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Signaling advertising by multiproduct firms

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Abstract We consider the use of advertising expenses as quality signals in multiproduct firms, extending previous results on single product firms. In our model, a firm introduces sequentially two products whose qualities are positively correlated. We investigate whether there exist information spillovers from the first to the second market. We show that, when correlation is high, the equilibrium in market 2 depends on the quality reputation the firm has gained in market 1. Moreover, if a firm with a high-quality product 1 wants to separate from its low-quality counterpart, it needs to advertise more in this market than if the qualities of the two products are unrelated. This advertising level signals not only high quality in the first market, but also the likely quality of the second product. Thus, advertising in the first market has information spillovers in the second market.

Keywords Quality signaling · Advertising · Multiproduct firms

JEL Classification L14 · L15 · M37

1 Introduction

Many firms in real world produce more than one good, and the quality of these goods is often correlated.[1] However economic literature has mainly dealt with the case of

[1] Common examples of this are car producers, soft drink producers, and others.

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single product firms. This paper extends the previous analysis on signaling advertising in monoproduct firms to multiproduct firms.

Advertising expenses may be used to indirectly signal the quality of experience goods, by spending an amount high enough to separate high-quality producers from low-quality ones. The incentive to use costly advertising signals changes if there are information spillovers across markets. Actually, signaling one good's quality reflects positively on the demand for the other goods, if their quality is believed to be correlated with the quality of the advertised good.

In this paper we consider a firm introducing sequentially two products in two distinct markets. The good introduced in each market may either be of low or high quality, and the qualities of the two products are correlated. Advertising can be used in both markets to signal quality. However, due to quality correlation, the advertising expenditure in the first market may influence the consumers' perception of the second product's quality.

Our model is quite interesting from a game-theoretic point of view. It is a signaling game but with some special features. The sender (the firm) learns its type gradually and may send a signal each time it gets more information about its own type. Thus we have a model with a sequence of signals: a sequential signaling model.

Although there is no paper directly connected with this work, it can be related to two branches of the literature: signaling advertising and quality signaling by multiproduct firms. The idea of using advertising expenditures as a signal of product quality was first presented by Nelson (1974). He identified three effects to support his argument: efficiency, repeat-business and match-products-to-buyers. According to the efficiency effect, demand expansion is most attractive to efficient firms (the ones who offer a better quality/price ratio).[2] Thus these firms will set low prices and advertise heavily to increase demand. The second reason why high-quality firms may advertise to signal quality is that high-quality products generate repeat purchases. Hence a high-quality firm gains more in creating goodwill. The last reason for advertising to be used as signal is that the firm has a greater incentive to send its ads to the consumers who value its product the most.

In the last three decades many authors have developed formal models where advertising is used as signal of product quality. Kihlstrom and Riordan (1984) show that dissipative advertising may be used to signal quality in a model where firms are competitive price takers, as long as marginal cost is sufficiently lower when quality is high (efficiency effect) or there exists a repeat-business effect which overwhelms an eventual marginal-cost advantage of low-quality firms (repeat-business effect). In Kihlstrom and Riordan (1984) prices are not used as signals of product quality. However, many models consider the possibility of using both advertising and prices as signals of product quality. This raises the interesting issue of whether the two signals will be used in equilibrium. The answer depends on whether we assume that advertising is dissipative or demand enhancing and on the presence of effects such as repeat-business and informed consumers. In static models Overgaard (1991) and Zhao (2000) show that dissipative advertising is not used as a signal (signaling is

[2] Schmalensee (1978) argues the opposite. He defends that a high-quality firm is likely to have higher production costs. Thus, for a given price, a low-quality firm has a higher profit margin.

done exclusively through prices). However, this result is not valid in other settings. Milgrom and Roberts (1986), who developed the first multiple signals model, consider a two-period model so as to incorporate a repeat-business effect. They show that dissipative advertising may complement prices in order to achieve separation at minimal cost.[3] Linnemer (2002) obtains a similar result in a static model with a mixture of informed and uninformed consumers. Dissipative advertising may also be used to signal high quality in duopoly models, as demonstrated by Hertzendorf and Overgaard (2001) and Fluet and Garella (2002). In our model the only quality signal is dissipative advertising. The existing literature suggests that one could build a multiproduct firm model where both price and advertising are used as signals. Such a model would be necessarily complex due to the existence of multiple signals and multiple products. Our simplifying assumption allows us to focus on the effects upon advertising of having a multiproduct firm, without having to deal with multiple signals.

To our knowledge, Bagwell (1992) is the only paper sofar concerned with quality signaling in multiproduct firms. Our focus, however, is distinct from his: whereas Bagwell concentrates on differences between products of a given product line, we concentrate on differences between monoproduct and multiproduct firms. Moreover, there are several important modelling differences. Bagwell (1992) considers a product line which may either be of low or high quality, implicitly assuming perfect correlation for the quality of the various products in the product line, while we only assume that products are positively correlated. In Bagwell (1992) the issue is how to use the prices of the various products in the product line to signal the quality of the all product line, whereas in our case advertising in the first market, which signals quality of the first product, may also affect the consumer's beliefs about the second product quality.

The remainder of the article is organized as follows. The next section presents the model. In Sect. 3 we discuss the results for a single product firm. The next two sections consider the case where the firm introduces sequentially two products whose qualities are correlated. In Sect. 4 we derive the most reasonable perfect Bayesian equilibrium when the quality of the first product is observed before the introduction of the second product. On the other hand, Sect. 5 presents the most reasonable equilibrium when the quality of the first product is not observed before the second product is introduced. Conclusions are summarized in the final section.

2 Model

Consider a firm producing two goods (1 and 2), whose quality is determined by Nature and can be either high (H) or low (L). Quality is observable to the firm, but not to the consumer. In this incomplete information game, the firm may be of one of four possible types: (H_1, H_2), (H_1, L_2), (L_1, H_2), or (L_1, L_2), where the subscripts denote the market.

Qualities are known to be positively correlated. Let ρ be the degree of correlation. The joint density function, which is common knowledge, is the following:

[3] Empirical work conducted by Thomas et al. (1998) on Milgrom and Roberts' findings has confirmed the use of advertising as a signal of product quality for the U.S. automobile industry.

	H_2	L_2
H_1	$p^2 + \rho p(1-p)$	$p(1-p) - \rho p(1-p)$
L_1	$p(1-p) - \rho p(1-p)$	$(1-p)^2 + \rho p(1-p)$

The prior probability for a high-quality product is $p \in (0, 1)$. Once the quality of the first product is known, the posterior probability for a high-quality product 2 is $p + \rho(1-p)$ if the first product was of high quality, and it is $p - \rho p$ if the first product was of low quality. As expected, the probability of the second product being of high quality is revised upwards when the quality of the first product is high and revised downwards when the quality of the first product is low. Moreover, the revision in the prior probabilities is larger when ρ is larger.

Let $\pi(q, \mu)$ denote gross profits of a firm having true quality q and perceived to be selling a high-quality product with probability μ, where $\mu \in [0, 1]$. For simplicity, we assume that $\pi_1 = \pi_2 = \pi$. Let a denote advertising expenditures. Net profits are $\pi(q, \mu) - a$. Advertising has no direct impact on demand or gross profits, only an indirect effect through quality perception; consumers' decisions about how much to buy and their willingness to pay depend on the expectation they have about the quality of the good. It is assumed that $\pi(q, \mu)$ is increasing[4] and continuous in μ. In addition, we assume that the firm's gain from influencing beliefs increases with the actual quality of the product:

$$\pi(H, \mu) - \pi(H, \mu') > \pi(L, \mu) - \pi(L, \mu'), \quad \text{for all } \mu > \mu'.$$

This single-crossing condition can be justified if we interpret $\pi(q, \mu)$ as the net present value of present and future profits. Hence we are implicitly assuming that the products are sold for several periods. In a repeated purchase setup, a high-quality firm will gain more than a low-quality firm does by convincing consumers that its quality is high, since a larger fraction of its current clients will buy again the product in the future and/or new consumers will buy it as a consequence of word of mouth.

