

Potential output and inflation dynamics after the Great Recession

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Abstract Ever since the end of the Great Recession, the US economy has experienced a period of mild inflation, which contradicts with the output–inflation relationship depicted by a traditional Phillips curve. This paper examines how the permanent output loss during the Great Recession has affected the ability of the Phillips curve to explain US inflation dynamics. We find great similarity among several established trend–cycle decomposition methods: potential output declined substantially after the Great Recession. Due to the fact that a lower level of potential output implies a lesser deflationary pressure, we then show that the Phillips curve does predict a period of mild inflation. This finding is largely consistent with the observed data.

Keywords Trend–cycle decomposition · Phillips curve · Potential output · Inflation dynamics

JEL Classification C22 · E31 · E32

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1 Introduction

Other than the fact that it is the greatest economic downturn in post-WWII US history, the "Great Recession" associated with the onset of the sub-prime crisis in 2008 receives particular attention for the puzzling relationship between weak economic activities and moderate inflation afterward. In particular, Ball and Mazumder (2011) estimate a standard Phillips curve with the Congressional Budget Office's (CBO) measures of output gaps and find that the predicted deflation was absent during the period from 2008 to 2010. This "missing deflation" puzzle has prompted an array of research on the changing dynamics of US inflation.

Given a steady level of inflation expectation, the Phillips curve describes a positive relationship between inflation and output gap, usually measured by the deviation of real output from its potential, or trend, level. However, output gap is not directly observable and can be measured using different estimation methods, which show great variations in their results. Figure 1 shows two widely used estimates of output gap, the CBO output gap and the Hodrick–Prescott (HP) filtered gap. The differences between them are striking: the HP-filtered estimates indicate a substantial degree of potential output loss during and after the Great Recession, while the CBO estimates imply the opposite. The notion that recessions can have permanent effects is not novel (e.g., Hamilton 1989; Kim et al. 2005); thus the possibly substantial declines in the post-2008 era.

In this paper, our aim is to investigate the extent to which the missing deflation puzzle can be explained by the potential output loss during the Great Recession. Our empirical strategy is comprised of two steps. First, we will construct different potential output measures by several established methods. Second, we will study the inflation dynamics during the Great Recession through the lens of both backward- and forwardlooking Phillips curves.

In order to measure the potential output variations, we consider a wide range of established and commonly used filtering methods, including the HP filter, unobservedcomponent models, the Beveridge–Nelson decomposition and neoclassical growth



Fig. 1 The measures of the output gap and the potential output (2005:1–2013:3). These estimates were obtained using the sample from 1947:1

models. Except for the CBO estimates, all measures indicate that a substantial portion of the declines in the US real GDP after 2008 can be considered as permanent. That is, it is likely that the US economy suffered long-term damage to its potential output after the Great Recession (Bijapur 2012; Furceri and Mourougane 2012; Benati 2012; Reifschneider et al. 2013; Ball 2014; Fernald 2014). Our results also support the view that the USA has experienced a one-time, permanent shock to wealth (Bullard 2012).

Can the estimated potential output variations explain the missing deflation? Following Ball and Mazumder (2011), we conduct a dynamic simulation of inflation after 2008 with different output gap measures. We find that the traditional backward-looking Phillips curve with any of the measures considered except the CBO's predicts near-zero or positive inflation. In order to account for the uncertainty associated with different measures of the output gap, we construct a weighted-averaged dynamic simulation based on the Bayesian information criterion. As a result, both the observed moderate inflation and the severe economic downturn after 2008 can be reconciled to a great extent within a backward-looking Phillips curve, if the potential output variations are considered.

Several studies have pointed out that these puzzling inflation dynamics can be explained by well-anchored inflation expectation (Ball and Mazumder 2011; Matheson and Stavrev 2013) or by higher inflation expectation due to oil price (Coibion and Gorodnichenko 2015). Accordingly, we extend our model by incorporating the forward-looking inflation expectation based on the Survey of Professional Forecasters (SPF) and also considering the core CPI as an alternative inflation measure to remove the effect of oil price fluctuations. Our empirical results are consistent with existing findings: in our extended exercise, the inflation prediction improves substantially with the CBO's output gap. However, it still sits below the realized path. We then show that the model-averaged simulation accounting for possible lowered potential output does, in fact, capture the inflation dynamics more precisely, suggesting that the potential output variations play an important role in inflation dynamics after the Great Recession.

The outline for the remainder of the paper is as follows. In Sect. 2, we investigate the potential output variations using several trend–cycle decomposition methods. In Sect. 3, we re-assess the missing deflation puzzle using various measures of the potential output. Section 4 is our conclusion.

2 Potential output variations: a retrospective study

2.1 Methods for obtaining the potential output

In this section, we investigate how the potential output evolves, based on the following trend–cycle decomposition methods:

- The Hodrick–Prescott (HP) filter. The smoothing parameter is set to be 1600 in accordance with quarterly data.
- The unobserved-component (UC) models.
- The Beveridge–Nelson (BN) decomposition.
- The neoclassical growth models, including the one used by CBO.

The HP filter is a convenient mechanical detrending device applied for macroeconomic data, and the other methods hinge on some structural assumptions discussed in what follows.

