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Efficiency and compromise: a bid-offer–counteroffer mechanism with two players

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Abstract A bid-offer–counteroffer mechanism is proposed to solve a fundamental two-person decision choice problem with two alternatives. It yields a unique subgame perfect equilibrium outcome, and leads to an intuitive overall solution that offers a reconciliation between egalitarianism and utilitarianism. We then investigate the axiomatic foundation of the solution. Furthermore, we compare it with several conventional strategic approaches to this setting.

Keywords Decision choice · Bargaining · Conflict resolution · Counteroffer · Implementation

JEL Classification C71 · C72 · D62 · D70

1 Introduction

Two agents have conflicting interests in two decision alternatives but only one will be adopted. How to resolve the conflicts such that both agree on an efficient alternative? This situation captures a wide range of decision making problems in real life. While such examples are abundant, below we present a local public good problem for illustration.

A university is opening a new campus and will relocate two colleges there. Due to the capacity constraint as well as the construction process, the relocation can only be done sequentially. Both colleges prefer moving to the new campus earlier rather than later. The costs and benefits of the relocation are different to the two colleges, which can be clearly monitored and therefore are common knowledge. Which college should

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be relocated first? By which criterion? If a mechanism can help resolve the conflict by compensating the college that moves secondly, then how much is the compensation?

The underlying feature of the example is that there are two alternatives with one being more favorable to an agent and the other alternative more favorable to the other agent, and finally one alternative is supposed to be chosen.

While such a two-agent two-alternative situation has a bargaining flavor, it cannot be directly modeled into a standard bargaining problem a la Nash (1950) because it has no well specified disagreement point.[1] Then, if one would like to analyze the problem from the standard bargaining point of view, a possible way is to specify the likely disagreement point by imposing further conditions or information (e.g., using a lottery or taking an outside option such as with zero payoffs to both players to generate a disagreement point), but this may change the nature of the problem. Chun and Thomson (1990a,b,c) introduce bargaining problems with uncertain disagreement points and study the egalitarian solutions and Nash solution in such environments from an axiomatic perspective. Among others, Peters and van Damme (1991) also studies the role of disagreement point in characterizing bargaining solutions. Their seminal work provides important insights and a well established axiomatic framework to study bargaining problems, and hence, offers a useful perspective for one to analyze the current problem in the spirit of Nash. For a recent research along this line, we refer to Kıbrıs and Tapkı (2010).

A general binary choice problem with side payments and quasi-linear utilities has been axiomatically investigated by Moulin (1987), where two social choice functions, the egalitarian rule and the laissez-faire rule, are characterized. Specific public decision problems, mainly in the context of public good provision and cost sharing, have also been extensively studied in, among others, Moulin (1981, 1984, 1985), and Jackson and Moulin (1992). In particular, the current research is well inspired by and closely related to the ‘auctioning the leadership with differentiated bids’ mechanism in Moulin (1981). Recently, Green (2005) studies a family of solutions to such problems with two players, from an axiomatic perspective. Chambers and Green (2005) pushes this analysis further.

The current paper is primarily taking a strategic perspective towards a decentralized mechanism for the binary choice problem, and meanwhile we do explore the axiomatic foundations of the solution resulted from the mechanism.[2] To this end, we propose a bid-offer–counteroffer mechanism that naturally fits the context of the problem and explicitly allows players chances to fully exert their strength in negotiation. In the literature, attention has been well paid to a fair way of choosing a proposer, such as the ‘auctioning the leadership’ rule proposed by Moulin (1981) and the multi-bidding approach proposed by Pérez-Castrillo and Wettstein (2001, 2002). And we preserve

[1] More discussion on this issue can be found in Green (2005). We like to further note that while negotiation situations without disagreement point are common and numerous such examples can be seen from the literature mentioned in this section, there is an often neglected one: queueing problems (cf. Maniquet 2003) are one of such, as illustrated in the above example, where it is assumed that a queue has to be organized but no specific order is taken as a predetermined disagreement point.

[2] As Nash (1953) suggests, these two approaches are complementary. And as argued by Serrano (2005), the normative side of a solution forms a more appropriate ‘foundation’ as it studies the independent (from game forms) principles.

this nice design to endogenously select a proposer in our mechanism. However, the negotiation after that is usually set to be an ultimatum game where if an offer is rejected the game ends with a certain scenario. This does not cause problems in situations without externalities. But for binary choice problems, where externalities exist, such a design may not suffice any more. We then incorporate a full fledged round of negotiation into the mechanism such that players can reach an efficiently compromised outcome by offer and counteroffer, no matter who is selected to be proposer. We show that this intuitive mechanism yields a unique subgame perfect equilibrium (SPE) outcome for generic cases, and leads to a reasonable overall solution to all types of problems. Furthermore, in SPE the mechanism implements the efficient alternative and each player obtains a payoff no worse than the 'flip-a-coin' payoff (the average payoff of the two alternatives).

As a distinct feature of the mechanism, it solves an open problem in the literature by offering a natural reconciliation between egalitarianism and utilitarianism defined by Moulin (1985).[3] Moulin (1985) argues in favor of a utilitarian social choice function for situations where agents have no conflicting interests, and an egalitarian social choice function for other situations. Indeed here these two perspectives are reconciled within the bid-offer–counteroffer mechanism. The dominant alternative will be chosen in SPE when the mechanism is applied to the no-conflict situation, and the mechanism will generate an egalitarian outcome in SPE when it is applied to the conflicting cases. Apart from other interesting aspects of the mechanism, we view this property as a major one that selects the mechanism out of the others.

While the paper is mainly driven by a strategic consideration, we also investigate the underlying axiomatic foundation of the solution obtained. When the overall solution is restricted to the no-conflict situations, it is characterized by the properties of efficiency and no transfer when no conflict. When it is restricted to the conflicting cases, the solution is characterized by four properties: efficiency, balanced threat, no influence, and additivity. When we focus on the fact that the bid-offer–counteroffer mechanism implements the overall solution in SPE, this paper can also be seen as a specific concretization of the result (by analyzing a very abstract stage mechanism) in Moore and Repullo (1988) with respect to two players. With complete information, Moore and Repullo (1988) shows that any social choice function is subgame perfect implementable even with two agents if both of them have quasi-linear utilities, and the corresponding mechanism needs no more than three stages.[4] Here, the current paper exactly meets these conditions.

The rest of the paper is structured as follows. The formal model and related definitions are given in the next section. In Sect. 3, we introduce the bid-offer–counteroffer mechanism and show it has a unique SPE outcome, which gives rise to the overall solution. Axiomatic characterizations are discussed in the following section. Section 5 analyzes and compares with several conventional strategic approaches to our problem. The final section concludes by discussing possible extensions for future work.

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[3] I am grateful to Hervé Moulin for an inspiring comment regarding this issue, pointing out that egalitarianism and utilitarianism are reconciled within one mechanism, and also thank him for suggesting the term of overall solution.

[4] See also Cremer and McLean (1987) and Maskin (1999) for related results in implementation.

2 The model of a two-person decision problem

Formally, a *two-person decision choice problem with two alternatives* (in short, *two-person decision problem*) is given by a pair (d^A, d^B) where $d^A = (d_1^A, d_2^A)$, decision alternative A, is a vector in $\mathbb{R}^2$ (here d_1^A represents the payoff of decision alternative A to player 1), and so is d^B. The set of two-person decision problems is denoted by 2DP. A two-person decision problem (d^A, d^B) in matrix form is expressed as follows.

