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A Markov–switching vector equilibrium correction model of the UK labour market

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Abstract. There is a wide literature on the dynamic adjustment of employment and its relationship with the business cycle. In this paper we present a statistical model that offers a congruent representation of part of the UK labour market since the mid 1960s. We use a cointegrated vector autoregressive Markov-switching model in which some parameters change according to the phase of the business cycle. Output, employment, labour supply and real earnings are found to have a common cyclical component. The long run dynamics are characterized by one cointegrating vector relating unemployment to trend-adjusted real wages and output. Despite there having been many changes affecting this sector of the UK economy, the Markov-switching vector-equilibrium-correction model with three regimes (representing recession, normal growth, and high growth) provides a good characterization of the sample data, and performs well relative to alternative linear and non-linear models. The results of an impulse-response analysis highlight the dangers of using VARs when the constancy of the estimated coefficients has not been established, and demonstrate the advantages of generating regime dependent responses.

Key words: Business Cycles, Employment, Impulse-Response Analysis, Cointegration, Regime Shifts, Markov Switching.

JEL classification: E32, E37, C32, E24

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1. Introduction

There have been numerous studies analyzing the time series relationships between wages, prices, productivity and unemployment in the UK (recent examples include Clements and Mizon, 1991, Mizon, 1995, Sgherri and Wallis, 2000 and Marcellino and Mizon, 2000). Most of these studies have found evidence of structural change, and have modelled it via split-sample analysis with a single break occurring around 1979, possibly with additional dummy variables to capture the effects of other changes affecting the UK labour market.

As an alternative to this deterministic approach to structural change and regime shifts, in this paper we develop a small model of part of the UK labour market using a multivariate Markov-switching vector equilibrium correction model (MS-VECM). This methodology is well suited to model the domestic and international cyclical swings that affected the UK economy, and it allows for changing relationships among the labour market variables across different phases of the business cycle. The results we obtain are easily interpretable, from both an economic and an econometric perspective. First, we find an equilibrium relationship that indicates that unemployment increases with deviations of the real wage from its overall trend, and decreases during expansions. Second, switches in the regimes are closely related to changes in the phases of the UK business cycle: the first regime is associated with recessions, the second and the third regimes with periods of normal and sustained growth respectively. Third, the MS-VECM provides a congruent statistical representation for the data, and the restrictions that lead to a standard linear model are strongly rejected. Fourth, the MS-VECM performs well in forecasting.

Another important characteristic of our MS-VECM is that the contemporaneous covariance matrix of the residuals is also regime switching, and substantial differences across regimes emerge. Hence, standard impulse response analysis is likely to lead to severely misleading results. We show that this is indeed the case, and derive the appropriate response functions for each regime. Moreover, the standard approach focuses on the response of the system to Gaussian innovations, even though other shocks could affect the system. In particular, changes in the phase of the cycle is what some economists have in mind when they refer to 'cyclical shocks', namely, investigating the dynamics of some variables in the transition from boom to bust, or vice versa. Within our framework, we also present response functions for this type of shock.

The structure of the paper is the following. Section 2 describes the data and some of the major changes that have taken place in the UK labour market. The specification of a Vector Equilibrium Correction Model (VECM) with non-constant parameters for the period 1965–2001 is described in section 3, and the particular form of the MS-VECM used in this paper is presented in section 4, which also contains the empirical results: firstly for the cointegrated VAR in section 4.1, and then for the MS-VECM in section 4.2. Section 5 contains a comparison of the impulse response functions of the alternative models, and illustrates the susceptibility of such analyses to non-constant parameters. The forecasting performance of the alternative models is evaluated in section 6. Section 7 summarizes and provides conclusions.

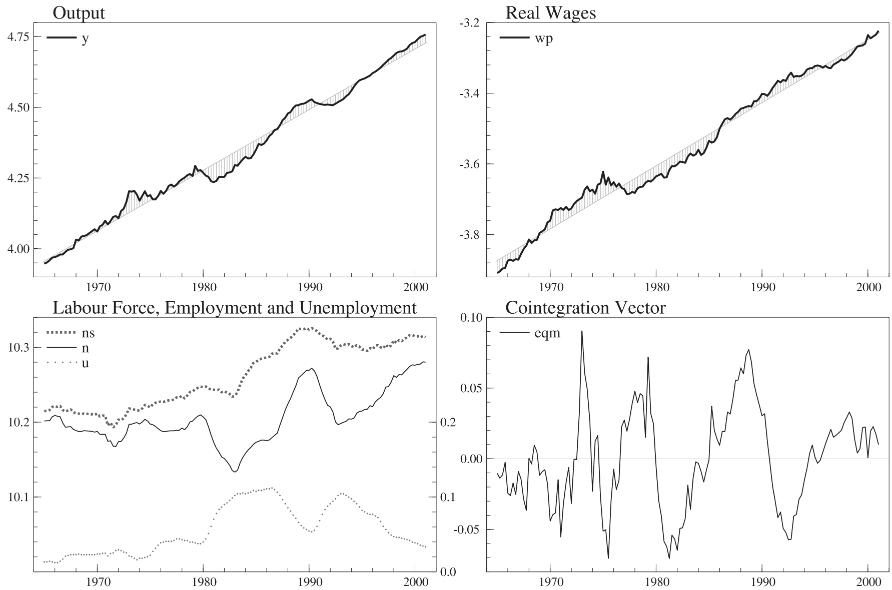

Fig. 1. The variables under analysis and the restricted equilibrium.

2. The data

The seasonally adjusted quarterly data for the UK are similar to those used in Clements and Mizon (1991), Mizon (1995) and Marcellino and Mizon (2000), extended to cover the period 1965(1)–2001(1). The original sources for these data are "Economic Trends" and "Monthly Digest of Statistics" published by the UK Office of National Statistics – more details are contained in the data appendix A.

The output variable, y_t, is the log of total constant price value added. Employment and the labour force, n_t and ns_t, are the logs of the number of employees and the total labour force in the whole economy respectively. The earnings variable, e_t, is the log of the ratio of wages and salaries to the number of employees. The price variable, p_t, is the log of the value added deflator.[1] The real wage, wp_t, is given by the log of real earnings ($wp_t = e_t - p_t$). This broad definition of the real wage is in line with earlier studies by Hall (1986) and Hall (1989). The variables are graphed in Figure 1. The first two panels of which show the strong trend in both output and real wages. The third panel shows the evolution of employment and the labour force, as well as their difference $u_t = (ns_t - n_t)$ as a measure of unemployment. The fourth panel graphs the restricted equilibrium relationship that is estimated in section 4.1 and used in section 4.2.

Notable events affecting the UK labour market in the sample period include the following. There was a big increase in real wages in 1975 associated with the ending of a period of statutory wage and price control. Unemployment increased strongly throughout most of the sample period, with some

[1] Similar results are obtained with the retail price index.

business cycle fluctuations. The decline in 1966 is related to the introduction of the "selective employment tax", which aimed to increase employment in manufacturing industries, though it was subsequently reduced by 50% in 1971. The substantial reductions in unemployment in 1974 and 1988/89 were mainly the delayed consequences of pre-election expansionary policies. 1974 was a turbulent year in the UK labour market with numerous strikes and the 3 Day Week restrictions leading to a change of government, followed by a strong increase in unemployment. There was also a major policy regime shift in late 1979 from broadly Keynesian full-employment to monetarist anti-inflation policies. A number of legislative changes were introduced in the early 1980s with the general effect of liberalizing the labour market. As a result of this and the tight monetary policy adopted to reduce the aggregate rate of inflation, there was a substantial increase in unemployment in the early and late 1980s. In addition, throughout the sample period there was an increase in female labour participation, which in turn led to more part-time working. The UK economy experienced a recession in the early 1990s, and sterling was forced out of the European Exchange Rate Mechanism in September 1992. Following the resulting devaluation there was steady growth in GDP, an increase in employment, and since the size of the labour force changed little from 1993 there was a steady reduction in unemployment to the end of the sample period. Overall there was less volatility in wages, prices, GDP, employment and unemployment post-1980 than pre-1980. Hence difficulties are likely to be experienced in attempting to develop VAR models with constant parameters for a small number of labour market variables. However, a MS-VECM may be better able to represent these events via changes in some of its parameters across regimes.

