Int J Game Theory (2014) 43:975–990
DOI 10.1007/s00182-014-0414-1

Existence of Edgeworth and competitive equilibria and fuzzy cores in coalition production economies

Jiuqiang Liu · Xiaodong Liu

Accepted: 27 February 2014 / Published online: 13 March 2014

Abstract Debreu and Scarf (Int Econ Rev 4:235–246, 1963) proved that for an exchange economy or a production economy with the same production set for all coalitions, under some standard assumptions, an Edgeworth equilibrium is a competitive equilibrium, and Florenzano (J Math Anal Appl 153:18–36, 1990) proved that for such an economy, any allocation in the fuzzy core is an Edgeworth equilibrium. These results are extended to coalition production economies where each coalition can have a different production set. In fact, we establish the coincidence of the fuzzy core, the set of Edgeworth equilibria, and the set of competitive equilibria in a coalition production economy under some standard assumptions. We then prove the existence of the fuzzy core in such a coalition production economy by using a fuzzy extension of Scarf's core existence theorem, thereby establishing the existence of Edgeworth equilibria and competitive equilibria in such economies.

Keywords Cores · Balanced collections · NTU games · Competitive equilibrium · Edgeworth equilibrium · Coalition production economy

J. Liu (✉) · X. Liu
School of Management Engineering, Xi'an University of Finance and Economics,
Xi'an 710100, Shaanxi, People Republic of China
e-mail: jliu@emich.edu

X. Liu
e-mail: xdliu@xaufe.edu.cn

J. Liu
Department of Mathematics, Eastern Michigan University,
Ypsilanti, MI 48197, USA

1 Introduction

Cores and equilibria are important solutions for cooperative games and economies. The core of a cooperative game is the set of feasible allocations that can not be improved upon by any coalition of players. In 1967, Scarf (1967) proved the following fundamental result about the existence of cores in non-transferable utility (NTU) games: Any balanced NTU game V has a non-empty core.

Since Scarf's result was published, there have been a number of advances in the study of conditions that ensure the non-emptiness of the core. Recently, Liu and Liu (2013) derived a necessary and sufficient condition for an NTU fuzzy game to have a non-empty fuzzy core which implies the fuzzy extension of Scarf's theorem: Any balanced NTU fuzzy game V has a non-empty fuzzy core.

In 1963, Debreu and Scarf (1963) proved a remarkable result which gives a connection between the cores and the competitive equilibria in exchange economies by showing that, when the set of economic agents is replicated, the set of core allocations of the replica economy converges to the set of competitive equilibria (implying that an Edgeworth equilibrium is a competitive equilibrium). Florenzano (1990) defined the fuzzy core of an economy as the set of allocations which can not be blocked by any fuzzy coalition and showed that for exchange economies (with no production involved) or production economies where every agent has his own production set, the asymptotic limit of the cores of replica economies coincides with the fuzzy core (implying that any allocation in the fuzzy core is an Edgeworth equilibrium).

In this paper, we deal with coalition production economies where every coalition can have its production set. We show that in a coalition production economy under some standard assumptions, an allocation in the fuzzy core is both an Edgeworth equilibrium and a competitive equilibrium, which extends the corresponding results of Debreu and Scarf (1963) and Florenzano (1990). In fact, we establish the coincidence of the fuzzy core, the set of Edgeworth equilibria, and the set of competitive equilibria in a coalition production economy under some standard assumptions. We then prove the existence of the fuzzy core in such a coalition production economy by using a fuzzy extension of Scarf's core existence theorem, thereby establishing the existence of Edgeworth equilibria and competitive equilibria in such economies.

2 Preliminaries

Let $N = \{1, 2, \dots, n\}$ be the set of all players. Any non-empty subset of N is called a *(crisp) coalition*. Throughout this paper, we denote the collection of all coalitions (non-empty subsets) of N by $\mathcal{N}$ and for any $a, b \in \mathbb{R}^n$, $a \geq b$ means $a_i \geq b_i$ for each $1 \leq i \leq n$, and $a \gg b$ means each coordinate $a_i > b_i$ for $1 \leq i \leq n$. For each $S \in \mathcal{N}$, denote e^S to be the vector in $\mathbb{R}^n$ with $e_i^S = 1$ if $i \in S$ and $e_i^S = 0$ if $i \notin S$. We use e^i for $e^{\{i\}}$ for each $i \in N$.

The notion of fuzzy coalitions was introduced by Aubin (1981) to reflect the situations where agents have the possibility to cooperate with different participation level (with values between 0 and 1), varying from non-cooperation (participation level 0) to full cooperation (participation level 1). A *fuzzy coalition* is a vector $s \in [0, 1]^N$, namely, $s = (s_1, s_2, \ldots, s_n)$ with $0 \leq s_i \leq 1$ for each $i \leq n$. The $i\text{-}th$ coordinate s_i of s is the *participation level* of player i in the fuzzy coalition s. We use $\mathcal{F}^N$ for the set of all non-zero fuzzy coalitions on player set N. Clearly, $\mathcal{F}^N$ is an infinite set. A crisp coalition $S \subseteq N$ corresponds in a canonical way to the fuzzy coalition e^S. For each $s \in \mathcal{F}^N$, we define the *carrier* of s by $car(s) = \{i \in N \mid s_i > 0\}$.

Let us recall the concepts of NTU games and NTU fuzzy games in coalitional form and other related concepts (see Liu and Liu 2013; Shapley and Vohra 1991; Zhou 1994).

Definition 2.1 A non-transferable utility (NTU) n-person game in coalition form (or, simply, a game) V is a mapping that maps each coalition S to a subset $V(S)$ of $\mathbb{R}^n$ and satisfies the following conditions:

(1) For each $\emptyset \neq S \subseteq N$, $V(S)$ is nonempty, closed, comprehensive (i.e.,if $x, y \in \mathbb{R}^n$ are such that $y \in V(S)$ and $x \leq y$, then $x \in V(S)$), bounded from above by $M > 0$ (in the sense that if $x \in V(S)$, then $x_i \leq M$ for all $i \in S$);
(2) For each $\emptyset \neq S \subseteq N$, $V(S)$ is cylindrical in the sense that if $x \in V(S)$ and $y \in \mathbb{R}^n$ such that $y_S = x_S$, then $y \in V(S)$;
(3) For every i, there is a $b_i > 0$ such that $V(\{i\}) = \{x \in \mathbb{R}^n | x_i \leq b_i\}$.

Given an NTU game V, a payoff vector $x \in V(N)$, and a coalition S, we say that S has an *objection* against x if there exists some $y \in V(S)$ such that $y_i > x_i$ for all $i \in S$.