2.1.1 UC model

The UC model has been a popular detrending method since it was developed in 1980s (Harvey 1985; Watson 1986; Clark 1987, etc). In general, the results obtained from the UC models vary with the assumptions for the econometric modeling of the trend, as well as with those for the cyclical components. In this paper, we adopt a version of the UC model used by Luo and Startz (2014), which nests a wide range of popular assumptions:

$$y_t = \tau_t + c_t \tag{1}$$

$$\tau_t = \mu_t + \tau_{t-1} + \eta_t \tag{2}$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \epsilon_t \tag{3}$$

In the above model, y_t is the logarithm of the quarterly real GDP, which consists of a trend component, τ_t , and a cyclical component, c_t . τ_t follows a random walk with a drift μ_t , which is interpreted as the mean growth rate of the real output. The mean growth rate is assumed to be constant except for a possible one-time structural change.¹ We assume an unknown break date to be estimated with other model parameters. The cyclical component, c_t , is assumed to follow a stationary AR(2) process. The two shocks, η_t and ϵ_t , are usually assumed to be jointly normally distributed. We denote this model simply as UC.

In addition, we extend the distributional assumption by specifying skew-normal distributions for the trend and cycle shocks. The skew-normal distribution has an extra shape parameter compared to the normal distribution. It collapses to normal distribution if the shape parameter is estimated to be zero.² We call this UC model with asymmetric shocks the UCSN model. Both models are estimated using Bayesian methods, as detailed in "Appendix A."³

2.1.2 BN decomposition

The BN (Beveridge and Nelson 1981) decomposition is another useful and general approach for trend–cycle decomposition. Given a reduced form ARIMA model, the

¹ Luo and Startz (2014) show that two breaks in the mean growth rate are not as likely as the one break assumption with quarterly real GDP data ended at 2013:3, and the detrending results are fairly similar to both assumptions if uncertainty in break dates is incorporated.

² A random variable *x* follows a multivariate skew-normal distribution, $SN(\overline{\Omega}, \alpha)$, with density given by $f_{SN}(x; \overline{\Omega}, \alpha) = 2\phi(x; \overline{\Omega})\Phi(\alpha' x)$, where $\phi(x; \overline{\Omega})$ is the pdf of the multivariate zero mean $N(0, \Omega)$ and $\Phi(.)$ is the cdf of the univariate N(0, 1) distribution.

³ Methodology and priors are mostly consistent with Luo and Startz (2014). Results are based on 20,000 effective samples after saving one of every 10 draws in 300,000 MCMC simulation and discarding the first 100,000 as burn-in samples.

BN trend is constructed according to the long-horizon conditional forecast of the time series minus any deterministic drift. Following Morley et al. (2003), we specify an ARIMA model for the quarterly growth rate of real GDP (Δy_t) as follows:

$$\Phi(L)(\Delta y_t - \mu) = \Psi(L)e_t,$$

where $\Phi(L)$ and $\Psi(L)$ denote lag polynomials with all roots outside the unit circle. The order of the lag polynomials is set to be 2. To account for the structural breaks in the mean growth rate, we allow μ to break at 1973:1 and 2006:1 (Perron and Wada 2009; Luo and Startz 2014). The model is cast into a state space form and estimated using the maximum likelihood estimation, and the BN trend component can be constructed based on the parameter estimates.

2.1.3 Neoclassical growth model

As for the neoclassical growth model, we first adopt the estimates provided by the CBO, due to its popularity in the empirical literature. In addition, we estimate the potential output based on CBO's method, but use the short-run unemployment rate instead of the total unemployment rate, to capture cyclicality of any variable.⁴ We denote the latter as CBO-SR. Use of the short-run unemployment rate is motivated by dramatic changes in the composition of the total unemployed since the onset of the Great Recession: the long-term unemployment rate had been stable at approximately 1% for six decades but skyrocketed to 4% in 2010.⁵ As the long-term unemployment can possibly be transformed into structural unemployment (Weidner and Williams 2011; Daly et al. 2012), the short-term unemployment rate may be a more representative indicator of business cycles.⁶

2.1.4 Brief assessment of the different methods

Potential output conceptually exists but is unobserved. Hence a degree of subjective judgment is usually needed as to what is a legitimate detrending method.

The methods we include are notably different in terms of prior views and assumptions. The HP filter is easy to apply but lacks consistency with some economic notions, in that it assumes the output gap is simply white noise. Also, the choice of the smoothing parameter is hard to justify in a transparent way. The BN approach allows the trend component to be nonlinear. However, without parameter instability, it usually implies small and noisy cycle components (Morley et al. 2003) that are inconsistent with eco-

⁴ The CBO's method uses the unemployment gap—the difference between the total unemployment rate and the NAIRU—as the key variable for obtaining the cyclical components of any variable, as the unemployment gap usually moves up and down closely with the business cycle. See "Appendix B" for details.

 $^{^{5}}$ The long-term unemployment is defined by the number of civilians unemployed for 27 weeks and longer.

⁶ Gordon (2013) also argues that the distinction between short-run and long-run unemployment may help to solve the "case of the missing deflation" after 2008 if the downward pressure on wages and the inflation rate comes mainly from the short-run unemployment.



Fig. 2 The measures of the output gap. Estimates were obtained using the US quarterly real GDP from 1947:3 to 2013:3

nomic intuitions.⁷ By contrast, the UC approaches and production function methods can be flexible in adapting a wide range of econometric assumptions and economic relationships. Nonetheless, detrending results are somewhat sensitive to specifications.