3. Modelling cointegrated systems with non-constant parameters

Clements and Hendry (1999) showed that unmodelled shifts in deterministic variables, such as intercepts and trends, are the major cause of forecast failure in econometric models, and that these shifts are detectable by conventional tests for parameter constancy such as those in Chow (1960). On the other hand, changes in short run adjustment coefficients and in the equilibrium coefficients are difficult to detect. Hence, we focus here on changes in the mean growth rates of the N variables in $\mathbf{x}_t$ and in the means of the r equilibrium relationships $\boldsymbol{\beta}'\mathbf{x}_t$. In particular, we consider a VECM for the $\mathsf{I}(1)$ variables $\mathbf{x}_t$ with intercept shifts introduced:

$$\Delta \mathbf{x}_t = \mathbf{v}(s_t) + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} + \mathbf{u}_t, \quad \mathbf{u}_t | s_t \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}), \tag{1}$$

where s_t denotes the unobservable regime indicator variable $s_t \in \{1, \dots, M\}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $N \times r$ matrices of rank r, and for simplicity only one period lags are introduced into the system and the error covariance matrix is assumed constant. Note that the intercept v is a function of the underlying state:

$$\mathbf{v}(s_t) = \mathbf{v}_{s_t} = \begin{cases} \mathbf{v}_1 & \text{if } s_t = 1 \\ & \vdots \\ \mathbf{v}_\mathrm{M} & \text{if } s_t = M. \end{cases} \tag{2}$$

and can be decomposed into:

$$\mathbf{v}(s_t) = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp'\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}_\perp'\mathbf{v}(s_t) + \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})^{-1}\boldsymbol{\beta}'\mathbf{v}(s_t)$$

$$= \boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s_t) + \boldsymbol{\alpha}\boldsymbol{\mu}(s_t)$$

when $\boldsymbol{\alpha}_\perp$ and $\boldsymbol{\beta}_\perp$ are $N \times (N - r)$ matrices such that $\boldsymbol{\alpha}_\perp'\boldsymbol{\alpha} = \mathbf{0}$ and $\boldsymbol{\beta}_\perp'\boldsymbol{\beta} = \mathbf{0}$. This means that there are $(N - r)$ linearly independent but state-dependent drifts $\boldsymbol{\delta}^*(s_t)$, and r linearly independent but state-dependent equilibrium means $\boldsymbol{\mu}(s_t)$ in the system. Hence, the process (1) can be represented as:

$$\Delta\mathbf{x}_t - \boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s_t) = \boldsymbol{\alpha}(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\mu}(s_t)) + \mathbf{u}_t, \quad \mathbf{u}_t|s_t \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}). \tag{3}$$

In (3), both $\Delta\mathbf{x}_t$ and $\boldsymbol{\beta}'\mathbf{x}_t$ are expressed as deviations about their regime- and time-dependent means, $\boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s)$ and $\boldsymbol{\mu}(s)$ respectively. Hence, each regime is characterized by an attractor of the system defined by the equilibrium value of the cointegration vector and the drift. Such a formulation is closely related to the notion of multiple equilibria in dynamic economic theory (see e.g. Cooper and John, 1988).

Two implications of the condition $\mathsf{E}[\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s)$ with $s_t = s_{t-1} = \cdots = s$ are worth noting. First, the mean growth rate of the equilibria $\boldsymbol{\beta}'\mathbf{x}_t$ is zero, i.e.,

$$\mathsf{E}[\Delta\boldsymbol{\beta}'\mathbf{x}_t \,|\, s] = \mathsf{E}[\boldsymbol{\beta}'\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\beta}'\mathsf{E}[\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\beta}'\boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s) = \mathbf{0}. \tag{4}$$

Second, if we interpret $\boldsymbol{\beta}_\perp'\mathbf{x}_t$ as stochastic trends in the system, then their mean growth rate in regime $s_t = s_{t-1} = \cdots = s$ is given by $\boldsymbol{\delta}^*(s)$, i.e.,

$$\mathsf{E}[\Delta\boldsymbol{\beta}_\perp'\mathbf{x}_t \,|\, s] = \mathsf{E}[\boldsymbol{\beta}_\perp'\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\beta}_\perp'\mathsf{E}[\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\beta}_\perp'\boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s) = \boldsymbol{\delta}^*(s).$$

Considering instead the stochastic trends represented by $\boldsymbol{\alpha}_\perp'\mathbf{x}_t$, as in Gonzalo and Granger (1995), then their expected change is

$$\mathsf{E}[\Delta\boldsymbol{\alpha}_\perp'\mathbf{x}_t \,|\, s] = \mathsf{E}[\boldsymbol{\alpha}_\perp'\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\alpha}_\perp'\mathsf{E}[\Delta\mathbf{x}_t \,|\, s] = \boldsymbol{\alpha}_\perp'\boldsymbol{\beta}_\perp\boldsymbol{\delta}^*(s) = \boldsymbol{\alpha}_\perp'\mathbf{v}(s).$$

When the changes in $\mathbf{v}(s_t)$ are due to a small number of deterministic shifts at known dates, their effects can be captured by including in the model an appropriate set of dummy variables. This is a common approach in empirical modelling of macroeconomic time series, and Clements and Mizon (1991) provide an example in the context of a small econometric model of the UK labour market. A similar approach can be adopted when there are changes in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as well as the intercepts. However, in this latter case, when the sub-samples permit, a valuable alternative is to conduct a split sample analysis of the data. Again with reference to the UK labour market, Marcellino and Mizon (2000) distinguish between the pre- and post-Thatcher period, finding evidence of substantial differences between the two sub-periods.

When the regime shifts are stochastic rather than deterministic both previous approaches can lead to biased, or at least inefficient, results. In this case, it is possible to enlarge the system by adding variables that are related to the regime shifts, such as policy variables, energy and raw material prices, and demographic and social indicators. Yet, it is difficult to jointly model the resulting enlarged set of variables, and conditioning on the regime shift related variables may not be valid and even if it were would not solve the forecasting problem (see Marcellino and Mizon, 2001a for more details).

Therefore, a multivariate generalization of the univariate Markov-switching model originally proposed by Hamilton (1989) provides a viable alternative. The general idea behind the class of MS models is that some of the parameters depend upon a stochastic, unobservable regime indicator variable $s_t \in \{1, \dots, M\}$. The stochastic process for generating the unobservable regimes is an ergodic Markov chain, defined by the transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, \dots, M\}. \tag{5}$$

By inferring the probabilities of the unobserved regimes conditional on an available information set, it is then possible to reconstruct the regimes. For an ergodic Markov chain, regime shifts are persistent if $p_{ij} \neq p_{ii}$ for some $i \neq j$, but not permanent if $p_{ii} \neq 1$ for all i. Further, (3) and (5) define a MSI-VECM (see Krolzig, 1997) when MSI refers to a Markov-switching intercept. The MSI-VECM exhibits equilibrium as well as error correction mechanisms: in each regime disequilibria are adjusted by the *vector equilibrium correction mechanism*; since the regimes themselves are generated by stationary, irreducible Markov chain; *errors* arising from regime shifts themselves are *corrected* towards the stationary distribution of the regimes.