	Player 1's payoff	Player 2's payoff
Decision alternative A	d_1^A	d_2^A
Decision alternative B	d_1^B	d_2^B

The current paper focuses on the set of generic problems, which, for simplicity, is still denoted by 2DP. The basic assumptions we impose to the two-person decision problems are the following: complete information, transferable utility (agents are allowed to make monetary transfers), quasi-linear utilities, risk neutrality, and agents' utilities are unknown to the social planner.

An *efficient alternative* (in terms of the total welfare of the two players), $e \in \{A, B\}$, is such that $d_1^e + d_2^e = \max\{d_1^A + d_2^A, d_1^B + d_2^B\}$.

A general type of two-person decision problems is that the two players have conflicting interests over the two decision alternatives. Formally, a problem $(d^A, d^B) \in$ 2DP is *undominated* if either $d_1^A > d_1^B$ and $d_2^B > d_2^A$ or $d_1^A < d_1^B$ and $d_2^B < d_2^A$.

In addition, we have the decision problems with a dominant alternative. If $d^A > d^B$, i.e., $d_1^A > d_1^B$ and $d_2^A > d_2^B$, then we say alternative A is a (strictly) *dominant* alternative. Similarly, if $d^B > d^A$, then alternative B is (strictly) dominant.

In some special cases, a decision problem may have a weakly dominant alternative. Decision alternative A is *weakly dominant* to alternative B if $d^A \geq d^B$ but $d^A \neq d^B$, that is, either $d_1^A = d_1^B$ while $d_2^A > d_2^B$ or $d_1^A > d_1^B$ while $d_2^A = d_2^B$. Since this is a non-generic case (the event happens with probability zero), we ignore such problems.

Below we provide two numerical examples for the undominated and dominant cases.

Example 2.1 (Undominated) Players, 1 and 2 face two decision alternatives, A and B.

	Player 1's payoff	Player 2's payoff
Decision alternative A	100	2
Decision alternative B	0	10

Example 2.2 (Dominant) Here, decision alternative B is dominant.

	Player 1's payoff	Player 2's payoff
Decision alternative A	60	2
Decision alternative B	100	10

A solution concept on 2DP is a function $f : 2\text{DP} \to \mathbb{R}^2$, where f_1 and f_2 are the payoffs of players 1 and 2, respectively, according to solution f. A solution concept f is *efficient* if, for any (d^A, d^B) in 2DP $f_1(d^A, d^B) + f_2(d^A, d^B) = \max\{d_1^A + d_2^A, d_1^B + d_2^B\}$. Below we define two solutions. For any $(d^A, d^B) \in$ 2DP, the *balanced threat* solution β is defined by

$$\beta_1(d^A, d^B) = \frac{d_1^A + d_1^B}{2} + \frac{\max\{d_1^A + d_2^A, d_1^B + d_2^B\} - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2};$$

$$\beta_2(d^A, d^B) = \frac{d_2^A + d_2^B}{2} + \frac{\max\{d_1^A + d_2^A, d_1^B + d_2^B\} - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}.$$

This is a type of equal surplus solution, as both players first get their respective average payoffs of the two alternatives, and then equally share the surplus from the total payoffs of the efficient alternative. For any $(d^A, d^B) \in$ 2DP, the *overall* solution o is defined by

$$o(d^A, d^B) = \begin{cases} (\beta_1(d^A, d^B), \beta_2(d^A, d^B)) & \text{if } (d^A, d^B) \text{ is undominated;} \\ (d_1^e, d_2^e) & \text{if } (d^A, d^B) \text{ is dominant.} \end{cases}$$

Note that the efficient alternative $e \in \{A, B\}$ is also the dominant alternative in a dominant problem. That is, when we apply the overall solution to a problem (d^A, d^B), it will generate the balanced threat solution if the problem is undominated, and will yield the dominant payoffs (d_1^e, d_2^e) if it is a dominant problem.

3 The bid-offer–counteroffer mechanism

In this section we propose the bid-offer–counteroffer mechanism. The key construct of the mechanism, learned from real-life negotiations, lies in the offer and counter-offer processes, where players can effectively bargain over the right of choosing a decision alternative, which generates an efficient outcome no matter who becomes the proposer.
Bid–offer–counteroffer mechanism The mechanism is played by two players, 1 and 2, and consists of three stages.

Stage 1 (bid)	Both players simultaneously bid to each other. So, $b_1 \in \mathbb{R}$ is the bid made by player 1 to player 2, and $b_2 \in \mathbb{R}$ is the bid made by player 2 to player 1. The player with the higher bid becomes the proposer, denoted as i, who pays the bid to the other player denoted as j. In case of equal bids, one player will be randomly selected as the proposer.
Stage 2 (offer)	The proposer i makes an offer $x_i \in \mathbb{R}$ to the other player j.
Stage 3 (counteroffer)	The non-proposer j decides to accept the offer, or reject it by making a counter-offer higher than the offer x_i.

Case 1: j Accepts the offer x_i. Then the proposer i pays the offer x_i to j and wins the right of choosing a decision alternative. Hence, the final payoff to the

proposer equals the payoff of the chosen decision alternative less the bid b_i and offer x_i made to the non-proposer j, whereas the final payoff to the non-proposer is the payoff of the chosen decision alternative plus the bid b_i and offer x_i. The game stops.

Case 2: j Rejects the offer x_i by making a counter-offer $y_j \in \mathbb{R}$ such that $y_j > x_i$ to the proposer i. As a result, j wins the right of choosing an alternative by paying y_j to i. Hence, the final payoff to i equals her payoff of the chosen decision alternative less the bid b_i but plus the counteroffer y_j made by j, whereas the final payoff to j is his payoff of the chosen alternative plus the bid b_i made by i and less the counteroffer y_j made to i. The game stops.

Here, we shall make it clear that throughout the paper only pure strategies are considered when players make transfers like bids and offers.

The basic idea underlying the mechanism is that using a bidding stage to decide (endogenously) who will initiate an offer. The offer and counteroffer stages are to decide who will choose a decision alternative. Stage 1 is a simultaneous competition, while stages 2 and 3 constitute a sequential competition to allocate the right of choosing an alternative to the player with a higher willingness to pay for this right. The argument for the proposer to accept the higher counteroffer is strengthened by the fact that she was already given the opportunity to offer first. The offer was aimed for the non-proposer to accept. Thus, it seems legitimate for her to accept a counteroffer that is higher than her offer. If she does not want to lose the right to the non-proposer, then she should have offered higher up to a level that the non-proposer would not compete. Moreover, such a design of competition ensures the mechanism is budget balanced: since offer and counteroffer are made to opponent players rather than outsiders like a social planner, there is no welfare loss.

The design of only one round of offer and counteroffer is inspired by real life observations. Quite often negotiations end after one round of offer and counteroffer. This happens even more frequently in housing and job markets when an outside option or a time constraint prevails. For business joint ventures, there is a popular exit mechanism called 'Texas Shootout' (cf. Brooks et al. 2010), a provision where one owner proposes a price and the other owner is compelled to either purchase the proposer's shares or sell her own shares at the proposed price. This clause, widely included in business contracts, and the 'divide-and-choose' mechanism in fair division problems are similar examples accommodating only one round of negotiation, where the proposer's offering strategy is constrained by the other player's choices, and no one is allowed for any space of improvement by making 'trial' offer or counteroffer. Note that the one time counteroffer idea is in the same spirit of the bilateral trading model of Myerson and Satterthwaite (1983) and especially their double auction mechanism where only one round of bargaining is allowed, and the one with higher price gets the good.