Furthermore, revisions between real-time and ex-post estimates affects results differently, depending on which of the various methods is used. It appears that the two-sided filters (HP and UC) give a higher weight to the end of sample observations, which are likely to be significantly revised (Cotis et al. 2004). Therefore, their estimates are likely to experience higher revisions than those produced by production function approaches at the last points of the estimation.

We would like to emphasize that we choose the set of methods we use because they are representative of the methods widely used in various literature. Cotis et al. (2004) summarizes properties of a wide range of detrending devices based on what they called "core requirements" or "user specific requirements," corresponding to criteria that are universally in consideration or only relevant when estimates of potential outputs and output gaps are used for specific purposes. We refer interested readers to their paper for an extensive discussion.

2.2 Comparing different potential output estimates

Figure 2 shows the six measures of the output gap considered in this paper for the financial crisis of 2008.⁸ For the Great Recession, the CBO and the CBO-SR exhibit the first and second deepest troughs. In other words, they suggest the least potential output loss during this period. The CBO estimate still exhibits a large negative gap until the end of our samples, four years after the Great Recession ended, about 4% below trend. However, all the other measures suggest that the output gap has been closed despite slightly different historical estimates.

 $[\]overline{7}$ It is not the case in our estimation as we consider structural breaks in trend growth rates.

⁸ Data are publicly available from FRED. Parameter estimates of all models are available by request.

The retrospective analysis points out that the potential output may have declined after the Great Recession. That is, a substantial proportion of the output loss can be considered as permanent or long term. Nonetheless, uncertainty remains as to which measure fits the data best. In the next section, we will account for this issue through the model averaging. Existing literature has suggested that the prolonged recession and subsequent slow recovery could have damaged the long-term capacity of the economy, as well as lowered the potential output through, e.g., less investment (Reifschneider et al. 2013) and R&D activities (Ouyang 2011 and Haltmaier 2012). Bullard (2012) argues that the economy was disrupted by a permanent, one-time shock to wealth, which lowered consumption and output. In fact, the long-term damages to the potential output of the US economy after the Great Recession have been found in many studies, e.g., Bijapur (2012), Furceri and Mourougane (2012), Benati (2012), Reifschneider et al. (2013), Ball (2014) and Fernald (2014). Furthermore, the lack of a period of temporary high-speed growth following the end of a recession (Ng and Wright 2013) also implies there could be some extent of long-term declines in the real output, at least to some extent.

3 Inflation dynamics after the great recession

The financial crisis, which wiped out people's wealth, forced households to reduce consumption and weakened corporations' abilities to invest. This can account for a decrease in aggregate demand and also create deflationary pressure. On the other hand, the deflationary pressure will be mitigated if there are long-term damages to the potential output, which suppress the output gap and possibly lead to mild inflation, as observed after the Great Recession. The analysis in the previous section indicates that a great proportion of the real output fluctuations after the Great Recession is likely attributed to the potential output movements. In this section, we investigate its implication on US inflation dynamics in the post-crisis era, through the lens of the Phillips curve.

3.1 The missing deflation and permanent output loss

Ball and Mazumder (2011) examine US inflation dynamics, with particular emphasis on the Great Recession, using an accelerationist Phillips curve as follows:

$$\pi_t = \pi_t^e + \alpha y_t + e_t, \tag{4}$$

where π_t is the annualized quarterly inflation of CPI, π_t^e is the expected inflation, and y_t is the measure of economic slack, which is measured by the output gap in this paper. We start with following Ball and Mazumder (2011) and assuming purely backward-looking expectations, i.e., $\pi^e = 0.25 \sum_{j=1}^{4} \pi_{t-j}$. Ball and Mazumder (2011)'s exercise, with the most recent data set, is as follows: we estimate Phillips curves (4) using the pre-crisis sample from 1981:3 to 2007:4 by ordinary least square



Fig. 3 The dynamic simulation of the CPI, based on the CBO output gap

and conduct dynamic simulations for the 2008–2013 period.⁹ The output gap to capture inflationary pressure is measured by the CBO output gap as in their paper.

Figure 3 illustrates the updated missing deflation puzzle. The dashed line represents the predicted inflation, which falls to a negative level according to the Phillips curve. However, the actual inflation rate has been consistently positive since the recession ended. Current explanations for the missing deflation have been focusing on a higher expectation of inflation (e.g., Coibion and Gorodnichenko 2015), instability of the slope coefficient (e.g., Matheson and Stavrev 2013; Murphy 2014) and a well-anchored inflation expectation (e.g., Ball and Mazumder 2011).

We provide an alternative point of view as follows: instead of being a "problem" that needs to be solved, the missing deflation may, in point of fact, contain valuable information about the potential output in recent years. As shown in Fig. 3, the negative CBO output gap has persisted long after the end of the Great Recession. As the results, the Phillips curve predicts a continuously declining inflation. Since the output gap may not be as large as what the CBO reports, given the evidence provided in Sect. 2, this "missing deflation" may reflect an under-estimated permanent effect of the Great Recession.