Markov-switching models of multiple time series (see Krolzig, 1997, for an overview) provide a powerful statistical tool for extracting the common component from a group of economic time series representing the business cycle. In their investigation of the interaction of the UK business cycle with changes in the industrial structure of the UK economy during the last three decades, Krolzig and Sensier (2000) propose a Markov-switching vector equilibrium correction model with three regimes representing recession, normal growth and high growth. In their model the regime shifts simultaneously affect the common growth rate and the sectoral equilibrium allocation of industrial production identifying a common cycle which is closely related to traditional datings of the UK business cycle. Hence, an MSI-VECM appears to be a promising alternative specification for a small model of the UK labour market.

4. An MSIH-VECM for real wages, output, and employment

Following Krolzig (1997) we adopt a Markov-switching vector equilibrium correction model with shifts in the drift $\boldsymbol{\delta}(s_t)$ and in the equilibrium mean $\boldsymbol{\mu}(s_t)$:

$$\begin{aligned} \Delta \mathbf{x}_t - \boldsymbol{\delta}(s_t) &= \boldsymbol{\alpha}(\boldsymbol{\beta}' \mathbf{x}_{t-1} - \boldsymbol{\mu}(s_t) - \boldsymbol{\gamma}(t-1)) \\ &\quad + \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k (\Delta \mathbf{x}_{t-k} - \boldsymbol{\delta}(s_t)) + \mathbf{u}_t, \end{aligned} \tag{6}$$

and the error variance is allowed to change across states $\mathbf{u}_t | s_t \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$. Krolzig (1997) denotes this model by MSIH-VECM, where the H refers to heteroscedasticity in the error process. Note that $\boldsymbol{\beta}'\boldsymbol{\delta}(s) = \mathbf{0}$ since $\mathsf{E}[\Delta \mathbf{x}_t \mid s] = \boldsymbol{\delta}(s) = \boldsymbol{\beta}_\perp \boldsymbol{\delta}^*(s)$ from (4). The vector $\mathbf{x}_t$ includes output (y_t), real wages (wp_t), employment (n_t), and the labour force (ns_t), where the latter is included in the system to capture some changes in demographic and social conditions, such as different birth rates and female participation rates. As discussed in section 2, other variables might affect and modify the relationships between y_t, wp_t, n_t and ns_t. Within the framework of the MSIH-VECM in (6) their effects are captured by $\boldsymbol{\delta}(s_t)$, $\boldsymbol{\mu}(s_t)$ and $\boldsymbol{\Sigma}(s_t)$. As in (5), the unobservable regime variable s_t is governed by a Markov chain with a finite number of states (3 in our case), defined by the transition probabilities p_{ij}.

Each regime, m, is associated with a particular attractor (μ_m, δ_m). Regime shifts in δ_m are interpreted as changes in the state of the business cycle, regime shifts in μ_m as changes in the equilibrium mean $\boldsymbol{\mu}(s_t)$. Note that changes in the equilibrium mean's deterministic trend, $\boldsymbol{\gamma}t$, are not considered. In the results presented below we find one equilibrium relationship indicating that unemployment (proxied by $ns - n$) increases with 'excess real wages' and decreases with 'excess income'. Hence, changes in μ_m reflect changes in equilibrium unemployment, real wages and income, the latter two trend adjusted.

We now estimate the MSIH-VECM in (6) using the data described in section 2. The estimation method used is the two-stage procedure suggested by Krolzig (1996): first we investigate the cointegration properties of the system; then we present the results from estimating the MSIH-VECM. On the first stage, the cointegration properties of the MS-VECM can be analyzed by applying Johansen's maximum likelihood procedure (see Johansen, 1995) to a finite-order vector autoregressive approximation of the underlying DGP. On the second stage, conditional on the estimated cointegrated matrix, the maximum likelihood estimation of the parameters of the MS-VECM can be based on a version of the Expectation-Maximization (EM) algorithm discussed in Hamilton (1990) and Krolzig (1997). The computations are carried out with the MSVAR class for Ox, see Krolzig (1998) and Doornik (1999).

4.1. Cointegration analysis

The cointegration properties of the data are studied within a linear VAR representation using the maximum likelihood procedure of Johansen (1995) for the sample period 1965(1) to 2001(1). The VAR(p) is here considered as an approximation of the VARMA representation of a MSI-VAR process. Thus the following cointegration analysis is a limited information maximum likelihood technique.

Starting with a VAR(6) and deleting lags which were not significant according to a likelihood ratio (LR) test, led us to a VAR(4). Although the AIC information criterion leads to the choice of a VAR(2) there is more evidence of mis-specification in the VAR(2) than the VAR(4). The Johansen procedure for cointegration analysis as implemented in PcGive (see Johansen, 1988 and Doornik and Hendry, 2001) is then applied to the VECM representation of a VAR with four lags of $\mathbf{x}_t$, a constant, and a linear trend restricted to lie in the cointegration space:

$$\Delta\mathbf{x}_t = \boldsymbol{\nu} + \sum_{k=1}^{3} \boldsymbol{\Gamma}_k \Delta\mathbf{x}_{t-k} + \boldsymbol{\alpha}(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\gamma}(t-1)) + \mathbf{u}_t. \tag{7}$$

The results of the cointegration tests are shown in Table 1, with the trace test statistics accepting the hypothesis $r = 1$ at the 10%. There is no remaining autocorrelation (vector AR 1–5 test: $F(80, 396) = 0.91$ with a p-value of 0.69) but there is strong evidence of non-normality in the residuals (vector normal-

Table 1. Johansen cointegration likelihood ratio test

eigenvalue	H_0 : rank $= r$	trace test	pvalue
0.1743	$r = 0$	60.711	[0.088]
0.1389	$r \leq 1$	33.699	[0.307]
0.0690	$r \leq 2$	12.621	[0.767]
0.0178	$r \leq 3$	2.536	[0.913]

ity test $\chi^2(8) = 51.34$ with a p-value of 0.00). The latter is most likely the result of the many outliers present in the estimated VAR. Although this can result in under-estimation of cointegrating rank, our subsequent results which allow for Markov switching between three separate regimes suggest that this is not the case.

We then identify the cointegrating vector by applying a set of non-rejected restrictions, the LR test for which is $\chi^2(2) = 1.55$ with a p-value of 0.46. The equilibrium indicates that unemployment (proxied by $ns - n$) increases with positive deviations of the real wage from a linear trend, and with negative deviations of output from a linear trend. After mean adjustment this equilibrium has the form:

$$eqm_t = ns_t - n_t - \underset{(0.19)}{0.85}\ (wp_t - \underset{(-)}{0.0048}\,t) + \underset{(0.21)}{1.91}\ (y_t - \underset{(-)}{0.0056}\,t) - \underset{(0.003)}{10.89} \tag{8}$$

The coefficients of the trends are restricted to coincide with the average growth of the variables over the period under analysis, about 2% per year for the real wage and 2.2% for output. Hence, unemployment decreases with the output gap as predicted by Okun's law, and increases when the accumulated growth of the real wage exceeds its average level (which, it is worth noting, coincides with the average growth of productivity ($y - n$), about 2% per year).