Definition 2.2 The core of a game V, denoted by $C(V)$, consists of all payoff vectors in $V(N)$ that have no objections against them, namely,

$$C(V) = V(N) \setminus [\cup_{S \in \mathcal{N}} int(V(S))], \tag{2.1}$$

where $int(D)$ is the interior of a set D.

Definition 2.3 A collection $\mathcal{B}$ of non-empty subsets (coalitions) of N is balanced if there exist positive numbers λ_S for $S \in \mathcal{B}$ such that

$$\sum_{S \in \mathcal{B}} \lambda_S e^S = e^N. \tag{2.2}$$

The numbers λ_S are called *balancing coefficients*.

Definition 2.4 An NTU game V is *balanced* if $\cap_{S \in \mathcal{B}} V(S) \subseteq V(N)$ for every balanced collection $\mathcal{B}$ of coalitions.

In 1967, Scarf (1967) proved the following fundamental result about the existence of cores in non-transferable utility (NTU) games.

Theorem 2.5 (Scarf 1967) *Any balanced NTU game V has a non-empty core.*

The following concept of NTU fuzzy games and related concepts introduced in Liu and Liu (2013) are natural extensions to the corresponding concepts for NTU games above.

Definition 2.6 A non-transferable utility (NTU) n-person fuzzy game V in coalition form is a mapping from $\mathcal{F}^N$ to subsets of $\mathbb{R}^n$ which satisfies the following conditions:

(1) For each $s \in \mathcal{F}^N$, $V(s)$ is nonempty, closed, comprehensive (i.e.,if $x, y \in \mathbb{R}^n$ are such that $y \in V(s)$ and $x \leq y$, then $x \in V(s)$), bounded from above by $M > 0$ (in the sense that if $x \in V(s)$, then $x_i \leq M$ for all $i \in car(s)$);
(2) For each $s \in \mathcal{F}^N$, $V(s)$ is cylindrical in the sense that if $x \in V(s)$ and $y \in \mathbb{R}^n$ such that $y_i = x_i$ for each $i \in car(s)$, then $y \in V(s)$;
(3) For every i, there is a $b_i > 0$ such that $V(e^i) = \{x \in \mathbb{R}^n | x_i \leq b_i\}$;

Given an NTU fuzzy game V, a payoff vector $x \in V(e^N)$, and a fuzzy coalition s, we say that s has an *objection* against x if there exists some $y \in V(s)$ such that $y_i > x_i$ for all $i \in car(s)$.

Definition 2.7 The fuzzy core of an NTU fuzzy game V, denoted by $C_F(V)$, consists of all payoff vectors in $V(e^N)$ that have no objections against them, i.e.,

$$C_F(V) = V(e^N) \setminus [\cup_{s \in \mathcal{F}^N} int(V(s))]. \tag{2.3}$$

Definition 2.8 A finite collection $\mathcal{B}$ of fuzzy coalitions from $\mathcal{F}^N$ is *balanced* if there exist positive numbers λ_s for $s \in \mathcal{B}$ such that

$$\sum_{s \in \mathcal{B}} \lambda_s s = e^N. \tag{2.4}$$

The numbers λ_s for $s \in \mathcal{B}$ are balancing coefficients.

Clearly, (2.4) is equivalent to the following:

$$\sum_{s \in \mathcal{B}} \lambda'_s \frac{s}{\sum_{i \in car(s)} s_i} = \frac{e^N}{n}, \tag{2.5}$$

where each $\lambda'_s = \frac{\sum_{i \in car(s_i)}}{n} \lambda_s$.

Definition 2.9 An NTU fuzzy game V is *balanced* if $\cap_{s \in \mathcal{B}} V(s) \subseteq V(e^N)$ for every balanced collection $\mathcal{B}$ of fuzzy coalitions.

Let Δ^N be the standard simplex:

$$\Delta^N = \{x \in \mathbb{R}^n | x_i \geq 0 \quad \text{for each } i \in N \quad \text{and} \quad \sum_{i=1}^n x_i = 1\}.$$

For each $\emptyset \neq S \subseteq N$, denote

$$\Delta^S = \{x \in \Delta^N | x_i = 0 \quad \text{for each } i \notin S\} = \{x \in \Delta^N | \sum_{i \in S} x_i = 1\}.$$

Denote

$$\Delta^* = (\Delta_s)_{s\in\mathcal{F}^N}, \quad \text{where each } \Delta_s = \Delta^{car(s)}.$$

Then $\pi = (\pi_s)_{s\in\mathcal{F}^N} \in \Delta^*$ means that $\pi_s \in \Delta_s = \Delta^{car(s)}$ for every $s \in \mathcal{F}^N$.

Billera (1970) extended the concept of balanced collections to π-balanced collection and recently, Liu and Liu (2013) extended the concept of π-balanced collection of coalitions to the following concept of π-balanced collections of fuzzy coalitions.

Definition 2.10 Let $\pi \in \Delta^*$ be given, with $\pi_{e^N} \gg 0$. A finite collection $\mathcal{B}$ of fuzzy coalitions is π-balanced if there exist positive numbers λ_s for $s \in \mathcal{B}$ such that

$$\sum_{s\in\mathcal{B}} \lambda_s \pi_s = \pi_{e^N}, \tag{2.6}$$

where $\pi_s \in \Delta^{car(s)}$ for each $s \in \mathcal{B}$ and the positive numbers λ_s for $s \in \mathcal{B}$ are called balancing coefficients.

Definition 2.11 Let $\pi \in \Delta^*$ be given, with $\pi_{e^N} \gg 0$. An NTU fuzzy game V is *π-balanced* if $\cap_{s\in\mathcal{B}} V(s) \subseteq V(e^N)$ for every π-balanced collection $\mathcal{B}$ of fuzzy coalitions.

The next result is Corollary 3.22 from Liu and Liu (2013).

Theorem 2.12 (Liu and Liu 2013) *Any π-balanced NTU fuzzy game V has a non-empty fuzzy core.*

Note from (2.5) and (2.6) that a balanced NTU fuzzy game V is π-balanced for the special $\pi \in \Delta^*$ with $\pi_s = \frac{s}{\sum_{i\in car(s)} s_i}$ for each $s \in \mathcal{F}^N$. The following fuzzy extension of Scarf's core existence theorem (Theorem 2.5) follows immediately from Theorem 2.12.

Theorem 2.13 *Any balanced NTU fuzzy game V has a non-empty fuzzy core.*

In the next section, we will apply Theorem 2.13 to show the existence of fuzzy cores and Edgeworth equilibria in coalition production economies.