Winning to be proposer does not automatically lead the player to choose a decision alternative. But this right of 'speaking' first is still rather important. Like in the standard ultimatum bargaining game, the proposer here has the advantage of making an offer in her best interest at stage 2. Yet this advantage is sufficiently restricted as the non-proposer can counteroffer. Hence, these two forces together result in a unique equilibrium outcome while adequately reflect players' overall strength in negotiation. One can establish the logic in an alternative way. At the intermediate level, player 1 (also 2) wants to choose a favorable alternative, and therefore may like to pay player 2 to buy his agreement. But if player 2 does not agree, he can pay a higher amount to player 1 to win the right of choosing an alternative. Then, at the upper level, players bid to decide who can propose first.

By assuming a smallest money unit θ used in making transfers such that bids b_1, b_2, offer x_i and counteroffer y_j are divisible by θ,[5] we obtain the following main result.

Theorem 3.1 *For any* $(d^A, d^B) \in 2DP$ *with* $\theta \to 0$*, the bid-offer–counteroffer mechanism yields a unique SPE outcome as prescribed by* $o(d^A, d^B)$.

Proof We first focus on the case of undominated problems.

Undominated problem Consider an undominated two-person decision problem (d^A, d^B) with $d_1^A > d_1^B$, $d_2^B > d_2^A$, and $d_1^A - d_1^B > d_2^B - d_2^A$ (so $d_1^A - d_1^B \geq (d_2^B - d_2^A) + \theta$). Thus, we know that $\max\{d_1^A + d_2^A, d_1^B + d_2^B\} = d_1^A + d_2^A$. (Note that other cases such as $d_1^A - d_1^B < d_2^B - d_2^A$ or $d_1^A - d_1^B = d_2^B - d_2^A$ can be proved along the same line as follows and are therefore omitted.) We first construct a strategy profile as follows and show it is an SPE and it leads to the SPE payoffs as specified in Theorem 3.1.

At stage 1, player 1 makes bid $b_1 = \frac{1}{2}\left(\left(d_1^A - \frac{d_2^B - d_2^A}{2}\right) - \left(d_1^A - \frac{d_1^A - d_1^B}{2}\right)\right) = \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}$ to player 2, and player 2 makes bid $b_2 = \frac{1}{2}\left(\left(d_2^A + \frac{d_1^A - d_1^B}{2}\right) - \left(d_2^A + \frac{d_2^B - d_2^A}{2}\right)\right) = \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}$.

At stage 2, if player 1 is chosen as the proposer, player 1 offers $x_1 = \frac{d_2^B - d_2^A}{2}$ to player 2; if player 2 becomes the proposer, player 2 offers $x_2 = \frac{d_1^A - d_1^B}{2} - \theta$ to player 1.

At stage 3, if player 2 is the non-proposer, player 2 accepts x_1 if $x_1 \geq \frac{d_2^B - d_2^A}{2}$ and, otherwise, rejects it by making a counteroffer $y_2 = x_1 + \theta$; if player 1 is the non-proposer, player 1 accepts x_2 if $x_2 \geq \frac{d_1^A - d_1^B}{2}$ and otherwise, if $x_2 \leq \frac{d_1^A - d_1^B}{2} - \theta$, player 1 rejects the offer, and then makes a counteroffer $y_1 = x_2 + \theta$ to player 2.

To see this strategy profile does constitute an SPE, we firstly look at the subgame with respect to stage 3. If player 1 is the proposer offering $x_1 \geq \frac{d_2^B - d_2^A}{2}$ at stage 2, then at stage 3, player 2, the non-proposer, will accept it and finally get $d_2^A + b_1 + x_1$. He has no incentive to reject it by making a higher counteroffer because that would at

[5] Such 'smallest money unit' assumptions have been widely used in literature and particularly in auction models such as Ausubel (2006), Sun and Yang (2009), and Roth and Sotomayor (1990, p.171), as continuous bids often lead to no equilibrium. A detailed investigation on this assumption in bargaining literature has been given in van Damme et al. (1991). For ease of exposition, we assume that all payoffs d_k^a, where $a = A, B$ and $k = 1, 2$, are integers (like main units of denominations such as Euro, Dollar and Pounds) whereas θ is a sufficiently small number (like subunits of cent or penny). This guarantees all the involved transfers are divisible by θ.

most yield $d_2^B + b_1 - (x_1 + \theta)$ to player 2, which is strictly less than $d_2^A + b_1 + x_1$ as shown below.

$$d_2^B + b_1 - (x_1 + \theta) \leq d_2^B + b_1 - \left(\frac{d_2^B - d_2^A}{2} + \theta\right)$$
$$< b_1 + \frac{d_2^B + d_2^A}{2} = d_2^A + b_1 + \frac{d_2^B - d_2^A}{2} \leq d_2^A + b_1 + x_1.$$

On the contrary, if x_1 is lower than $\frac{d_2^B - d_2^A}{2}$, implying that $x_1 \leq \frac{d_2^B - d_2^A}{2} - \theta$, then player 2's best response is to reject it by making the counteroffer $y_2 = x_1 + \theta$. The resulting final payoff $d_2^B + b_1 - (x_1 + \theta)$ is higher than that (i.e., $d_2^A + b_1 + x_1$) of accepting x_1:

$$d_2^B + b_1 - (x_1 + \theta) \geq d_2^B + b_1 - \left(\frac{d_2^B - d_2^A}{2} - \theta + \theta\right)$$
$$> b_1 + \frac{d_2^B + d_2^A}{2} - \theta = d_2^A + b_1 + \frac{d_2^B - d_2^A}{2} - \theta \geq d_2^A + b_1 + x_1.$$

Now suppose that player 2 is the proposer offering $x_2 \leq \frac{d_1^A - d_1^B}{2} - \theta$ to player 1. At stage 3, player 1, as the non-proposer, will reject the offer by making a counteroffer $y_1 = x_2 + \theta$ to player 2. The final payoff to player 1 for her to do so is $d_1^A + b_2 - (x_2 + \theta)$, which can be readily shown, in a similar way as above, that it is higher than her final payoff $d_1^B + b_2 + x_2$ if she simply accepts x_2. Note that here player 1 cannot do better because θ is the minimal allowable transfer unit. On the other hand, if player 2's offer is higher than $\frac{d_1^A - d_1^B}{2} - \theta$, which implies that $x_2 \geq \frac{d_1^A - d_1^B}{2}$, then player 1 will accept the offer and obtain the final payoff $d_1^B + b_2 + x_2$. Also similar to the above, one can readily show that any deviation by making a higher counteroffer $y_1 \geq x_2 + \theta$ can only make her worse off.