For our purpose, price is considered exogenous and not used as quality signal. The only signaling variable is advertising expense. This allows us to concentrate on the effects upon advertising choices of producing more than one good.

The sequencing of the game is as follows: In stage 1 Nature chooses the quality of the first product. In stage 2, after observing the quality of the first product, the firm chooses advertising level a_1. In stage 3, after observing a_1, consumers decide on how much to buy of the first product. In stage 4, Nature chooses the quality of the second product. In stage 5, after observing the quality of the second product, the firm chooses advertising level a_2. Finally, in stage 6, after observing a_2, consumers form their expectations about the quality of product 2 and decide on how much to buy of it. Consumers only learn the quality of the two products afterwords.

One important feature of our model's timing is that consumers decide on how much to buy of the second product before learning the true quality of the first product. One case where this assumption is reasonable is when the first product is a durable good. For such a durable good it is natural to assume that quality is not known immediately

[4] No similar assumption is made about true quality, because production costs may be increasing in q.

after purchase. Characteristics such as durability will only be known many periods after the purchase is made. If the decision on how much to buy of the second product was taken after knowing the quality of the first product, the level of advertising for product 1, a_1, would be irrelevant in the second product purchasing decision. Since the consumers would learn if product 1 was H_1 or L_1 before the second product was introduced, the posterior beliefs would be $p + \rho(1 - p)$ if H_1 was observed and $p - \rho p$ if L_1 was observed, regardless of the value of a_1. Our timing was chosen because it allows a richer informational spillovers' analysis. In our setup it may happen that a_1 affects the consumer's beliefs in market 2, thus we may have advertising spillovers. However, for comparison purposes, we will also describe in Sect. 4 what would happen if the consumer learns the quality of good 1 immediately after purchase.

Another interesting property of our model's timing is that the firm learns its type gradually (quality of product 1 is observed before quality of product 2) and sends signals every time it learns more about its type. Thus we have a sequence of signals. To the best of our knowledge, the idea of sequential signaling has not been used before.[5] This type of model is technically more complex than a simple signaling game, but provides us interesting insights.

The firm's strategy is described by (a_1, a_2), where $a_k (k = 1, 2)$ is the advertising level in market k, a_1 being contingent on the quality of the first product (H_1 or L_1), and a_2 being contingent on the firm's type, as well as on the past action a_1. When the firm chooses a_2, the only relevant issue is how this decision affects the profit in market 2. However, when the firm chooses a_1, the effect on both markets has to be taken into account. Consumers' expectations about the quality of product 1 are based on a_1. The posterior probability of good 1 being of high quality given a_1 is denoted by $\mu_1(a_1)$. For product 2 the prior probability is revised twice, after a_1 is observed, and then after a_2 is observed. Let the first revised probability be $\mu_{21}(a_1)$ and the second one be $\mu_{22}(a_2)$. We will look for pure-strategy Perfect Bayesian Equilibria (henceforth PBE).

3 Monoproduct firm—equilibrium selection

In the single-product case it is easy to show that there exist both separating and pooling PBE's. In a separating equilibrium, the two firms choose different advertising levels $a^L \neq a^H$. Thus, when a^L or a^H are observed, consumers learn the quality of the product. It is easy to show, by contradiction, that $a^L = 0$ in any separating equilibrium. On the other hand, a^H has to be such that the L firm does not want to choose a^H even if it is perceived as being of high quality, while also being optimal for the H producer:

$$\pi(L, 0) \geq \pi(L, 1) - a^H \quad \text{and} \quad \pi(H, 1) - a^H \geq \pi(H, 0)$$

[5] The idea of sequential signaling can be applied to any signaling model, by assuming that the sender learns his type gradually instead of knowing his type in a precise manner immediately. Moreover, the receiver has a less precise knowledge than the sender.

Under the sorting condition there exists a continuum of separating equilibria[6] with $a^L = 0$ and $a^H \in [\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)]$. There is also a set of pooling equilibria where both types choose the same level of advertising $\tilde{a} \in [0, \pi(L,p) - \pi(L,0)]$, supported by the belief that any firm deviating from $\tilde{a}$ is L with probability one.

To solve the problem of multiplicity of equilibria, a usual procedure is to impose restrictions on off-the-equilibrium path beliefs. One popular criterion is the Cho and Kreps (1987) *intuitive criterion*. According to this criterion a type should not be expected to play an equilibrium dominated action (an action such that the best payoff that can be obtained by playing that action is lower than the equilibrium payoff). Thus, if the uninformed player observes that action he should put zero probability on types for whom that action is dominated in equilibrium. This may change the uninformed player's best response, which in turn may induce some other type to deviate from the proposed equilibrium. If this happens, the equilibrium does not survive the intuitive criterion.

In the single product case, it is easy to show that the least-cost separating equilibrium (the one with $a^L = 0$ and $a^H = \pi(L,1) - \pi(L,0) = \underline{a}^H$) is the only PBE which survives the Cho and Kreps (1987) refinement. Surprisingly, the result that the pooling equilibria do not survive the intuitive criterion does not depend on the prior probability of each type. Even if the probability of the H firm is arbitrarily close to 1, the result holds. However, in this case, the pooling equilibrium with $\tilde{a} = 0$ seems more reasonable from an economic point of view than the separating equilibrium. If the consumer is almost sure that the product has high quality, why spend money just to convince him that this is the case? Thus, the intuitive criterion may eliminate equilibria which look more reasonable than the surviving equilibrium.

In this paper we do not use the intuitive criterion since it seems to be too strong a refinement. Instead we use the weaker *domination criterion*. According to this criterion, if an action is strictly dominated for a certain type, then the uninformed player should put probability zero on this type when observing that action. Note that the domination criterion only assumes common knowledge of rationality whereas the intuitive criterion requires common knowledge of the equilibrium being played. The next lemma describes the set of equilibria that survive the domination criterion.

Lemma 1 *If* $\pi(H,1) - [\pi(L,1) - \pi(L,0)] > \pi(H,p)$, *then there exists a unique equilibrium that survives the domination criterion: the least-cost separating equilibrium* ($a^L = 0$ *and* $a^H = \underline{a}^H = \pi(L,1) - \pi(L,0)$)*. However, if* $\pi(H,p) > \pi(H,1) - [\pi(L,1) - \pi(L,0)]$ *the set of equilibria surviving the domination criterion is constituted by the least-cost separating equilibrium and the set of pooling equilibria with* $\tilde{a} \leq [\pi(L,1) - \pi(L,0)] - [\pi(H,1) - \pi(H,p)]$.

Proof Notice that any level of $a \in (\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)]$ is a strictly dominated action for the L firm, but not for the H one. Thus if such level of a is observed, consumers should put probability zero on the firm being L, that is $\mu = 1$. However, these beliefs imply that a H firm will never spend more than

[6] The following beliefs can be used to support the separating equilibria as PBE: if $a < a^H$, the posterior belief for μ is 0; if $a \geq a^H$ the posterior belief for μ is 1.

$\underline{a}^H = \pi(L, 1) - \pi(L, 0)$. Thus the least-cost separating equilibrium is the only separating PBE which survives the domination criterion.

What about the pooling equilibria? Do they survive the domination criterion? The answer depends on the prior probability. When $\pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] > \pi(H, p)$ no pooling equilibria survives the domination criterion. Since advertising levels $a > \underline{a}^H$ lead to beliefs $\mu = 1$, it cannot be optimal for the H firm to advertise $\widetilde{a} \geq 0$ if that leads consumers to maintain their prior beliefs, $\mu(\widetilde{a}) = p$. In other words, the H firm prefers to choose $\underline{a}^H$ and be perceived as being of high quality than to choose $\widetilde{a}$ and be «pooled» with an L firm. Consequently, when $\pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] > \pi(H, p)$ the unique equilibrium surviving the domination criterion is the least-cost separating equilibrium.

