Int J Game Theory (2005) 33: 505–514
DOI: 10.1007/s00182-005-0211-y

International Journal of Game Theory

A characterization of the position value*

Marco Slikker

Department of Technology Management, Technische Universiteit Eindhoven, P.O. Box 513 5600 MB, Eindhoven, The Netherlands.
(E-mail: M.Slikker@tm.tue.nl)

Received: September 2003/Accepted: July 2005

Abstract. We characterize the position value for arbitrary communication situations. The two properties involved in the characterization are component efficiency, which is standard, and balanced link contributions, which is in the same spirit as balanced contributions. Since the Myerson value can be characterized by component efficiency and balanced contributions a comparison between the two allocation rules based on characterizing properties can be made.

Key words: Communication situation, position value, characterization.

JEL classification numbers. C71

1. Introduction

The study of cooperative games with restricted cooperation possibilities is well documented. The first to model restricted cooperation by means of an undirected graph was Myerson (1977). He introduced communication situations, which consist of a cooperative game and an undirected graph. The vertices in the graph correspond to the players in the cooperative game and the edges between the players correspond to bilateral communication possibilities. For these situations, Myerson (1977) introduced and characterized an allocation rule, the so-called Myerson value. Later on, alternative characterizations, some valid on restricted sets of communication situations only, were given in Myerson (1980) and Borm et al. (1992).

* The author thanks Anne van den Nouweland, two anonymous referees, an associate editor and the editor for useful suggestions and comments.

The main contribution of Meessen (1988) and Borm et al. (1992) was the introduction of an alternative rule for communication situations. This rule is called the position value. Borm et al. (1992) provided a characterization of this rule for communication situations with trees as the underlying graphs only. They concluded by stating the open problem how to characterize the position value for the class of all communication situations. As far as we know, no satisfactory answer to this question has been given so far. We remark that Slikker (2005) proved that for reward communication situations, there exist a unique *link potential* and a unique *player potential*, where reward communication situations can be seen as a superset of the set of communication situations. The marginal contributions of the players to these potentials correspond to the position value and the Myerson value, respectively. This is in the same spirit as the potential for the Shapley value of Mas-Colell (1989), who remarked (Remark 2.5) that the existence of a unique potential can be seen as a new characterization. This would contradict the statement we just made. Though useful, e.g., for proving results for general classes of games, we feel that the existence of a unique potential does not seem to be generally accepted as a (solid) characterization. Finally, for the sake of completeness, we remark that the potential of Slikker (2005) can be restricted to communication situations only, thereby perhaps losing some of its original appeal.

Yet another rule for communication situations has been introduced by Hamiache (1999). This rule is defined as the only rule satisfying five properties. The driving force behind this value is a consistency property. As compared to several other consistency properties, this one does not consider reduced games but so-called associated games. For details we refer to Hamiache (1999). Some deficiencies in this paper are pointed out and addressed by Bilbao et al. (2005).

The main contribution of this paper is a characterization of the position value for arbitrary communication situations. The two properties involved in this characterization are *component efficiency* and *balanced link contributions*. *Component efficiency* states that the profit obtained by a component (a maximal set of connected players) should be divided among its members. This property is standard in the earlier characterizations of the position value and the Myerson value. *Balanced link contributions* deals with the payoff difference a player experiences if another player breaks one of his links. The benefit a player contributes to another player is defined as the sum of these differences over the links of the first player. *Balanced link contributions* states that the contribution of a player to the payoff of another player equals the reverse contribution. This property is in the same spirit as *balanced contributions*, which in conjunction with *component efficiency* characterizes the Myerson value (See Myerson (1980)). This allows for a solid comparison between the values based on underlying properties.

The recent literature on communication situations or, more generally, on networks in cooperative situations increasingly concentrates on network formation. Slikker and van den Nouweland (2001) provided a recent review of both the cooperative, axiomatic approaches to allocation rules and of the formation issues.

The setup of the remainder of this paper is as follows. In Section 2 we introduce notation and definitions. This section contains the formal descriptions of the position value and the Myerson value. Our main result, a characterization of the position value, can be found in Section 3. We conclude in Section 4 with a comparison of the position value and the Myerson value based on characterizing properties.

2 Preliminaries

In this section we present notation and definitions.