We now turn to the subgame starting from stage 2. Given the above analysis of the subgame with respect to stage 3, we see that if player 1 is chosen as the proposer, at stage 2 player 1 will indeed offer $x_1 = \frac{d_2^B - d_2^A}{2}$ to player 2. Player 1 would not increase the offer as otherwise, her final payoff would be reduced because player 2 will accept any offer from player 1 that is no less than $\frac{d_2^B - d_2^A}{2}$. Meanwhile, player 1 has no incentive to lower the offer, either, because that would not increase her final payoff. Any offer x_1' that is lower than $\frac{d_2^B - d_2^A}{2}$ would incur a rejection and counteroffer $x_1' + \theta$ from player 2, which will then result in the final payoff $d_1^B - b_1 + x_1' + \theta$ to player 1. We can show as follows that this payoff is less than $d_1^A - b_1 - x_1$ that is the final payoff when player 1 offers $x_1 = \frac{d_2^B - d_2^A}{2}$ to player 2.

$$d_1^B - b_1 + (x_1' + \theta) \leq d_1^B - b_1 + \left(\frac{d_2^B - d_2^A}{2} - \theta + \theta\right)$$

$$= d_1^B - b_1 + \frac{d_2^B - d_2^A}{2} < d_1^A - b_1 - \frac{d_2^B - d_2^A}{2} = d_1^A - b_1 - x_1.$$

The second inequality follows from the fact that $d_1^A - d_1^B > d_2^B - d_2^A$.

On the other hand, if player 2 is the proposer, at stage 2 player 2 will indeed offer $x_2 = \frac{d_1^A - d_1^B}{2} - \theta$ to player 1 in equilibrium. Given the equilibrium strategies with respect to stage 3, it is obvious to see that player 2 has no incentive to lower the offer as that would make him worse off finally. Player 2 has no incentive to increase the offer to x_2' that is higher than $\frac{d_1^A - d_1^B}{2} - \theta$ either, as otherwise player 1 will accept the offer made by player 2, which will result in the final payoff $d_2^B - b_2 - x_2'$ to player 2. Below we show that this payoff cannot be higher than $d_2^A - b_2 + x_2 + \theta$ that is the final payoff when player 2 does not deviate.

$$\begin{aligned} d_2^B - b_2 - x_2' &< d_2^B - b_2 - \left(\frac{d_1^A - d_1^B}{2} - \theta \right) \\ &\leq d_2^A - b_2 + \left(\frac{d_1^A - d_1^B}{2} - \theta \right) + \theta = d_2^A - b_2 + x_2 + \theta. \end{aligned}$$

The second inequality follows from the fact that $d_1^A - d_1^B \geq (d_2^B - d_2^A) + \theta$.

Hence, the SPE of the subgame starting from stage 2 will generate the following final payoffs: if player 1 is the proposer, player 1 receives the final payoff $d_1^A - b_1 - \frac{d_2^B - d_2^A}{2}$, and player 2 obtains $d_2^A + b_1 + \frac{d_2^B - d_2^A}{2}$ finally. On the other hand, if player 2 is the proposer, then player 1 receives the final payoff $d_1^A + b_2 - \frac{d_1^A - d_1^B}{2}$, and player 2 obtains $d_2^A - b_2 + \frac{d_1^A - d_1^B}{2}$ finally.

Then, we check the equilibrium strategy of each player at stage 1. One can readily see that by taking such bidding strategy, player 1 will get the same final payoff $\frac{d_1^A + d_1^B}{2} + \frac{d_1^A + d_2^A - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}$ irrespective of who is selected as proposer. So will player 2: $\frac{d_2^A + d_2^B}{2} + \frac{d_1^A + d_2^A - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}$. No one would like to deviate from the bidding strategy: lowering the bid cannot increase the final payoff, whereas increasing the bid can only make the player worse off although it will guarantee the player to be the proposer.

Thus, we have shown this strategy profile indeed constitutes an SPE, and the SPE payoffs are as specified in the theorem.

Next, we show that the mechanism has one unique SPE payoff vector by a series of claims.

Claim (a) In any SPE at stage 3, if player 2 is the non-proposer, player 2 accepts any offer x_1 from player 1 if $x_1 \geq \frac{d_2^B - d_2^A}{2}$ and rejects it by making a counteroffer $y_2 = x_1 + \theta$ if $x_1 \leq \frac{d_2^B - d_2^A}{2} - \theta$; if player 1 is the non-proposer, player 1 accepts x_2 if $x_2 \geq \frac{d_1^A - d_1^B}{2}$ and otherwise, if $x_2 \leq \frac{d_1^A - d_1^B}{2} - \theta$, player 1 rejects it by counteroffering $y_1 = x_2 + \theta$ to player 2.

The proof of this claim is the same as above to show that the strategies at stage 3 are indeed an SPE of the subgame.

Claim (b) In any SPE at stage 2, if player 1 is the proposer, player 1 offers $x_1 = \frac{d_2^B - d_2^A}{2}$ to player 2; if player 2 becomes the proposer, player 2 offers $x_2 = \frac{d_1^A - d_1^B}{2} - \theta$ to player 1.
There does not exist any other SPE strategy at this stage and the proof is the same as above to show that the strategies at stage 2 are indeed an SPE of the subgame.

Claim (c) In any SPE at stage 1, the bidding profile (b_1, b_2) of the two players must be one of the following four forms: (b^*, b^*), $(b^*, b^* - \theta)$, $(b^* - \theta, b^*)$, $(b^* - \theta, b^* - \theta)$, where $b^* = \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}$.
We will show that these four bidding profiles are indeed SPE strategies and there exists no other SPE bidding profiles. There are two cases: $b_1 = b_2$ or $b_1 \neq b_2$. Moreover, here recall that following stages 2 and 3, if player 1 is the proposer, the final payoffs to players 1 and 2 are $d_1^A - b_1 - \frac{d_2^B - d_2^A}{2}$ and $d_2^A + b_1 + \frac{d_2^B - d_2^A}{2}$, respectively; and if player 2 is the proposer, the final payoffs to players 1 and 2 are $d_1^A + b_2 - \frac{d_1^A - d_1^B}{2}$ and $d_2^A - b_2 + \frac{d_1^A - d_1^B}{2}$, respectively.

Case 1: $b_1 = b_2$, that is, $(b_1, b_2) = (b, b)$. If this is an SPE bidding profile, it requires that no player has incentive to make any deviation of the bidding. This implies that the following four inequalities must hold. The first inequality means that player 1 has no incentive to increase the bid; the second means she has no incentive to decrease the bid; the third inequality means that player 2 has no incentive to increase the bid; and the final one means he has no incentive to lower the bid.

$$d_1^A - (b+\theta) - \frac{d_2^B - d_2^A}{2} \leq \frac{1}{2}\left(d_1^A - b - \frac{d_2^B - d_2^A}{2} + d_1^A + b - \frac{d_1^A - d_1^B}{2}\right);$$

$$d_1^A + b - \frac{d_1^A - d_1^B}{2} \leq \frac{1}{2}\left(d_1^A - b - \frac{d_2^B - d_2^A}{2} + d_1^A + b - \frac{d_1^A - d_1^B}{2}\right);$$

$$d_2^A - (b+\theta) + \frac{d_1^A - d_1^B}{2} \leq \frac{1}{2}\left(d_2^A + b + \frac{d_2^B - d_2^A}{2} + d_2^A - b_2 + \frac{d_1^A - d_1^B}{2}\right);$$

$$d_2^A + b + \frac{d_2^B - d_2^A}{2} \leq \frac{1}{2}\left(d_2^A + b + \frac{d_2^B - d_2^A}{2} + d_2^A - b_2 + \frac{d_1^A - d_1^B}{2}\right).$$

Solving these four inequalities, we have

$$\frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4} - \theta \leq b \leq \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}$$

$$\Rightarrow b^* \in \left\{ \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4} - \theta,\ \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4} \right\}.$$