From an economic point of view, the equilibrium is coherent with models of imperfectly competitive goods and labour markets, where the wage is set in a bargaining process between unions and firms, and the latter determine the level of employment given the wage, and the price level given the wage and demand, see e.g. Kaufman (1994). This theoretical framework is more strictly followed by Sgherri and Wallis (2000) in the development of a small scale structural model of the UK, to be used for the evaluation of monetary policy rules.

Equation (9) reports the estimated coefficients of the equilibrium for each variable in the VECM – the estimated lag coefficients and constants have been omitted for simplicity. The only coefficient of the equilibrium that appears to be statistically significant, that in the GDP equation, has an appropriate sign. However, the equilibrium indirectly affects wp_t, n_t and ns_t through their dependence on lags of y_t.

$$\begin{aligned} \widehat{\Delta wp_t} &= \underset{(0.04)}{0.05}\ eqm_{t-1}, & \widehat{\Delta ns_t} &= \underset{(0.014)}{0.002}\ eqm_{t-1}, \\ \widehat{\Delta n_t} &= \underset{(0.015)}{0.002}\ eqm_{t-1}, & \widehat{\Delta y_t} &= -\underset{(0.04)}{0.14}\ eqm_{t-1}. \end{aligned} \tag{9}$$

Table 2. Persistence of regimes

	Ergodic Probability	Duration	Observations
Regime 1	0.1231	2.73	17.8
Regime 2	0.6355	16.62	89.0
Regime 3	0.2414	4.82	35.2

4.2. The MS-VECM

The cointegration results from the last section are now used in the second stage of our analysis. We adopt an MSIH-VECM with 3 regimes and 2 lags (MSIH(3)-VECM(2)) model, with shifts in the intercept $\mathbf{v}$ and the error variance $\boldsymbol{\Sigma}$. Two lags is the outcome of an HQ model selection procedure for the lag length p, which is coherent with the result for the VAR in levels, while the number of regimes is fixed a priori, with the aim of capturing periods of low, normal, and high growth. Hence, the resulting model is

$$\Delta\mathbf{x}_t = \mathbf{v}(s_t) + \boldsymbol{\alpha}(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\gamma}(t-1)) + \boldsymbol{\Gamma}\Delta\mathbf{x}_{t-1} + \mathbf{u}_t, \tag{10}$$

where $(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\gamma}(t-1))$ is the mean-adjusted full sample estimate as defined in (8) and $\mathbf{u}_t|s_t \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$.

The estimated parameters of the MS-VECM defined in equation (10) using data from 1965(4) to 2000(1) are presented in Table 4. Note that regime 1 is characterized by contraction in output, employment and labour supply, while all three variables expand in regime 3. Interestingly, regime 2 is associated with jobless growth: output increases at a rate of 2.4% a year while employment stagnates. The matrix of transition probabilities is given by

$$\mathbf{P} = \begin{bmatrix} 0.6333 & 0.0731 & 0.2936 \\ 0.0383 & 0.9398 & 0.0218 \\ 0.0862 & 0.1211 & 0.7927 \end{bmatrix},$$

where $p_{ij} = \Pr(s_{t+1} = j \mid s_{t-1} = i)$. These estimates indicate that regime shifts are persistent, with no regime being permanent. However, there is a 94% probability of staying in the normal growth regime, which has an estimated duration of 4 years: Recessions on the other hand have duration of 8 months and only occur in steady state 12% of the time.

The resulting regime probabilities are given in Figure 2: Regime 1 depicts very clearly the double dip recessions of 1973:2–1974:1 [0.9987]/1974:4–1975:2 [0.9498] and 1979:1–1979:1 [0.9750]/1979:4–1980:4 [0.9846], and the recession in the early nineties, 1990:3–1991:2 [0.8832]. The figures in square brackets are the average regime probabilities for the quoted periods. Regime 1 also captures an outlier in 1971:1 [1.0000]. Whilst regime 3 characterizes high-growth episodes – Regime 2 represents by default 'normal' growth. It is particularly noticeable that the period up to 1985 was characterized by 'boom and bust' policies, whereas that after 1985 was one of normal growth with the exception of the recession of the early 1990s. As in the Hamilton (1989) model of the US business cycle, the regime classification of the MS-VECM is close to traditional business cycle datings. In figure 2 the contraction periods issued by

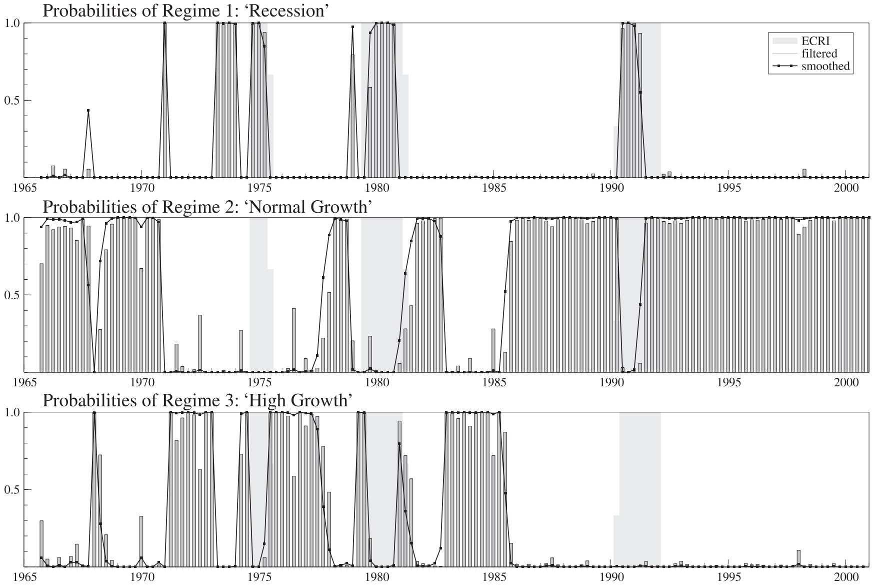

Fig. 2. Regime probabilities of the MS-VECM.

the Economic Cycle Research Institute (ECRI) are superimposed on the smoothed and filtered probabilities of the MS-VECM. The dating of the cycle is very similar to the one produced by the ECRI: using the ex-post (smoothed) regime probabilities, the coverage of UK recessions by regime 1 and expansions by regimes 2 and 3 is 0.9005. The timing of the major recessions is also similar to that obtained following the method suggested by Breunig and Pagan (2001) and Harding and Pagan (2001). We conclude that the MSIH(3)-VECM(2) model is a valid statistical representation of the UK business cycle and that the detection of business cycle turning points is very precise.

Table 3 reports the results of Wald specification tests regarding the significance of the regime shifts in the intercepts of the equations for the individual variables separately, and in the system as a whole, respectively. Under the null hypothesis of $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3$ etc., the unrestricted regime-dependent variances ensure the statistical identification of the model under the null hypothesis. The tests are nuisance parameter free so that classical likelihood theory can be invoked, and the asymptotic null distribution of the Wald test is $\chi^2(q)$ where q is the number of linearly independent restrictions. The regime-shifts in equilibrium means and mean growth rates of the system are statistically highly significant. There is not only strong support for recurring recessions and expansions, but also for the presence of a third regime. Running tests for the equations of the system separately helps to characterize the regimes. In the case of recessions (regime 1) versus normal growth (regime 2), the hypothesis $v_{k1} = v_{k2}$ can be rejected very strongly for Δy_t, Δn_t and Δns_t. Comparing the second and the third regime, there is a significant change in the intercepts of all variables, though this is less strong for those of Δy_t and Δwp_t. Finally, it is worth noting that real wage growth has the least evidence for being affected by regime shifts.