To support our point of view, we re-estimate model (4) above with all output gap measures reported in Sect. 2. To reduce the effect of the supply side, we also apply our analysis to core CPI. The empirical results are presented in Table 1. All estimates based on different measures of the output gap are reasonable; however, their implications for the post-2008 inflation are heterogeneous. Figure 4 shows that, apart from the CBO gap which predicts deflation in the post-crisis period, the other five output gaps suggest a mild inflation rate, ranging from 0 to 4%, at the end of 2013.

To account for the uncertainty associated with the measures of the output gap, we take an average of the dynamic simulations weighted across different output gaps as the following:

⁹ There are two reasons for using data from 1981: first, we will incorporate the SPF inflation forecast, which is only available since 1981, as a proxy for forward-looking expectation. Secondly, the inflation dynamics in this sample period has been relatively stable than that in 1970s, so parameter instability is not a major concern. The empirical results are similar using sample starting from 1985.

Output gap	СВО	HP	UC	UCSN	BN	CBO-SR
Inflation rate	based on CPI					
α	0.2204	0.4082	0.2974	0.2208	0.2383	0.1929
	(0.0824)	(0.1407)	(0.1133)	(0.1112)	(0.0840)	(0.0753)
LLK	-201.8007	-201.2341	-202.2751	-204.4210	-201.4158	-201.8902
Weight	0.1712	0.3017	0.1065	0.0125	0.2516	0.1565
Inflation rate	based on core (CPI				
α	0.2240	0.4238	0.2920	0.2095	0.2197	0.1912
	(0.0810)	(0.1405)	(0.1161)	(0.1131)	(0.0897)	(0.0742)
LLK	-146.3124	-143.8000	-148.6730	-154.6417	-148.2838	-147.3767
Weight	0.0719	0.8865	0.0068	0.0000	0.0100	0.0248

Table 1 Estimated slope of the pure backward-looking Phillips curve

Model: $\pi_t = 0.25 \sum_{i=1}^{4} \pi_{t-i} + \alpha y_t + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$. The Newey–West robust standard errors are in parentheses. LLK stands for log likelihood. The estimation sample is from 1981:3 to 2007:4



Fig. 4 The dynamic simulations on inflation with the backward-looking Phillips curve based on estimates reported in Table 1

$$\tau_t = \sum_{j=1}^6 w_j \tau_{j,t}, \quad \text{for} \quad t \ge 2008.$$

The weight, w_i , is constructed according to

$$w_j = \frac{\exp(BIC_j)}{\sum_{j=1}^6 \exp(BIC_j)},$$

where BIC is the Bayesian information criteria (BIC) defined as $LLK - 0.5k \ln(T)$, LLK is the log likelihood, k is the number of regressors, and T is the sample size.¹⁰

¹⁰ The model averaging technique we adopt has been used in many studies. See, for example, Garratt et al. (2008) and Morley and Piger (2012). One can also follow Garratt et al. (2014) for a more



Fig. 5 The model-averaged dynamic simulations on inflation with the backward-looking Phillips curve

Table 1 reports the weight for each model. The highest weight, 0.3017, is assigned to the HP gap for the CPI inflation, but the weights for other gap measures are not negligible. The CBO gap is assigned a weight of 0.1712, considerable but not dominating. As a result, the weighted average of the simulated inflation shown in Fig. 5 shifts toward the positive range. The model-averaged simulation indicates moderate and positive inflation for most of the simulation period, which is in sharp contrast to the simulation based on the CBO output gap. Results and point estimates in the Phillips curve are similar for the core CPI inflation. The model-averaged simulated inflation is brought upward to be mostly around zero, rather than being negative. Over 88% of the model weight, as reported in Table 1, is given to the HP gap, while the other gaps play relatively minor roles.

3.2 Incorporating forward-looking inflation expectation

Despite the remarkable improvement in the Phillips curve prediction through considering permanent output loss, the performance of the pure backward-looking Phillip curve (4) is not fully satisfactory: it predicts near-zero inflation, which is not consistent with the mild inflation after 2011. One possible reason is that the simple backward-looking inflation expectation does not capture the expectations of producers well enough. We now examine the Phillips curve incorporating both forward- and backward-looking inflation expectation as follows.

$$\pi_t = \theta \pi_t^F + (1 - \theta) \pi_t^L + \alpha y_t + e_t,$$
(5)

Footnote 10 continued

sophisticated averaging technique where weights are based on the ability of each specification to provide accurate probabilistic forecasts of inflation.