However, when $\pi(H, p) > \pi(H, 1) - [\pi(L, 1) - \pi(L, 0)]$ there are pooling equilibria which survive the domination criterion. In fact, for any pooling advertising level such that $\pi(H, p) - \widetilde{a} > \pi(H, 1) - [\pi(L, 1) - \pi(L, 0)]$ the high quality firm prefers to choose $\widetilde{a}$ and be «pooled» with an L firm than to choose $\underline{a}^H$ and be perceived as being of high quality. Consequently, all pooling equilibria with $\tilde{a} \leq [\pi(L, 1) - \pi(L, 0)] - [\pi(H, 1) - \pi(H, p)]$ survive the domination criterion. □

Hence the domination criterion does not necessarily select a unique equilibrium. When $\pi(H, p) > \pi(H, 1) - [\pi(L, 1) - \pi(L, 0)]$ there exist multiple PBE's satisfying the domination criterion: the least-cost separating equilibrium and a set of pooling equilibria. In this case, although there are several equilibria with reasonable off-the-equilibrium path beliefs, one of them is more natural because all players are better off if that equilibrium is played. This equilibrium is the unique Pareto efficient equilibrium in the set of equilibria satisfying the domination criterion: the pooling equilibrium where both types choose $a = 0$. A possible justification for the selection of the Pareto efficient equilibrium is a coalition-proof argument. The Pareto efficient equilibrium is the unique surviving equilibrium which is immune to deviations by the coalition of all players. From hereon we define the *most reasonable PBE* as follows:

Definition 1 The most reasonable PBE are the Pareto efficient equilibria in the set of PBE that survive the domination criterion.

In order to characterize the most reasonable equilibria we need the following intermediate result:

Lemma 2 *There exists a unique p^* such that:*

$$\pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] = \pi(H, p^*)$$

Proof By the sorting condition when $p = 0, \pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] - \pi(H, p) > 0$. In addition, when $p = 1, \pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] - \pi(H, p) < 0$. Since, by assumption, $\pi(H, p)$ is increasing and continuous in p, there exists a unique p^* such that $\pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] - \pi(H, p) = 0$. □

Note that the value of p^* depends on the difference $\big(\pi(H, \mu) - \pi(H, \mu')\big) - \big(\pi(L, \mu) - \pi(L, \mu')\big)$ where $\mu > \mu'$. When the two types are very different the value of p^* is high, when they are similar the value of p^* is low.

Let us now characterize the most reasonable equilibria in the single product case, as a function of prior beliefs (Fig. 1 illustrates this result):

Proposition 1 *Let p^* be as defined above. Then the unique most reasonable PBE in the single product case is:*

(i) *The least-cost separating equilibrium ($a^L = 0$ and $a^H = \underline{a}^H = \pi(L, 1) - \pi(L, 0)$) when $p < p^*$.*
(ii) *The pooling equilibrium with $a^H = a^L = 0$ when $p > p^*$.*

Proof The result is immediate from the two previous lemmas and the definition of most reasonable equilibria. For values of p below p^* the most reasonable equilibrium is the least-cost separating equilibrium as it is the unique equilibrium satisfying the domination criterion. For values above p^*, there are multiple PBE surviving the domination criterion but the unique Pareto efficient PBE in this set is the pooling equilibrium where both types pool at $a = 0$. □

In the following sections we extend the monoproduct firm results for a firm producing two goods, whose qualities are believed to be positively correlated. The firm can influence the perception of quality in one market through the advertising amount spent in the other market.

4 Equilibria when quality of good 1 is observed

In this section and the following one, we derive the equilibrium levels of a_1, assuming that the continuation equilibrium is the most reasonable one, both on-the-equilibrium path and off-the-equilibrium path. In other words, we restrict our analysis to PBE's with reasonable continuation equilibria.

In this section we analyze what would happen if the consumer learns the quality of the first product before observing a_2. In this case, consumers' beliefs after they learn the quality of product 1 but before observing a_2 are given by $\mu_2(H_1) = p + \rho(1 - p)$ if the first product is of high quality, while $\mu_2(L_1) = p - \rho p$ if the first product is of low quality. Notice that these beliefs do not depend on a_1.

Since the continuation equilibrium does not depend on a_1, the optimal a_1 for each type of product 1 is determined only by what happens in the first market. Consequently, the conditions which define the separating and pooling equilibria in market 1 are precisely the same as in the monoproduct case, and the most reasonable equilibrium is determined in the same manner. Depending on the prior probability and the qualities' correlation, we may have four types of equilibria in the complete game (Fig. 2 illustrates the four regions where each type of equilibrium holds):

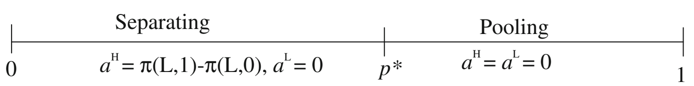

Fig. 1 Most reasonable equilibrium as a function of prior beliefs

Proposition 2 *If the consumer learns the quality of good 1 before observing a_2:*

(i) *When $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$ the most reasonable PBE is the least-cost separating equilibrium in both markets. That is, $a_2^H = \underline{a}^H$ and $a_2^L = 0$ regardless of a_1 and $a_1^H = \underline{a}^H$ and $a_1^L = 0$.*

(ii) *When $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$ in the most reasonable equilibrium H_1 and L_1 separate in market 1 (type H_1 just needs to advertise $a_1^{H_1} = \underline{a}^H$). If the firm is H_1, then in the second market there is pooling ($a_2^H = a_2^L = 0$). If the firm is L_1, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

(iii) *When $p > p^*$ and $\rho < \frac{p-p^*}{p}$ the most reasonable equilibrium is the pooling one with $a^H = a^L = 0$ in both markets.*

(iv) *When $p > p^*$ and $\rho > \frac{p-p^*}{p}$ we have pooling in market 1. If the firm is H_1, then in the second market there is pooling ($a_2^H = a_2^L = 0$). If the firm is L_1, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

Proof The continuation equilibrium depends on whether H_1 or L_1 is observed and on whether $\mu_2(L_1)$ and $\mu_2(H_1)$ are below or above p^* (which depend on p and ρ). Thus a_1 does not influence the continuation equilibrium in the second market and consequently the most reasonable equilibrium in market 1 is as defined in Proposition 1, i.e. depends only on whether $p \gtrless p^*$. The rest of the result follows from $\mu_2(L_1) = p - \rho p$ and $\mu_2(H_1) = p + \rho(1 - p)$. □

The previous result tells us that when quality correlation is low ($\rho < \frac{p^*-p}{1-p}$ for $p < p^*$ or $\rho < \frac{p-p^*}{p}$ for $p > p^*$) the multiproduct firm just replicates in each market the behavior of a single product firm.

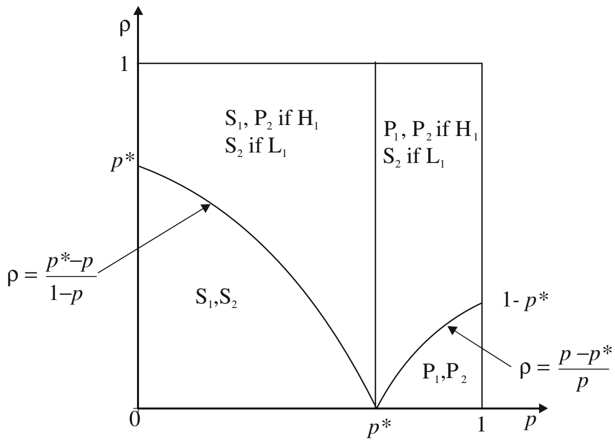

Fig. 2 The most reasonable equilibrium when the consumer learns the quality of good 1 before a_2 is observed

On the other hand, when quality correlation is high, the reputation acquired in the first market[7] influences the equilibrium in the second market. When the first product is of high quality, consumers attribute a very high probability to the second product being of high quality too. Thus, in the second market the firm does not need to advertise, as it can exploit the reputation created with the first product. On the contrary, when the first product is of low quality, the reputation of the firm after the quality of the first product is observed is low. Thus, if a firm happens to have a second product of high quality, the firm will want to advertise high enough to credibly signal that this is the case. This shows that with high correlation the behavior of a multiproduct firm is quite different from the behavior of a single product firm.