A *TU-game* (a cooperative game with transferable utilities) is a pair (N, v), where $N = \{1, \dots, n\}$ denotes the set of players and $v : 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$ the characteristic function. A game (N, v) is *zero-normalized* if $v(\{i\}) = 0$ for all $i \in N$. For a coalition $S \subseteq N$, $v_{|S}$ denotes the restriction of v to the player set S, i.e., $v_{|S}(T) = v(T)$ for each coalition $T \subseteq S$. The pair $(S, v_{|S})$ is a game with player set S.

Let N be a set of players and let $R \in 2^N \backslash \{\emptyset\}$. The *unanimity game* (N, u_R) is the game defined by $u_R(S) = 1$ if $R \subseteq S$ and $u_R(S) = 0$ otherwise (see Shapley (1953)). Every game (N, v) can be written as a linear combination of unanimity games in a unique way, i.e., $v = \sum_{R \in 2^N \backslash \{\emptyset\}} \lambda_R(v) u_R$. The *Shapley value* *Sh* of a game (N, v) is now easily described by

$$Sh_i(N, v) = \sum_{R \subseteq N:\, i \in R} \frac{\lambda_R(v)}{|R|} \text{ for all } i \in N^1.$$

A (*communication*) *graph* is a pair (N, L) where the vertices in N represent players and edges in $L \subseteq L^N = \{\{i, j\} \mid \{i, j\} \subseteq N,\ i \neq j\}$ represent bilateral (communication) links. Two players i and j are *directly connected* if $\{i, j\} \in L$. Two players i and j are *connected* (directly or indirectly) if $i = j$ or there exists a path between players i and j, i.e., there exists a sequence of players $(i_1, i_2, \dots, i_t)$ such that $i_1 = i$, $i_t = j$, and $\{i_k, i_{k+1}\} \in L$ for all $k \in \{1, 2, \dots, t-1\}$. The notion of connectedness induces a partition of the player set into communication components, two players being in the same *communication component* if they are connected. The set of communication components of (N, L) is denoted by N/L. Furthermore, denote the subgraph on the vertices in coalition $S \subseteq N$ by $(S, L(S))$, where $L(S) = \{\{i, j\} \in L \mid \{i, j\} \subseteq S\}$, and the partition of S into communication components according to graph $(S, L(S))$ by S/L. Finally, for all $L \subseteq L^N$ and all $i \in N$, let

$$L_i = \left\{ l \in L \;\middle|\; i \in l \right\} \tag{1}$$

be the set of links in (N, L) player i belongs to.

Myerson (1977) studied *communication situations* (N, v, L) where (N, v) is a TU-game and (N, L) a communication graph.[2] An *allocation rule* on a class D of communication situations is a function φ that assigns a payoff vector $\varphi(N, v, L) \in \mathbb{R}^N$ to all $(N, v, L) \in D$. Myerson (1977) introduced the *graph-restricted game* (N, v^L), where

[1] $|R|$ denotes the cardinality of set R.

[2] Hamiache (1999) refers to a communication situation as a game with communication structure. We stick to the name that is used in most of the literature.

$$v^L(S) = \sum_{C \in S/L} v(C) \text{ for all } S \subseteq N.$$

So, a coalition is split into communication components and the value of this coalition in the graph-restricted game is then defined as the sum of the values of the communication components in the original game. The Shapley value of the game (N, v^L) is usually referred to as the *Myerson value* of the communication situation (N, v, L), i.e.,

$$\mu(N, v, L) = Sh(N, v^L).$$

An alternative rule for communication situations, the *position value*, is introduced by Meessen (1988) and Borm et al. (1992). Let (N, v, L) be a communication situation. For the game (N, v), let the associated *link game* (L^N, r^v) be defined by

$$r^v(A) = \sum_{C \in N/A} v(C) \text{ for all } A \subseteq L^N.$$

This link game can be seen as a TU-game in which the players can be identified with pairs of players in the original game. Hence, the link game can be written as a unique linear combination of unanimity games, i.e.,

$$r^v = \sum_{A \subseteq L^N} \lambda_A(r^v) u_A.$$

The restriction of (L^N, r^v) to L can be described similarly, i.e.,

$$r^v_{|L} = \sum_{A \subseteq L} \lambda_A(r^v) u_A.$$

We refer to the pair $(N, r^v_{|L})$ as the *restricted link game* associated with communication situation (N, v, L). Borm et al. (1992) referred to the restricted link game as the *arc game* and defined it directly. We chose a two-stage introduction, which shows immediately that for all $A \subseteq L^N$ coefficient $\lambda_A(r^v_{|L}) = \lambda_A(r^v)$ does not depend on $L \supseteq A$. This facilitates comparing payoffs for two communication situations that differ in the underlying graph only.

