively expected goodwill of the firm, which is the central element of the approach. Goodwill is defined as the difference between the market value and the book value of equity (Ellis 2001). A combination of this resulting vector of current and future goodwill with typical forecasted accounting data, namely book values of equity and net income, provides a practicable method to calculate both the corporate value and the value created within a certain period. The results are completely consistent with alternative standard DCF methods, which make the same assumptions. This congruence proves the equivalence to these widely accepted techniques of valuation. Hence, the paper provides a self-contained solution to the problem of how practitioners can directly apply the residual income approach with changing debt levels to derive values and performance measures from the prevalent rolling forecast systems based on traditional accounting terms.

The article is structured as follows. In Sect. 2, the principles of valuation and performance measurement based on residual income are discussed with reference to the existing literature in this field. Section 3 represents the conceptual core of the article, providing a matrix-based approach for the purposes mentioned above, namely valuation and performance measurement. This is illustrated in Sect. 4 using a numerical example. A concluding discussion on the pros and cons of the method can be found in Sect. 5.

2 Valuation and performance measurement by residual incomes

Within this problem setting, residual income concepts are a well-established tool for purposes of valuation and value-based performance measurement (e.g. Bromwich and Walker 1998; Cornell 2013; Koller et al. 2015; O'Hanlon and Peasnell 2002; Ohlson 2005; Penman 2013; Pinto et al. 2015; Young and O'Byrne 2001). They are based on traditional accounting information and provide results that are equivalent to contemporary DCF-based valuation techniques. In general, residual income is calculated by subtracting a capital charge from a specific profit measure.

$$\begin{aligned} Residual\,income &= Income\,earned \;-\; Income\,required \\ &= Profit\,before\,cost\,of\,capital - Cost\,of\,capital \times Capital\,base \end{aligned} \tag{1}$$

In theory and practice, manifold variations of this approach are used. Some of the most important versions are for instance the concepts of economic value added (EVA), cash value added (CVA) and economic profit (EP), which use the weighted average cost of capital (WACC) and provide an entity-based view of the company including both equity and debt (Copeland et al. 2000; Lewis 1994; Stern et al. 1995; Stewart 1991). From an equity-based view, the residual income (*RI*) can be defined similarly as the remaining part of the net income (*NI*) after subtracting the cost of equity according to (2). The cost of equity is calculated here by multiplying the book value of equity (*E*) at the beginning of a period with the return rate r^l that is required by the shareholders of the levered company (Cheng 2005; Feltham and Ohlson 1995; Lundholm and O'Keefe 2001; Ohlson 2002; Peasnell 1982; Penman and Sougiannis 1998):

$$RI_t = NI_t - r_t^l \cdot E_{t-1} \tag{2}$$

The knowledge of these residual incomes of future periods enables the calculation of their present values by discounting them with the cost of capital. In the context of EVA, the expression market value added (*MVA*) was coined for this term (Stewart 1991). Adding the discounted residual income to the book value of the capital base gives the market value of a company at a certain point of time. Thus, in a general sense, *MVA* can be defined as the difference between the market value and the book value. The expression *MVA* will be retained in the case of the equity approach as discussed here and conforms to the goodwill of a firm (Ellis 2001; Schultze and Weiler 2010). The value of equity can then be written according to (3).[1]

$$V_t^{E(RI)} = \sum_{s=t+1}^{\infty} \frac{RI_s}{\prod_{q=t+1}^{s}\left(1+r_q^l\right)} + E_t = MVA_t + E_t \tag{3}$$

This value of equity based on residual income, $V^{E(RI)}$, is identical to the results of a DCF-based valuation if the requirements of the clean surplus relation (CSR) are met (Edwards and Bell 1961; Peasnell 1982; Preinreich 1937). This means that all the changes in the book value of equity that are not caused by transactions with stockholders have to be reflected completely in the net income. If forecasts of future residual incomes based on an accounting system of this type exist, the residual income can be used as a valuation tool in accordance with contemporary DCF-based valuation.

As mentioned before, variants of residual income are also very popular tools in the context of value-based performance measurement (Chen and Dodd 1997). A positive value of the periodical residual income or its positive improvement, derived from a comparison with that of the previous period, gives some indication of the creation of additional value (Bromwich and Walker 1998; Stewart 2009; Young and O'Byrne 2001). However, the residual income of a single period does not measure the creation of value itself. On the contrary, it often emphasises short-term results, discouraging investments that are profitable in the long run (O'Byrne 2016). Therefore, metrics for created value have to consider a dynamic context (Schultze and Weiler 2010). An accurate quantification of this aspect provides the so-called net value created (*NVC*) of a certain period, which is conditional on new projects or unexpected earnings. A simple realisation of already-existing plans does not lead to additional value (O'Byrne 2016). In the context of DCF-based valuation, with its specific variants of equity- or entity-based definitions of relevant cash flows (C) the appropriate costs of capital (k) and the resulting corporate values (V), the net value created in a certain period t (NVC_t) is defined in general as follows (Schueler et al. 2008):

$$\begin{aligned} NVC_t^{DCF} &= \underbrace{C_{t|t} - C_{t|t-1}}_{\textit{Deviation from expectation}} + \underbrace{V_{t|t} - V_{t|t-1}}_{\textit{Revision of expectations}} \\ &= \underbrace{C_{t|t} + V_{t|t} - V_{t-1|t-1}}_{\textit{Economic income}} - k_t \cdot V_{t-1|t-1} \end{aligned} \tag{4}$$

This shows the so-called economic income of this period after charging the costs of capital at the initial value and can be calculated as the earned cash flow plus the change in value that exceeds the bare time effect of simply waiting for one period (e.g. Brealey et al. 2014). This newly created value comes from changes in future expectations or from a deviating realisation of the expected current cash flow. Based on the residual

[1] The expectations operator is omitted in this article for the ease of presentation.

income, the net value created has to be written equivalently in the following way (Schueler and Krotter 2008):

$$NVC_t^{RI} = \underbrace{RI_{t|t} - RI_{t|t-1}}_{Deviation\,from\,expectation} + \underbrace{MVA_{t|t} - MVA_{t|t-1}}_{Revision\,of\,expectations} \tag{5}$$

This definition is equivalent to other measures of economic success, such as the residual economic income (Schultze and Weiler 2010) or the excess return (O'Byrne 2016), which monetarily quantify the performance actually achieved in relation to the existing plans. In particular, the *NVC* compares the current residual income RI_t and the discounted residual incomes MVA_t known in period t with the expectations of them seen from the previous period. Since $RI_{t|t}$ and $RI_{t|t-1}$ use identical costs of capital and capital bases, their difference can be replaced by the difference in the net income (Schueler and Krotter 2008), so the *NVC* can also be written as:

$$NVC_t^{RI} = \underbrace{NI_{t|t} - NI_{t|t-1}}_{Deviation\,from\,expectation} + \underbrace{MVA_{t|t} - MVA_{t|t-1}}_{Revision\,of\,expectations} \tag{6}$$

If the expectations are met exactly, the residual incomes and discounted residual incomes seen from the point of time t are equal to the values seen from one period earlier. In this case only the existing expectations are fulfilled and no value is destroyed or created additionally, so the *NVC* would be zero.