It should be noted that if the quality of the first product is learned before observing a_2, then there are no signaling spillovers. The level of advertising in the first market does not affect the consumers perception of good 2's quality. Consumers perception is only influenced by whether their previous experience with product 1 was good or bad.

5 Equilibria when quality of good 1 is not observed

As usual in dynamic games, we analyze first the continuation equilibria after a_1 is observed, and then proceed backwards to derive the equilibrium levels of a_1. As mentioned above we restrict our analysis to PBE's with reasonable continuation equilibria. Notice that the level of a_1 only influences the profit in the second product through the beliefs.

Let $\mu_{21}(a_1)$ be the probability that product 2 is of high quality given a_1. This is equal to

$$\begin{aligned}\mu_{21}(a_1) &= \Pr(H_2|H_1) \bullet \Pr(H_1|a_1) + \Pr(H_2|L_1) \bullet \Pr(L_1|a_1)\\ &= (p + \rho(1-p))\,\mu_1(a_1) + (p - \rho p)\,(1 - \mu_1(a_1))\end{aligned}$$

One can interpret $\mu_{21}(a_1)$ as the beliefs before a_2 is observed. The continuation game after a_1 is observed looks as a one-product signaling game where $\mu_{21}(a_1)$ are the prior beliefs.

From the monoproduct analysis we know that we may have separating and pooling equilibria in market 2. When $\mu_{21}(a_1) < p^*$, the most reasonable continuation equilibrium is the least-cost separating one, with $a_2^L = 0$ and $a_2^H = \underline{a}^H$. On the other hand, if the previous condition fails, the most reasonable continuation equilibrium is the pooling equilibrium with $a_2^L = a_2^H = 0$. Since the most reasonable continuation equilibrium depends on the prior beliefs, p, and on the correlation between the two products' qualities, ρ, our presentation is organized according to the values of these two parameters.

[7] This reputation is measured by the posterior beliefs $\mu_2(H_1)$ or $\mu_2(L_1)$.

5.1 Low prior and low correlation: $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$

Under these circumstances, if we proceed backwards to study the existence of PBE we conclude that:

Lemma 3 *If $p < p^*$ and $\rho < \frac{p^*-p}{1-p}$, then the unique reasonable continuation PBE is the least-cost separating equilibrium ($a_2^H = \underline{a}^H$ and $a_2^L = 0$) for all a_1. Moreover there exist a continuum of separating equilibria in a_1 with $a_1^L = 0$ and $a_1^H \in [\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)]$ and a continuum of pooling equilibria in a_1 with $a_1^L = a_1^H = \widetilde{a}_1 \in [0, \pi(L,p) - \pi(L,0)]$.*

Proof Under the current assumptions $p+\rho(1-p) < p^*$. Note that the most favorable beliefs after a_1 is observed are that $\mu_1(a_1) = 1$ which leads consumers to update their beliefs about product 2 being of high quality from p to $p+\rho(1-p)$. This implies that $\mu_{21}(a_1) < p^*$ for all values of a_1. But then by Proposition 1 the unique reasonable continuation equilibrium in market 2 is the least-cost separating equilibrium.

Let us first prove the existence of separating equilibria in a_1. In a separating equilibrium types H_1 and L_1 choose different levels of a_1, $a_1^H \neq a_1^L$. If the two types separate, in equilibrium the consumer will learn the quality of product 1 just by observing a_1. As a consequence, on-the-equilibrium path posterior beliefs will be as if quality of product 1 was observed, $\mu_{21}(a_1^H) = p + \rho(1-p)$ and $\mu_{21}(a_1^L) = p - \rho p$. For other values of a_1 we may assume that the firm is interpreted to be a L_1 firm. By contradiction, it is easy to show that $a_1^L = 0$ in any separating equilibrium (if $a_1^L > 0$ a L_1 firm would gain by deviating to $a_1^L = 0$ as advertising expenditures would be lower and beliefs cannot be worse than for a_1^L). On the other hand, considering that the most reasonable continuation PBE is always the least-cost separating one, $a_1^{H_1}$ has to be such that:

$$\begin{aligned}&\pi(L,0) + ((1-p)+\rho p)\,\pi(L,0) + (p-\rho p)\left[\pi(H,1) - \underline{a}^H\right]\\&\quad\geq \pi(L,1) - a_1^{H_1} + ((1-p)+\rho p)\,\pi(L,0) + (p-\rho p)\left[\pi(H,1) - \underline{a}^H\right]\end{aligned}$$

and

$$\begin{aligned}&\pi(H,1) - a_1^{H_1} + ((1-p)-\rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]\\&\quad\geq \pi(H,0) + ((1-p)-\rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]\end{aligned}$$

Thus, the set of separating equilibria is given by:

$$a_1^H \in [\pi(L,1) - \pi(L,0), \pi(H,1) - \pi(H,0)] \quad \text{and} \quad a_1^{L_1} = 0.$$

This is precisely the same set as in the single market case.

Let us now prove the existence of PBE where both types choose the same $\widetilde{a}_1$. In this case, the posterior beliefs after $\widetilde{a}_1$ is observed are equal to the prior beliefs, $\mu_{21}(\widetilde{a}_1) = p$. If we assume that whenever $a_1 \neq \widetilde{a}_1$ is observed the firm is perceived to be L_1, we have $\mu_{21}(a_1) = p - \rho p$ for all other levels of a_1. For a pooling equilibrium to exist in the first product advertising level:

$$\pi(L, p) - \widetilde{a}_1 + ((1-p) + \rho p)\,\pi(L, 0) + (p - \rho p)\left[\pi(H, 1) - \underline{a}^H\right]$$
$$\geq \pi(L, 0) + ((1-p) + \rho p)\,\pi(L, 0) + (p - \rho p)\left[\pi(H, 1) - \underline{a}^H\right]$$

which is equivalent to

$$\widetilde{a}_1 \leq \pi(L, p) - \pi(L, 0).$$

This is the same condition that we obtain for pooling in just one market. □

The next result shows that there exists a unique equilibrium that survives the domination criterion.

Lemma 4 *If $p < p^*$ and $\rho < \frac{p^* - p}{1-p}$, then the unique equilibrium that survives the domination criterion is the least-cost separating equilibrium in both markets. That is, $a_1^H = \underline{a}^H$ and $a_1^L = 0$ and, in market 2, $a_2^H = \underline{a}^H$ and $a_2^L = 0$ regardless of a_1.*

Proof For type L_1, choosing a_1 larger than $\underline{a}^H = \pi(L, 1) - \pi(L, 0)$ is a dominated strategy. On the other hand choosing $a_1 \in (\underline{a}^H, \pi(H, 1) - \pi(H, 0)]$ is not dominated for a H_1 firm. Hence the domination criterion implies that $\mu_1(a_1) = 1$ for any a_1 in that interval. But then a H_1 firm can choose $a_1^{H_1}$ arbitrarily close to $\underline{a}^H$ to separate itself from the L_1 firm. Hence the unique separating PBE surviving the domination criterion is the least-cost separating equilibrium, with $a_1^{H_1} = \underline{a}^H$ and $a_1^{L_1} = 0$.