Case 2: $b_1 \neq b_2$. Then the only possibility is either $(b_1, b_2) = (b, b - \theta)$ or $(b_1, b_2) = (b - \theta, b)$. The difference between their bids cannot be higher than θ as otherwise the one who bids higher would lower the bid. Consider $(b_1, b_2) = (b, b - \theta)$. If this is in equilibrium, the following two inequalities must hold: player 1 has no incentive to lower the offer and player 2 has no incentive to increase the offer. (we already know that player 1 has no incentive to raise the offer, and obviously player 2 has no incentive to lower the offer).

$$\frac{1}{2}\left(d_1^A - (b-\theta) - \frac{d_2^B - d_2^A}{2} + d_1^A + (b-\theta) - \frac{d_1^A - d_1^B}{2} \right)$$
$$\leq d_1^A - b - \frac{d_2^B - d_2^A}{2};$$
$$\frac{1}{2}\left(d_2^A + b + \frac{d_2^B - d_2^A}{2} + d_2^A - b + \frac{d_1^A - d_1^B}{2} \right)$$
$$\leq d_2^A + b + \frac{d_2^B - d_2^A}{2}.$$

Solving these two inequalities, we obtain

$$\frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4} \leq b \leq \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4} \Rightarrow$$
$$b^* = \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}.$$

For the case that $(b_1, b_2) = (b - \theta, b)$, one can show it in a similar way as above and get

$$b^* = \frac{d_1^A - d_1^B}{4} - \frac{d_2^B - d_2^A}{4}.$$

Claim (d) In any SPE, the final payoff to player 1 is $\frac{d_1^A + d_1^B}{2} + \frac{d_1^A + d_2^A - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}$ and the final payoff to player 2 is $\frac{d_2^A + d_2^B}{2} + \frac{d_1^A + d_2^A - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}$. This follows from claims (a), (b) and (c).

Dominant problem Consider a problem (d^A, d^B) with B being a dominant alternative. (the opposite case that A is dominant can be proved analogously.) One can readily show that the following strategy profile is an SPE and leads to the SPE payoff vector specified in Theorem 3.1.

At stage 1, both players bid 0. At stage 2, the proposer offers 0 to the non-proposer. At stage 3, if player 2 is the non-proposer, player 2 accepts x_1 if $x_1 \geq 0$ and, otherwise, rejects it by making a counteroffer $y_2 = x_1 + \theta$; if player 1 is the non-proposer, player 1 accepts x_2 if $x_2 \geq 0$ and, otherwise, rejects it by making a counteroffer $y_1 = x_2 + \theta$.

Next, to show the uniqueness of the SPE outcome, one can readily verify the above described SPE with respect to stage 3 is the unique SPE at this stage. There are two types of SPE at stage 2. One is that a proposer can offer 0, and the other is to offer $-\theta$. And at stage 1, there are four SPE bidding profiles: $(0, 0)$, $(0, -\theta)$, $(-\theta, 0)$, $(-\theta, -\theta)$. All these SPE yield the same SPE payoffs that player 1 gets d_1^B finally and player 2's final payoff is d_2^B. It can be readily checked that there exists no other SPE. □

Following Theorem 3.1, one can immediately see that in any SPE of the bid-offer–counteroffer mechanism the efficient alternative always emerges as the chosen alternative, and no transfer is made to outside the game. Hence, without proof we state the following corollary with two important properties of the mechanism.

Corollary 3.2 *The bid-offer–counteroffer mechanism is efficient and budget balanced.*

Next, we show that the bid-offer–counteroffer mechanism is superior to a 'flip-a-coin' mechanism. Hence, active negotiation is Pareto improving over this expected outcome.

Corollary 3.3 *The SPE outcome of the mechanism yields a payoff that is no less than the average payoff of the two alternatives for every player.*

Proof By Theorem 3.1, for an undominated problem, in SPE player 1 (analogous for player 2) receives $\beta_1(d^A, d^B) = \frac{d_1^A + d_1^B}{2} + \frac{\max\{d_1^A + d_2^A, d_1^B + d_2^B\} - \frac{d_1^A + d_1^B}{2} - \frac{d_2^A + d_2^B}{2}}{2}$. If $d_1^A + d_2^A \geq d_1^B + d_2^B$, we have $\max\{d_1^A + d_2^A, d_1^B + d_2^B\} \geq \frac{d_1^A + d_2^A}{2} + \frac{d_1^B + d_2^B}{2}$, which implies that $\beta_1(d^A, d^B) \geq \frac{d_1^A + d_1^B}{2}$. This also holds when $d_1^A + d_2^A \leq d_1^B + d_2^B$. For dominant problems, it is obviously true. □

This corollary has an intuitive implication. Since for any decision choice problem both players have equal a priori rights of choosing an alternative, a natural and fair (despite inefficient) benchmark is to give both players equal chances to choose. Hence, taking player 1 for example, $\frac{d_1^A + d_1^B}{2}$ is her expected payoff under this random assignment mechanism. Then, it seems reasonable to require any 'real' solution (in the sense that players could actively negotiate to resolve the conflict via a mechanism) to be no worse than these expected payoffs.

To highlight the result of Theorem 3.1, we note a desirable feature of the bid-offer–counteroffer mechanism that it offers a natural reconciliation between egalitarianism and utilitarianism defined by Moulin (1985). That is, for an undominated problem, the solution we get in SPE is in the same spirit of Moulin's egalitarian social choice function (one concrete case is referred to as the equal sharing above a convex decision social choice functions, cf. Chun (1986, 2000)), whereas the solution for the dominant problems is in the same spirit of a utilitarian social choice function by Moulin (1985).

4 Characterization

In this section, we study a solution from the axiomatic perspective. Note that the undominated and dominant problems, as two different types, may require different properties due to their own distinctive features. For dominant problems, since players have no conflict with the dominant alternative (as any rational player will act so), we require a solution to simply select the dominant alternative such that the players get the respective payoffs.

Dominance: A solution concept f on 2DP is of dominance if for any (d^A, d^B) in 2DP $f(d^A, d^B) = (d_1^A, d_2^A)$ when $d^A > d^B$, and $f(d^A, d^B) = (d_1^B, d_2^B)$ when $d^B > d^A$.

One readily verifies that the overall solution suggested in Theorem 3.1 satisfies the property of dominance. In addition, one can take a slightly different perspective to characterize the solution by using efficiency and the property of *no transfer when no conflict*.

No transfer when no conflict: A solution concept f on 2DP satisfies no transfer when no conflict if for any (d^A, d^B) in 2DP $\left(f_1(d^A, d^B), f_2(d^A, d^B)\right) \in \left\{(d_1^A, d_2^A), (d_1^B, d_2^B)\right\}$ when $d^A > d^B$ or $d^B > d^A$.

Here we provide an intuitive discussion on dominance and no transfer when no conflict. In real-world negotiations, when a dominant alternative (hence a uniquely efficient one) is available, we can understand that an agent may renounce the privilege of being the proposer (conditional on the second agent agreeing to be proposer, otherwise, she would reclaim it) and leave the right of making a choice to the second agent. In that case, the second agent will choose the efficient alternative anyway, whereas it is not credible for him to demand any transfer from the first agent by threatening to adopt the dominated alternative. Of course, in case he does not make a choice but gives the right back to the first agent, she will be happy to choose the efficient alternative, too. Both agents would strictly prefer avoiding the intervention of the chance of nature as the expected outcome is strictly dominated by the efficient alternative. Alternatively, one can consider a mechanism that a social planner asks the agents to reveal their preferences and then implement the efficient alternative without any transfer. In this case, no agent would have incentive to misrepresent his or her preference. The reason we specifically studied the bid-offer–counteroffer with respect to this situation is not only for completeness of analysis, but also to show that the same mechanism can apply to both types of problems and obtain reasonable solutions.