For both purposes, valuation and performance measurement, *MVA*—the company's goodwill—represents a central term. The matrix approach presented in Sect. 3 contains a procedure to calculate the current and expected amounts of *MVA* directly and simultaneously. However, another aspect has to be considered first. To calculate the *RI* and *MVA*, the knowledge of r^l as the cost of equity of a levered corporation is necessary. It can be calculated in general by considering the following relation (Holthausen and Zmijewski 2014; Koller et al. 2015):

$$r_t^l = r_t^u + \left(r_t^u - i_t^D\right) \cdot \frac{V_{t-1}^D}{V_{t-1}^E} - \left(r_t^u - k_t^{TS}\right) \cdot \frac{V_{t-1}^{TS}}{V_{t-1}^E} \tag{7}$$

The variables V^E, V^{TS} and V^D reflect the market values of equity, tax shields and debt. The latter is usually assumed to be equal to its book value ($V^D = D$). The term i^D stands for the interest rate on debt and the term r^u for the required returns from an unlevered company, reflecting the systematic risk inherent in the operating business. The variable k^{TS} is the appropriate discount rate for tax shields. These tax shields quantify the tax savings caused by interest payments based on the corporate tax rate τ. Thus, the value of the tax shields has to be calculated in general by discounting them as follows:

$$V_t^{TS} = \sum_{s=t+1}^{\infty} \frac{TS_s}{\prod_{q=t+1}^{s}\left[1 + k_q^{TS}\right]} = \sum_{s=t+1}^{\infty} \frac{\tau \cdot i_s^D \cdot D_{s-1}}{\prod_{q=t+1}^{s}\left[1 + k_q^{TS}\right]} \tag{8}$$

Finding the appropriate discount rates for the tax shields k^{TS} is an ongoing process of developing and differentiating the underlying assumptions and is sometimes subject to controversial scientific debates (e.g. Arzac and Glosten 2005; Cooper and Nyborg 2006; Fernandez 2004; Fieten et al. 2005). Depending on the assumed financing policy and the resulting risk structure of the tax shields, different valuation approaches have been developed (e.g. the overviews by Fernandez 2006 or Ansay 2010). In wide parts of the literature, the two following idealised financing policies are used to reduce the discount rates of the tax shields either to the interest rate on debt or to the required return of an unlevered company:

(a) So-called autonomous financing implies that debt is predetermined and known in fixed scheduled amounts (Kruschwitz and Loeffler 2006). In this case, the tax shields are often assumed to carry the same risk as the debt itself, and therefore k^{TS} is set equal to i^D (e.g. Inselbag and Kaufhold 1997; Luehrman 1997; Myers 1974). This assumption is usually linked with the adjusted present value approach (APV) of discounted cash flow valuation.
(b) If the assumption is made that debt follows a debt-to-value relation and is rebalanced continuously, an appropriate discount rate for the tax shields k^{TS} is seen in the required returns from an unlevered company r^u (e.g. Harris and Pringle 1985; Ruback 2002; Taggart 1991). The reasoning behind this assumption is that assets and debt financing depend on operating cash flows and are assumed to have the same characteristics of risk. This assumption often accompanies implicitly the weighted average cost of capital approach (WACC) and explicitly the capital cash flow approach (CCF).

Both idealised financing policies with these derived discount rates for tax shields are standard nowadays in many leading textbooks on corporate finance and valuation (e.g. Berk and DeMarzo 2017; Brealey et al. 2014; Koller et al. 2015; Pratt and Grabowski 2014). However, they contain strongly simplifying assumptions, and the empirical evidence suggests that a mixture of the two policies mentioned above explains actual financing decisions better than a single one does (Dierkes and Schaefer 2016). Accordingly, other proposed financing policies often fall between these two poles, with tax shields discounted partly by i^D and partly by r^u. Such are, for instance, assumed target debt-to-value relations with rebalancing only once a year (Arzac and Glosten 2005; Miles and Ezzell 1980; Taggart 1991), a financing policy based on book values (Fernandez 2008) or combinations of autonomous and debt-to-value financed elements (Dierkes and Schaefer 2016; Ruback 2002). More detailed discussions of further effects on the value of tax shields, such as personal taxes, riskiness of debt or other interactions with interest and tax payments, require additional assumptions and raise valuation issues of higher complexity, which are beyond the scope of this paper (e.g. Ansay 2010; Arzac and Glosten 2005; Cooper and Nyborg 2008; Grinblatt and Liu 2008). Caused by their prevalence in theory and practice, only the two idealised, extreme prototypes of financing policies mentioned above will be considered in the following, which covers a possible range of values. Thus, if the expected or fixed scheduled amounts of debt and the resulting interest payments are known from the corporate planning, the value of tax shields can be calculated directly for these two policies according to (8) using either i^D or r^u as discount rates k^{TS}. However, the latter will even become obsolete, because the related term vanishes in the approach presented in the following.

The only parameter of (7) that has not been specified so far is the value of equity V^E. Due to the fact that the market value of equity V^E itself is needed to adjust the levered cost of equity r^l, a circularity problem is apparent. This interdependency between the value and the discount rate has been treated by authors and practitioners in different ways (Mejía-Peláez and Vélez-Pareja 2011). Sometimes the problem has simply been ignored by postulating a constant cost of capital or the use of an assumed fixed target capital structure has been recommended (Benninga and Sarig 1996; Brealey et al. 2014; Koller et al. 2015; Penman 2013; Pratt and Grabowski 2014). To find suitable weights of debt and equity, the first orientation on current share prices or multiples could be used (Koller et al. 2015; Pratt and Grabowski 2014). Differing results could then be readjusted by iterative valuations. Further, the use of results from other valuation techniques, such as the APV, is suggested (Berk and DeMarzo 2017), which however makes the additional use of r^l and WACC obsolete to a certain extent. Periodical readjustment of the cost of equity for changing leverage is seen as a complex issue (Koller et al. 2015). Possible solutions can again be found in iterative procedures, which can be realised manually or automatically (Pratt and Grabowski 2014). Due to the existing level of IT, spreadsheet solutions nowadays provide powerful tools to find asymptotic solutions for such circularity problems (Ansay 2010; Tham and Vélez-Pareja 2004; Vélez-Pareja and Tham 2009; Wood and Leitch 2004). Further, analytical solutions to that problem can be attained in a recursive way (Mejía-Peláez and Vélez-Pareja 2011) or simultaneously based on a matrix approach (Casey 2004). The latter will be used in the following section. It is based on the residual income, incorporating a permanent readjustment of the cost of equity to account for changing debt levels under consideration of the two basic financing policies, which is necessary to be equivalent to other DCF approaches like the APV or CCF (Lundholm and O'Keefe 2001; Ruback 2002). This aspect is rarely tackled in the context of residual income and even less in the field of value-based management. Usually the problem is avoided by simply postulating a constant cost of capital or assuming a constant target level of leverage (O'Byrne 2016; O'Hanlon and Peasnell 2002; Schueler and Krotter 2008; Schultze and Weiler 2010). Although this might be convenient practically, it does not reflect the reality when the firm's debt financing is already quantified by financial planning, either in fixed scheduled or in expected amounts with varying debt levels. The following section proposes a solution to this issue.

3 Matrix approach based on residual incomes

The basis of the matrix approach presented in this section was introduced by Casey (2004) in conjunction with the WACC approach to DCF-valuation and is alternated here for residual incomes and other financing policies. The central term of the solution supposed is *MVA* as the present value of future expected residual incomes. It can be written in a recursive way for a certain point in time t as follows:

$$MVA_t = \frac{RI_{t+1} + MVA_{t+1}}{1 + r^l_{t+1}} = \frac{NI_{t+1} - r^l_{t+1} \cdot E_t + MVA_{t+1}}{1 + r^l_{t+1}} \tag{9}$$

Combining this formula with the readjustments of the cost of equity (7) enables the formation of a set of recursive equations, considering the planning period and the terminal value, as shown in detail in "Appendices 1 and 2". This set of equations can be solved simultaneously using fundamental matrix algebra.