Table 4. ML estimation results for the MS-VECM, 1965(4)–2001(1)

	Δy_t	Δn_t	Δns_t	Δwp_t
Regime-dependent intercepts ($\times 10^2$)				
v_1	−0.888 (0.197)	−0.498 (0.055)	−0.278 (0.079)	0.414 (0.373)
v_2	0.607 (0.089)	−0.016 (0.042)	−0.015 (0.041)	0.690 (0.132)
v_3	0.991 (0.237)	0.271 (0.062)	0.327 (0.061)	0.291 (0.261)
Short-run dynamics				
Δy_{t-1}	−0.213 (0.082)	−0.105 (0.030)	−0.115 (0.032)	−0.176 (0.121)
Δy_{t-2}	0.052 (0.072)	0.036 (0.021)	0.009 (0.024)	−0.032 (0.116)
Δn_{t-1}	0.151 (0.435)	1.072 (0.173)	0.425 (0.175)	0.059 (0.629)
Δn_{t-2}	0.569 (0.383)	−0.271 (0.168)	−0.342 (0.166)	−0.526 (0.558)
Δns_{t-1}	0.391 (0.461)	−0.801 (0.192)	−0.338 (0.189)	0.294 (0.660)
Δns_{t-2}	−0.311 (0.391)	0.326 (0.178)	0.382 (0.171)	0.804 (0.556)
Δwp_{t-1}	0.137 (0.067)	0.138 (0.029)	0.126 (0.028)	−0.131 (0.100)
Δwp_{t-2}	0.060 (0.062)	0.046 (0.023)	0.051 (0.024)	0.097 (0.091)
Equilibrium correction				
eqm_{t-1}	−0.054 (0.024)	0.033 (0.012)	0.030 (0.011)	0.034 (0.034)
Standard errors ($\times 10^2$)				
σ_1	0.6950	0.1599	0.2778	1.2872
σ_2	0.4671	0.3020	0.2712	0.7311
σ_3	1.1963	0.3054	0.2899	1.0982
Regime 1: correlation				
Δy_t	1.0000	−0.5479	−0.2741	0.6186
Δn_t	−0.5479	1.0000	0.7359	−0.7246
Δns_t	−0.2741	0.7359	1.0000	−0.4411
Δwp_t	0.6186	−0.7246	−0.4411	1.0000
Regime 2: correlation				
Δy_t	1.0000	0.3602	0.2334	0.1615
Δn_t	0.3602	1.0000	0.9354	−0.2595
Δns	0.2334	0.9354	1.0000	−0.2420
Δwp_t	0.1615	−0.2595	−0.2420	1.0000
Regime 3: correlation				
Δy_t	1.0000	0.0050	−0.1686	0.4343
Δn_t	0.0050	1.0000	0.8997	0.1752
Δns_t	−0.1686	0.8997	1.0000	0.0332
Δwp_t	0.4343	0.1752	0.0332	1.0000

Fitting	MS-VECM	linear VECM	MS-DVAR	linear DVAR
logLik	2374.9886	2317.9406	2361.6400	2304.3041
AIC	−32.2674	−31.9428	−32.1358	−31.8071
HQ	−31.5569	−31.5199	−31.4591	−31.4180
SC	−30.5189	−30.9020	−30.4705	−30.8496

From Table 4 it is clear that not only are the estimated intercepts $\mathbf{v}(s_t)$ different across regimes, but there are also changes in $\boldsymbol{\Sigma}(s_t)$ – note in particular the changes in $cov(u_{\Delta y}, u_{\Delta wp})$. This suggests that the correlations between the variables, conditional on the past, differ across regimes, and so the use of a constant parameter VECM could lead to severely misleading results. A likelihood ratio test of the linear VECM(2) against the MSIH(3)-VECM(2) strongly rejects the linearity hypothesis ($\text{LR}(28) = 114.10$), even when the upper bound of Davies (1977) is invoked. Further, the AIC and the HQ criterion favour of the non-linear VECM.

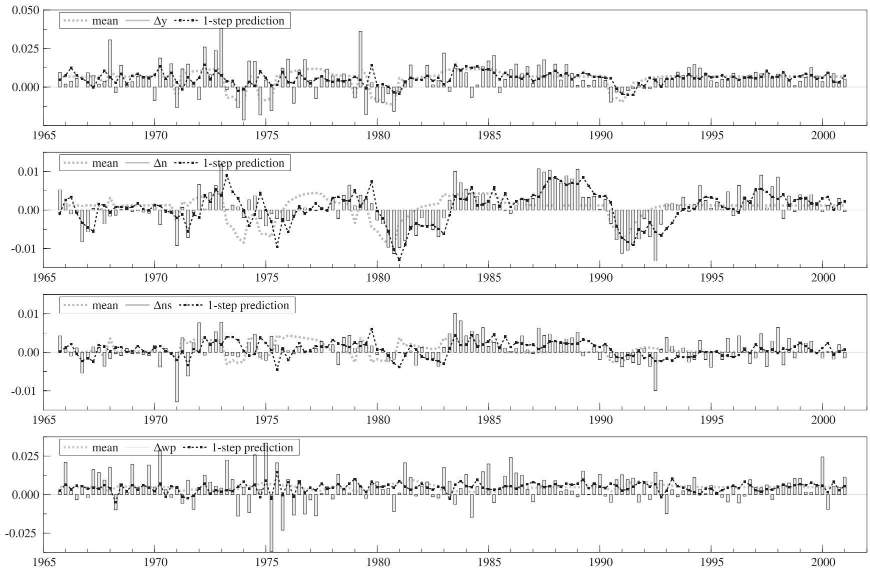

Fig. 3. Fit of the MS-VECM.

We note also that the estimated adjustment coefficients for the MSIH(3)-VECM(2) show that all variables other than real wages respond to the equilibrium, which contrasts with the results for the VECM(3) in (9). The estimated response of labour supply to the equilibrium term though has a perverse sign.

Additional evidence in favour of the MSIH(3)-VECM(2) is provided by the plots of the actual, one-step predictions, and mean values for each variable given in Figure 3 where the fit of the model is seen to be good. Also Figures 4 and 5 show the residuals to be non-autocorrelated, homoscedastic, and normally distributed, in contrast to those of the VAR(4), and so provide further support for the model. Thus the regime inference of our model appears to be based on a congruent econometric model of the inter-relationship between real wages, income, employment and labour supply. As the regime inference is a by-product of the parameter estimation of the MS-VECM, our model overcomes the problems in Acemoglu and Scott (1994) where conditional models of employment dynamics in the UK are considered, but the probability of a recession is derived from a non-congruent Markov-switching model of UK output growth.

5. Impulse-response analysis

In this section we analyze the dynamic properties of the MSIH-VAR underlying the MSIH-VECM by calculating the impulse response functions (IRF). First we compare the results for the MSIH-VAR and a linear VAR. We then show that the differences reported in sub-section 5.1 are essentially replicated when Choleski decompositions for various orderings of the variables are used. Finally, we follow the approach in Krolzig and Toro (1998) to evaluate changes in the MSIH-VAR responses across regimes.