Below we offer properties that seem relevant to a solution for the type of undominated problems where conflict arises between the players.

Balanced threat (or equal impact): A solution concept f satisfies the balanced threat property if for any (d^A, d^B) in 2DP with $d_1^A - d_2^A = d_2^B - d_1^B$ (which is equivalent to the condition of $d_1^B - d_2^B = d_2^A - d_1^A$ or $d_1^A + d_1^B = d_2^A + d_2^B$), $f_1(d^A, d^B) = f_2(d^A, d^B)$.

An intuitive interpretation of this property can be as follows. Suppose player 1 prefers alternative A while player 2 prefers B. Then, $d_1^A - d_2^A$ measures the relative (to player 2) well-off of player 1 if her preferred choice is adopted; similarly, $d_2^B - d_1^B$ is for player 2. When these two relative well-offs are equal, it implies that both players can generate equal impact to the other following their choices on alternatives. Hence, they should get the same payoff.

No influence: A solution concept f is said of possessing the no influence property if for any (d^A, d^B) in 2DP where $d_1^A = d_1^B$ and $d_2^A = d_2^B$, $f_1(d^A, d^B) = d_1^A$ and $f_2(d^A, d^B) = d_2^B$.

This is a rather weak property. Since every player will get the same payoff whichever alternative is chosen, the two alternatives are essentially identical. Consequently, no player can effectively threaten the other by adopting a 'different' alternative. Hence, each player will just get what an alternative generates to him and cannot demand any transfer.

Additivity: A solution concept f on 2DP is additive if for any two problems (d^A, d^B) and $(\bar{d}^A, \bar{d}^B)$ in 2DP such that $\max\{(d_1^A+\bar{d}_1^A)+(d_2^A+\bar{d}_2^A), (d_1^B+\bar{d}_1^B)+(d_2^B+\bar{d}_2^B)\} = \max\{d_1^A + d_2^A, d_1^B + d_2^B\} + \max\{\bar{d}_1^A + \bar{d}_2^A, \bar{d}_1^B + \bar{d}_2^B\}$, $f(d^A + \bar{d}^A, d^B + \bar{d}^B) = f(d^A, d^B) + f(\bar{d}^A, \bar{d}^B)$.

Like the standard additivity axiom in the literature, here it simply means that the solution of a third problem as the sum of two given problems should be equal to the sum of the solutions of the two given problems. The condition we imposed there basically implies that the two given problems have the same efficient alternative: if A is the efficient alternative for the first problem, so is for the second, and hence for the third. This condition therefore ensures that all these three problems are the same in structure, which makes the property weaker than without the condition.

Theorem 4.1 *The balanced threat solution is the unique solution concept on 2DP that satisfies the properties of efficiency, balanced threat, no influence, and additivity.*

Proof It is easy to verify that the balanced threat solution β satisfies these four properties. Now we show if a solution concept f satisfies these four properties, then f necessarily is β. Suppose (d^A, d^B) is given as follows.

$$(d^A, d^B) = \begin{pmatrix} d_1^A & d_2^A \\ d_1^B & d_2^B \end{pmatrix}.$$

Suppose that $(d_1^A - d_2^A) - (d_2^B - d_1^B) = z$. Thus, $d_1^A - (d_2^A + \frac{z}{2}) = (d_2^B + \frac{z}{2}) - d_1^B$. Then, we construct a second problem $(\bar{d}^A, \bar{d}^B)$ and a third problem $(d^A + \bar{d}^A, d^B + \bar{d}^B)$ as follows,

$$(\bar{d}^A, \bar{d}^B) = \begin{pmatrix} 0 & \frac{z}{2} \\ 0 & \frac{z}{2} \end{pmatrix}, \quad (d^A + \bar{d}^A, d^B + \bar{d}^B) = \begin{pmatrix} d_1^A & d_2^A + \frac{z}{2} \\ d_1^B & d_2^B + \frac{z}{2} \end{pmatrix}.$$

By efficiency and balanced threat, for the problem $(d^A + \bar{d}^A, d^B + \bar{d}^B)$ we have

$$f_1(d^A+\bar{d}^A, d^B+\bar{d}^B) = f_2(d^A+\bar{d}^A, d^B+\bar{d}^B) = \frac{\max\{d_1^A+d_2^A+\frac{z}{2}, d_1^B+d_2^B+\frac{z}{2}\}}{2}.$$

By no influence, for the second decision problem $(\bar{d}^A, \bar{d}^B)$ we have $f_1(\bar{d}^A, \bar{d}^B) = 0$ and $f_2(\bar{d}^A, \bar{d}^B) = \frac{z}{2}$. According to efficiency, for the first decision problem

(d^A, d^B) we know that $f_1(d^A, d^B) + f_2(d^A, d^B) = \max\{d_1^A + d_2^A, d_1^B + d_2^B\}$. Using additivity, we have $f_1(d^A + \bar{d}^A, d^B + \bar{d}^B) = f_1(d^A, d^B) + f_1(\bar{d}^A, \bar{d}^B)$. Since $f_1(\bar{d}^A, \bar{d}^B) = 0$, we obtain that

$$\begin{aligned} f_1(d^A, d^B) &= f_1(d^A+\bar{d}^A, d^B+\bar{d}^B) \\ &= \frac{\max\{d_1^A+d_2^A+\frac{z}{2}, d_1^B+d_2^B+\frac{z}{2}\}}{2} \\ &= \frac{\max\{d_1^A+d_2^A, d_1^B+d_2^B\}+\frac{z}{2}}{2} \\ &= \frac{d_1^A+d_1^B}{2}+\frac{\max\{d_1^A+d_2^A, d_1^B+d_2^B\} - \frac{d_1^A+d_1^B}{2} - \frac{d_2^A+d_2^B}{2}}{2} = \beta_1(d^A, d^B). \end{aligned}$$

By efficiency, one can readily verify that $f_2(d^A, d^B) = \beta_2(d^A, d^B)$. □

5 Comparison with conventional approaches

In this section we examine and compare several conventional methods in the literature that could be applied to our model. Given the purpose for comparison only, the detailed and complete proofs of the related results are skipped but can be obtained from the author.

Consider a simple simultaneous bidding mechanism where the player with the higher bid chooses an alternative by paying the bid to the other player. (Winner is randomly selected in case of equal bids.) Unfortunately, this mechanism has multiple SPE outcomes when the smallest money unit θ applies. This indeterminacy is due to that the bid mingles the right of suggesting a proposal with the right of finally choosing an alternative. Below we construct a mechanism to separate the two components.

The bid-offer mechanism The mechanism consists of three stages. Stage 1 is the same bidding stage, where the winner is denoted by i, and the other one is denoted by j. At stage 2, i makes an offer $x_i \in \mathbb{R}$ to j. At stage 3, j accepts or rejects the offer. In case of acceptance, i pays x_i to j, and then chooses an alternative. Hence, both players receive the payoffs corresponding to the chosen alternative. If j rejects the offer, then an alternative will be randomly chosen, and leads to the corresponding payoffs.