Two cases are considered. First, a financing policy is assumed when fixed debt schedules are known (autonomous financing). In this case i^D is used as discount rate for the tax shield (Inselbag and Kaufhold 1997; Luehrman 1997; Myers 1974). If no growth in the terminal value is assumed (alternatively, with growth MVA_S has to be calculated according to (17a); see "Appendix 2"), this provides the following set of equations in the typical structure $A \cdot \bar{v} = \bar{b}$ with (conforming to "Appendix 3"):

$$\mathrm{A} = \begin{pmatrix} 1 + r^u_{t+1} & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 + r^u_{t+2} & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1 + r^u_{t+3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 + r^u_{S-1} & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 + r^u_S \end{pmatrix}$$

$$\bar{v} = \begin{pmatrix} MVA_t \\ MVA_{t+1} \\ MVA_{t+2} \\ \vdots \\ MVA_{S-2} \\ MVA_{S-1} \end{pmatrix} \quad \bar{b} = \begin{pmatrix} b_t \\ b_{t+1} \\ b_{t+2} \\ \vdots \\ b_{S-2} \\ b_{S-1} \end{pmatrix} \tag{10}$$

with:

$$b_s = NI_{s+1} - r^u_{s+1} \cdot E_s - \left(r^u_{s+1} - i^D_{s+1}\right) \cdot \left(V^D_s - V^{TS}_s\right) \quad \text{for} \quad s = t, \ldots, S-2$$

$$b_{S-1} = NI_S - r^u_S \cdot E_{S-1} - \left(r^u_S - i^D_S\right) \cdot \left(V^D_{S-1} - V^{TS}_{S-1}\right) + MVA_S$$

$$MVA_S = \frac{NI_{S+1} - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot \left(V^D_S - V^{TS}_S\right)}{r^u_{S+1}} - E_S$$

$$V^{TS}_t = \sum_{s=t+1}^{\infty} \frac{\tau \cdot i^D_s \cdot D_{s-1}}{\prod_{q=t+1}^{s} \left[1 + i^D_q\right]}$$

The inverse of matrix A provides discount factors for the modified residual incomes in vector $\bar{b}$. Multiplying this inverted matrix A with the vector $\bar{b}$ results in a vector $\bar{v}$, which contains all the MVA_t of the whole planning period, as Eq. (11) shows.

$$\mathrm{A}^{-1} \cdot \begin{pmatrix} b_t \\ b_{t+1} \\ b_{t+2} \\ \vdots \\ b_{S-2} \\ b_{S-1} \end{pmatrix} = \begin{pmatrix} MVA_t \\ MVA_{t+1} \\ MVA_{t+2} \\ \vdots \\ MVA_{S-2} \\ MVA_{S-1} \end{pmatrix} \tag{11}$$

This MVA_t vector can now be used either for valuation purposes by adding E_t or for periodical performance measurement to compare it with previous expectations of MVA. The results are completely equivalent to the standard APV approach because of the identical assumptions. This will be demonstrated using a numerical example in the next section.

The second case considers financing with budgeted incomes and balance sheets, in which debt is quantified as an expected and therefore risky amount. Assuming that the debt values and resulting interest tax shields have the same systematic risk as the firm's underlying cash flows and that debt is rebalanced continuously to a target leverage, r^u is assumed be the appropriate discount rate for the expected, risky tax shields (Harris and Pringle 1985; Ruback 2002; Taggart 1991). The related set of equations, without considering terminal growth, is presented in Eq. (12) and deviated in detail in "Appendix 4" (including terminal growth).

$$\mathrm{A} = \begin{pmatrix} 1+r^u_{t+1} & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1+r^u_{t+2} & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1+r^u_{t+3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+r^u_{S-1} & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1+r^u_S \end{pmatrix} \quad \bar{v} = \begin{pmatrix} MVA_t \\ MVA_{t+1} \\ MVA_{t+2} \\ \vdots \\ MVA_{S-2} \\ MVA_{S-1} \end{pmatrix} \quad \bar{b} = \begin{pmatrix} b_t \\ b_{t+1} \\ b_{t+2} \\ \vdots \\ b_{S-2} \\ b_{S-1} \end{pmatrix} \tag{12}$$

with:

$$b_s = NI_{s+1} - r^u_{s+1} \cdot E_s - \left(r^u_{s+1} - i^D_{s+1}\right) \cdot V^D_s \quad \text{for} \quad s = t, \ldots, S-2$$

$$b_{S-1} = NI_S - r^u_S \cdot E_{S-1} - \left(r^u_S - i^D_S\right) \cdot V^D_{S-1} + MVA_S$$

$$MVA_S = \frac{RI_{S+1}}{r^l_{S+1}} = \frac{NI_{S+1} - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot V^D_S}{r^u_{S+1}} - E_S$$

The equations simplify under this financing policy, because the term V^{TS} vanishes and does not have to be calculated separately as before. Hence, the assumption of this financing policy is the more convenient variant of this RI-based matrix approach, because all the required parameters except r^u can be taken directly from budgeted income statements and balance sheets. Using the same procedure as described above, the resulting vector of *MVA* is completely in line with the WACC approach (Harris and Pringle 1985) and the CCF approach (Ruback 2002) using identical assumptions.

Although this matrix-based formulation of the valuation problem may seem unfamiliar at first, it is easy to handle with ordinary spreadsheet software, as an example will demonstrate in the following section.

4 Numerical example

The example presented in this section relies on the typical assumption made when following the APV approach, that is, autonomous debt financing. Further, a company is assumed that continuously compiles four-year rolling forecasts concerning its income statements and balance sheets. The last planning period is used always as an estimation for all the subsequent years. In the first step, a base case will be introduced, in which the following data (Tables 1, 2) have been budgeted so far at time $t = 0$.

The unlevered cost of capital r^u for that business, which is the case of being entirely equity financed, is assumed to be constant at 0.1, and the interest rate paid for debt i^D is always 0.05. The expected interest payments equal the claimed return of the creditors, so the value of debt equals its book value. Because these rates do not necessarily have to be constant over time, all the formulae have a time index. The corporate tax rate

Table 1 Budgeted income statements (base case)

	$t = 1$	2	3	4…
Revenues	500.00	550.00	580.00	600.00
Costs (excluding depreciation)	300.00	325.00	340.00	350.00
Depreciation	140.00	160.00	190.00	200.00
EBIT	60.00	65.00	50.00	50.00
Interest ($i^D = 0.05$)	10.00	12.50	14.00	15.00
EBT	50.00	52.50	36.00	35.00
Corporate tax ($\tau^{TS} = 0.3$)	15.00	15.75	10.80	10.50
Net income	35.00	36.75	25.20	24.50

Table 2 Budgeted balance sheets (base case)

	$t = 0$	1	2	3	4…
Net working capital	50.00	55.00	58.00	60.00	60.00
Net fixed assets	280.00	320.00	360.00	370.00	370.00
Total net assets	330.00	375.00	418.00	430.00	430.00
Equity	110.00	99.00	111.00	100.00	100.00
Long-term provisions	20.00	26.00	27.00	30.00	30.00
Debt	200.00	250.00	280.00	300.00	300.00
Total equity and liabilities	330.00	375.00	418.00	430.00	430.00

τ is 0.3, and the interest payments are assumed to be completely tax deductible. All payments are considered to occur at the end of a period.

From the budgeted balance sheets and income statements (Tables 1, 2), including expected future interest payments, the tax shields can be calculated by multiplying them with the tax rate τ. Their value results according to assumed autonomous debt financing by discounting them at i^D according to (8) (Table 3).

Considering the last period of the rolling forecast to be representative of all the later years without growth, the terminal value MVA_3 can be calculated (see Eq. 17c in "Appendix 2"):

$$MVA_3 = \frac{NI_4 - \left(r_4^u - i_4^D\right) \cdot \left(D_3 - V_3^{TS}\right)}{r_4^u} - E_3$$
$$= \frac{24.50 - (0.10 - 0.05) \cdot (300.00 - 90.00)}{0.1} - 100.00 = 40.00$$

Next, the values of the vector $\bar{b}$ have to be calculated according to (10) and the matrix A has to be compiled:

$$\mathrm{A} = \begin{pmatrix} 1.10 & -1 & 0 \\ 0 & 1.10 & -1 \\ 0 & 0 & 1.10 \end{pmatrix}$$
$$\bar{b} = \begin{pmatrix} 35.00 - 0.10 \cdot 110.00 - (0.10 - 0.05) \cdot (200.00 - 87.63) \\ 36.75 - 0.10 \cdot 99.00 - (0.10 - 0.05) \cdot (250.00 - 89.01) \\ 25.20 - 0.10 \cdot 111.00 - (0.10 - 0.05) \cdot (280.00 - 89.71) + 40.00 \end{pmatrix}$$
$$= \begin{pmatrix} 18.38 \\ 18.80 \\ 44.59 \end{pmatrix}$$

Multiplication of the inverted matrix A with the calculated vector $\bar{b}$ immediately gives the sought MVA as a vector.

$$\mathrm{A}^{-1} \cdot \bar{b} = \bar{v} = \begin{pmatrix} MVA_0 \\ MVA_1 \\ MVA_2 \end{pmatrix} = \begin{pmatrix} 65.75 \\ 53.94 \\ 40.53 \end{pmatrix}$$

Adding the budgeted amounts of equity leads to the value of equity $V^{E(RI)}$ at each point within the planning period (Table 4).