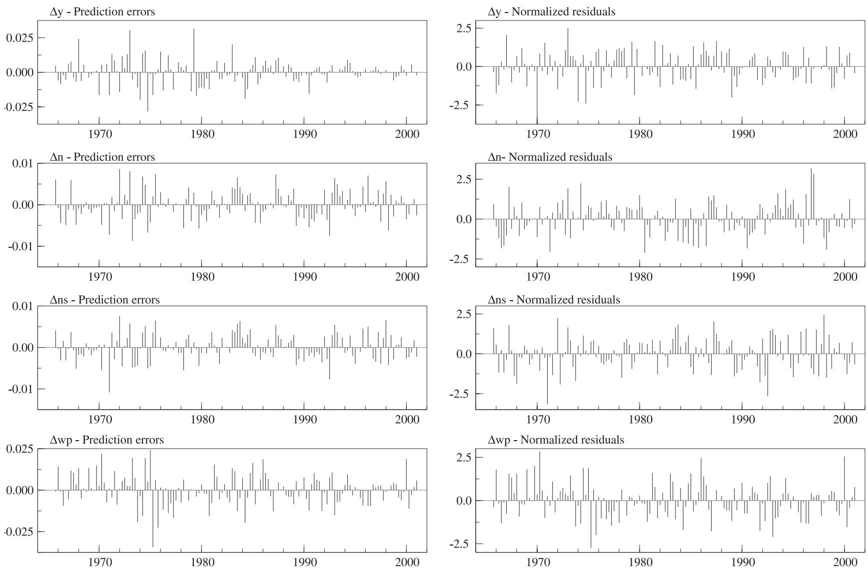

Fig. 4. Residuals of the MS-VECM.

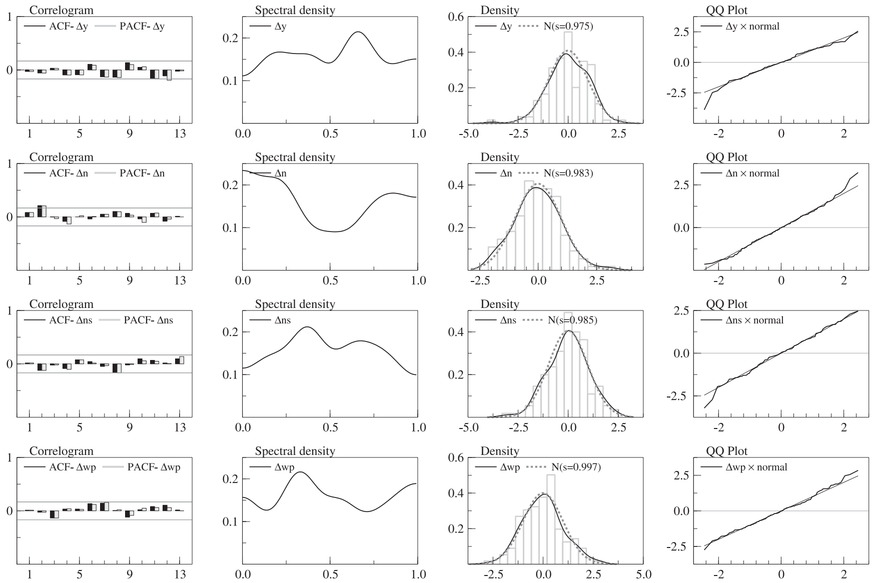

Fig. 5. Statistical properties of the normalized residuals.

5.1. MSIH-VAR and VAR

In Figure 5 we compare the results for the MSIH-VAR and a standard VAR. Cumulative impulse response functions (IRFs) are calculated for unit impulses to the innovations for each of the variables, without orthogonalizing

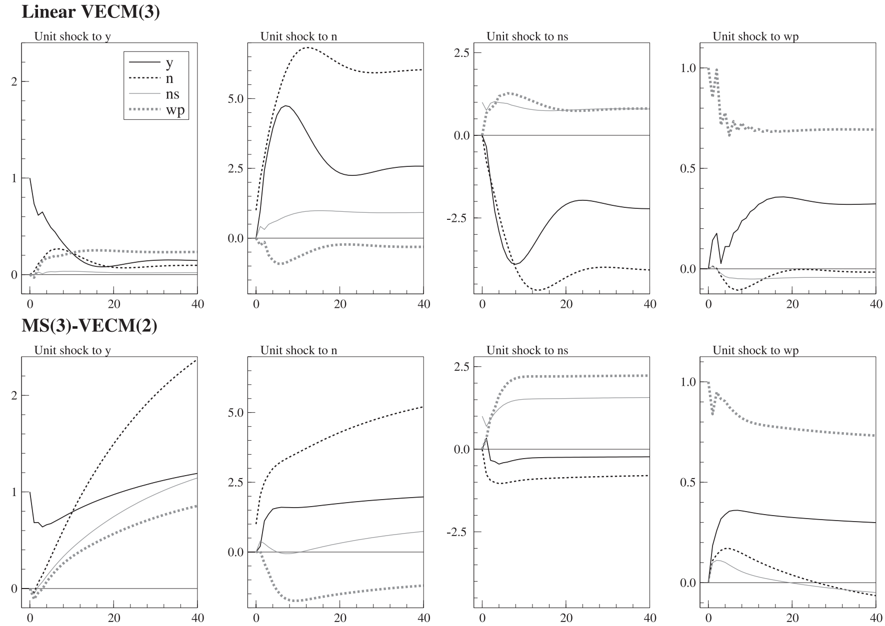

Fig. 6. Impulse Response Functions, MS-VAR and standard VAR.

the variables. We do this both for simplicity and because we want to focus on possible differences in the responses between models and across regimes.

Inspection of Figure 6 reveals that the shape and timing of each variable's response to unit impulses in each of the innovations are very different for the two models. Hence, the MS properties have a noticeable effect on the estimated long run relationships between the variables. We might expect further differences to emerge if orthogonalized innovations were used. However, these results should be interpreted with care because, from the previous section, the covariance matrix of the error term is regime dependent, so that orthogonalized IRFs will differ across regimes. In the next subsection we evaluate whether this is the case.

5.2. Keynesian and classical orthogonalizations

We consider two alternative orderings of the variables: *y*-*n*-*wp*-*ns* and *ns*-*n*-*y*-*wp*. The former can be related to a standard Keynesian model where increased demand leads to increased production, which requires an increase in labour demand. Unemployment falls as only a part of the employment growth is sustained by an increased labour force participation. As nominal wages tend to be more sluggish than prices, the real wage falls initially, over time wages adjust to the increased level of labour productivity. The latter ordering is more in line with a classical model in which changes in the labour supply drive labour input and production, which in turn affect the real wage.

Analyzing orthogonalized IRFs is standard in the case of linear VARs (see, *inter alia*, Hamilton, 1994, §11.4). For the MSIH-VECM proposed in section 4, the presence of regime-dependent heteroscedasticity requires special

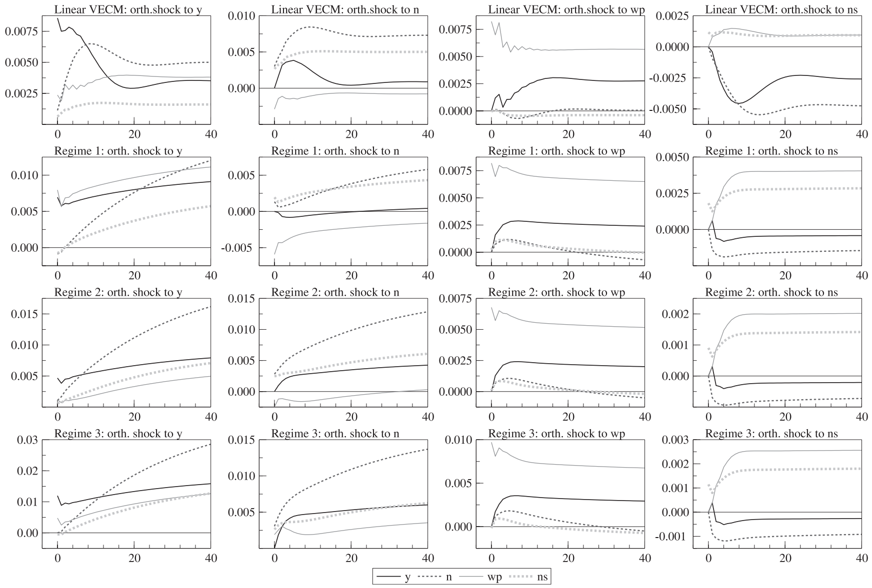

Fig. 7. Impulse Response Functions: Keynesian Orthogonalization.

attention. As the covariance matrix of the error vector is regime dependent, so are the corresponding Choleski decompositions and the associated orthogonalized impulses. Thus for each of the two orderings of the variables, we get three IRFs describing the response of the variables dependent on the state of the system when the shock occurs.