3 Existence of fuzzy core and Edgeworth equilibrium in a coalition production economy

In 1963, Debreu and Scarf (1963) proved a remarkable result which gives a connection between the cores and the competitive equilibria in exchange economies. Florenzano (1990) defined the fuzzy core of an economy as the set of allocations which can not be blocked by any fuzzy coalition and showed that for certain production economies, including exchange economies, the asymptotic limit of the cores of replica economies coincides with the fuzzy core. In this section, we will show that in a coalition production economy with some standard assumptions, any allocation in the fuzzy core is an

Edgeworth equilibrium and we will apply Theorem 2.13—the fuzzy extension of Scarf's core existence theorem—to prove the non-emptiness of fuzzy cores in certain coalition production economies from which the existence of Edgeworth equilibria follow.

We first recall the concept of a coalition production economy given in Inoue (2013). For simplicity, we assume that the preference orderings are representable by real valued concave (or quasi-concave) utility functions, which can be used to approximate rather general preference relations arbitrarily closely according to Billera (1974) (also see Allouch and Florenzano 2004 and Allouch and Predtetchinski 2008). In the following models, every agent plays two roles as a consumer and as a member of production units.

A *coalition production economy* $\mathcal{E} = (\mathbb{R}^L, (X^i, u^i, w^i)_{i\in N}, (Y^S)_{S\in\mathcal{N}})$ with n agents is a collection of the commodity space $\mathbb{R}^L$, where L is the set of commodities, agents' characteristics $(X^i, u^i, w^i)_{i\in N}$, and coalitions' production sets $(Y^S)_{S\in\mathcal{N}}$. The triple (X^i, u^i, w^i) is agent i's characteristics as a consumer: $X^i \subseteq \mathbb{R}^L$ is his consumption set, $u^i : X^i \to \mathbb{R}$ is his utility function, and $w^i \in \mathbb{R}^L$ is his endowment vector. The set $Y^S \subseteq \mathbb{R}^L$ is the production set of the firm (coalition) S for which every agent $i \in S$ works and Y^S consists of all production plans that can be achieved through a joint action by the members of S. We use $Y = Y^N$ for the total production possibility set of the economy.

An *exchange economy* is a coalition production economy with $Y^S = \{0\}$ for every $S \in \mathcal{N}$.

When dealing with replica of an economy $\mathcal{E}$, we need some special conditions on the production possibility sets $(Y^S)_{S\in\mathcal{N}}$. The key assumption is that when $y \in Y^S$, $cy \in Y^S$ for any nonnegative constant c. Since inputs into production appear as negative components of $y \in Y^S$ and outputs as positive components, we must have $Y^S \cap \mathbb{R}^L_+ = \{0\}$ (*impossibility of free production*) for any production set Y^S, where $\mathbb{R}^L_+$ is the nonnegative orthant of the commodity space $\mathbb{R}^L$. Here are some common assumptions:

- **(P.1)** $Y^S = \{0\}$ for all $S \in \mathcal{N}$ (exchange economies, see Allouch and Florenzano 2004; Debreu and Scarf 1963);
- **(P.2)** Y is a convex cone with vertex at the origin and $Y^S = Y$ for all $S \in \mathcal{N}$ (see Debreu and Scarf 1963);
- **(P.3)** Y^S is a convex cone containing the origin (a special case is that Y^j is a convex cone with vertex at the origin for each $j \in N$ and $Y^S = \sum_{j\in S} Y^j$ for every $S \in \mathcal{N}$).

Clearly, (P.3) contains (P.2) which contains (P.1). Any coalition production economy $\mathcal{E} = (\mathbb{R}^L, (X^i, u^i, w^i)_{i\in N}, (Y^S)_{S\in\mathcal{N}})$ satisfying that each Y^S is a convex cone with vertex at the origin and $Y^S \neq Y^T$ when $S \neq T$ would be an example of a coalition production economy satisfying (P.3) but not (P.1) or (P.2).

We make the following assumptions on consumption sets, utility functions, and the sets of attainable allocations here:

- **(A.1)** For every agent $i \in N$, $X^i \subseteq \mathbb{R}^L$ is non-empty, closed, and convex, and $w^i \in X^i$.
- **(A.2)** For each $i \in N$, $u^i : X^i \to \mathbb{R}$ is continuous and quasi-concave;

(A.3) for each $S \in \mathcal{N}$, $Y^S \subseteq \mathbb{R}^L$ is nonempty and closed, and the set $F_{\mathcal{E}}(S)$ of feasible (attainable) S-allocations is nonempty and compact, where

$$F_{\mathcal{E}}(S) = \{(x^i)_{i \in S} | x^i \in X^i \quad \text{for each } i \in S \quad \text{and} \quad \sum_{i \in S}(x^i - w^i) \in Y^S\}.$$

The set of all attainable allocations of the economy $\mathcal{E}$ is

$$F(\mathcal{E}) = F_{\mathcal{E}}(N) = \{(x^i)_{i \in N} | x^i \in X^i \quad \text{for each } i \in N \quad \text{and} \quad \sum_{i \in N}(x^i - w^i) \in Y^N = Y\}$$

which is non-empty and compact.

Note that for each $S \in \mathcal{N}$, $F_{\mathcal{E}}(S) \neq \emptyset$ if and only if $\left(\sum_{i \in S} X^i\right) \cap \left(\sum_{i \in S} w^i + Y^S\right) \neq \emptyset$, and $0 \in Y^S$ implies that $(w^i)_{i \in S} \in F_{\mathcal{E}}(S)$.

Any coalition production economy $\mathcal{E}$ generates an NTU game $V_{\mathcal{E}} : \mathcal{N} \mapsto \mathbb{R}^n$ by defining, for each $S \in \mathcal{N}$,

$$V_{\mathcal{E}}(S) = \{v \in \mathbb{R}^S | \text{ there exists } (x^i)_{i \in S} \in F_{\mathcal{E}}(S) \text{ such that } v_i \leq u^i(x^i) \\ \text{for every } i \in S\},$$

where $\mathbb{R}^S = \{x \in \mathbb{R}^n | x_i = 0 \text{ for each } i \in N \setminus S\}$. Inoue (2013) proved that an NTU game is generated by a coalition production economy if and only if it is compactly generated (i.e., for every $S \in \mathcal{N}$, there exists a nonempty compact subset C_S of $\mathbb{R}^S$ such that $V(S) = C_S - \mathbb{R}^S_+$).