These are exactly the same results as those produced by the standard APV variant of DCF valuation (Myers 1974). In this case the value of equity has to be calculated according to (13) with the resulting values shown in Table 5. The detailed calculation

Table 3 Values of tax shields

	$t = 0$	1	2	3…
V^{TS}	87.63	89.01	89.71	90.00

Table 4 Values of equity based on the residual income (matrix approach)

	$t = 0$	1	2	3…
MVA	65.75	53.94	40.53	40.00
Equity (*book value*)	110.00	99.00	111.00	100.00
$V^{E(RI)}$	175.75	152.94	151.53	140.00

Table 5 Values of equity based on the APV approach

		$t = 0$	1	2	3…
	V^u	288.11	313.93	341.82	350.00
+	V^{TS}	87.63	89.01	89.71	90.00
–	V^D	200.00	250.00	280.00	300.00
=	$V^{E(APV)}$	175.75	152.94	151.53	140.00

of the underlying free cash flows (FCF_t) is given in "Appendix 5", and the values of the tax shields equal those in Table 3.

$$V_t^{E(APV)} = V_t^u + V_t^{TS} - V_t^D = \sum_{s=t+1}^{\infty} \frac{FCF_s}{\prod_{q=t+1}^{s}\left[1 + r_q^u\right]} + \sum_{s=t+1}^{\infty} \frac{TS_s}{\prod_{q=t+1}^{s}\left[1 + i_q^D\right]} - D_t \quad (13)$$

As stated before, the second useful property of this matrix approach appears in the context of performance measurement. Looking at the budgeted net incomes and the calculated *MVA*, the net value created (*NVC*) of each period can be calculated according to (6). If all the expectations of the base case are exactly met as budgeted in $t = 0$, the *NVC* of each period is zero. This is the correct result, because additional value is created or destroyed only if the situation changes in an unexpected way. Therefore, the following modification of the original base case will be introduced. Close to the end of period 2, the management launches a new restructuring programme. Staff training and process re-engineering cause additional expenses in periods 2 and 3. However, these actions reduce the operating expenses for staff and materials in year 3 and further years. The project is financed partly by retained earnings and partly by increased debt by 30 in period 2, which will be repaid at the end of period 3. The higher interest payment in period 3 causes an increased tax shield too.

If the restructuring project had been planned as a separate investment project, the creation of value could have been calculated using the net present value (*NPV*). This means discounting the project cash flows at appropriate rates except interest or principal payments (e.g. Brealey et al. 2014). This implies 0.1 for the operating cash flows and 0.05 for the tax shields on interest. From the shareholders' viewpoint, operating and financing decisions are combined here (buying the restructuring programme and selling a financial contract, Table 6).

Table 6 Net present value of the new restructuring project

	$t = 2$	3	4…
Own expenses of the restructuring programme	– 40.00	– 20.00	
Reduction of operating expenses (material/staff)		5.00	10.00
Project cash flows before taxes	– 40.00	– 15.00	10.00
Project cash flows after taxes ($\tau = 0.3$)	– 28.00	– 10.50	7.00
NPV from operations (0.1)	26.09		
Cash flows from financing before taxes ($i^D = 0.05$)	30.00	– 31.50	0
Cash flows from financing after taxes ($\tau^{\mathrm{TS}} = 0.3$)	30.00	– 31.05	0
NPV from financing (0.05)	0.43		
Total NPV of the new restructuring project	26.52		

Table 7 Net income of period t expected at time s (considering the restructuring project)

$NI_{t\vert s}$	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$NI_{t\vert t} - NI_{t\vert t-1}$
$t = 1$	*35.00*	*35.00**				0.00
$t = 2$	36.75	*36.75*	*8.75**			– 28.00
$t = 3$	25.20	25.20	*13.65*	*13.65**		0.00
$t = 4$	24.50	24.50	31.50	*31.50*	*31.50**	0.00
…	…	…	…	…	…	

Italic values give the deviation from expectation

*Realised NI of period t known in $s = t$; other values in the column are the budgeted NI for period t viewed from s

The effort and gain of that project reduce the net income in period 2 by 28.00 (from the originally expected 36.75 to 8.75) and in period 3 by 11.55 (from the expected 25.20 to 13.65, thereof operations 10.5 and 1.05 interest after taxes). The later years, starting in period 4, will benefit from the restructuring project, which is reflected in a net income increased by 7.00 (from the budgeted 24.50 to 31.50). These changes are observable for the first time on the realised balance sheet and in the income statement and the rolling budgeting at the end of period 2.

If the presented matrix approach was performed continuously in combination with the rolling budgeting procedure, it would show the additional value created immediately in each period. Caused by the restructuring programme starting in period 2, the net income of this period is reduced due to the additional expenses for the project. The deviation from expectations of the current period $NI_{t|t} - NI_{t|t-1}$ can be taken directly from this, as shown in Table 7. The future expectations would change at the same time, reflected in the annually made rolling forecasts at time $s = 2$.

If additionally the matrix approach is performed every year, calculating *MVA* as a vector based on actualised rolling forecasts, the following values will be available. A revision of future expectations $MVA_{t|t} - MVA_{t|t-1}$ will be apparent immediately (Table 8).

Table 8 Market value added (goodwill) of period t expected at time s (considering the restructuring project)

$MVA_{t\|s}$	$s=0$	$s=1$	$s=2$	$s=3$	$s=4$	$MVA_{t\|t} - MVA_{t\|t-1}$
$t=1$	*53.94*	*53.94*				0.00
$t=2$	40.53	*40.53*	*95.05*			54.52
$t=3$	40.00	40.00	*110.00*	*110.00*		0.00
$t=4$	…	40.00	110.00	*110.00*	*110.00*	0.00
…	…	…	…	…	…	

Italic values give the revision of expectations

Combining the deviation of the realised *NI* from its forecast with the changes in the expected *MVA* to the $NVC^{(RI)}$ according to (6) shows the additional value created in total. For period 2 this gives the correct amount of 26.52, equal to a separately performed NPV analysis.

$$NVC_2^{(RI)} = \underbrace{NI_{2|2} - NI_{2|1}}_{Deviation\ from\ expectation} + \underbrace{MVA_{2|2} - MVA_{2|1}}_{Revision\ of\ expectations}$$
$$= -28.00 + 54.52 = 26.52$$

The first term could be analysed further with the common tools of variance analysis for costs and revenues to quantify the single effects that changed the net income of the current period (e.g. Bhimani et al. 2012). This analysis is complemented by the effect of changed future expectations. If later years actually perform according to these changed expectations, again the *NVC* of these future periods will be zero. If later years do not achieve the intended cost reductions or provide even better results, the changes in value will become transparent as soon as this information is available—either from surprising actual net incomes or from changed rolling forecasts. Hence, rolling forecasts in terms of traditional accounting combined with the matrix approach presented here will continuously answer the questions regarding value and its additional creation.

Since the assumptions made for this numerical example are those of the standard APV approach, it can be stated further that the additional value created could also be calculated via this method. In this case the effects on free cash flows and tax shields would have to be analysed separately, considering the changes in the actual realised performance and in the expectations. "Appendix 6" shows the details. Achieving an identical result underlines the correctness of the method presented above. The degree of effort and complexity seems not to be lower when using the APV due to the separate elements of this approach. Admittedly, it provides greater transparency of the single effects from financing and operations.

5 Conclusions on the pros and cons of the method

The aim of this article is to provide neither more accurate values nor values that could not have been computed via alternative DCF methods. These well-established valuation techniques form a solid theoretical fundament and are a reliable benchmark.