Starting with the IRFs for the Keynesian ordering given in Figure 7, the main difference across regimes to a shock to the y equation innovation appears to be in the response of wp. The reactions to a shock in wp are rather limited and similar in all three regimes. Note that there appears to be a positive long run effect on productivity $(y - n)$ in the linear VAR, but less so in the MSIH-VECM. A shock to ns has similar effects across the three regimes, but is less marked than in the linear VAR.

Turning to IRFs for the classical ordering given in Figure 8, the response to a shock in y are similar to the former case, but with fewer changes across regimes. The effects of an impulse to wp are also very similar across the three regimes. However, there are very marked differences in the responses to shocks in ns and n, and each set is very different from the corresponding Keynesian set.

In summary, the most relevant result for our purposes is the finding of substantial differences in the IRFs that emerge across regimes and relative to the linear VAR, independently of the chosen orthogonalization. In the next subsection we explore an alternative approach to analyze the source and propagation of shocks.

5.3. IRF of regime shifts

Standard IRF analysis focuses on the response of the system to Gaussian innovations, but other shocks could affect the system. In particular, changes in

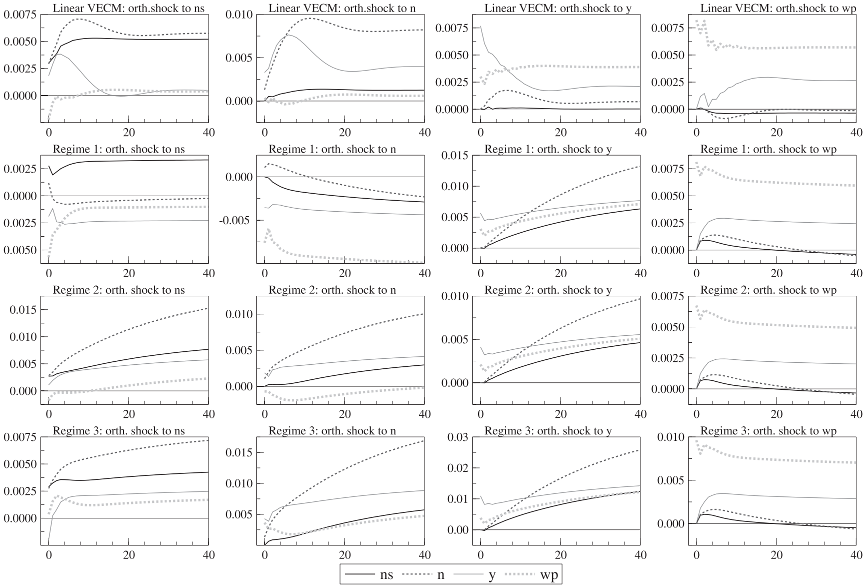

Fig. 8. Impulse Response Functions: Classical Orthogonalization.

the phase of the cycle is probably what some economists have in mind when they refer to 'cyclical shocks', for example, investigating the dynamics of some variables in the transition from boom to bust, or vice versa. Within the MSIH-VAR framework we can deal with both types of shock.

The impulse responses with respect to transitions of the state variables depend on the properties of the VAR, combined with those of the hidden Markov chain. In fact, these effects are due to: (i) changes in the current state and hence changes to the conditional expectation of a future regime; and (ii) the autoregressive transmission of intercept shifts. A formal mathematical derivation of these responses is presented in the appendix of Krolzig and Toro (1998).

In Figure 9 we characterize the behaviour of the variables for the system for each of the three regimes when compared to the ergodic regime probability distribution, and present the responses to changes in the phase of the cycle. A first characteristic that emerges from the IRFs on the diagonal of Figure 9 is the difference in the responses of *n* and *ns*. Indeed, there is increasing unemployment in the recession (regime 1), and decreasing unemployment in the high growth regime. Further all variables react negatively in a recession, but they all respond positively in an expansion. These asymmetries in behaviour can be given a theoretical rationale – see Krolzig and Toro (1998) for an overview of some theoretical models.

Consider now the path taken by the variables when there is a change in regime. We observe that moving from a recession to a period of normal or of high growth leads to all variables reacting positively and in a permanent manner to the shocks, though real wages are least affected. Also, the transitions from a high growth period to a normal period or to a recession are similar. Further, the transitions from normal growth to both recession and to

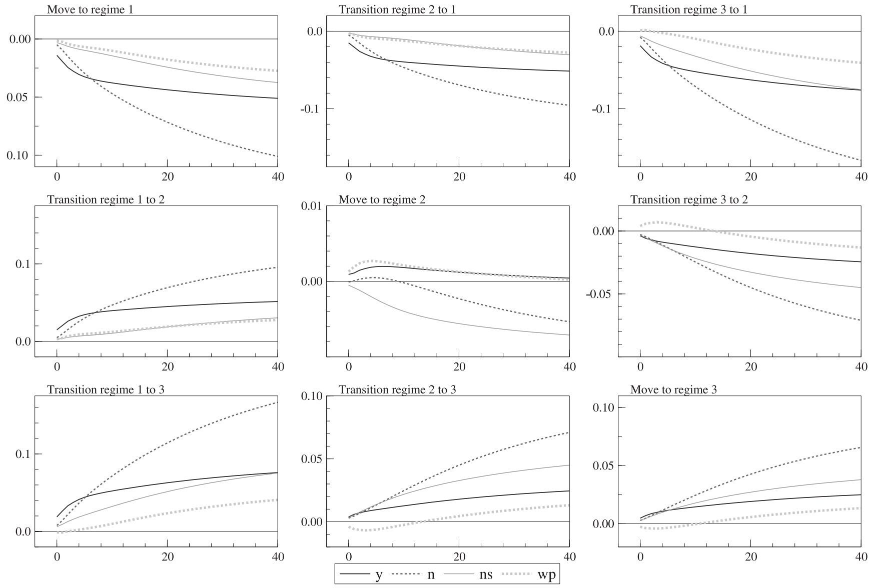

Fig. 9. The Response of the System after Shifts in Regime.

high growth are very different. Finally, transitions from normal growth to high growth is the mirror image (but with negative coefficients) of the responses to a change from high to normal growth. This results from the fact that in our model the mean and variance of the process are state-dependent, but the autoregressive parameters are constant across regimes.

6. Forecasting performance

From the above results the MS-VECM appears to provide a good representation for the in-sample behaviour of all four variables wp_t, y_t, ns_t and n_t, and a significant improvement with respect to standard linear specifications – see the statistics on model fit in Table 4. While regime-switching models tend to be superior to linear models in capturing certain features of the business cycle, their superiority from a forecasting perspective is often less convincing (see, *inter alia*, Clements and Krolzig, 1998). We now evaluate whether the MS-VECM of the UK labour market performs as well out in forecasting.