Note that the market game V defined by Billera (1974) is an NTU game generated by a market - a special coalition production economy $\mathcal{E}$ satisfying $Y^S = \sum_{i \in S} Y^i = \sum_{i \in S}(X^i - \{w^i\})$ for each $S \in \mathcal{N}$. While Billera (1974) proved that a market game is always totally balanced and has non-empty core, it is not the case for an NTU game arising from a general coalition production economy.

Florenzano (1989) gave the following concept of balanced economy, where Y is the total production possibility set of the economy.

Definition 3.1 A coalition production economy $\mathcal{E}$ is *balanced* if $\sum_{S \in \mathcal{B}} \lambda_S Y^S \subseteq Y$ for every balanced collection $\mathcal{B}$ of coalitions with balancing coefficients λ_S, $S \in \mathcal{B}$.

We say that an allocation $x = (x^i)_{i \in N}$ in a coalition production economy $\mathcal{E}$ is blocked by a coalition S if there is an attainable S-allocation $(y^i)_{i \in S}$ such that $u^i(y^i) > u^i(x^i)$ for all $i \in S$. The core $C(\mathcal{E})$ of an economy $\mathcal{E}$ is the set of all attainable allocations that can not be blocked by any coalition. Boehm (1974) proved the following result and later Florenzano (1989) extended it to the case where the commodity space is infinite-dimensional.

Theorem 3.2 (Boehm 1974; Florenzano 1989) *If a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) is balanced, then it has a non-empty core $C(\mathcal{E})$.*

Next, we derive the existence of fuzzy cores in certain coalition production economies which will be used to show the existence of Edgeworth equilibrium and competitive equilibrium. Florenzano (1990) defined the fuzzy core $C_F(\mathcal{E})$ of such an economy $\mathcal{E}$ to be the set of all attainable allocations which can not be blocked by any fuzzy coalition. The following formal definition for a fuzzy core is an extension of the fuzzy core for an exchange economy defined by Allouch and Predtetchinski (2008), where we have replaced Δ^N by the set of all non-zero fuzzy coalitions $\mathcal{F}^N$ (clearly, $\Delta^N \subseteq \mathcal{F}^N$). Denote $X = \prod_{i \in N} X^i$.

Definition 3.3 An allocation $x \in F(\mathcal{E})$ is an element of the fuzzy core $C_F(\mathcal{E})$ of the economy $\mathcal{E}$ if there exists no $s \in \mathcal{F}^N$ and no $y \in X$ such that $\sum_{i \in N} s_i(y^i - w^i) \in Y^{car(s)}$ and $u^i(x^i) < u^i(y^i)$ for each $i \in car(s)$.

Clearly, the fuzzy core $C_F(\mathcal{E})$ of an economy $\mathcal{E}$ is a subset of its core $C(\mathcal{E})$.

Definition 3.4 A coalition production economy $\mathcal{E}$ is *strongly balanced* if $\sum_{s \in \mathcal{B}} \lambda_s Y^{car(s)} \subseteq Y$ for every balanced collection $\mathcal{B}$ of fuzzy coalitions with balancing coefficients λ_s, $s \in \mathcal{B}$.

Clearly, if a coalition production economy $\mathcal{E}$ satisfies (P.1) or (P.2), then it is strongly balanced. We now prove the non-emptiness of fuzzy cores by applying Theorem 2.13.

Theorem 3.5 *If a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) and (P.3) is strongly balanced, then its fuzzy core $C_F(\mathcal{E})$ is non-empty.*

Proof We define an NTU fuzzy game V from the economy $\mathcal{E}$ as follows: For each $s \in \mathcal{F}^N$, define

$$V(s) = \left\{ v \in \mathbb{R}^n \,\middle|\, \begin{array}{l} \text{there exists } (x^i)_{i \in car(s)} \in F_{\mathcal{E}}(car(s)) \\ \text{such that } \sum_{i \in N} s_i(x^i - w^i) \in Y^{car(s)} \\ \text{and } v_i \leq u^i(x^i) \text{ for each } i \in car(s) \end{array} \right\}.$$

It is not difficult to check that V satisfies the conditions for an NTU fuzzy game given in Definition 2.6. In fact, Condition (2) in Definition 2.6 holds as it is clear from the definition of $V(s)$ that each $V(s)$ is cylindrical in the sense that if $x \in V(s)$ and $y \in \mathbb{R}^n$ such that $y_i = x_i$ for each $i \in car(s)$, then $y \in V(s)$. Also, it is clear that each $V(s)$ is closed and comprehensive (i.e.,if $x, y \in \mathbb{R}^n$ are such that $y \in V(s)$ and $x \leq y$, then $x \in V(s)$). For each $s \in \mathcal{F}^N$, $V(s)$ is nonempty because $(w^i)_{i \in car(s)} \in F_{\mathcal{E}}(car(s))$ and $0 \in Y^{car(s)}$ imply that $v \in V(s)$ with $v_i = u^i(w^i)$ for each $i \in car(s)$. By (A.2) and (A.3), u^i is continuous for each $i \in N$ and $F_{\mathcal{E}}(car(s))$ is compact for each $s \in \mathcal{F}^N$. It follows that for each $s \in \mathcal{F}^N$, there exists $(x^i)_{i \in car(s)} \in F_{\mathcal{E}}(car(s))$ such that $u^i(x^i)$ is maximum for each $i \in car(s)$ which implies that $V(s)$ is bounded from above. Thus, Condition (1) in Definition 2.6 holds. Similarly, one can verify Condition (3) in Definition 2.6.

Note that

$$V(e^N) = \left\{ v \in \mathbb{R}^n \,\middle|\, \begin{array}{l} \text{there exists } x \in X \text{such that } \sum_{i \in N}(x^i - w^i) \in Y^N = Y \text{ and} \\ v_i \leq u^i(x^i) \text{for each} i \in N \end{array} \right\}.$$

By (2.3), it follows that the fuzzy core of V coincides with the fuzzy core of the economy $\mathcal{E}$, that is,

$$C_F(V) = V(e^N) \setminus [\cup_{s\in\mathcal{F}^N}\, int(V(s))] = C_F(\mathcal{E}).$$

By Theorem 2.13, in order to show $C_F(\mathcal{E}) \neq \emptyset$, it suffices to show that the NTU fuzzy game V is balanced. Let $\mathcal{B}$ be any balanced collection of fuzzy coalitions with λ_s, $s \in \mathcal{B}$, being balancing coefficients. Then, by (2.4), we have $\sum_{s\in\mathcal{B}} \lambda_s s = e^N$ which implies that

$$\sum_{s\in\mathcal{B}} \lambda_s s_i = 1 \quad \text{for each } i \in N.$$

Let $v \in \cap_{s\in\mathcal{B}} V(s)$. Then for each $s \in \mathcal{B}$, $v \in V(s)$ implies that there exists $x_s \in X$ such that $(x_s^i)_{i\in car(s)} \in F_{\mathcal{E}}(car(s))$, $\sum_{i\in N} s_i(x_s^i - w^i) \in Y^{car(s)}$, and $v_i \leq u^i(x_s^i)$ for each $i \in car(s)$.