Theorem 5.1 *For any $(d^A, d^B) \in 2DP$, with smallest money unit θ, the bid-offer mechanism yields multiple SPE outcomes.*

Why again indeterminacy arises? See that an alternative will be randomly chosen in case of rejection. So the reservation payoff for each player is his or her expected payoff of the two alternatives happening with equal probability. Suppose $d_1^A - d_1^B > d_2^B - d_2^A$, then if player 2 is the proposer, he would rather make an offer be rejected and leave the choice to nature. Thus, the efficient alternative may not be chosen. If player 1 is the proposer, she would make an offer be accepted and implement the efficient alternative. Such asymmetric outcomes make it impossible to obtain a unique SPE outcome if the bids are discrete in θ.

Hence, a successful mechanism to the problem implies that the efficient outcomes should be realized no matter who is the proposer. One way to achieve it is to allow for more interaction between both players after the bidding stage a la Rubinstein (1982).

Alternating-offer bargaining Players 1 and 2 alternate offers. In round one, player 1 proposes an alternative and offers $x \in \mathbb{R}$ to player 2. Player 2 then accepts or rejects. If he accepts, then the alternative is chosen such that payoffs are realized and the offer is paid to player 2. In case of rejection, the game breaks down with probability $\gamma \in (0, 1)$ and nature randomly chooses an alternative; and with probability $1 - \gamma$ it enters the second round where player 2 suggests an alternative and offers $y \in \mathbb{R}$ to player 1. Then player 1 accepts or rejects. Rejection will incur the same break-down probability. The game continues in this fashion until an agreement is reached.

Theorem 5.2 *For any* $(d^A, d^B) \in 2DP$*, in the limit, as* $\gamma \to 0$*, the payoffs obtained by the two players 1 and 2 respectively in the unique SPE of the alternating-offer game converge to the following,* $\frac{d_1^A+d_1^B}{2} + \frac{1}{2}\Big(\max\{d_1^A + d_2^A, d_1^B + d_2^B\} - \frac{d_1^A+d_1^B}{2} - \frac{d_2^A+d_2^B}{2}\Big)$ *for player 1 and* $\frac{d_2^A+d_2^B}{2} + \frac{1}{2}\Big(\max\{d_1^A + d_2^A, d_1^B + d_2^B\} - \frac{d_1^A+d_1^B}{2} - \frac{d_2^A+d_2^B}{2}\Big)$ *for player 2.*

When applying to dominant problems like Example 2.2, this alternating-offer bargaining approach has a restriction, as it violates the property of no transfer when no conflict: the SPE payoffs (92, 18) requiring player 1 to transfer eight to player 2 when dominant alternative *B* is finally chosen. This is due to the specification of the model where in case of breakdown, the chance of nature takes effect, and 'perturbed' the interaction between the two players.[6]

In a similar setting as the current paper, Moulin (1981) has commented on the issue of using a lottery in determining a disagreement point. He pointed out that implementing an efficient and just (anonymous and neutral) decision could be more difficult when no decision or lottery can play the role of a status quo (i.e., a disagreement point). To tackle the problem, he constructed the 'auctioning the leadership with differentiated bids' (ALDB) mechanism, which is of rich implication to our context and can well be applied here.

The ALDB mechanism It has three stages. At stage 1, players bid, but the one with a lower bid becomes the proposer, and the other player pays this bid to the proposer. So the bid here is actually a demand.[7] In case of equal bids, a proposer will be selected randomly. At stage 2, the proposer makes an offer to the non-proposer. At stage 3, the non-proposer accepts or rejects the offer. If accepting it, then the proposer pays the offer, chooses an alternative so that the final payoffs are realized, and the game stops. Rejection leads the non-proposer to choose an alternative such that its payoffs are realized. The game stops.

[6] The SPE payoffs of the alternating-offer game can be obtained via a mechanism involving no repetition of making offers. That is, one can modify the bid-offer mechanism such that after the bidding stage, the proposer suggests a combination of an alternative and a monetary transfer, which the other player can accept or reject, and still nature chooses an alternative at random in case of rejection.

[7] The equilibrium outcome will not change if we use the protocol that the player with higher bid becomes the proposer and pays the bid to the opponent. Then, in equilibrium players will make negative bids.

Theorem 5.3 *For any $(d^A, d^B) \in 2DP$, the ALDB mechanism yields a unique SPE payoff vector, called the ALDB solution, $\mu(d^A, d^B)$, which is defined by*

$$\mu_1(d^A, d^B) = \max\left\{d_1^A, d_1^B\right\} + \frac{1}{2}\Big(\max\left\{d_1^A + d_2^A, d_1^B + d_2^B\right\} - \max\left\{d_1^A, d_1^B\right\} - \max\left\{d_2^A, d_2^B\right\}\Big);$$

$$\mu_2(d^A, d^B) = \max\left\{d_2^A, d_2^B\right\} + \frac{1}{2}\Big(\max\left\{d_1^A + d_2^A, d_1^B + d_2^B\right\} - \max\left\{d_1^A, d_1^B\right\} - \max\left\{d_2^A, d_2^B\right\}\Big).$$

A solution concept f satisfies *equal concession* if for any (d^A, d^B) in $2DP$, $\max\{d_1^A, d_1^B\} - f_1(d^A, d^B) = \max\{d_2^A, d_2^B\} - f_2(d^A, d^B)$.

Theorem 5.4 *The ALDB solution is the unique solution concept for all two-person decision problems that satisfies efficiency and equal concession.*

The ALDB mechanism conveniently overcomes the issues present in the previous mechanisms of this section. Its well-characterized solution serves for a benchmark for other explorations. As an important feature, this mechanism and its solution favor the player with a higher payoff of the alternatives: irrespective of $\min\{d_1^A, d_1^B\}$ and $\min\{d_2^A, d_2^B\}$, so long as $\max\left\{d_1^A, d_1^B\right\} \geq \max\left\{d_2^A, d_2^B\right\}$, then $\mu_1(d^A, d^B) \geq \mu_2(d^A, d^B)$. Hence, this naturally gives rise to its opposite variant, called the equal well-being solution, following an idea making players equally well off based on their respective least payoffs. *The equal well-being solution* $\nu(d^A, d^B)$ of a decision problem $(d^A, d^B) \in 2DP$ is defined by

$$\nu_1(d^A, d^B) = \min\left\{d_1^A, d_1^B\right\} + \frac{1}{2}\Big(\max\left\{d_1^A + d_2^A, d_1^B + d_2^B\right\} - \min\left\{d_1^A, d_1^B\right\} - \min\left\{d_2^A, d_2^B\right\}\Big);$$

$$\nu_2(d^A, d^B) = \min\left\{d_2^A, d_2^B\right\} + \frac{1}{2}\Big(\max\left\{d_1^A + d_2^A, d_1^B + d_2^B\right\} - \min\left\{d_1^A, d_1^B\right\} - \min\left\{d_2^A, d_2^B\right\}\Big).$$

Apparently, both the ALDB solution and the equal well-being solution take polar views. Thus, the average of these two solutions offers a natural compromise. Indeed, this will lead to and further help justify the balanced threat solution introduced earlier in the paper.

6 Conclusion

This paper studies a fundamental decision choice problem using both strategic and axiomatic approaches. The bid-offer–counteroffer mechanism consists of three elements that work well and cohere logically: bid to initiate an offer, offer to win decision right, and counteroffer to rule out unreasonable offers. Offer and counteroffer work together in making a solution reflect the comparison of negotiation strength between the two players, while guaranteeing the efficient outcome. Among those desirable features discussed in the paper, we highlight that the mechanism leads to a reconciliation between egalitarianism and utilitarianism introduced in Moulin (1985).