The position value attributes to each player in a communication situation (N, v, L) half of the value of each link he is involved in, where the value of a link is defined as the payoff the Shapley value attributes to this link in the associated restricted link game $(L, r^v_{|L})$. Formally, the *position value* π of communication situation (N, v, L) with (N, v) zero-normalized is defined by

$$\pi_i(N, v, L) = \sum_{l \in L_i} \frac{1}{2} Sh_l(L, r^v_{|L}) \quad \text{for all } i \in N. \tag{2}$$

Throughout this work we restrict ourselves to communication situations with a zero-normalized underlying TU-game.[3]

[3]For communication situations with arbitrary (N, v), the position value of a player is defined as the sum of his individual value and his position value of the communication situation with the zero-normalization of (N, v) as the underlying game. Our results hold for this more general setting, but would require additional notation, which would only distract from the main result.

3. The characterization

In this section we prove the main result of this paper, a characterization of the position value for communication situations. As compared to earlier characterizations, no condition on the underlying graph is required.

Consider the following properties for a rule φ defined on a class of communication situations D. The first property is standard and dates back to Myerson (1977).

Component efficiency (CE): For all $(N, v, L) \in D$ and all $C \in N/L$,

$$\sum_{i \in C} \varphi_i(N, v, L) = v(C). \tag{3}$$

The second property deals with the gains players contribute to each other. The total contribution of a player to the payoff of another player is defined as the sum over all links of the first player of the payoff difference the second player experiences if such a link is broken. It states that the total contribution of a player to the payoff of another player is equal to the reverse total contribution.

Balanced link contributions (BLC): For all $(N, v, L) \in D$ and all $i, j \in N$,

$$\sum_{l \in L_j} \left[\varphi_i(N, v, L) - \varphi_i(N, v, L \backslash \{l\})\right] = \sum_{l \in L_i} \left[\varphi_j(N, v, L) - \varphi_j(N, v, L \backslash \{l\})\right]. \tag{4}$$

Before we prove that the position value satisfies these two properties, we provide an alternative description of the position value. Recall from Section 2 that the link game (L^N, r^v) associated with a game (N, v) can be written as a unique linear combination of unanimity games, i.e.,

$$r^v = \sum_{A \subseteq L^N} \lambda_A(r^v) u_A.$$

The position value of communication situation (N, v, L) can now be described in terms of the unanimity coefficients of the associated (restricted) link game (cf. Slikker (2005)), since for all $i \in N$,

$$\pi_i(N, v, L) = \sum_{l \in L_i} \frac{1}{2} Sh_l(L, r^v_{|L}) = \sum_{l \in L_i} \frac{1}{2} \sum_{A \subseteq L:\ l \in A} \frac{\lambda_A(r^v)}{|A|} = \sum_{A \subseteq L} \frac{1}{2} \lambda_A(r^v) \frac{|A_i|}{|A|}, \tag{5}$$

where the second equality follows by the description of the Shapley value in Section 2. Using this expression, we can easily show the following lemma.

Lemma 3.1. The position value satisfies *component efficiency and balanced link contributions.*

Proof: *Component efficiency* is proven by Borm et al. (1992). To prove *balanced link contributions*, let (N, v, L) be a communication situation and let $i, j \in N$ be two distinct players. Then

$$\sum_{l\in L_j}\Big[\pi_i(N,v,L)-\pi_i(N,v,L\backslash\{l\})\Big] = \frac{1}{2}\sum_{l\in L_j}\left[\sum_{A\subseteq L}\lambda_A(r^v)\frac{|A_i|}{|A|}-\sum_{A\subseteq L\backslash\{l\}}\lambda_A(r^v)\frac{|A_i|}{|A|}\right]$$

$$=\frac{1}{2}\sum_{l\in L_j}\sum_{A\subseteq L:\,l\in A}\lambda_A(r^v)\frac{|A_i|}{|A|}$$

$$=\frac{1}{2}\sum_{A\subseteq L}|A_j|\lambda_A(r^v)\frac{|A_i|}{|A|}$$

$$=\frac{1}{2}\sum_{A\subseteq L}\lambda_A(r^v)\frac{|A_i|\cdot|A_j|}{|A|}$$

$$=\sum_{l\in L_i}\Big[\pi_j(N,v,L)-\pi_j(N,v,L\backslash\{l\})\Big],$$

where the first equality follows by definition, the second to fourth equalities by rearranging terms, and the last equality follows by the same arguments as for the first four equalities (note that the expression after the fourth equality sign is symmetric in i and j). ■

The following example illustrates the lemma. Additionally, it shows that the Myerson value and the value of Hamiache do not satisfy *balanced link contributions*.