The matrix approach shows an alternative way for practitioners to apply the residual income approach directly to achieve equivalent results if identical assumptions are made. If the debt financing is assumed to be fixed scheduled, it is equivalent to the standard APV approach, as underlined by the numerical example. If the debt amounts are planned, considering that they are risky but correlated with the operating cash flows, the method can be formulated equivalently to the CCF approach and the WACC approach based on the assumptions made by Ruback (2002) and Harris and Pringle (1985) as shown in "Appendix 4". However, the possibility of integrating these two basic financing policies does not release the user from making a choice between them. Considering both of these idealised poles would disclose the range of possible values for varying financing strategies, which could also be of interest. All the arguments for or against these assumptions concerning the financing pattern will be the same as for the equivalent DCF techniques and will be left to the specific literature. If debt is not planned in fixed scheduled or at least expected amounts, and only a target leverage ratio is assumed, the matrix approach as presented or any other direct calculation of the equity-based residual income cannot be applied. In this case neither NI nor E and D will be quantified in the forecasts, because of their circular dependence on the corporate values sought. However, this might rather be the case for early-stage project valuation, because corporate planning usually incorporates quantifications of interest and debt (Ruback 2002). These planned debt values do not necessarily reflect a constant debt-to-value proportion, which is necessary to use a constant WACC or constant cost of equity. Many practitioners and textbooks make this convenient assumption to avoid the difficulty of readjusting the cost of capital considering its circularity (Ansay 2010). In the base case of the numerical example, the correct cost of equity ranges from 13% in the first period to 18% in the fourth period given the assumed autonomous debt financing (see Table 9). In this case the usage of a constant cost of capital in the managerial accounting is very likely to produce incorrect results. The integrated readjusting of the cost of capital is a clear advantage of the method presented here.

Even if the correct cost of capital is known, the calculated residual income of certain periods on their own could result in misleading interpretations. The periodic values of the base case are shown in Table 9.

The single residual incomes are positive for all the periods, meaning that the realised return on equity is always higher than the required one. However, the periodical improvements of the residual income (ΔRI) show different positive and negative amounts. Nevertheless, all this indicates less whether or not additional value was created. In the case of the example, this would mean that, if new management was hired at the beginning of period 1 and ran the company according to the original budget

Table 9 Cost of equity and residual income of the base case (without the restructuring project)

	$t = 1$	2	3	4
r^l	0.13	0.15	0.16	0.18
RI	20.48	21.64	7.13	7.00
ΔRI $(= RI_t - RI_{t-1})$		1.16	− 14.51	− 0.13

(base case), it would simply fulfil the existing expectations. The net value created of each period always equals zero in this case. The presented matrix approach does not even report the periodical amounts of both the cost of capital and the residual income. It focuses directly on *MVA*, which is the essential term to provide correct information about value and its periodical creation in combination with net incomes or book equity. Figure 1 summarises this conceptual frame.

Instead of cash flows, the presented approach relies on accounting terms, which could be seen as a further advantage of the method, because practitioners are usually very familiar with them. The budgeting procedures of forecasting and control especially are often performed in these terms, rather than considering the relevant cash flows for certain DCF methods. From this reliance on the accounting system, an influence from accounting choices and policy or from the degree of conservatism of the accounting system could be suspected. The annual income and book values could be over- or understated. However, for the calculated value of equity $V^{E(RI)}$ as well as the periodical net value created *NVC*, all these aspects have no influence at all. Because of the complementary effects between income and equity and the interrelation between periods, these possible periodical distortions offset each other completely. The answers are always in line with those from the cash flows, as shown above.

If the net value creation of a certain period is found to be unequal to zero, it means that the situation became better or worse than expected, showing the consequences for the shareholders. This should trigger detailed analyses of the budgeting process and of the actual and planned net incomes (Schueler and Krotter 2008). Windfall gains or losses could be separated, and the traditional methods of variance analysis for revenues and expenses (Bhimani et al. 2012) could be further steps. Since the creation and destruction of value often stem from permanently ongoing changes of operating revenues or expenses, frequently no separate investment appraisals, such as an NPV analysis for the restructuring project in the numerical example, are conducted. The presented approach is able to deliver this information periodically at an aggregated level for the whole company ($NVC_t = \sum NPV_t$ of all 'projects' initialized in period t).

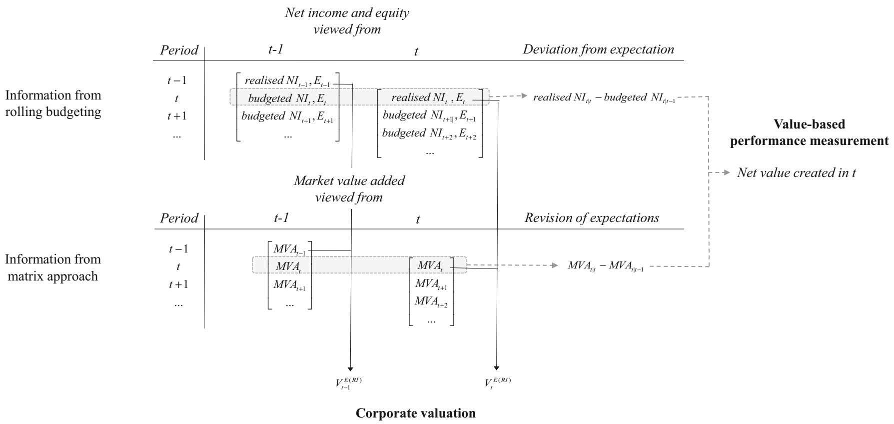

Fig. 1 Conceptual framework of the RI-based matrix approach

A much more important question is whether the underlying accounting system is in strict compliance with the clean surplus relation (CSR), which is a fundamental condition for the correctness of the results. Contemporary accounting systems do not fulfil this claim in each case. Possible distortions can occur for instance from currency conversions or other components of the so-called 'other comprehensive income' (OCI), which is an aspect of increasing practical importance in several accounting systems (e.g. Black 2016; Detzen 2016; Doni et al. 2017). In this case, corrections of 'dirty surplus elements' in the forecasted income and the equity could become necessary. However, the method presented here does not require clean surplus accounting in the past. Hence, any book value of equity from a certain accounting system can be used as a starting point, as long as the forecasts are strictly in line with the clean surplus relation (Palepu et al. 2004). Referring to the typical planning procedures for budgeted balance sheets and income statements, these aspects seem to be negligible.

All valuation methods, DCF- or RI-based, rely on forecasts. Only if this information is available can the procedures be performed. Companies often spend considerable time and effort on compiling budgeted balance sheets and income statements. If this is realised as a rolling planning instrument, as in many companies (Heupel and Schmitz 2015; Libby and Lindsay 2010; Rickards and Ritsert 2012), it will provide a directly applicable database for the matrix approach presented. This includes the necessity to control the budgetary data for self-interest-led effects from agents to avoid so-called budgetary slack (Kirby et al. 1991; Libby and Lindsay 2010; Osband and Reichelstein 1985; Weitzman 1976). This is not a specific problem of the matrix approach but applies to all performance measurement tools that compare actual and planned values. Such kinds of negative influences undermine the validity of performance analyses in each case but seem to be much more serious and difficult to discover with growing planning horizons. The information content of *MVA* would suffer in particular. DCF-based valuation techniques would in contrast require additional effort to derive specific cash flows from these data, including the possibility of making additional mistakes. On the other hand, this of course provides a deeper insight into the operating and investing processes of the company and helps to prove the plausibility of the planning, which is useful in reducing such problems.

As capital market-based information, the required return of an unlevered company r^u is needed in the RI-based matrix approach. This term is not observable by itself and has to be calculated by unlevering the empirical returns of specific companies or from peer group data (e.g. Pratt and Grabowski 2014). As this aspect is also an element of any other valuation technique mentioned (APV, CCF and WACC including relevering to the target leverage), it will be left to the literature and not discussed here as a specific (dis-)advantage.

A final comment can be made regarding the circularity problem of valuation and the presented matrix-based solution. Even though such a calculation is rare in accounting and valuation, it simply compiles recursive equations into a consistent set, which can be solved simultaneously. Since similar techniques are used in the field of cost accounting to derive internal transfer prices, some users will be familiar with the matrix algebra applied here. Calculations can be realised easily by ordinary spreadsheets. These software tools usually offer alternatively the possibility for an iterative solution of the circularity problem. Because the number of iterations is automatically limited, it provides an approximate solution with small differences. If some tricky aspects of the building of spreadsheet models (e.g. deviation by zero caused by unfortunate initial values, etc.) are avoided, an advanced user will probably be indifferent between a matrix-based and an iterative solution. The third option, however, simply ignoring the information on planned debt values in favour of the convenient assumption of a target leverage to reach constant costs of capital, seems not to be a preferable method. This article tries to show an applicable recourse for that.