We also need to show that no pooling equilibrium survives the domination criterion. Since the pooling equilibrium with $\widetilde{a}_1 = 0$ leads to the highest payoffs among all pooling equilibria, it is enough to prove that this equilibrium cannot be sustained once we impose the domination criterion. Since $\mu_1 = 1$ for any $a_1 \in (\pi(L, 1) - \pi(L, 0), \pi(H, 1) - \pi(H, 0)]$, a type H_1 gains by deviating from the pooling equilibrium to a_1 slightly above $\underline{a}^H$ since

$$\pi(H, 1) - \underline{a}^H + ((1-p) - \rho(1-p))\,\pi(L, 0) + (p + \rho(1-p))\left[\pi(H, 1) - \underline{a}^H\right]$$
$$> \pi(H, p) + ((1-p) - \rho(1-p))\,\pi(L, 0) + (p + \rho(1-p))\left[\pi(H, 1) - \underline{a}^H\right]$$

But this is equivalent to

$$\pi(H, 1) - [\pi(L, 1) - \pi(L, 0)] > \pi(H, p),$$

which holds for $p < p^*$. Since $p + \rho(1-p) < p^*$ implies $p < p^*$ the previous condition is verified. Thus the pooling equilibria do not survive the domination criterion. □

Hence the unique equilibrium that survives the domination criterion is the least-cost separating equilibrium in each market. But this means that the multiproduct firm just replicates in each market the single product firm. The total level of advertising that a firm of type (H_1, H_2) needs to do in order to separate itself in both markets is the same that is needed for separation to be possible in two independent product markets (that is, with zero quality correlation, $\rho = 0$). In this case, there are no information spillovers: a_1^H reveals that the firm is H_1, but this does not give any advantage to the firm in market 2. Since the prior is low and correlation is also low, the posterior beliefs are low even when the firm reveals being H_1. But then the consumers expect the firm to advertise $\underline{a}^H$ in market 2 when product 2 is of high quality, regardless of what happened in market 1. As a consequence, advertising in market 1 does not affect the equilibrium in market 2 and the level of advertising needed to separate H_1 from L_1 is the same as for a single product firm.

5.2 Low prior and high correlation: $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$

In this case, the sets of separating and pooling PBE are the following ones:

Lemma 5 *If $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$, then in any separating equilibrium in $a_1(a_1^H \neq a_1^L)$ the most reasonable continuation equilibrium following a_1^H is the pooling equilibrium with $a_2 = 0$ whereas the most reasonable continuation equilibrium following a_1^L is the least-cost separating equilibrium. Moreover there exist a continuum of separating equilibria in a_1 with $a_1^L = 0$ and $a_1^H \in [\underline{a}_1^{H_1}, \overline{a}_1^{H_1}]$ where $\underline{a}_1^{H_1}$ and $\overline{a}_1^{H_1}$ are given by:*

$$\begin{aligned}\underline{a}_1^{H_1} &= \pi(L,1) - \pi(L,0) + ((1-p) + \rho p)\,[\pi(L, p+\rho(1-p)) - \pi(L,0)] \\ &\quad + (p - \rho p)\left[\pi(H, p+\rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right)\right]\end{aligned} \tag{1}$$

and

$$\begin{aligned}\overline{a}_1^{H_1} &= \pi(H,1) - \pi(H,0) + ((1-p) - \rho(1-p))\,[\pi(L, p+\rho(1-p)) - \pi(L,0)] \\ &\quad + (p + \rho(1-p))\left[\pi(H, p+\rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right)\right]\end{aligned} \tag{2}$$

On the other hand, for any pooling equilibrium in $a_1(a_1^H = a_1^L = \widetilde{a}_1)$ the continuation equilibrium following $\widetilde{a}_1$ is the least-cost separating equilibrium. Additionally any $\widetilde{a}_1 \leq \pi(L,p) - \pi(L,0)$ can be sustained as a pooling PBE.

Proof If $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$ we know that $p + \rho(1-p) > p^* > p - \rho p$. But then by Proposition 1, if the firm reveals to be H_1, the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$, while if the firm reveals to be L_1, only the least-cost separating equilibrium is reasonable in the second market. On the other hand, if the two types choose the same advertising level, $\widetilde{a}_1$, then on-the-equilibrium path beliefs are $\mu_1(\widetilde{a}_1) = p$ and the most reasonable continuation equilibrium is the least-cost separating equilibrium. Off-the-equilibrium path, we assume that if $a_1 \neq a_1^H$ and $a_1 \neq a_1^L$ then consumers believe that the first product is of low quality, $\mu_1(a_1) = 0$, thus the reasonable continuation equilibrium is the least-cost separating equilibrium.

Considering the continuation equilibria and proceeding backwards, we conclude that, in order to separate, the H_1 firm must spend $a_1^{H_1}$ such that

$$\begin{aligned}
&\pi(L,0) + ((1-p)+\rho p)\,\pi(L,0) + (p-\rho p)\left[\pi(H,1) - \underline{a}^H\right] \\
&\quad \geq \pi(L,1) - a_1^{H_1} + ((1-p)+\rho p)\,\pi(L, p+\rho(1-p)) \\
&\qquad + (p-\rho p)\,\pi(H, p+\rho(1-p))
\end{aligned}$$

and

$$\begin{aligned}
&\pi(H,1) - a_1^{H_1} + ((1-p)-\rho(1-p))\,\pi(L, p+\rho(1-p)) \\
&\quad + (p+\rho(1-p))\,\pi(H, p+\rho(1-p)) \\
&\quad \geq \pi(H,0) + ((1-p)-\rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H\right]
\end{aligned}$$

The previous conditions take into account the expected profit in the second market, knowing the posterior probabilities and the continuation equilibrium associated with each level of a_1. They mean that a H_1 firm is willing to do $a_1^{H_1}$, but a L_1 firm prefers $a_1^{L_1} = 0$ to $a_1^{H_1}$, even if by doing $a_1^{H_1}$ it is perceived as high-quality in the first market. These two conditions define the lower and the upper bound in the values of a_1^H that can be supported in a separating equilibrium. Solving them with respect to a_1^H we obtain $\underline{a}_1^{H_1}$ and $\overline{a}_1^{H_1}$ defined by Eqs. 1 and 2, respectively.

The proof that any $\widetilde{a}_1 \leq \pi(L,p) - \pi(L,0)$ can be supported as a pooling equilibrium is precisely the same as in Lemma 3. □

The result shows that there are quite interesting new features in the characterization of the separating equilibria. In particular, the least-cost separating equilibrium in market 1 is given by $\underline{a}_1^{H_1}$ defined in Eq. 1. The amount that H_1 has to spend in order to separate equals the expected gain L_1 would achieve in both markets by pretending to be high-quality in the first and being perceived as such. This formula generalizes the one for a single product firm, since it includes, apart from the gain in the first market, the expected gain in the second market due to an increase in quality perception.[8]

Notice that when $\rho = 1$, $\underline{a}_1^{H_1} = 2\,[\pi(L,1) - \pi(L,0)]$. In this case, advertising $\underline{a}_1^{H_1}$ reveals that the two products are of high quality. The H_1 firm just advertises for the first product, but this signals high quality in both markets.

As expected, the fact that a firm is multiproduct makes a difference. In particular, if the firm has shown being H_1 in the first market, it is more likely that the continuation

[8] The expected gain is clearly positive, since we are assuming $p + \rho(1-p) > p^*$, which is equivalent to assuming:

$$\pi(H, p+\rho(1-p)) > \pi(H,1) - \underline{a}^H \quad \Leftrightarrow \quad \pi(H, p+\rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right) > 0.$$

equilibrium is a pooling equilibrium than in the case where markets are unrelated. Moreover, the higher quality correlation is (the closer ρ is to 1) the higher the likelihood of the previous result. Thus, in this case one can really speak of informational spillovers, since the advertising level in the first market has implications on the beliefs in both markets.

The next lemma characterizes the most reasonable PBE in this case.