Example 3.1. Consider the communication situation (N,v,L) with $N=\{1,2,3\}$, $v=u_{\{1,2\}}$, and $L=\{\{1,2\},\{1,3\},\{2,3\}\}$ (see also Borm et al. (1992)).

Let $a=\{1,3\}$, $b=\{2,3\}$, and $c=\{1,2\}$. Then

$$r^v(A)=\begin{cases}0 & \text{if } A\in\{\emptyset,\{a\},\{b\}\};\\ 1 & \text{otherwise.}\end{cases}$$

The payoffs for several (sub-)graphs according to the position value (π) are given in Table 1. For example, the payoffs for (sub-)graph $(N,\{\{1,3\},\{2,3\}\})$ are determined in two steps. First, the links $a=\{1,3\}$ and $b=\{2,3\}$ are both attributed a value equal to $\frac{1}{2}$ according to the Shapley value of the game $(\{a,b\},r^v_{|\{a,b\}})$ (Note that both a and b are needed to connect players 1 and 2, i.e., $r^v_{|\{a,b\}}=u_{\{a,b\}}$). Subsequently, player 1 gets half of the value of a, player 2 half of the value of b, and finally, player 3 gets half of the value of a and half of the value of b. Hence, the payoffs to the players are $(\frac{1}{4},\frac{1}{4},\frac{1}{2})$.

Consider the complete graph (N,L). Then, the total contribution of player 1 to player 2 equals $(\frac{5}{12}-\frac{1}{4})+(\frac{5}{12}-\frac{1}{2})=\frac{1}{12}$, by breaking the link with player 2 and that with player 3, respectively. Obviously, the reverse contribution is the same since players 1 and 2 are symmetric.

The total contribution of player 1 to player 3 equals $(\frac{1}{6}-\frac{1}{2})+(\frac{1}{6}-0)=-\frac{2}{12}$, by breaking the link with player 2 and that with player 3, respectively. The reverse contribution of player 3 to player 1 equals $(\frac{5}{12}-\frac{1}{2})+(\frac{5}{12}-\frac{1}{2})=-\frac{2}{12}$ as well.

This illustrates that according to the position value the summations of contributions of links are balanced indeed.

Table 1. Position values, Myerson values and Hamiache values for several graphs

graph A	$\pi(N,v,A)$	$\mu(N,v,A)$	$\mathrm{Ham}(N,v,A)$
{{1, 2}, {1, 3}}	$(\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2},0)$
{{1, 2}, {2, 3}}	$(\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2},0)$
{{1, 2}, {2, 3}}	$(\frac{1}{4},\frac{1}{4},\frac{1}{2})$	$(\frac{1}{3},\frac{1}{3},\frac{1}{3})$	$(\frac{1}{4},\frac{1}{4},\frac{1}{2})$
L	$(\frac{5}{12},\frac{5}{12},\frac{1}{6})$	$(\frac{1}{2},\frac{1}{2},0)$	$(\frac{1}{2},\frac{1}{2},0)$

The payoffs according to the Myerson value (μ) and the Hamiache value (Ham) can be found in Table 1 as well. We remark that both values have the property that for a communication situation with a complete underlying graph the payoffs correspond to the Shapley value of the underlying game. The payoffs according to the Hamiache value can be computed using Lemma 1, Theorem 2, and Table 1 of Hamiache (1999).

Consider the complete graph (N,L). According to the Myerson value the total contribution of player 1 to player 3 equals $-\frac{1}{3}+0=-\frac{1}{3}$, while the reverse contribution equals $0+0=0$. According to the value of Hamiache these contributions are $-\frac{1}{2}+0=-\frac{1}{2}$ and $0+0=0$. We conclude that the total link contributions need not be balanced according to the Myerson value or the Hamiache value. $\diamond$

We will use lemma 3.1 in the following theorem.