СВО	HP	UC	UCSN	BN	CBO-SR
ased on CPI					
0.8881	0.8695	0.9577	0.9159	0.8818	0.9133
(0.1057)	(0.1075)	(0.0993)	(0.1289)	(0.1028)	(0.1085)
0.2106	0.3710	0.3423	0.2347	0.2244	0.1981
(0.0419)	(0.1048)	(0.0569)	(0.0987)	(0.0463)	(0.0366)
-186.0679	-186.1239	-183.8150	-188.4587	-185.8229	-185.1147
0.0648	0.0613	0.6169	0.0059	0.0828	0.1682
ased on core Cl	PI				
0.6624	0.6426	0.6976	0.7735	0.6912	0.6784
(0.1534)	(0.1359)	(0.1404)	(0.1329)	(0.1402)	(0.1513)
0.1148	0.2654	0.1581	0.1110	0.1056	0.0912
(0.0625)	(0.1051)	(0.0762)	(0.0739)	(0.0644)	(0.0575)
-133.8145	-130.0877	-133.5717	-135.4023	-134.6447	-134.6173
0.0223	0.9251	0.0284	0.0046	0.0097	0.0100
	CBO ased on CPI 0.8881 (0.1057) 0.2106 (0.0419) -186.0679 0.0648 ased on core CP 0.6624 (0.1534) 0.1148 (0.0625) -133.8145 0.0223	CBO HP ased on CPI 0.8695 0.1057) 0.1075) 0.2106 0.3710 0.0419) 0.1048) -186.0679 -186.1239 0.0648 0.0613 ased on core CFF 0.6624 0.1534) 0.1359) 0.1148 0.2654 (0.0625) (0.1051) -133.8145 -130.0877 0.0223 0.9251	CBO HP UC ased on CPI 0.8695 0.9577 0.8881 0.8695 0.0993) 0.1057) (0.1075) 0.3423 0.0419 0.1048) 0.3659 -186.0679 -186.1239 -183.8150 0.0648 0.0613 0.6169 ased on core CVI 0.13591 0.1404) 0.1534) 0.13591 0.1404) 0.1148 0.2654 0.1581 (0.0625) (0.1051) (0.0762) -133.8145 -130.0877 -133.5717 0.0223 0.9251 0.0284	CBO HP UC UCSN ased on CPI 0.8695 0.9577 0.9159 0.8881 0.8695 0.0993) (0.1289) 0.1057) (0.1075) 0.3423 0.2347 0.0419) (0.1048) (0.0569) (0.0987) -186.0679 -186.1239 -183.8150 -188.4587 0.0648 0.0613 0.6169 0.0059 ased on core CVI U U U 0.6624 0.6426 0.6976 0.7735 (0.1534) (0.1359) (0.1404) (0.1329) 0.1148 0.2654 0.1581 0.1110 (0.0625) (0.1051) (0.0762) (0.0739) -133.8145 -130.0877 -133.5717 -135.4023 0.0223 0.9251 0.0284 0.0046	CBO HP UC UCSN BN ased on CPI

Table 2 Estimated coefficients of the forward-looking Phillips curve

Model: $\pi_t = \theta \pi_t^F + (1 - \theta) \pi_t^L + \alpha y_t + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$. $\pi_t^L = 0.25 \sum_{i=1}^4 \pi_{t-i}$. The Newey-West robust standard errors are in parentheses. LLK stands for log likelihood. The estimation sample is from 1981:3 to 2007:4

where π_t^F is the forward-looking expectation term and $\pi_t^L = \sum_{i=1}^4 \pi_{t-i}/4$ is a backward-looking expectation. If $\theta = 0$, the model collapses to the simple model used by Ball and Mazumder (2011).

We use the SPF forecasts of the CPI inflation as a proxy for π_t^F in both analyses with the CPI and the core CPI inflation.¹¹ Table 2 reports the estimates of the Phillips curve (5), and Fig. 6 reports the results from the dynamic simulations.

Our results are consistent with the existing literature, in that the forward-looking inflation expectation plays an important role. The estimated θ is around 0.9 for CPI inflation and around 0.7 for core CPI inflation. However, although greatly improved, the Phillips curve based on the CBO gaps still fails to capture the inflation dynamics for most of the post-Great Recession period: it predicts 0–0.5% inflation rates, which are mostly below the observed numbers (Fig. 7).

On the other hand, the Phillips curve based on other gap measures performs reasonably well in comparison. The predicted inflation rates lie mostly between 1 and 2%. As well, we observe notice that the differences among the predictions with different output gaps are not as dramatic as those with the backward-looking Phillips curve. This result is not surprising, considering the dominating role of the forward-looking component and the stable SPF inflation forecast after the Great Recession.

As in Table 2, the major model weight shifts toward to the UC gap with CPI inflation and to the HP gap with core CPI inflation, while the weight assigned to the CBO's

¹¹ The SPF forecasts of the core CPI inflation are not available until 2007.



Fig. 6 The dynamic simulations on inflation with forward-looking inflation expectation



Fig. 7 The model-averaged dynamic simulations on inflation with forward-looking inflation expectation

gap is below 10% in both cases. Therefore, our model-averaged prediction follows the actual data quite closely.

3.3 Wage inflation dynamics

In this subsection, we investigate whether there is missing wage deflation and, if there is, how potential output variations explain wage inflation dynamics. A wage Phillips curve connects wage inflation to unemployment gap, and we adopt the same structure as (4). The measure of wages is based on earnings data for production and nonsupervisory workers from the Establishment Survey. We obtain measures of unemployment gap corresponding to our measures of the output gap through a gap version of Oknu's law: $u_t = -0.5y_t$, which comes from a general finding that for every 1% increase in



Fig. 8 The dynamic simulations on wage inflation with backward-looking inflation expectation

Output gap	СВО	HP	UC	UCSN	BN	CBO-SR
Inflation rate	based on CPI					
α	-0.0868	-0.1023	-0.1037	-0.0636	-0.0876	-0.0763
	(0.0390)	(0.0728)	(0.0493)	(0.0360)	(0.0393)	(0.0362)
LLK	-262.9597	-264.7572	-263.6507	-264.8204	-263.3386	-263.2239
Weight	0.3054	0.0506	0.1530	0.0475	0.2091	0.2345

 Table 3 Estimated slope of the pure backward-looking wage Phillips curve

Model: $\pi_t^w = 0.25 \sum_{i=1}^4 \pi_{t-i}^w + \alpha u_t + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$. The Newey–West robust standard errors are in parentheses. LLK stands for log likelihood. Estimation sample is 1964:1 to 2007:4

the unemployment gap, the US GDP will be roughly an additional 2% lower than its potential GDP. The estimation sample spans from 1964:1 to 2013:3.