Take x with $x^i = \sum_{s\in\mathcal{B}} \lambda_s s_i x_s^i$ for each $i \in N$. For each $s \in \mathcal{B}$, $(x_s^i)_{i\in car(s)} \in F_{\mathcal{E}}(car(s))$ implies $x_s^i \in X^i$ for every $i \in car(s)$. Then X^i is convex and $\sum_{s\in\mathcal{B}} \lambda_s s_i = 1$ together imply $x^i \in X^i$ for each $i \in N$, and so $x \in X$. Moreover, for each $i \in N$, since $\sum_{s\in\mathcal{B}: i\in car(s)} \lambda_s s_i = \sum_{s\in\mathcal{B}} \lambda_s s_i = 1$ and u^i is quasi-concave, we have

$$v_i \leq \min_{s\in\mathcal{B}: i\in car(s)} u^i(x_s^i) \leq u^i\left(\sum_{s\in\mathcal{B}: i\in car(s)} \lambda_s s_i x_s^i\right) = u^i\left(\sum_{s\in\mathcal{B}} \lambda_s s_i x_s^i\right) = u^i(x^i) \quad \text{and}$$

$$\sum_{i\in N}(x^i - w^i) = \sum_{i\in N}\left(\sum_{s\in\mathcal{B}} \lambda_s s_i x_s^i - \sum_{s\in\mathcal{B}} \lambda_s s_i w^i\right) = \sum_{s\in\mathcal{B}} \lambda_s \left(\sum_{i\in N} s_i(x_s^i - w^i)\right).$$

Set $y^s = \sum_{i\in N} s_i(x_s^i - w^i)$ for each $s \in \mathcal{B}$. Since $\mathcal{E}$ is strongly balanced and $y^s \in Y^{car(s)}$ for each $s \in \mathcal{B}$, we have

$$\sum_{i\in N}(x^i - w^i) = \sum_{s\in\mathcal{B}} \lambda_s y^s \in Y.$$

It follows that $v \in V(e^N)$. Thus V is balanced. By Theorem 2.13, $C_F(\mathcal{E}) = C_F(V) \neq \emptyset$ and the theorem follows. □

In an effort to connect the two concepts of core and competitive equilibrium in exchange economies (more generally, coalition production economies satisfying (P.2)), Debreu and Scarf (1963) considered r-fold replica of an economy. For each positive integer r, the r-fold replica of the economy $\mathcal{E}$, denoted by $\mathcal{E}_r$, is defined to be the economy composed of r subeconomies identical to $\mathcal{E}$ with a set of consumers

$$N_r = \{(i, q) | i = 1, \ldots, n \text{ and } q = 1, \ldots, r\}.$$

The first index of consumer (i, q) refers to the type of the individual and the second index distinguishes different individuals of the same type. It is assumed that all consumers of type i are identical in terms of their consumption sets, endowments, and utility functions. Let S be a non-empty subset of N_r. An allocation $(x^{(i,q)})_{(i,q)\in S}$ is S-attainable in the economy $\mathcal{E}_r$ if

$$\sum_{(i,q)\in S} (x^{(i,q)} - w^{(i,q)}) \in Y^{S'}, \tag{3.1}$$

where $S' = \{i \in N | (i, q) \in S\}$, $x^{(i,q)} \in X^i$ and $w^{(i,q)} = w^i$ for every q. Thus, (3.1) can be written as

$$\sum_{i\in S'} \sum_{q\in S(i)} x^{(i,q)} - \sum_{i\in S'} |S(i)| w^i \in Y^{S'}, \tag{3.2}$$

where $S(i) = \{q \in \{1, 2, \ldots, r\} | (i, q) \in S\}$ and $|S(i)|$ denotes the number of elements in $S(i)$.

Define $s = (s_i)_{i\in N}$ by setting $s_i = \frac{|S(i)|}{r}$ for each $i \in N$, and define x_s by letting $x_s^i = \frac{1}{|S(i)|} \sum_{q\in S(i)} x^{(i,q)}$. Then, under the assumption (P.3) (or (P.1) or (P.2)), (3.2) becomes

$$\sum_{i\in S'} s_i x_s^i - \sum_{i\in S'} s_i w^i \in Y^{S'}. \tag{3.3}$$

For each $i \in S'$, since each $x^{(i,q)} \in X^i$ for $1 \le q \le r$ and X^i is convex,

$$x_s^i = \frac{1}{|S(i)|} \sum_{q\in S(i)} x^{(i,q)} \in X^i.$$

Thus, we have shown that an allocation $(x^{(i,q)})_{(i,q)\in S}$ is S-attainable in the economy $\mathcal{E}_r$ means

$$\sum_{i\in S'} s_i x_s^i - \sum_{i\in S'} s_i w^i \in Y^{S'} \tag{3.4}$$

with $S' = \{i \in N | (i, q) \in S\}$, s_i being a rational number and $x_s^i \in X^i$ for each $i \in S'$. Note that since $car(s) = S'$, (3.4) is the same as

$$\sum_{i\in N} s_i (x_s^i - w^i) \in Y^{car(s)}. \tag{3.5}$$

For each positive integer r, the r-fold repetition of an allocation $x = (x^i)_{i\in N}$ is defined to be

$$(x^{(i,q)})_{(i,q)\in N_r}, \quad \text{where } x^{(i,q)} = x^i \quad \text{for each } 1 \le q \le r \quad \text{and} \quad \text{every } i \in N.$$

The following definition of an Edgeworth equilibrium was given in Florenzano (1990).

Definition 3.6 An *Edgeworth equilibrium* in an economy $\mathcal{E}$ is an attainable allocation $x \in F(\mathcal{E})$ such that for any positive integer r, the r-fold repetition of x belongs to the core of the r-fold replica $\mathcal{E}_r$ of the original economy $\mathcal{E}$. We will denote by $C^E(\mathcal{E})$ the set of all Edgeworth equilibria of $\mathcal{E}$.

As it is easily proved by Florenzano (1990) that for exchange economies (coalition production economies satisfying (P.1)) or production economies where every agent has his own production set, an Edgeworth equilibrium can also be defined as an attainable allocation which can not be blocked by any fuzzy coalition with rational rates of participation. By a similar argument, one can see that the same is true for coalition production economies satisfying (P.2) or (P.3) and we have the following proposition.