For possible extensions of the model, one interesting direction is about the scenario with more decision alternatives. There may exist two ways to address it. One is that all the alternatives are available to both players so that either player can choose any alternative should she win the right of making a choice. Then, to get a similar result as in the current paper, our conjecture for constructing a relevant mechanism is to appropriately play the bid-offer–counteroffer mechanism twice, with the first time generating the second best outcome dependent on each player's highest possible payoff, and the second time leading to the efficient solution built on this second best outcome. The other way is in a similar spirit as in de Clippel and Eliaz (2011) such that each player will have a set of alternatives that the other player cannot choose from. Then, the problem becomes even more strategic. An immediately related extension along this line is to introduce cooperation, with which the alternatives become essentially variable disagreement points. That is, on top of those disagreement points there is an efficient payoff for players to negotiate.

No doubt that generalizing the analysis to situations with more than two players is interesting as well, but can be highly challenging. Here we offer a very preliminary idea, based on three players for illustration, which might be useful for future investigation. Players firstly participate in a multi-bidding stage a la Pérez-Castrillo and Wettstein (2001, 2002) to select the first proposer. Suppose player 1 wins and then she waits. Next, the remaining two players, 2 and 3, play the bid-offer–counteroffer mechanism. Here, they use offer and counteroffer to compete for becoming the representative of the coalition $\{2, 3\}$, rather than directly for the right of choosing an alternative. And then, player 1 makes offer to player 2 (suppose he becomes the representative) and if he rejects, he can make a counteroffer to player 1. The winner at this stage will have the right to finally choose an alternative. The other players will receive the payoff associated to this alternative besides the transfers (bid and offer) made at the relevant stages.

Naturally one may consider studying all such problems with incomplete information. Here please note that while one can tackle the problem from a mechanism design point of view,[8] there is an increasingly interesting and demanding analytical framework, i.e., axiomatic studies under incomplete information (cf. de Clippel 2010 and de Clippel et al. 2010), which would help us to gain further insights.

Finally, concerning applications, the suggested mechanism may apply to the strategic studies for the bankruptcy problem and the NIMBY problem,[9] as well as queueing problems. It can also be used to analyze concrete economic settings like industrial organization and the competition policy. In particular, it will shed light on the research of endogenous timing in duopoly markets from both theoretical

[8] One possible way seems to apply the idea of Qin and Yang (2009) to this context, where now the essential challenge is to handle the externalities of decision alternatives.

[9] Surveys for these two problems can be found in Thomson (2003) and Thomson (2010), respectively.

(cf. Hamilton and Slutsky 1990 and van Damme and Hurkens 1999) and experimental (Fonseca et al. 2006) perspectives.

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References

Ausubel LM (2006) An efficient dynamic auction for heterogeneous commodities. Am Econom Rev 96:602–629
Brooks F, Landeo C, Spier KE (2010) Trigger happy or gun shy? Dissolving common-value partnerships with Texas shootouts. RAND J Econom 41:649–673
Chambers C, Green J (2005) Additive rules for the quasi-linear bargaining problem. Working Paper, Harvard University, Cambridge
Chun Y (1986) The solidarity axiom for quasi-linear social choice problems. Soc Choice Welf 3:297–320
Chun Y (2000) Agreement, separability and other axioms for quasi-linear social choice problems. Soc Choice Welf 17:507–521
Chun Y, Thomson W (1990a) Bargaining problems with uncertain disagreement points. Econometrica 58:951–959
Chun Y, Thomson W (1990b) Egalitarian solutions and uncertain disagreement points. Econom Lett 33:29–33
Chun Y, Thomson W (1990c) Nash solution and uncertain disagreement points. Games Econom Behav 2:213–223
Cremer J, McLean RP (1987) Full extraction of the surplus in Bayesian and dominant strategy auctions. Econometrica 56:1247–1257
de Clippel G (2010) Egalitarianism under incomplete information. Working Paper 2010–2014, Brown University, Providence
de Clippel G, Pérez-Castrillo D, Wettstein D (2010) Egalitarian equivalence under asymmetric information. Working Paper 2010–2015, Brown University, Providence
de Clippel G, Eliaz K (2011) On the strategic disclosure of feasible options in bargaining. Working Paper 2011–2015, Brown University, Providence
Fonseca M, Müller W, Normann H-T (2006) Endogenous timing in Duopoly: experimental evidence. Int J Game Theory 34:443–456
Green J (2005) Compensatory transfers in two-player decision problems. Int J Game Theory 33:159–180
Hamilton J, Slutsky S (1990) Endogenous timing in Duopoly games: Stackelberg or Cournot equilibria. Games Econom Behav 2:29–46
Jackson M, Moulin E (1992) Implementing a public project and distributing its costs. J Econom Theory 57:125–140
Kıbrıs Ö, Tapkı İG (2010) Bargaining with nonanonymous disagreement: monotonic rules. Games Econom Behav 68:233–241
Maniquet F (2003) A characterization of the Shapley value in queueing problems. J Econom Theory 109:90–103
Maskin E (1999) Nash equilibrium and welfare optimality. Rev Econom Stud 66:23–38
Moore J, Repullo R (1988) Subgame perfect implementation. Econometrica 56:1191–1220
Moulin E (1981) Implementing just and efficient decision making. J Public Econom 16:193–213
Moulin E (1984) The conditional auction mechanism for sharing a surplus. Rev Econom Stud 51:157–170
Moulin E (1985) Egalitarianism and utilitarianism in quasi-linear bargaining. Econometrica 53:49–67
Moulin E (1987) The pure compensation problem: egalitarianism versus Laissez-fairism. Quart J Econom 102:769–783

Myerson R, Satterthwaite M (1983) Efficient mechanisms for bilateral trading. J Econom Theory 29:265–281
Nash JF (1950) The bargaining problem. Econometrica 18:155–162
Nash JF (1953) Two person cooperative games. Econometrica 21:128–140
Peters H, van Damme E (1991) Characterizing the Nash and Raiffa bargaining solutions by disagreement point axioms. Math Oper Res 16:447–461
Pérez-Castrillo D, Wettstein D (2001) Bidding for the Surplus: A non-cooperative approach to the Shapley value. J Econom Theory 100:274–294
Pérez-Castrillo D, Wettstein D (2002) Choosing Wisely: a multibiding approach. Am Econom Rev 92:1577–1587
Qin C.-Z, Yang C.-L (2009) Make a guess: a robust mechanism for King Solomon's dilemma. Econom Theory 39:259–268
Roth AE, Sotomayor MA (1990) Two-sided matching: a study in game–theoretic modeling and analysis. Cambridge University Press, Cambridge
Rubinstein A (1982) Perfect equilibrium in a bargaining model. Econometrica 50:97–109
Serrano R (2005) Fifty years of the Nash program, 1953–2003. Investig Econom 29:219–258
Sun N, Yang Z (2009) A double-track adjustment process for discrete markets with substitutes and complements. Econometrica 77:933–952
Thomson W (2003) Axiomatic and game–theoretic analysis of bankruptcy and taxation problems: a survey. Math Soc Sci 45:249–297
Thomson W (2010) The theory of fair allocation. Princeton University Press, Princeton
van Damme E, Hurkens S (1999) Endogenous Stackelberg leadership. Games Econom Behav 28:105–129
van Damme E, Selten R, Winter E (1991) Alternating bid bargaining with a smallest money unit. Games Econom Behav 2:188–201