Appendix 1: Recursive formulation of MVA including the adjustment of the levered return

Inserting Eqs. (2) and (7) in (9) gives:

$$MVA_t = \frac{NI_{t+1} - \left(r^u_{t+1} + \left(r^u_{t+1} - i^D_{t+1}\right) \cdot \frac{V^D_t}{E_t+MVA_t} - \left(r^u_{t+1} - k^{TS}_{t+1}\right) \cdot \frac{V^{TS}_t}{E_t+MVA_t}\right) \cdot E_t + MVA_{t+1}}{1 + r^u_{t+1} + \left(r^u_{t+1} - i^D_{t+1}\right) \cdot \frac{V^D_t}{E_t+MVA_t} - \left(r^u_{t+1} - k^{TS}_{t+1}\right) \cdot \frac{V^{TS}_t}{E_t+MVA_t}} \tag{14}$$

The equation can be rearranged and simplified to:

$$\left(1 + r^u_{t+1}\right) \cdot MVA_t - MVA_{t+1} = NI_{t+1} - r^u_{t+1} \cdot E_t - \left(r^u_{t+1} - i^D_{t+1}\right) \cdot V^D_t + \left(r^u_{t+1} - k^{TS}_{t+1}\right) \cdot V^{TS}_t \tag{15}$$

If k^{TS} is specified as $k^{TS} = i^D$ (autonomous financing), this gives:

$$\left(1 + r^u_t\right) \cdot MVA_t - MVA_{t+1} = NI_{t+1} - r^u_{t+1} \cdot E_t - \left(r^u_{t+1} - i^D_{t+1}\right) \cdot \left(V^D_t - V^{TS}_t\right) \tag{15a}$$

If debt is rebalanced continuously at a certain debt-to-value ratio with $k^{TS} = r^u$, this gives:

$$\left(1 + r^u_{t+1}\right) \cdot MVA_t - MVA_{t+1} = NI_{t+1} - r^u_{t+1} \cdot E_t - \left(r^u_{t+1} - i^D_{t+1}\right) \cdot V^D_t \tag{15b}$$

Appendix 2: Terminal values

Starting at time S, which is the end of the planning period, the terminal value MVA_S is typically defined as a perpetuity, which sometimes implies a constant growth rate g. In this case the terminal value MVA_S would have to be defined as:

$$MVA_S = \frac{RI_{S+1}}{r^l_{S+1} - g} = \frac{NI_{S+1} - \left(r^u_{S+1} + \left(r^u_{S+1} - i^D_{S+1}\right) \cdot \frac{V^D_S}{E_S+MVA_S} - \left(r^u_{S+1} - k^{TS}_{S+1}\right) \cdot \frac{V^{TS}_S}{E_S+MVA_S}\right) \cdot E_S}{r^u_{S+1} + \left(r^u_{S+1} - i^D_{S+1}\right) \cdot \frac{V^D_S}{E_S+MVA_S} - \left(r^u_{S+1} - k^{TS}_{S+1}\right) \cdot \frac{V^{TS}_S}{E_S+MVA_S} - g} \tag{16}$$

The general Eq. (16) can be rearranged and grouped as:

$$MVA_S = \frac{NI_{S+1} - r^u_{S+1} \cdot E_S - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot V^D_S + \left(r^u_{S+1} - k^{TS}_{S+1}\right) \cdot V^{TS}_S}{r^u_{S+1} - g} \tag{17}$$

In the special case of autonomous financing ($k^{TS} = i^D$), it can be specified and rearranged as:

$$MVA_S = \frac{NI_{S+1} - r^u_{S+1} \cdot E_S - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot \left(V^D_S - V^{TS}_S\right)}{r^u_{S+1} - g} \tag{17a}$$

With continuously rebalanced debt-to-value financing ($k^{TS} = r^u$), this gives:

$$MVA_S = \frac{NI_{S+1} - r^u_{S+1} \cdot E_S - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot V^D_S}{r^u_{S+1} - g} \tag{17b}$$

A further simplification occurs if no growth is assumed after the planning period ($g = 0$). In the two cases considered here, this gives:

$$if\ \left(k^{TS} = i^D\right) : MVA_S = \frac{RI_{S+1}}{r^l_{S+1}} = \frac{NI_{S+1} - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot \left(V^D_S - V^{TS}_S\right)}{r^u_{S+1}} - E_S \tag{17c}$$

$$if\ \left(k^{TS} = r^u\right) : MVA_S = \frac{RI_{S+1}}{r^l_{S+1}} = \frac{NI_{S+1} - \left(r^u_{S+1} - i^D_{S+1}\right) \cdot V^D_S}{r^u_{S+1}} - E_S \tag{17d}$$

Appendix 3: Matrix approach to autonomous debt financing ($k^{TS} = i^D$)

The recursive equations of the planning period (15a) and the terminal value above form a set of equations in the typical structure $A \cdot \bar{v} = \bar{b}$ with:

$$\text{A} = \begin{pmatrix} 1+r^u_{t+1} & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1+r^u_{t+2} & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1+r^u_{t+3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+r^u_{S-1} & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1+r^u_S \end{pmatrix} \bar{v} = \begin{pmatrix} MVA_t \\ MVA_{t+1} \\ MVA_{t+2} \\ \vdots \\ MVA_{S-2} \\ MVA_{S-1} \end{pmatrix} \bar{b} = \begin{pmatrix} b_t \\ b_{t+1} \\ b_{t+2} \\ \vdots \\ b_{S-2} \\ b_{S-1} \end{pmatrix} \tag{18}$$

with:

$$b_s = NI_{s+1} - r^u_{s+1} \cdot E_s - \left(r^u_{s+1} - i^D_{s+1}\right) \cdot \left(V^D_s - V^{TS}_s\right) \quad \text{for} \quad s = t, \ldots, S-2$$

$$b_{S-1} = NI_S - r^u_S \cdot E_{S-1} - \left(r^u_S - i^D_S\right) \cdot \left(V^D_{S-1} - V^{TS}_{S-1}\right) + MVA_S$$

The terminal value MVA_S has to be calculated according to (17a) or (17c) depending on the growth expectation.

Appendix 4: Matrix approach to continuously rebalanced debt-to-value financing ($k^{TS} = r^u$)

The recursive equations of the planning period (15b) and the terminal value form a set of equations in the typical structure $A \cdot \bar{v} = \bar{b}$ with:

$$A = \begin{pmatrix} 1+r^u_{t+1} & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1+r^u_{t+2} & -1 & \cdots & 0 & 0 \\ 0 & 0 & 1+r^u_{t+3} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+r^u_{S-1} & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1+r^u_S \end{pmatrix} \quad \bar{v} = \begin{pmatrix} MVA_t \\ MVA_{t+1} \\ MVA_{t+2} \\ \vdots \\ MVA_{S-2} \\ MVA_{S-1} \end{pmatrix} \quad \bar{b} = \begin{pmatrix} b_t \\ b_{t+1} \\ b_{t+2} \\ \vdots \\ b_{S-2} \\ b_{s-1} \end{pmatrix} \tag{19}$$

with:

$$b_s = NI_{s+1} - r^u_{s+1} \cdot E_s - \left(r^u_{s+1} - i^D_{s+1}\right) \cdot V^D_s \quad \text{for} \quad s = t, \ldots, S-2$$
$$b_{S-1} = NI_S - r^u_S \cdot E_{S-1} - \left(r^u_S - i^D_S\right) \cdot V^D_{S-1} + MVA_S$$

The terminal value MVA_S has to be calculated according to (17b) or (17d) depending on the growth expectation.

Appendix 5: Free cash flows of the numerical example's base case (APV)

The free cash flows of the APV approach assume all-equity financing (Table 10).