Theorem 3.1. *The position value is the unique allocation rule on the domain of all communication situations that satisfies component efficiency* and *balanced link contributions.*

Proof: Lemma 3.1 states that the position value satisfies CE and BLC. Conversely, suppose φ satisfies CE and BLC. We show that $\varphi=\pi$. The proof is by induction on $|L|$.

First note that for all (N,v,L) with $|L|=0$ it follows directly that φ and π coincide, since the two rules satisfy CE.

Secondly, let $k\geq 1$ and suppose that φ and π coincide for all (N,v,L) with $|L|\leq k-1$. Let (N,v,L) be such that $|L|=k$. We will show that for all $C\in N/L$ and all $i\in C$ $\varphi_i(N,v,L)=\pi_i(N,v,L)$. This equality follows directly from CE if $|C|=1$. Let $C\in N/L$ with $|C|\geq 2$. Without loss of generality denote $C=\{1,2,\ldots,c\}$. By BLC and CE, we have the following system of equalities,[4]

$$|L_2|\varphi_1(L)-|L_1|\varphi_2(L)=\sum_{l\in L_2}\varphi_1(L\backslash\{l\})-\sum_{l\in L_1}\varphi_2(L\backslash\{l\});$$

$$\ldots$$

$$|L_c|\varphi_1(L)-|L_1|\varphi_c(L)=\sum_{l\in L_c}\varphi_1(L\backslash\{l\})-\sum_{l\in L_1}\varphi_c(L\backslash\{l\});$$

$$\sum_{i\in C}\varphi_i(L)=v(C).$$

[4]We suppress N and v and write, for example, $\varphi(L)$ instead of $\varphi(N,v,L)$.

The first $c-1$ equalities come from BLC applied to pairs $\{1,j\}$ for all $j \in \{2,\ldots,c\}$. Note that since $|C| \geq 2$ and $j \in C$ for all $j \in \{1,\ldots,c\}$, $|L_j| \geq 1$ for all $j \in \{1,\ldots,c\}$. The last equality comes from CE.

By the induction hypothesis it follows that the system above is equivalent to

$$|L_2|\varphi_1(L) - |L_1|\varphi_2(L) = \sum_{l \in L_2} \pi_1(L\backslash\{l\}) - \sum_{l \in L_1} \pi_2(L\backslash\{l\});$$

$$\ldots$$

$$|L_c|\varphi_1(L) - |L_1|\varphi_c(L) = \sum_{l \in L_c} \pi_1(L\backslash\{l\}) - \sum_{l \in L_1} \pi_c(L\backslash\{l\});$$

$$\sum_{i \in C} \varphi_i(L) = v(C).$$

It is a straightforward exercise to show that this is a regular system in c variables, $\varphi_1(L),\ldots,\varphi_c(L)$. Consequently, it has a unique solution. Since the position value satisfies BLC and CE, $(\pi_1(L),\ldots,\pi_c(L))$ is a solution, and, hence, it is the unique solution.

We conclude that $\varphi(N,v,L) = \pi(N,v,L)$ for all (N,v,L) with $|L| = k$. ■

4. Discussion

Theorem 3.1 provides a characterization of the position value for arbitrary communication situations. This is in line with the work of Borm et al. (1992), in which characterizations are provided in case the underlying graph is a tree. Furthermore, on the same restricted class, they provided a similar characterization for the Myerson value. The two characterizations share three properties. The first two are *component efficiency*, already encountered, and *additivity*, which states that the sum of the payoffs to a player in two communication situations with the same underlying graph equals the payoff this player receives in the communication situation with the same graph, but with the sum of the two original games as the underlying game. Thirdly, both characterizations involve *superfluous link property*, which states that deleting a link that is a zero player in the associated restricted link game should not influence the eventual payoffs. Additionally, the characterization of the position value uses *link anonymity*, which states that if the value of a set of links in the restricted link game depends on the number of links only and not on their identities, then the payoffs to a player should be proportional to the number of links he is involved in. On the other hand, in the characterization of the Myerson value, *point anonymity* is used. *Point anonymity* states that if the value of a coalition in the graph-restricted game depends on the number of non-isolated players in this coalition only and not on their identities, then all non-isolated players should get the the same payoff. On the set of communication situations with a tree as underlying graph, the difference between the position value and the Myerson value can be traced to the difference between *link anonymity* and *point anonymity*.[5]