Lemma 6 *When $p < p^*$ and $\rho > \frac{p^*-p}{1-p}$ the most reasonable PBE depends on whether the following condition is true:*

$$\pi(H,1) - \underline{a}^H > \pi(H,p) + \rho\,[(\pi(H,1) - \pi(H,p+\rho(1-p))) - (\pi(L,1) - \pi(L,p+\rho(1-p)))] \tag{3}$$

(i) *If condition (3) holds, then the most reasonable is the least-cost separating equilibrium in market 1 with $a_1^L = 0$ and $a_1^H = \underline{a}_1^H$ defined in Eq. 1. If $a_1 \geq \underline{a}_1^H$ then in the second market there is pooling, $a_2^H = a_2^L = 0$. If $a_1 < \underline{a}_1^H$, then in the second market there is separation ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

(ii) *If condition (3) does not hold, then the most reasonable equilibrium involves pooling in market 1 ($a_1^H = a_1^L = 0$) and separation in market 2 ($a_2^H = \underline{a}^H$ and $a_2^L = 0$).*

Proof As usual, it is easy to show that the unique separating equilibrium in market 1 that survives the domination criterion is the least-cost separating equilibrium with $a_1^H = \underline{a}_1^{H_1}$, thus we omit the proof.

The domination criterion implies that $\mu_1 = 1$ for $a_1 > \underline{a}_1^{H_1}$. With these beliefs, a type H_1 gains by deviating from the pooling equilibrium with $\widetilde{a}_1 = 0$ as long as

$$\begin{aligned}\pi(H,1) &- \underline{a}_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L,p+\rho(1-p)) \\ &+ (p+\rho(1-p))\,\pi(H,p+\rho(1-p)) \\ &> \pi(H,p) + ((1-p) - \rho(1-p))\,\pi(L,0) + (p+\rho(1-p))\left[\pi(H,1) - \underline{a}^H)\right]\end{aligned}$$

Substituting $\underline{a}_1^{H_1}$ in the previous expression and simplifying we get

$$\begin{aligned}\pi(H,1) - \underline{a}^H + \rho&\left[\pi(H,p+\rho(1-p)) - \left(\pi(H,1) - \underline{a}^H\right)\right. \\ &- (\pi(L,p+\rho(1-p)) - \pi(L,0))] > \pi(H,p)\end{aligned}$$

Since $\underline{a}^H = \pi(L,1) - \pi(L,0)$, this is equivalent to

$$\begin{aligned}\pi(H,1) - \underline{a}^H - \rho\,[(\pi(H,1) &- \pi(H,p+\rho(1-p))) - (\pi(L,1) \\ &- \pi(L,p+\rho(1-p)))] > \pi(H,p)\end{aligned}$$

which is equivalent to condition (3). Thus the pooling equilibrium with $\widetilde{a}_1 = 0$ does not survive the domination criterion. But then no other pooling equilibria satisfies the domination criterion. Hence the unique equilibrium satisfying the domination criterion is the least-cost separating equilibrium in market 1.

If condition (3) is not satisfied, the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion and it is better for both types (L_1 and H_1) than the least-cost separating equilibrium. Thus, using Pareto optimality to select the most reasonable equilibrium, the pooling equilibrium $\widetilde{a}_1 = 0$ is the most reasonable one. In this case, the continuation equilibrium is the separating one. □

Notice that, by the sorting condition,

$$[(\pi(H, 1) - \pi(H, p + \rho(1 - p))) - (\pi(L, 1) - \pi(L, p + \rho(1 - p)))] > 0$$

This implies that the condition for the least-cost separating equilibrium to be the most reasonable one is stricter than if the two markets were independent. The intuition is that with information spillovers, the amount that H_1 has to advertise in order to separate itself from his low quality counterpart is higher. Thus, it is more difficult for H_1 to gain by deviating from the pooling equilibrium with $\widetilde{a}_1 = 0$.

5.3 High prior and high correlation: $p > p^*$ and $\rho > \frac{p-p^*}{p}$

Under these circumstances, if we proceed backwards to study the existence of PBE we conclude that:

Lemma 7 *If $p > p^*$ and $\rho > \frac{p-p^*}{p}$, then in any separating equilibrium in $a_1(a_1^H \neq a_1^L)$ the most reasonable continuation equilibrium following a_1^H is the pooling equilibrium with $a_2 = 0$, whereas the most reasonable continuation equilibrium following a_1^L is the least-cost separating equilibrium. Moreover there exist a continuum of separating equilibria in a_1 with $a_1^L = 0$ and $a_1^H \in [\underline{a}_1^{H_1}, \overline{a}_1^{H_1}]$ where $\underline{a}_1^{H_1}$ and $\overline{a}_1^{H_1}$ are defined by Eqs. 1 and 2, respectively.*

On the other hand, in any pooling equilibrium in $a_1(a_1^H = a_1^L = \widetilde{a}_1)$ the continuation equilibrium following $\widetilde{a}_1$ is the pooling equilibrium with $a_2 = 0$ and any $\widetilde{a}_1 \leq \overline{a}_1$ can be sustained as a pooling PBE, where $\overline{a}_1$ is given by:

$$\begin{aligned}\overline{a}_1 = \pi(L, p) - \pi(L, 0) + ((1 - p) + \rho p)\,[\pi(L, p) - \pi(L, 0)] \\ + (p - \rho p)\left[\pi(H, p) - \left[\pi(H, 1) - \underline{a}^H\right]\right]\end{aligned} \tag{4}$$

Proof When $p > p^*$ and $\rho > \frac{p-p^*}{p}$ we know that $p + \rho(1 - p) > p^* > p - \rho p$. But then by Proposition 1, if the firm reveals being H_1, the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$, while if the firm reveals being L_1, only the least-cost separating equilibrium is reasonable in the second market. On the other hand, if the two types choose the same advertising level, $\widetilde{a}_1$, then on-the-equilibrium path beliefs are $\mu_1(\widetilde{a}_1) = p$ and the most reasonable continuation equilibrium is is the pooling equilibrium with $a_2 = 0$ since $p > p^*$.

To derive the set of separating equilibria in a_1, the proof is precisely the same as in Lemma 5, since the continuation equilibria are the same. To prove the existence of pooling equilibria in a_1, assume that whenever $a_1 \neq \widetilde{a}_1$ is observed the firm is perceived as having a low quality product 1, thus the continuation equilibrium following a deviation is the least-cost separating equilibrium. But then choosing $a_1 = \widetilde{a}_1$ is optimal for a L_1 firm as long as:

$$\begin{aligned}&\pi(L,p) - \widetilde{a}_1 + ((1-p) + \rho p)\,\pi(L,p) + (p - \rho p)\,\pi(H,p)\\ &\quad\geq \pi(L,0) + ((1-p) + \rho p)\,\pi(L,0) + (p - \rho p)\left[\pi(H,1) - \underline{a}^H\right]\end{aligned} \tag{5}$$

The corresponding condition for a H_1 firm is less restrictive, hence the previous condition defines the set of pooling equilibria. Solving with respect to $\widetilde{a}_1$ we obtain $\overline{a}_1$ as defined by Eq. 4. □

Since the set of separating equilibria coincides with the set identified in the previous section, all the remarks we pointed out in the previous section apply here as well. In particular, it should be noted that the minimum amount that H_1 has to spend in order to separate equals the expected gain L_1 would achieve in both markets by pretending to be high quality in the first and being perceived as such.

It is also worthwhile noticing that the set of pooling equilibria is larger than in a single product case. This is due to the fact that a deviation from the pooling equilibrium leads posterior beliefs to deteriorate from p to $p - \rho p$, which affects the expected profits in market 2. Since deviations are less attractive, more levels of a_1 can be sustained as a pooling equilibrium in market 1.

Let us now identify the most reasonable PBE in this case.

Lemma 8 *If $p > p^*$ and $\rho > \frac{p-p^*}{p}$, then the most reasonable PBE involves pooling with zero advertising in both markets. That is, $a_1^H = a_1^L = 0$ and $a_2^H = a_2^L = 0$.*

Proof As usual one can show that the unique separating equilibrium that survives the domination criterion is the least-cost separating one. Additionally, to verify if any pooling equilibrium survives the domination criterion it is enough to check whether that is true for the pooling equilibrium with $a_1^H = a_1^L = 0$.