Table 10 Free cash flows of the unlevered company

		$t = 1$	2	3	4…
=	Net income	35.00	36.75	25.20	24.50
+	Depreciation	140.00	160.00	190.00	200.00
+	Changes in long-term provisions	6.00	1.00	3.00	0.00
–	Changes in net working capital	5.00	3.00	2.00	0.00
–	Investments in fixed assets	180.00	200.00	200.00	200.00
+	Interest	10.00	12.50	14.00	15.00
–	Tax shield ($\tau^{TS} \cdot i^D_t \cdot D_{t-1}$)	3.00	3.75	4.20	4.50
=	FCF	3.00	3.50	26.00	35.00

Table 11 Net value created by the restructuring project based on the APV

	$t = 1$	2	3	4…
$FCF_{t\|t}$	3.00	– 2450	15.50	42.00
$FCF_{t\|t-1}$	3.00	3.50	15.50	42.00
${V_{t\|t}}^u$	313.93	395.91	395.91	420.00
${V_{t\|t-1}}^u$	313.93	341.82	395.91	420.00
NVC^u	0.00	26.09	0.00	0.00
$TS_{t\|t}$	3.00	3.75	4.65	4.5
$TS_{t\|t-1}$	3.00	3.75	4.65	4.5
${V_{t\|t}}^{TS}$	89.42	90.14	90.00	90.00
${V_{t\|t-1}}^{TS}$	89.42	89.71	90.00	90.00
NVC^{TS}	0.00	0.43	0.00	0.00
Total $NVC^{(APV)}$ $(= NVC^u + NVC^{TS})$	0.00	26.52	0.00	0.00

Appendix 6: Calculation of the NVC based on the APV approach

In analogy to (4), the net value creation can be calculated with reference to the APV approach as follows (Schueler et al. 2008):[2]

$$NVC_t^{(APV)} = \underbrace{FCF_{t|t} - FCF_{t|t-1} + V_{t|t}^u - V_{t|t-1}^u}_{NVC\ from\ operations} + \underbrace{TS_{t|t} - TS_{t|t-1} + V_{t|t}^{TS} - V_{t|t-1}^{TS}}_{NVC\ from\ tax\ shields} \tag{20}$$

If the expectations are met exactly, the net value creation of such periods is always zero. The changes in the realised and expected operating cash flows, financing and tax shields caused here by the new restructuring project occur in period 2 (Table 11).

References

Amato, N. (2013). Forward roll. How companies can move beyond traditional budgeting. *Journal of Accountancy, 216,* 26–30.

Ansay, T. (2010). *Firm valuation: Tax shields and discount rates*. Working paper (20 May 2010). https://ssrn.com/abstract=1610143. Accessed August 7, 2017.

Arzac, E., & Glosten, L. (2005). A reconsideration of tax shield valuation. *European Financial Management, 11,* 453–460. https://doi.org/10.1111/j.1354-7798.2005.00292.x.

Banham, R. (2012). Freed from the budget. *CFO, 28,* 41–46.

Benninga, S., & Sarig, O. (1996). *Corporate finance: A valuation approach*. New York, NY: McGraw-Hill.

Berk, J. B., & DeMarzo, P. M. (2017). *Corporate finance* (4th ed.). Harlow: Pearson.

Bhimani, A., Horngren, C. T., Datar, S. M., & Rajan, M. V. (2012). *Management and cost accounting* (5th ed.). Harlow: Pearson.

[2] From the equity-based viewpoint of the shareholders, this term would have to be completed by another term considering cash flows to debt and values of debt. However, under the assumptions made here, specifically that the claimed and paid interest rates are identical, this term becomes zero, because unexpected principal payments in t then always exactly offset unexpected changes in the value of debt.

Bhimani, A., & Soonawalla, K. (2005). From conformance to performance: The corporate responsibilities continuum. *Journal of Accounting and Public Policy, 24,* 165–174. https://doi.org/10.1016/j.jaccpubpol.2005.03.001.

Black, D. E. (2016). Other comprehensive income: A review and directions for future research. *Accounting and Finance, 56,* 9–45. https://doi.org/10.1111/acfi.12186.

Brealey, R. A., Myers, S. C., & Allen, F. (2014). *Principles of corporate finance* (11th ed.). Maidenhead: McGraw Hill.

Bromwich, M., & Walker, M. (1998). Residual income past and future. *Management Accounting Research, 9,* 391–419. https://doi.org/10.1006/mare.1998.0091.

Casey, C. (2004). Neue Aspekte des Roll Back-Verfahrens in der Unternehmensbewertung. *Zeitschrift für Betriebswirtschaft, 74,* 139–163.

Chen, S., & Dodd, J. L. (1997). Economic value added (EVA™): An empirical examination of a new corporate performance measure. *Journal of Managerial Issues, 9,* 318–333.

Cheng, Q. (2005). What determines residual income? *The Accounting Review, 80,* 85–112.

Cooper, I. A., & Nyborg, K. G. (2006). The value of tax shields is equal to the present value of tax shields. *Journal of Financial Economics, 81,* 215–225. https://doi.org/10.1016/j.jfineco.2005.07.003.

Cooper, I. A., & Nyborg, K. G. (2008). Tax-adjusted discount rates with investor taxes and risky debt. *Financial Management, 37,* 365–379. https://doi.org/10.1111/j.1755-053X.2008.00016.x.

Copeland, T., Koller, T., & Murrin, J. (2000). *Valuation* (3rd ed.). New York, NY: Wiley.

Cornell, B. (2013). Discounted cash flow and residual earnings valuation: A comparison in the context of valuation disputes. *Business Valuation Review, 31,* 157–164. https://doi.org/10.5791/12-00013R1.1.

Detzen, D. (2016). From compromise to concept? A review of 'other comprehensive income'. *Accounting and Business Research, 46,* 760–783. https://doi.org/10.1080/00014788.2015.1135783.

Dierkes, S., & Schaefer, U. (2016). Corporate taxes, capital structure, and valuation: Combining Modigliani/Miller and Miles/Ezzell. *Review of Quantitative Finance and Accounting, 48,* 363–383. https://doi.org/10.1007/s11156-016-0554-4.

Doni, F., Rossetti, S., & Verona, R. (2017). Performance reporting choices after the adoption of IAS 1 revised: Comparative evidence from Europe and the USA. *International Journal of Economics and Financial Issues, 7,* 558–574.

Edwards, E. O., & Bell, P. W. (1961). *The theory and measurement of business income*. Berkeley, CA: University of California Press.

Ellis, M. (2001). Goodwill accounting: Everything has changed and nothing has changed. *Journal of Applied Corporate Finance, 14,* 103–112. https://doi.org/10.1111/j.1745-6622.2001.tb00442.x.

Feltham, G., & Ohlson, J. (1995). Valuation and clean surplus accounting for operating and financial activities. *Contemporary Accounting Research, 11,* 689–731. https://doi.org/10.1111/j.1911-3846.1995.tb00462.x.

Fernandez, P. (2004). The value of tax shields is not equal to the present value of tax shields. *Journal of Financial Economics, 73,* 145–165. https://doi.org/10.1016/j.jfineco.2002.10.001.

Fernandez, P. (2006). *The correct value of tax shields—An analysis of 23 theories*. Working paper IESE Business School (May 2006). http://www.iese.edu/research/pdfs/DI-0628-E.pdf. Accessed December 15, 2017.

Fernandez, P. (2008). *Levered and unlevered beta*. Working paper. https://ssrn.com/abstract=303170. Accessed August 15, 2017. https://doi.org/10.2139/ssrn.303170.

Fieten, P., Kruschwitz, L., Löffler, A., Laitenberger, J., Tham, J., Velez-Pareja, I., et al. (2005). Comment on: 'The value of tax shields is not equal to the present value of tax shields'. *The Quarterly Review of Economics and Finance, 45,* 184–187. https://doi.org/10.1016/j.qref.2004.07.004.

Grinblatt, M., & Liu, J. (2008). Debt policy, corporate taxes, and discount rates. *Journal of Economics Theory, 141,* 225–254. https://doi.org/10.1016/j.jet.2007.09.009.

Hahn, G. J., & Kuhn, H. (2012). Designing decision support systems for value-based management: A survey and an architecture. *Decision Support Systems, 53,* 591–598. https://doi.org/10.1016/j.dss.2012.02.016.

Harris, R. S., & Pringle, J. J. (1985). Risk-adjusted discount rates—Extensions from the average-risk case. *Journal of Financial Research, 8,* 237–244.

Heupel, T., & Schmitz, S. (2015). Beyond budgeting—A high-hanging fruit: The impact of managers' mindset on the advantages of beyond budgeting. *Procedia Economics and Finance, 26,* 729–736. https://doi.org/10.1016/S2212-5671(15)00831-X.