Proposition 3.7 *For a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) and (P.3), every allocation in the fuzzy core of $\mathcal{E}$ is an Edgeworth equilibrium, that is, $C_F(\mathcal{E}) \subseteq C^E(\mathcal{E})$.*

Proof Recall that the fuzzy core $C_F(\mathcal{E})$ of an economy $\mathcal{E}$ is the set of all attainable allocations which can not be blocked by any fuzzy coalition. It suffices to show that for a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) and (P.3), if an attainable allocation $x \in F(\mathcal{E})$ can not be blocked by any fuzzy coalition with rational rates of participation, then x is an Edgeworth equilibrium.

Suppose, to the contrary, that $x \in F(\mathcal{E})$ can not be blocked by any fuzzy coalition with rational rates of participation, but x is not an Edgeworth equilibrium. Then there exists $r > 0$, such that the r-fold repetition of x is blocked by a coalition $S \subseteq N_r$ in the r-fold replica $\mathcal{E}_r$ through an S-attainable partial allocation $(\overline{x}^{(i,q)})_{(i,q)\in S}$. By (3.1), (3.4), and (3.5), there exists a fuzzy coalition s with each s_i being a rational number such that

$$\sum_{i\in N} s_i(\overline{x}^i_s - w^i) \in Y^{car(s)},$$

where $S(i) = \{q \in \{1, 2, \ldots, r\} | (i, q) \in S\}$, $\overline{x}^i_s = \frac{1}{|S(i)|}\sum_{q\in S(i)} \overline{x}^{(i,q)} \in X^i$, $s_i = \frac{|S(i)|}{r}$ for each $i \in N$, and $car(s) = S' = \{i \in N | (i, q) \in S\}$. For each $i \in S'$, since $u^i(\overline{x}^{(i,q)}) > u^i(x^i)$ for every $q \in S(i)$ and u^i is quasi-concave,

$$u^i(x^i) < min_{q\in S(i)}\{u^i(\overline{x}^{(i,q)})\} \le u^i\left(\frac{1}{|S(i)|}\sum_{q\in S(i)} \overline{x}^{(i,q)}\right) = u^i(\overline{x}^i_s).$$

It follows that x is blocked by the fuzzy coalition s with rational rates of participation through the s-attainable partial vector $(\overline{x}^i_s)_{i\in S'=car(s)}$, a contradiction. □

We remark here that under an additional assumption that Y^S is open for each $S \in \mathcal{N}$, we have $C^E(\mathcal{E}) = C_F(\mathcal{E})$. In fact, suppose that there is $x \in C^E(\mathcal{E})$ such that

$x \notin C_F(\mathcal{E})$. Then there exists a fuzzy coalition $s \in \mathcal{F}^N$ such that x is blocked by s, that is, there exists $y \in X$ such that $\sum_{i \in N} s_i(y^i - w^i) \in Y^{car(s)}$ and $u^i(x^i) < u^i(y^i)$ for each $i \in car(s)$. Since $Y^{car(s)}$ is open, we can choose a fuzzy coalition s' with each s'_i being a rational number such that $car(s') = car(s)$ and each s'_i is close enough to s_i so that $\sum_{i \in N} s'_i(y^i - w^i) \in Y^{car(s)}$. As $u^i(x^i) < u^i(y^i)$ for each $i \in car(s) = car(s')$, x is blocked by the fuzzy coalition s' with rational rates of participation which implies that $x \notin C^E(\mathcal{E})$ by the earlier remark, a contradiction.

In the next section, we will establish the coincidence of the fuzzy core, the set of Edgeworth equilibria, and the set of competitive equilibria in a coalition production economy under some standard assumptions.

The following existence theorem for Edgeworth equilibrium follows immediately from Theorem 3.5 and Proposition 3.7.

Theorem 3.8 *If a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) and (P.3) is strongly balanced, then the set of Edgeworth equilibria $C^E(\mathcal{E})$ is non-empty.*

Note that if a coalition production economy $\mathcal{E}$ satisfies (P.1) or (P.2) , then it is strongly balanced. Since exchange economies are special coalition production economies which satisfy $Y^S = \{0\}$ for every $S \in \mathcal{N}$ (i.e., (P.1)) and (P.3) contains (P.1) and (P.2), Theorem 3.8 implies the next result proved by Allouch and Florenzano (2004).

Theorem 3.9 (Allouch and Florenzano 2004) *For an exchange economy $\mathcal{E}$ satisfying (A.1)–(A.3), the set of Edgeworth equilibria $C^E(\mathcal{E})$ is non-empty.*

Also, since (P.2) implies (P.3) and a coalition production economy $\mathcal{E}$ satisfying (P.2) is strongly balanced, Theorem 3.8 has the following immediate consequence.

Theorem 3.10 *If a coalition production economy $\mathcal{E}$ satisfies (A.1)–(A.3) and (P.2), then its set of Edgeworth equilibria $C^E(\mathcal{E})$ is non-empty.*

We conclude this section with the following remark: There are a number of papers in the literature dealing with existence of Edgeworth equilibria, see for example Aliprantis et al. (1987), Allouch (2012), Allouch and Florenzano (2004), and Florenzano (1990). They worked on either exchange economies - special coalition production economies with $Y^S = \{0\}$ for all $S \in \mathcal{N}$ (Allouch and Florenzano 2004) or coalition production economies satisfying (P.2) (Allouch 2012) or coalition production economies where every agent i has his own closed and convex production set Y^i containing 0 and $Y = \sum_{i \in N} Y^i$ (Aliprantis et al. 1987; Florenzano 1990), which is essentially a special case of (P.3). We consider coalition production economies with a production set Y^S for each coalition $S \in \mathcal{N}$ satisfying (P.3) which includes (P.1) and (P.2). Thus, our result Theorem 3.8 either implies existing existence theorems for Edgeworth equilibrium (see Theorem 3.9) or gives consequences such as Theorem 3.10 which will be used to show the existence of competitive equilibrium in the next section.

4 Existence of competitive equilibrium in coalition production economy

Throughout this section, we assume that $\mathcal{E}$ is a coalition production economy described in the previous section and denote $X = \prod_{i \in N} X^i$.

We first recall the concept of competitive equilibrium given in Debreu and Scarf (1963).