First, we find that there is missing wage deflation. Figure 8 indicates that simulated wage inflation according to the CBO gaps is far below the actual observations. Consistent with our previous results, other gap measures do not imply deflation. Note that the CBO-SR gaps perform very well in explaining wage inflation. Second, we find that wage inflation dynamics are not puzzling if we account for permanent output variations properly. According to Table 3, more weights are put on the CBO gap, compared to estimation results from the price Phillips curve, but about 70% of the model weight is placed on other gap measures. Therefore, the model-averaged simulation shown in Table 3 closely captures the wage inflation dynamics.

4 Conclusion

Our empirical results suggest that the seemingly broken Phillips curve relationship after the Great Recession may be recovered to a great extent if we take permanent output losses into account. We have applied several popular detrending methods to estimate the output gap and found that a large portion of the declines in the real GDP during the Great Recession seems to be permanent. As a result, the Phillips curve with all measures of the output gap, except the CBO's estimates, predicts mild inflation. A likelihood-based model averaging has been used to account for model uncertainty associated with the choice of detrending method. Finally, we have shown that the model-averaged prediction is largely in line with the observed data.

Given the complicated nature of the economy after the Great Recession, the puzzling post-crisis inflation dynamics may well result from more than one factor. We have shown that well-anchored inflation expectations truly do work to improve Phillips curve prediction substantially; nonetheless the potential output variations play an important role in explaining the post-crisis inflation dynamics.

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Appendices

A Bayesian inference of the UC model

A.1 Normal innovations

The model containing (1)–(3) can be rewritten into state space form:

$$y_{t} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_{t}$$
(A.1)
$$x_{t} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} 1(t > Tb) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_{1} & \phi_{2} \\ 0 & 1 & 0 \end{bmatrix} x_{t-1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_{t} \\ \epsilon_{t} \end{bmatrix}$$
(A.2)

where $x_t = [\tau_t, c_t, c_{t-1}]'$.

In order to ensure that the estimated covariance matrix is positive semi-definite, we decompose the covariance matrix in the following way:

$$\begin{bmatrix} \sigma_{\eta}^2 & \rho \sigma_{\eta} \sigma_{\epsilon} \\ \rho \sigma_{\eta} \sigma_{\epsilon} & \sigma_{\epsilon}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$
(A.3)

and directly estimate $\{\sigma_1, \sigma_2, b\}$ instead of the covariance matrix parameters $\{\sigma_\eta, \sigma_\epsilon, \rho\}$. The posterior samples for the covariance parameters are obtained through transformation.

Independent proper priors are specified for all parameters. σ_1^2 and σ_2^2 are assumed to have independent inverse gamma priors $IG(100, 0.5)^{12}$. These priors are diffuse

$$f(x; s^{-2}, \nu) = \left(\frac{2s^{-2}}{\nu}\right)^{-\frac{\nu}{2}} \frac{1}{\Gamma\left(\frac{\nu}{2}\right)} x^{-\frac{\nu}{2}-1} exp\left(-\frac{\nu}{2s^{-2}x}\right)$$

where $\Gamma(\cdot)$ is the gamma function.

¹² We follow Koop (2003) for the definition of inverse gamma (IG) distribution. If x > 0 follows inverse gamma distribution $IG(s^{-2}, \nu)$, the probability density function of x is defined as:

and do not have finite moments. Therefore, a heavy weight will be put on sample information. We assume somewhat informative normal priors for $\phi_1 + \phi_2 \sim N(0.5, 1)$, $\phi_2 \sim N(-0.5, 1)$ and $b \sim N(0, 1)$. Imposing truncations for $\mu \in [0, 2]$ and $d \in [0, 2]$ [-1, 1], ¹³ the uniform priors for μ and d are over the truncated areas giving truncated normal posteriors accordingly. While the above priors are broadly consistent with estimates from the literature, our estimation results are robust to more diffuse priors.¹⁴ Lastly, a flat proper prior is assumed for Tb such that all dates from 1947:1 to 2013:2 have equal probability to be the break date in the mean growth rate. Therefore, the joint prior density is the product of all the above marginal prior densities.