We know that for type L_1, advertising more than $\underline{a}_1^{H_1}$ defined in Eq. 1 is a dominated strategy, but then if such a_1 is observed, posterior beliefs should be $\mu_1 = 1$. However with these beliefs a type H_1 gains by deviating from the pooling equilibrium as long as

$$\begin{aligned}&\pi(H,1) - \underline{a}_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p))\\ &\quad + (p + \rho(1-p))\,\pi(H, p + \rho(1-p))\\ &\quad \geq \pi(H,p) + ((1-p) - \rho(1-p))\,\pi(L,p) + (p + \rho(1-p))\,[\pi(H,p)]\end{aligned}$$

Substituting $\underline{a}_1^{H_1}$ defined in Eq. 1, the previous condition is equivalent to the next expression being negative:

$$\begin{aligned}&(1 + p(1-\rho))\left[\pi(H,p) - (\pi(H,1) - \underline{a}^H)\right] + ((1-p) + \rho p)\,[\pi(L,p) - \pi(L,0)]\\ &\quad - \rho\,[(\pi(H, p+\rho(1-p)) - \pi(H,p)) - (\pi(L, p+\rho(1-p)) - \pi(L,p))]\end{aligned} \tag{6}$$

In this expression, the first two terms are positive whereas the third term is negative. In the third term, the largest value that the expression inside parentheses can take is:

$$(\pi(H,1)-\pi(H,p))-(\pi(L,1)-\pi(L,p)) = -\left[\pi(H,p)-\left(\pi(H,1)-\underline{a}^H\right)\right] + (\pi(L,p)-\pi(L,0)).$$

Thus, a lower bound for expression (6) is:

$$(1+p+\rho(1-p))\left[\pi(H,p)-(\pi(H,1)-\underline{a}^H)\right] \\ +(1-p)(1-\rho)\left[\pi(L,p)-\pi(L,0)\right]$$

But, for $p > p^*$ this is positive. Thus condition (6) can never be negative. Consequently, the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion. In addition it is Pareto optimal, so it is the most reasonable equilibrium. □

5.4 High prior and low correlation: $p > p^*$ and $\rho < \frac{p-p^*}{p}$

In this last scenario the set of PBE is as follows:

Lemma 9 *If $p > p^*$ and $\rho < \frac{p-p^*}{p}$, the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$ for all a_1. Moreover there exist a continuum of separating equilibria in a_1 with $a_1^L = 0$ and $a_1^H \in [\underline{a}_1^{H_1}, \overline{a}_1^{H_1}]$ where $\underline{a}_1^{H_1}$ and $\overline{a}_1^{H_1}$ are given by:*

$$\underline{a}_1^{H_1} = \pi(L,1)-\pi(L,0)+((1-p)+\rho p)\left[\pi(L,p+\rho(1-p))-\pi(L,p-\rho p)\right] \\ +(p-\rho p)\left[\pi(H,p+\rho(1-p))-\pi(H,p-\rho p)\right] \tag{7}$$

and

$$\overline{a}_1^{H_1} = \pi(H,1)-\pi(H,0) \\ +((1-p)-\rho(1-p))\left[\pi(L,p+\rho(1-p))-\pi(L,p-\rho p)\right] \\ +(p+\rho(1-p))\left[\pi(H,p+\rho(1-p))-\pi(H,p-\rho p)\right] \tag{8}$$

In addition, any $\widetilde{a}_1 \leq \overline{a}_1$ can be sustained as a pooling PBE, where $\overline{a}_1$ is given by:

$$\overline{a}_1 = \pi(L,p)-\pi(L,0)+((1-p)+\rho p)\left[\pi(L,p)-\pi(L,p-\rho p)\right] \\ +(p-\rho p)\left[\pi(H,p)-\pi(H,p-\rho p)\right] \tag{9}$$

Proof If $p > p^*$ and $\rho < \frac{p-p^*}{p}$, we have $p-\rho p > p^*$. This means that posterior beliefs μ_{21} are above p^* even if the firm reveals being L_1. Hence the most reasonable continuation equilibrium is the pooling equilibrium with $a_2 = 0$, for all a_1.

As usual one can show that in any separating equilibrium $a_1^L = 0$. When $a_1 = \underline{a}_1^{H_1}$, the firm reveals being H_1 and the posterior beliefs are $\mu_{21} = p+\rho(1-p)$. When

$a_1^L = 0$, the consumer knows the firm is L_1 and $\mu_{21} = p - \rho p$. A separating equilibrium exists in the first product advertising level if:

$$\begin{aligned}&\pi(L,0) + ((1-p)+\rho p)\,\pi(L,p-\rho p) + (p-\rho p)\,\pi(H,p-\rho p)\\&\quad\geq \pi(L,1) - a_1^{H_1} + ((1-p)+\rho p)\,\pi(L,p+\rho(1-p))\\&\qquad + (p-\rho p)\,\pi(H,p+\rho(1-p))\end{aligned}$$

and

$$\begin{aligned}&\pi(H,1) - a_1^{H_1} + ((1-p)-\rho(1-p))\,\pi(L,p+\rho(1-p))\\&\qquad + (p+\rho(1-p))\,\pi(H,p+\rho(1-p))\\&\quad\geq \pi(H,0) + ((1-p)-\rho(1-p))\,\pi(L,p-\rho p) + (p+\rho(1-p))\,\pi(H,p-\rho p)\end{aligned}$$

These two conditions define the lower and the upper bound on the values that $a_1^{H_1}$ can take in a separating equilibrium. Solving with respect to $a_1^{H_1}$ we get $\underline{a}_1^{H_1}$ and $\overline{a}_1^{H_1}$ as defined by Eqs. 7 and 8, respectively.

To prove the existence of pooling equilibria in a_1, assume that whenever $a_1 \neq \widetilde{a}_1$ is observed the firm is perceived to be L_1, thus $\mu_{21}(a_1) = p - \rho p$ for all other levels of a_1. But then choosing $a_1 = \widetilde{a}_1$ is optimal for a L_1 firm as long as:

$$\begin{aligned}&\pi(L,p) - \widetilde{a}_1 + ((1-p)+\rho p)\,\pi(L,p) + (p-\rho p)\,\pi(H,p)\\&\quad\geq \pi(L,0) + ((1-p)+\rho p)\,\pi(L,p-\rho p) + (p-\rho p)\,\pi(H,p-\rho p)\end{aligned}$$

The corresponding condition for a H_1 firm is less restrictive, hence the previous condition defines the set of pooling equilibria. Solving with respect to $\widetilde{a}_1$ we obtain $\overline{a}_1$ as defined by Eq. 9. □

Once again the characterization of the separating equilibria differs from the single product case. In particular, in the least-cost separating equilibrium, the level of advertising needed by H_1 to separate itself from L_1 is greater than the level needed for separation in market 1 if the two products were independent. This level of advertising is equal to the expected gain of L_1 in both markets by pretending to be H_1. The gain in the first market is $\pi(L,1) - \pi(L,0)$, the expected gain in the second market is:

$$\begin{aligned}&((1-p)+\rho p)\,[\pi(L,p+\rho(1-p)) - \pi(L,p-\rho p)]\\&\quad + (p-\rho p)\,[\pi(H,p+\rho(1-p)) - \pi(H,p-\rho p)]\end{aligned}$$

and reflects the expected increase in profits through the posterior beliefs. By imitating H_1, L_1 changes beliefs from $p - \rho p$ to $p + \rho(1-p)$.

Furthermore, like in the case analyzed in the previous subsection, the set of pooling equilibria is larger than in a single product case since a deviation from the pooling equilibrium leads posterior beliefs to deteriorate from p to $p - \rho p$.

The next lemma describes the most reasonable PBE when priors are high and correlation is low.