Definition 4.1 For an attainable allocation $x \in X$ and a price vector $p \neq 0$, the couple (x, p) is a *competitive equilibrium* (Walrasian equilibrium) of a coalition production economy $\mathcal{E}$ if the profit is maximized on Y (which is zero under the assumption (P.3) as remarked below) and for each $i \in N$, x^i satisfies the preferences of the i-th consumer under the constraint $p \cdot x^i \leq p \cdot w^i$, that is, for each $i \in N$,

$$p \cdot x^i = p \cdot w^i \quad \text{and} \quad u^i(\overline{x}^i) > u^i(x^i) \text{ imply } p \cdot \overline{x}^i > p \cdot x^i.$$

We remark here that, under the assumption (P.3), we have that for any price vector p and for each $S \in \mathcal{N}$, $p \cdot y \leq 0$ for any $y \in Y^S$. For otherwise, if $p \cdot y > 0$ for some $y \in Y^S$, then $cy \in Y^S$ for any $c \geq 0$ as the production set Y^S is a convex cone and the profit $p \cdot (cy) = c(p \cdot y)$ approaches to the positive infinity as c approaches to the positive infinity, which is impossible. Thus the maximum profit $sup\ p \cdot Y^S$ at price p on Y^S is zero for each $S \in \mathcal{N}$.

The following fact shows that a competitive allocation is in the fuzzy core for a coalition production economy, which follows similarly from a standard argument for the well-known fact that a competitive allocation is in the core by Debreu and Scarf (1963).

Proposition 4.2 *For a coalition production economy $\mathcal{E}$ satisfying (A.1)–(A.3) and (P.3), any competitive equilibrium of $\mathcal{E}$ is in the fuzzy core of $\mathcal{E}$.*

Proof Suppose that (x, p) is a competitive equilibrium but x is not in the fuzzy core $C_F(\mathcal{E})$. Then there exists a fuzzy coalition $s \in \mathcal{F}^N$ such that x is blocked by s, that is, there exists $y \in X$ such that $\sum_{i \in N} s_i(y^i - w^i) \in Y^{car(s)}$ and $u^i(x^i) < u^i(y^i)$ for each $i \in car(s)$. By the remark above, we have $p \cdot \sum_{i \in N} s_i(y^i - w^i) \leq 0$. On the other hand, since x is competitive, $u^i(x^i) < u^i(y^i)$ implies that $p \cdot y^i > p \cdot x^i = p \cdot w^i$ for each $i \in car(s)$. Thus, $p \cdot \sum_{i \in N} s_i(y^i - w^i) > 0$, a contradiction. □

We make the following additional assumptions for the existence of competitive equilibria.

- **(A.4)** *Local nonsatiation:* For each $i \in N$, let $x^i \in X^i$ be an arbitrary commodity bundle. Then there exists $\overline{x}^i \in X^i$ arbitrarily close to x^i such that $u^i(\overline{x}^i) > u^i(x^i)$;
- **(A.5)** For each $i \in N$, u^i is strongly convex, i.e., for all x^i and $\overline{x}^i$ such that $u^i(x^i) \geq u^i(\overline{x}^i)$ and all α, $0 < \alpha < 1$, $u^i(\alpha x^i + (1-\alpha)\overline{x}^i) > u^i(\overline{x}^i)$.
- **(A.6)** For each $i \in N$, w^i is an interior point in X^i.

Given an economy $\mathcal{E}$, an allocation $x = (x^1, \ldots, x^n) \in X$ is in the core of the r-fold replica economy $\mathcal{E}_r$ of $\mathcal{E}$ means the r-fold repetition of x is in the core of $\mathcal{E}_r$. The following theorem is proved by Debreu and Scarf (1963), where convex preferences are replaced by quasi-concave utility functions.

Theorem 4.3 (Debreu and Scarf 1963) *Let $\mathcal{E}$ be a coalition production economy satisfying (P.2) (or (P.1)), (A.1), (A.2), (A.4)–(A.6). If an allocation $x = (x^1, \ldots, x^n)$ is in the core of the r-fold replica economy $\mathcal{E}_r$ for all r, then x is a competitive equilibrium.*

Recall that an Edgeworth equilibrium is an allocation $x = (x^1, \ldots, x^n)$ which is in the core of the r-fold replica economy $\mathcal{E}_r$ for all r. Theorem 4.3 simply says that every Edgeworth equilibrium in such an economy is a competitive equilibrium. Since condition (P.2) (or (P.1)) implies (P.3), the next theorem follows immediately from Proposition 3.7 and Theorems 3.5 and 4.3.

Theorem 4.4 *Let $\mathcal{E}$ be a coalition production economy satisfying (P.2) (or (P.1)) and (A.1)–(A.6). Then every allocation in the fuzzy core is a competitive equilibrium and the set of competitive equilibria of $\mathcal{E}$ is non-empty.*

Theorem 4.3 can be extended to the following form with condition (P.2) replaced by (P.3). The following proof follows standard decentralization arguments and is based heavily on the ideas from the proof for Theorem 4.3 by Debreu and Scarf (1963). The main difference is at the first part where extra efforts are needed to overcome the difficulty caused by the fact that different coalitions may have different production sets under assumption (P.3). Note that the assumption each u^i is continuous in (A.2) implies that for each $i \in N$ and any $x^i \in X^i$, the two sets $\{x' \in X^i | u^i(x') > u^i(x^i)\}$ and $\{x' \in X^i | u^i(x') < u^i(x^i)\}$ are open in X^i.

Theorem 4.5 *Let $\mathcal{E}$ be a coalition production economy satisfying (P.3), (A.1), (A.2), (A.4)–(A.6). If an allocation $x = (x^1, \ldots, x^n)$ is in the core of the r-fold replica economy $\mathcal{E}_r$ for all r (i.e., an Edgeworth equilibrium), then x is a competitive equilibrium.*

Proof Let $x = (x^1, \ldots, x^n)$ be an allocation which is in the core of the r-fold replica economy $\mathcal{E}_r$ for all r. For each $i \in N$, define

$$A_i(x^i) = \{z \in \mathbb{R}^L | w^i + z \in X^i \text{ and } u^i(w^i + z) > u^i(x^i)\}.$$

Then for every $i \in N$, $A_i(x^i)$ is non-empty by (A.4) and is open by (A.2).

For each $r \geq 1$, let S_j^r be a coalition in an r-fold replica economy $\mathcal{E}_r$ with q_{ij}^r members of type i, where $1 \leq q_{ij}^r \leq r$, and denote $Q_j^r = (q_{1j}^r, q_{2j}^r, \ldots, q_{nj}^r)$. Define

$$A^r(Q_j^r, x) = \sum_{i \in N} q_{ij}^r A_i(x^i) \quad \text{and}$$

$$A^r(x) = \text{ convex hull of } \cup_{Q_j^r} A^r(Q_j^r, x) = \cup\{\sum_{j \in \mathcal{F}} \lambda_j A^r(Q_j^r, x)\},$$

where the last union is taken over all finite convex combinations.