Gibbs sampling approach is used to draw posterior samples for parameters including the break date. Sampling the joint posterior distribution of parameters can be conducted by sequential sampling from the conditional distributions. Details on the Gibbs sampling are summarized as follows:

Define $\theta = [\mu, \phi_1, \phi_2, \sigma_\eta, \sigma_\epsilon, \rho, d]$. Let $(\cdot)^{(k)}$ denote the *k*th posterior draw of the latent variable x_t or the parameters. Y denotes all the observed quarterly log real GDP $\{y_1, y_2, \ldots, y_T\}$. The *k*th step in our Gibbs sampler involves the following blocks:

- Draw $\{x_t^{(k)} : t = 1, ..., T\} \sim f(x_1, ..., x_T | Y, \theta^{(k-1)}, Tb^{(k-1)})$ obtained from the simulation smoother developed by Durbin and Koopman (2002). We then obtain $\tau_t^{(k)}$ and $c_t^{(k)}$ as the first two elements in $x_t^{(k)}$, as well as the residual terms $[\hat{\eta}_t, \hat{\epsilon}_t]'$. Draw $[\phi_1^{(k)}, \phi_2^{(k)}] \sim f(\phi_1, \phi_2 | Y, x_t^{(k)}, \sigma_{\epsilon}^{(k-1)})$ given that the second row in (A.2)
- has the following regression form:

$$c_t = \begin{bmatrix} c_{t-1} & c_{t-2} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} + \epsilon_t \tag{A.4}$$

Stack (A.4) by time, we have

$$Y_c = C\Phi + \epsilon$$

where $Y_c = \left\{ c_3^{(k)}, \dots, c_T^{(k)} \right\}'$, $\epsilon = \{\epsilon_3, \dots, \epsilon_T\}' \stackrel{i.i.d.}{\sim} N(0, 1/h_{\epsilon})$ with $h_{\epsilon} =$ $(\sigma_{\epsilon}^{(k-1)})^{-2}, \Phi = \{\phi_1 + \phi_2, \phi_2\}'$ and

$$C = \begin{bmatrix} c_2 & c_1 - c_2 \\ \vdots \\ c_{t-1} & c_{t-2} - c_{t-1} \\ \vdots \\ c_{T-1} & c_{T-2} - c_{T-1} \end{bmatrix}$$

¹³ Such truncation is assumed to avoid unreasonable draws for μ and d. As noted in de Pooter et al. (2008), μ and d are nearly unidentified when the samples of ϕ_1 and ϕ_2 get very close to the nonstationary region. In this case, arbitrary real values for μ and d can be drawn and cause the Gibbs sampler to have difficulty in moving away from the nonstationary region.

¹⁴ As robustness check, setting the variance for the normal priors to be 10 and priors for σ_1^2 and σ_2^2 to be IG(100, 0.1) gives similar results.

Given multivariate normal prior $N(\Phi_0, V_{\Phi 0})$ for Φ , the posterior distribution follows a multivariate normal distribution $N(\tilde{\Phi}, \tilde{V}_{\Phi})$, where

$$\tilde{V}_{\boldsymbol{\Phi}} = \left(V_{\boldsymbol{\Phi}0}^{-1} + h_{\boldsymbol{\epsilon}}C'C\right)^{-1} \tag{A.5}$$

$$\tilde{\Phi} = \tilde{V}_{\Phi} \left(V_{\Phi 0}^{-1} \Phi_0 + h_{\epsilon} C' Y_c \right)$$
(A.6)

The posterior samples for $\left[\phi_1^{(k)}, \phi_2^{(k)}\right]$ must guarantee the stationarity of the process. Therefore, we discard nonstationary draws and regenerate new ones until they meet the stationary requirement.

- Draw $[\mu^{(k)}, d^{(k)}] \sim f(\mu, d|Y, x_t^{(k)}, \sigma_{\eta}^{(k-1)})$ given the regression in the first row of (A.2):

$$\tau_t - \tau_{t-1} = \begin{bmatrix} 1 & 1(t > Tb) \end{bmatrix} \begin{bmatrix} \mu \\ d \end{bmatrix} + \eta_t$$
(A.7)

Stack (A.7) by time, we have

$$Y_{\tau} = D \begin{bmatrix} \mu \\ d \end{bmatrix} + \eta$$

where $Y_{\tau} = \left\{ \tau_2^{(k)} - \tau_1^{(k)}, \dots, \tau_T^{(k)} - \tau_{T-1}^{(k)} \right\}', \eta = \{\eta_2, \dots, \eta_T\}' \stackrel{i.i.d.}{\sim} N(0, 1/h_{\eta})$ with $h_{\eta} = (\sigma_{\eta}^{(k-1)})^{-2}$ and

$$D = \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 1(t > Tb) \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$$

Given uniform priors $\mu \sim [0, 2]$ and $d \sim [-1, 1]$, the posterior distribution follows a truncated multivariate normal distribution. It is equivalent to sampling the posterior from $N(\tilde{M}, \tilde{V}_M)$ and discard the samples out of the valid region, where

$$\tilde{V}_M = (h_n D'D)^{-1} \tag{A.8}$$

$$\tilde{M} = h_{\eta} \tilde{V}_M D' Y_{\tau} \tag{A.9}$$

- Draw
$$\left[\sigma_{1}^{(k)}, \sigma_{2}^{(k)}\right] \sim f(\sigma_{1}, \sigma_{2}|Y, x_{t}^{(k)}, \mu^{(k)}, d^{(k)}, \phi_{1}^{(k)}, \phi_{2}^{(k)}, b^{(k-1)})$$
. Define $\eta_{t}^{*} = \eta_{t} \sim N(0, \sigma_{1}^{2})$ and $\epsilon_{t}^{*} = -b\eta_{t} + \epsilon_{t} \sim N(0, \sigma_{2}^{2})$, and we have the following:

$$B^{-1}\begin{bmatrix}\hat{\eta}_t\\\hat{\epsilon}_t\end{bmatrix} = \begin{bmatrix}\hat{\eta}_t\\-b\hat{\eta}_t+\hat{\epsilon}_t\end{bmatrix} = \begin{bmatrix}\hat{\eta}_t^*\\\hat{\epsilon}_t^*\end{bmatrix} \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\sigma_1^2 & 0\\0 & \sigma_2^2\end{bmatrix}\right)$$
(A.10)

where

$$B = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$$

According to (A.10),

$$\begin{bmatrix} \eta_t^* \\ \epsilon_t^* \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right)$$
(A.11)