Krolzig (2000) developed a general approach to predict multiple time series subject to Markovian shifts in the regime. Consider the MS(M)-VECM($p-1$) model

$$\Delta \mathbf{x}_t = \mathbf{M}\boldsymbol{\xi}_t + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\alpha}\boldsymbol{\gamma}(t-1) + \sum_{k=1}^{p-1} \boldsymbol{\Gamma}_k \Delta \mathbf{x}_{t-k} + \mathbf{u}_t,$$

where $\mathbf{M} = [\mathbf{v}_1 : \cdots : \mathbf{v}_M]$ and $\boldsymbol{\xi}_t$ is the M-dimensional state vector consisting of indicator variables $I(s_t = i) = 1$ for $s_t = i$ and 0 otherwise. The one-step predictor can be derived from the corresponding MSIH(M)–VAR(p) representation given by

$$\mathbf{x}_t = \mathbf{M}\boldsymbol{\xi}_t - \boldsymbol{\alpha\gamma}(t-1) + \sum_{k=1}^{p} \mathbf{A}_k \mathbf{x}_{t-k} + \mathbf{u}_t, \tag{11}$$

where $\mathbf{A}_1 = \mathsf{I}_\mathsf{N} + \boldsymbol{\alpha\beta}' + \boldsymbol{\Gamma}_1$ and $\mathbf{A}_j = \boldsymbol{\Gamma}_j - \boldsymbol{\Gamma}_{j-1}$ for $1 < j \leq p$ with $\boldsymbol{\Gamma}_p = \mathbf{0}_N$. Hence we have that

$$\mathsf{E}[\mathbf{x}_{t+1} \mid \mathbf{x}_t, \ldots, \mathbf{x}_0] = \mathbf{MP}\hat{\boldsymbol{\xi}}_{t|t} - \boldsymbol{\alpha\gamma} t + \sum_{k=1}^{p} \mathbf{A}_k \mathbf{x}_{t+1-k}, \tag{12}$$

where $\mathbf{P}$ is the transposed matrix of transition probabilities and $\hat{\boldsymbol{\xi}}_{t|t}$ is the vector of filtered regime probabilities at time t. The predictor for the MSIH(M)-DVAR($p-1$) model follows from (12) by setting $\boldsymbol{\alpha} = \mathbf{0}$.

We compare 5 models: an MS-VECM and a linear VECM, plus an MS-DVAR and a standard VAR in first differences (DVAR), and finally a VAR in the second differences of variables (DDVAR), which is a forecasting device that exploits the fact that very few variables accelerate/decelerate indefinitely. In fact, DVARs and DDVARs can provide good forecasts during periods of structural changes, in particular when there are shifts in equilibrium means and mean growth rates – see Clements and Hendry (1999) for a general exposition and Marcellino and Mizon (2001b) for an example relative to the Italian labour market.

The models are estimated using data for the periods 1965(4)–1991(1) and 1965(4)–1998(1), and produce forecasts for the periods 1991(2)–2001(1) and 1998(2)–2001(1) respectively. The models are compared on the basis of the root mean square prediction errors (RMSPE) and the mean absolute prediction error (MAPE) for each of the 12 and 40 quarters forecast horizons. This choice leaves a sufficiently long estimation period to guarantee structural stability of the MS-VECM as a business cycle model, reasonable long-run properties, and strong convergence of the EM estimation procedure.

Table 5 reports the MAPE and RMSPE of the one-step prediction errors in forecasting y_t, n_t, n_t, and wp_t for each of the five models. From Table 5 it is evident that the MSIH(3)-VECM(2) performs well for both forecast periods. Over the 1998(2)–2001(1) period, the MS-VECM performs extremely well for all four variables. It consistently dominates the linear VECM. The outcome of the DDVAR is overall disappointing.

Hence we conclude that the MSIH-VECM provides a good characterization of the four variables modelled in a period of much change, can forecast well, and has an appealing economic interpretation.

7. Conclusions

In this paper we have estimated an MSIH-VECM for a small set of UK labour market variables, and found that it provides a good characterization of the sample data over the period 1965(4)–2001(1) despite there having been many changes affecting this sector of the UK economy. In addition, the switches between the three regimes are closely related to changes in the phases of the UK business cycle. Having found evidence for the existence of three

Table 5. One-step prediction errors

1998(2)–2001(1)	MAPE ($\times 10^2$)				RMSPE ($\times 10^2$)			
Model	*y*	*n*	*ns*	*wp*	*y*	*n*	*ns*	*wp*
MSI(3)-VECM(2)	0.2686*	0.2179*	0.2063*	0.5639*	0.3392*	0.2774*	0.2395*	0.7633*
MSI(3)-DVAR(2)	0.4245	0.2508	0.2480	0.6448	0.4909	0.3094	0.2891	0.8437
Linear VECM(3)	0.3042	0.2482	0.2512	0.5851	0.3685	0.3024	0.2777	0.7855
Linear DVAR(3)	0.2985	0.2486	0.2513	0.6026	0.3641	0.3029	0.2778	0.8011
Linear DDVAR(2)	0.3897	0.2648	0.2589	0.6536	0.4762	0.3384	0.3040	0.8756

1991(2)–2001(1)	MAPE ($\times 10^2$)				RMSPE ($\times 10^2$)			
Model	*y*	*n*	*ns*	*wp*	*y*	*n*	*ns*	*wp*
MSI(3)-VECM(2)	0.3889	0.2692*	0.3056	0.6062	0.5049	0.3343*	0.3676	0.8088
MSI(3)-DVAR(2)	0.5020	0.2944	0.3116	0.5636*	0.6183	0.3673	0.3804	0.7304*
Linear VECM(3)	0.4394	0.2835	0.2940	0.6487	0.5191	0.3463	0.3361	0.8241
Linear DVAR(3)	0.3236*	0.2806	0.2907*	0.6604	0.4182*	0.3450	0.3328*	0.8367
Linear DDVAR(2)	0.4078	0.3091	0.2919	0.7689	0.4791	0.3767	0.3460	0.9945

Note: The model with the lowest prediction error is indicated by*.

separate regimes the results in this paper also highlight the dangers of using impulse response analysis for VARs when the constancy of the estimated coefficients has not been established. This supports the arguments in Ericsson, Hendry and Mizon (1998) and Hendry and Mizon (2000) that great care is needed in using and interpreting impulse response functions. Reassuringly, the MSIH-VECM forecasts better than the linear VECM and models in first and second differences of variables, thus indicating the importance of representing some of the changes. Indeed, our analysis of a small model of the UK labour market illustrates that MS-VECMs do not only offer valuable economic insights having economically interpretable equilibria and regimes shifts that are closely related to changes in the phases of the business cycle, they are also useful statistical forecasting devices.

Data appendix

The data are quarterly seasonally adjusted for the period 1965(1) to 2001(1). From the basic series described in Table A1 the following variables were

Table A1. Definitions of variables

			Source (Mnemonic)
WS	wages and salaries	£m at current prices, SA	E Trends (ROYJ)
EE	number of employees in employment	000s, SA	E Trends (BCAJ)
ET	size of total workforce in industry	000s, SA	E Trends (DYDC)
Y	Gross Value Added at basic prices	index, 1995 prices, SA	E Trends (CGCE)
P	Implicit deflator of GVA	SA	E Trends (CGBV)
U	Unemployment benefit claimants	% of total labour force, SA	E Trends (BCJE)

constructed: log earnings $e = \log(WS/EE)$; log producer price $p = \log(P)$; log real wages $wp = (e - p)$; log labour force $ns = \log(ET/(1 - 0.01 * U))$.

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