By standard decentralization arguments, similar to that in the proof for Theorem 4.3 in Debreu and Scarf (1963), one can show that for any $r \geq 1$, $A^r(x) \cap Y = \emptyset$, with the following key idea: for otherwise, under the assumptions (A.2) and (A.5), one could find, for k sufficiently large, a coalition $S \subseteq N_k$ in the k-fold replica economy $\mathcal{E}_k$ which blocks the k-fold repetition of the allocation $x = (x^1, \ldots, x^n)$ through an S-attainable partial allocation $(\overline{x}^{(i,q)})_{(i,q)\in S}$ with $S' = \{i \in N|(i,q) \in S\} = N$ (where S-attainable means that $\sum_{(i,q)\in S}(\overline{x}^{(i,q)} - w^{(i,q)}) \in Y^{S'} = Y^N = Y$ by (3.1)), contradicting the assumption that $x = (x^1, \ldots, x^n)$ is in the core of the r-fold replica economy $\mathcal{E}_r$ for all r.

Now, by the separating hyperplane theorem, the two convex sets $A^r(x)$ and Y are separated by a hyperplane: there exists a nonzero $p \in \mathbb{R}^L$ such that

$$p \cdot z \geq 0 \quad \text{for all points } z \in A^r(x), \tag{4.1}$$

$$p \cdot y \leq 0 \quad \text{for all points } y \in Y.$$

We claim that for each $i \in N$, $p \cdot z \geq 0$ for all $z \in A_i(x^i)$. Suppose there exists $z^k \in A_k(x^k)$ such that $p \cdot z^k < 0$. For any $Q^r_j = (q^r_{1j}, q^r_{2j}, \ldots, q^r_{nj})$ with $1 \leq q^r_{ij} \leq r$ and any $z^i \in A_i(x^i)$ for $i \in N$, by (4.1), $\sum_{i\in N} q^r_{ij} z^i \in A^r(Q^r_j, x) = \sum_{i\in N} q^r_{ij} A_i(x^i)$ implies that $p \cdot \sum_{i\in N} q^r_{ij} z^i \geq 0$. But, choosing q^r_{kj} sufficiently large (this requires r to be large enough) while fixing other q^r_{ij}'s would lead to $p \cdot \sum_{i\in N} q^r_{ij} z^i < 0$, a contradiction.

Thus, we have that for each $i \in N$, $p \cdot z \geq 0$ for all $z \in A_i(x^i)$ which implies that for any $x' = w^i + z \in X^i$,

$$u^i(x') > u^i(x^i) \text{ implies} p \cdot x' \geq p \cdot w^i.$$

Now, since w^i is an interior point of X^i for each $i \in N$, by standard arguments, similar to that given in Debreu and Scarf (1963), one can show that (x, p) is a competitive equilibrium. □

The next coincidence theorem follows immediately from Propositions 3.7 and 4.2, and Theorem 4.5.

Theorem 4.6 *Let $\mathcal{E}$ be a coalition production economy satisfying (P.3) and (A.1)–(A.6). Then the fuzzy core $C_F(\mathcal{E})$, the set $C^E(\mathcal{E})$ of Edgeworth equilibria, and the set of competitive equilibria of $\mathcal{E}$ coincide.*

As an immediate consequence of Theorems 3.5 and 4.6, we have the following existence result for competitive equilibrium.

Theorem 4.7 *Let $\mathcal{E}$ be a coalition production economy satisfying (P.3) and (A.1)–(A.6). If $\mathcal{E}$ is strongly balanced, then the set of competitive equilibria of $\mathcal{E}$ is non-empty.*

5 Concluding remark

Following the ideas from Debreu and Scarf (1963) and Florenzano (1990), we have extended Debreu and Scarf's result that an Edgeworth equilibrium is a competitive equilibrium and Florenzano's result that any allocation in the fuzzy core is an Edgeworth equilibrium to a wider class of coalition production economies under some standard assumptions. We believe that the same should be true for other economies with some necessary assumptions. Therefore the existence of fuzzy cores can be used to establish the existence of competitive equilibria.

Acknowledgments The authors would like to thank the referees for their many helpful and inspiring suggestions and comments which resulted significant improvements to the paper. In particular, we thank one referee for one comment which made an important improvement in the presentation of the proof for Theorem 3.5 and the other referee for one comment which leads to Proposition 4.2 and the current more important version of Theorem 4.6 - the coincidence theorem of the fuzzy core, the set of Edgeworth equilibria, and the set of competitive equilibria of a coalition production economy $\mathcal{E}$. We also thank the Science Foundation of Shaanxi Province, P. R. China (2011JM9011) for the support.

References

Aliprantis CD, Brown DJ, Burkinshaw O (1987) Edgeworth equilibra in production economies. J Econ Theory 43:252–291

Allouch N (2012) A core-equilibrium convergence in a public goods economy. J Public Econ Theory 12:857–870

Allouch N, Florenzano M (2004) Edgeworth and Walras equilibra of an arbitrage-free exchange economy. Econ Theory 23:353–370

Allouch N, Predtetchinski A (2008) On the non-emptiness of the fuzzy core. Int J Game Theory 37:203–210

Aubin JP (1981) Cooperative fuzzy games. Math Oper Res 6:1–13

Billera LJ (1970) Some theorems on the core of an n-person game without side-payments. SIAM J Appl Math 18:567–579

Billera LJ (1974) On games without side payments arising from a general class of markets. J Math Econ 1:129–139

Boehm V (1974) The core of an economy with production. Rev Econ Stud 41:429–436

Debreu G, Scarf H (1963) A limit theorem on the core of an economy. Int Econ Rev 4:235–246

Florenzano M (1989) On the non-emptiness of the core of a coalition production economy without ordered preferences. J Math Anal Appl 141:484–490

Florenzano M (1990) Edgeworth equilibria, production economy without ordered preferences. J Math Anal Appl 153:18–36

Inoue T (2013) Representation of non-transferable utility games by coalition production economies. J Math Econ 49:141–149

Liu J, Liu X (2013) A necessary and sufficient condition for an NTU fuzzy game to have a non-empty fuzzy core. J Math Econ 49:150–156

Scarf H (1967) The core of an N-person game. Econometrica 35:50–69

Shapley LS, Vohra R (1991) On Kakutani's fixed point theorem, the K-K-M-S theorem and the core of a balanced game. Econ Theory 1:108–116

Zhou L (1994) A theorem on open coverings of a simplex and Scarf's core existence theorem through Brouwer's fixed point theorem. Econ Theory 4:473–477