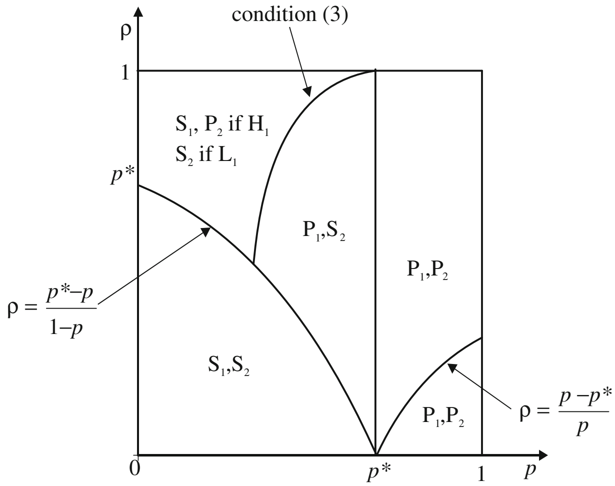

Fig. 3 The most reasonable equilibrium when the consumer does not learn the quality of good 1 before a_2 is observed

Lemma 10 *If $p > p^*$ and $\rho < \frac{p-p^*}{p}$, then the most reasonable PBE involves pooling with zero advertising in both markets. That is, $a_1^H = a_1^L = 0$ and $a_2^H = a_2^L = 0$.*

Proof As usual one can show that the unique separating equilibrium that survives the domination criterion is the least-cost separating one. Additionally, to verify if any pooling equilibrium survives the domination criterion it is enough to check whether that is true for the pooling equilibrium with $a_1^H = a_1^L = 0$.

Any level of $a_1 > \underline{a}_1^{H_1}$ defined by Eq. 7 is a dominated strategy for type L_1, thus $\mu_1 = 1$ following any $a_1 > a_1^{H_1}$. With these beliefs a type H_1 would want to deviate from $\widetilde{a}_1 = 0$ as long as:

$$\begin{aligned}
&\pi(H, 1) - \underline{a}_1^{H_1} + ((1-p) - \rho(1-p))\,\pi(L, p + \rho(1-p)) \\
&\quad + (p + \rho(1-p))\,\pi(H, p + \rho(1-p)) \\
&\qquad \geq \pi(H, p) + ((1-p) - \rho(1-p))\,\pi(L, p) + (p + \rho(1-p))\,\pi(H, p)
\end{aligned}$$

As in the previous case, this condition can never hold. Consequently, the pooling equilibrium with $\widetilde{a}_1 = 0$ survives the domination criterion. In addition it is Pareto optimal, so it is the most reasonable equilibrium. □

5.5 Summary of results

The next Proposition summarizes the results on the most reasonable equilibrium. These results are illustrated in Fig. 3.

Proposition 3 *If the consumer does not learn the quality of good 1 before observing a_2:*

(i) *When* $p < p^*$ *and* $\rho < \frac{p^*-p}{1-p}$ *the most reasonable PBE is the least-cost separating equilibrium in both markets. That is,* $a_2^H = \underline{a}^H$ *and* $a_2^L = 0$ *regardless of* a_1 *and in equilibrium* $a_1^H = \underline{a}^H$ *and* $a_1^L = 0$.

(ii) *When* $p < p^*$ *and* $\rho > \frac{p^*-p}{1-p}$*, if condition* (3) *holds then the most reasonable equilibrium involves separation in market 1, with* $a_1^L = 0$ *and* a_1^H *defined by Eq.* 1 *where* $a_1^H > \underline{a}^H$*. If* $a_1 \geq a_1^H$ *then in the second market there is pooling,* $a_2^H = a_2^L = 0$*. If* $a_1 = 0$*, then in the second market there is separation* ($a_2^H = \underline{a}^H$ *and* $a_2^L = 0$).

(iii) *When* $p < p^*$ *and* $\rho > \frac{p^*-p}{1-p}$*, if condition* (3) *does not hold then the most reasonable equilibrium involves pooling in market 1* ($a_1^H = a_1^L = 0$) *and separation in market 2* ($a_2^H = \underline{a}^H$ *and* $a_2^L = 0$)

(iv) *When* $p > p^*$*, the most reasonable equilibrium is pooling in both markets* ($a_1^H = a_1^L = 0$ *and* $a_2^H = a_2^L = 0$).

When ρ is low we get the same results as when the quality of the first product is observed. That is, the multiproduct firm just replicates in each market the behavior of a single product firm.

When ρ is high ($\rho > \frac{p^*-p}{1-p}$ for $p < p^*$ or $\rho > \frac{p-p^*}{p}$ for $p > p^*$), there are some similarities, but there are also important differences with the case where the quality of the first product is observed. As when the quality of the first product is observed, when ρ is high and p is sufficiently low, the most reasonable equilibrium still involves separation in the first market, whereas the continuation equilibrium in the second market depends on whether the firm revealed to be H_1 or L_1. Like before, a firm who advertises enough to credibly signal that it has a high-quality first product will exploit in market 2 its reputation. On the other hand, if a firm revealed being L_1 and consequently has a bad reputation, then if the firm has a high-quality second product, it will advertise enough in order to demonstrate that the second product is in fact of high quality.

However, the amount that a H_1 firm has to advertise in order to separate itself from a L_1 firm is higher than when the quality of good 1 is observed. What happens is that a L_1 firm has greater incentives to mimic the behavior of a H_1 firm, because revealing being H_1 brings benefits to the firm not only in the first market, but also in the second, through the improvement in the quality expectations of good 2. Then a H_1 firm needs to advertise a higher amount in order to credibly signal that its first product is of high quality. Here advertising in the first market has information spillovers in the second market and to create a reputation of being H_1, a firm has to advertise sufficiently.

In addition, when $p > p^*$ and $\rho > \frac{p-p^*}{p}$ in the most reasonable equilibrium none of the types advertises in both markets and consequently, the consumer's posterior beliefs are equal to his prior beliefs, p. This contrasts with the case where the quality of the first product is observed, where there was no advertising in market 1, but consumers revised their beliefs about product 2, because they learned the quality of good 1 immediately after consumption.

Finally, for high ρ and intermediate p, we now have firms pooling in the first market and separating in the second market. This may happen because separation in the first market is «too expensive » and both types, L_1 and H_1, are better off by not advertising even if doing so leads consumers not to change their beliefs regarding product 1.

6 Conclusion

This paper considered a model where a firm introduces sequentially two products with positively correlated qualities. Signaling advertising may be used in both markets. In this model we investigate whether there exist information spillovers from the first to the second market.

When the correlation between the two products is low, we have shown that the multiproduct firm just replicates in each market the behavior of a single product firm. In this case, knowing the quality of good 1 has a very small influence on the consumers' perception of product 2's quality, hence it does not affect the equilibrium advertising levels in market 2.

When the correlation between the two products is high, the equilibrium in market 2 depends heavily on what consumers have learned about product 1 prior to the introduction of the second product. If consumers have learned that product 1 is of high quality then, in the second market, the firm will exploit its reputation and will not advertise. On the contrary, if a firm has shown to have a low-quality product 1 and hence has acquired a bad reputation, then if the firm has a high-quality second product, it will advertise high enough in order to demonstrate to the consumer that the second product is in fact of high quality. Thus, if quality correlation is high, when the firm chooses its advertising level in the first market it has to take into account the impact of its decision in market 2.

In terms of advertising expenses in the first market, the results depend crucially on whether the quality of the first product is observed by the consumers before the second product is introduced or not. If the quality of the first product is observed, the consumers learn the true quality of product 1 regardless of the advertising level in market 1. Hence the advertising level in market 1 does not have signaling spillovers in market 2 and, consequently, the equilibrium in market 1 is the same as in a single product firm.

On the other hand, when the quality of the first product is not observed, the advertising level in the first market influences beliefs about the second product quality. In this case, if a firm with a high-quality first product wishes to separate from its low-quality counterpart, it will need to advertise a higher amount than if the qualities of the two products were unrelated. This advertising level signals not only high quality in the first market, it also signals that it is very likely that product 2 will be of high quality. In the particular case where the two products are perfectly correlated, a high-quality firm ends up advertising only in the first market, but the amount spent in this market is the same as what is needed for separation in two independent markets.

In reality we observe at times levels of advertising for one product which seem excessive, given the demand for that good. Our model provides an explanation for this phenomena. It may well be that, by advertising a very high amount, the firm is also signaling the quality of the products that it will introduce in the future.

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