Holthausen, R. W., & Zmijewski, M. E. (2014). *Corporate valuation: Theory, evidence and practice*. Cambridge: Cambridge Business Publishers.

Hope, J., & Fraser, R. (2003). Who needs budgets? *Harvard Business Review, 81,* 108–115.

Inselbag, I., & Kaufhold, H. (1997). Two DCF approaches for valuing companies under alternative financing strategies. *Journal of Applied Corporate Finance, 10,* 114–122. https://doi.org/10.1111/j.1745-6622.1997.tb00132.x.

Ittner, C. D., & Larcker, D. F. (2001). Assessing empirical research in managerial accounting. *Journal of Accounting and Economics, 32,* 349–410. https://doi.org/10.1016/S0165-4101(01)00026-X.

Kirby, A. J., Reichelstein, S., Sen, P. K., & Paik, T. Y. (1991). Participation, slack, and budget-based performance evaluation. *Journal of Accounting Research, 29,* 109–128. https://doi.org/10.2307/2491031.

Koller, T., Goedhart, M., & Wessels, D. (2015). *Valuation. Measuring and managing the value of companies* (6th ed.). Hoboken, NJ: Wiley.

Kruschwitz, L., & Loeffler, A. (2006). *Discounted cash flow*. Chichester: Wiley.

Leon, L., Rafferty, P., & Herschel, R. (2012). Replacing the annual budget with business intelligence driver-based forecasts. *Intelligent Information Management, 4,* 6–12. https://doi.org/10.4236/iim.2012.41002.

Lewis, T. (1994). *Steigerung des Unternehmenswertes—Total value management*. Landsberg/Lech: Moderne Industrie.

Libby, T., & Lindsay, R. M. (2010). Beyond budgeting or budgeting reconsidered? *Management Accounting Research, 21,* 56–75. https://doi.org/10.1016/j.mar.2009.10.003.

Luehrman, T. A. (1997). Using APV (adjusted present value): A better tool for valuing operations. *Harvard Business Review, 75,* 145–154.

Lundholm, R., & O'Keefe, T. (2001). Reconciling value estimates from the discounted cash flow model and the residual income model. *Contemporary Accounting Research, 18,* 311–335.

Malmi, T., & Ikäheimo, S. (2003). Value based management practices—Some evidence from the field. *Management Accounting Research, 14,* 235–254. https://doi.org/10.1016/S1044-5005(03)00047-7.

Mejía-Peláez, F., & Vélez-Pareja, I. (2011). Analytical solution to the circularity problem in the discounted cash flow valuation framework. *Innovar, 21,* 55–68.

Messer, R. (2017). Budgets and other lies: Evidence of bias in financial planning. *Business Horizons, 60,* 447–453. https://doi.org/10.1016/j.bushor.2017.03.010.

Miles, J. A., & Ezzell, J. R. (1980). The weighted average cost of capital, perfect capital markets, and project life: A clarification. *Journal of Financial and Quantitative Analysis, 15,* 719–730.

Myers, S. (1974). Interactions of corporate financing and investment decisions: Implications for capital budgeting. *Journal of Finance, 29,* 1–25. https://doi.org/10.1111/j.1540-6261.1974.tb00021.x.

O'Byrne, S. F. (2016). A better way to measure operating performance (or why the EVA math really matters). *Journal of Applied Corporate Finance, 28,* 68–86. https://doi.org/10.1111/jacf.12194.

O'Hanlon, J., & Peasnell, K. (2002). Residual income and value-creation: The missing link. *Review of Accounting Studies, 7,* 229–245. https://doi.org/10.1023/a:1020230203952.

Ohlson, J. (2002). Discussion of 'Residual income and value-creation: The missing link'. *Review of Accounting Studies, 7,* 247–252. https://doi.org/10.1023/A:1020234304861.

Ohlson, J. (2005). On accounting-based valuation formulae. *Review of Accounting Studies, 10,* 323–347. https://doi.org/10.1007/s11142-005-1534-4.

Osband, K., & Reichelstein, S. (1985). Information-eliciting compensation schemes. *Journal of Public Economics, 25,* 107–115. https://doi.org/10.1016/0047-2727(85)90031-3.

Palepu, K., Healy, P., & Bernard, V. (2004). *Business valuation and analysis using financial statements* (3rd ed.). Mason: South-Western.

Peasnell, K. (1982). Some formal connections between economic values and yields and accounting numbers. *Journal of Business Finance Accounting, 9,* 361–381. https://doi.org/10.1111/j.1468-5957.1982.tb01001.x.

Penman, S. (2013). *Financial statement analysis and security valuation* (5th ed.). New York, NY: McGraw Hill.

Penman, S., & Sougiannis, T. (1998). A comparison of dividend, cash flow, and earnings approaches to equity valuation. *Contemporary Accounting Research, 15,* 343–383. https://doi.org/10.1111/j.1911-3846.1998.tb00564.x.

Pinto, J. E., Henry, E., Robinson, T. R., & Stowe, J. D. (2015). *Equity asset valuation* (3rd ed.). Hoboken, NJ: Wiley.

Pratt, S. P., & Grabowski, R. J. (2014). *Cost of capital* (5th ed.). Hoboken, NJ: Wiley.

Preinreich, G. (1937). Valuation and amortization. *Accounting Review, 12,* 209–226.
Reichelstein, S. (1997). Investment decisions and managerial performance evaluation. *Review of Accounting Studies, 2,* 157–180.
Rickards, R. C., & Ritsert, R. (2012). Rediscovering rolling planning: Controller's roadmap for implementing rolling instruments in SMEs. *Procedia Economics and Finance, 2,* 135–144. https://doi.org/10.1016/S2212-5671(12)00073-1.
Ruangviset, J., Jiraporn, P., & Kim, J. C. (2014). How does corporate governance influence corporate social responsibility? *Procedia-Social and Behavioral Science, 143,* 1055–1057. https://doi.org/10.1016/j.sbspro.2014.07.554.
Ruback, R. S. (2002). Capital cash flows: A simple approach to valuing risky cash flows. *Financial Management, 31,* 85–103.
Schueler, A., Bauer, G., & Krotter, S. (2008). Unternehmenswertorientierte Performance-Messung mit Cashflows und Residualgewinnen. *Control and Management, 52,* 336–345. https://doi.org/10.1007/s12176-008-0097-1.
Schueler, A., & Krotter, S. (2008). The link between residual income and value created for levered firms: A note. *Management Accounting Research, 19,* 270–277. https://doi.org/10.1016/j.mar.2008.02.003.
Schultze, W., & Weiler, A. (2010). Goodwill accounting and performance measurement. *Managerial Finance, 36,* 768–784. https://doi.org/10.1108/03074351011064645.
Stern, J. M., Stewart, G. B., & Chew, D. H. (1995). The EVA® financial management system. *Journal of Applied Corporate Finance, 8,* 32–46.
Stewart, B. (1991). *The quest for value*. New York, NY: HarperCollins.
Stewart, B. (2009). EVA momentum: The one ratio that tells the whole story. *Journal of Applied Corporate Finance, 21,* 74–86. https://doi.org/10.1111/j.1745-6622.2009.00228.x.
Taggart, R. A. (1991). Consistent valuation and cost of capital—Expressions with corporate and personal taxes. *Financial Management, 20,* 8–20.
Tham, J., & Vélez-Pareja, I. (2004). *Principles of cash flow valuation: An integrated market-based approach*. Boston, MA: Academic Press.
Vélez-Pareja, I., & Tham, J. (2009). Market value calculation and the solution of circularity between value and the weighted average cost of capital WACC. *Revista de Administração Mackenzie, 10,* 101–131. https://doi.org/10.1590/S1678-69712009000600007.
Weitzman, M. L. (1976). The new Soviet incentive model. *The Bell Journal of Economics, 7,* 251–257. https://doi.org/10.2307/3003200.
Wood, J. S., & Leitch, G. (2004). Interactions of corporate financing and investment decisions: The financing present value ('FPV') approach to evaluating investment projects that change capital structure. *Managerial Finance, 30,* 16–37. https://doi.org/10.1108/03074350410768886.
Young, S. D., & O'Byrne, S. F. (2001). *EVA and value-based management*. New York, NY: McGraw Hill.