The characterization in this paper clears the way for a comparison between the position value and the Myerson value on the class of all communication situations. The characterization of the Myerson value that comes closest to the characterization in this paper is one that can be traced back to Myerson (1980). The two properties involved are *component efficiency* and *balanced contributions*. We define this latest property for a rule φ defined on a class of communication situations D:

Balanced contributions (BC): For all $(N, v, L) \in D$ and all $i, j \in N$,

$$\varphi_i(N, v, L) - \varphi_i(N, v, L \backslash L_j) = \varphi_j(N, v, L) - \varphi_j(N, v, L \backslash L_i).$$

The difference between the position value and the Myerson value comes down to a difference in measuring contributions. For the Myerson value, this contribution is measured by the payoff difference a player experiences if all the links of the other player are removed. For the position value, this contribution is measured by the sum of the payoff differences a player experiences if one of the links of the other player is removed.

By rewriting the contribution for the Myerson value, the two properties can be compared on an even more detailed level. Let (N, v, L) be a communication situation and let $i, j \in N$. Write $L_j = \{l_1, \ldots, l_j\}$. Then

$$\begin{aligned}\varphi_i(N, v, L) - \varphi_i(N, v, L \backslash L_j) = {} & \Big(\varphi_i(N, v, L) - \varphi_i(N, v, L \backslash \{l_1\})\Big) \\ & + \Big(\varphi_i(N, v, L \backslash \{l_1\}) - \varphi_i(N, v, L \backslash \{l_1, l_2\})\Big) \\ & + \cdots + \Big(\varphi_i(N, v, L \backslash \{l_1, \ldots, l_{j-1}\}) \\ & - \varphi_i(N, v, L \backslash \{l_1, \dot{s}, l_j\})\Big).\end{aligned}$$

Hence, both the contribution for the position value and the one for the Myerson value can be seen as the sum of the payoff differences a player can inflict on another player by breaking one of his links. However, when breaking these links one-by-one, for the position value a broken link should be restored before breaking the next one, whereas this link should not be restored for the Myerson value.

Concluding, we stress that besides the fact that the characterization of the position value is interesting in itself, it provides us with the opportunity to make a solid comparison between characterizing properties of the position value and of the Myerson value on the class of all communication situations.

We end this discussion with a remark on the domain of the characterization. In this paper we characterized the position value on the domain of all communication situations. A careful analysis of the proof of the main result shows that one could restrict the analysis to smaller domains as well. The crucial requirement on the domain D is that for any $(N, v, L) \in D$ and any $A \subseteq L$, $(N, v, A) \in D$. For example, let (N, v') be an arbitrary TU-game. Then

[5] Borm et al. (1992) refer to *superfluous link property*, *link anonymity*, and *point anonymity* as *superfluous arc property*, *degree property*, and *communication ability property*, respectively. Our terminology is in line with Slikker and van den Nouweland (2001).

the proof still goes through on the domain of all communication situations that have (N, v') as the underlying TU-game, i.e., on the domain $D' = \{(N, v', L) \mid L \subseteq L^N\}$. As another example one could consider the same domain as the original characterization of Borm et al.(1992), i.e., the domain of all communication situations with a tree as the underlying communication graph.

References

Bilbao J, Jiménez N, López J (2005) A note on a value with incomplete information. Games and Economic Behavior (to appear)

Borm P, Owen G, Tijs S (1992) On the position value for communication situations. SIAM Journal on Discrete Mathematics 5:305–320

Hamiache G (1999) A value with incomplete information. Games and Economic Behavior 26:59–78

Hart S, Mas-Colell A (1989) Potential, value and consistency. Econometrica 57:589–614

Meessen R (1988) Communication games In Dutch. Master's thesis, Department of Mathematics, University of Nijmegen, The Netherlands

Myerson R (1977) Graphs and cooperation in games. Mathematics of Operations Research 2:225–229

Myerson R (1980) Conference structures and fair allocation rules. International Journal of Game Theory 9:169–182

Shapley L (1953) A value for n-person games. In: Tucker A, Kuhn H, (eds.) Contributions to the Theory of Games II, Princeton University Press, Princeton pp 307–317

Slikker M (2005) Link monotonic allocation schemes. International Game Theory Review (to appear)

Slikker M, van den Nouweland A (2001) Social and Economic Networks in Cooperative Game Theory. Kluwer Academic Publishers, Boston