We assume independent priors for $h_i^{-1} = \sigma_i^2 \sim IG(s_{i0}^{-2}, v_{i0})$ with i = 1, 2. It is equivalent to assume a Gamma prior $G(s_{i0}^{-2}, v_{i0})$ for h_i . The posterior of h_i , in this case, is $G(\tilde{s}_i^{-2}, \tilde{v}_i)$, where for i = 1, 2

$$\tilde{\nu}_i = T + \nu_{i0} \tag{A.12}$$

$$\tilde{s}_1^2 = \frac{\eta^{*'} \eta^* + \nu_{10} s_{10}^2}{\tilde{\nu}_1} \tag{A.13}$$

$$\tilde{s}_2^2 = \frac{\epsilon^{*'} \epsilon^* + \nu_{20} s_{20}^2}{\tilde{\nu}_2} \tag{A.14}$$

$$\eta^* = \left[\eta_1^{*(k)}, \dots, \eta_T^{*(k)}\right]'$$
(A.15)

$$\boldsymbol{\epsilon}^* = \left[\boldsymbol{\epsilon}_1^{*(k)}, \dots, \boldsymbol{\epsilon}_T^{*(k)}\right]' \tag{A.16}$$

- Draw $b^{(k)} \sim f(b|Y, x_t^{(k)}, \mu^{(k)}, d^{(k)}, \phi_1^{(k)}, \phi_2^{(k)}, \sigma_2^{(k)})$. Given the second row in (A.10), we have a standard regression to sample *b*:

$$\hat{\epsilon}_t = \hat{\eta}_t b + \epsilon_t^* \tag{A.17}$$

where $\epsilon_t^* \sim N(0, \sigma_2^2)$.

Stack (A.17) by time, we have

$$\hat{\epsilon} = Eb + e^*$$

where $\hat{\epsilon} = \left\{ \hat{\epsilon}_{1}^{(k)}, \dots, \hat{\epsilon}_{T}^{(k)} \right\}', E = \left\{ \hat{\eta}_{1}^{(k)}, \dots, \hat{\eta}_{T}^{(k)} \right\}'$ and $e^{*} = \left\{ \epsilon_{1}^{*}, \dots, \epsilon_{T}^{*} \right\}' \stackrel{i.i.d.}{\sim} N(0, 1/h_{s2})$ with $h_{s2} = (\sigma_{2}^{(k)})^{-2}$. $\hat{\eta}_{t}^{(k)}$ and $\hat{\epsilon}_{t}^{(k)}$ are residuals in the first two rows in (A.2).

Given normal prior $N(b_0, V_{b0})$ for *b*, the posterior distribution follows a normal distribution $N(\tilde{b}, \tilde{V}_b)$, where

$$\tilde{V}_b = \left(V_{b0}^{-1} + h_{s2}E'E\right)^{-1}$$
(A.18)

$$\tilde{b} = \tilde{V}_b \left(V_{b0}^{-1} b_0 + h_{s2} E' \hat{\epsilon} \right) \tag{A.19}$$

- Draw $Tb^{(k)} \sim f(Tb|Y, \theta^{(k)})$. According to Wang and Zivot (2000), given the flat proper prior assumed for Tb,

$$f(Tb|Y,\theta) = \frac{f(Y|Tb,\theta)f(Tb|\theta)}{f(Y|\theta)}$$

$$\propto f(Y|Tb,\theta)f(Tb)$$

$$\propto f(Y|Tb,\theta)$$
(A.20)

According to (A.20), we can draw *Tb* from a multinomial distribution where $f(Tb|Y, \theta) = \frac{f(Y|Tb,\theta)}{\sum_{t=1}^{T-1} f(Y|Tb=t,\theta)}$

A.2 Skew-normal innovations

A linear Gaussian state space model:

$$y_t = H\beta_t + \omega \epsilon_t^*, \quad \epsilon_t^* \sim N(0, \bar{\Omega}_m)$$
 (A.21)

$$\beta_t = F\beta_{t-1} + \sigma \eta_t^* \quad \eta_t^* \sim N(0, \bar{\Omega}_s) \tag{A.22}$$

where $\boldsymbol{\omega} = \text{diag}(\omega_1, \dots, \omega_d)$ and $\boldsymbol{\sigma} = \text{diag}(\sigma_1, \dots, \sigma_r)$. In what follows, matrix with upper bar represents a correlation matrix. A linear skew-normal state space model deviates the Gaussian one by assuming that

$$\epsilon_t^* \sim SN(\bar{\Omega}_m, \alpha_m), \text{ and } (A.23)$$

$$\eta_t^* \sim SN(\bar{\Omega}_s, \alpha_s). \tag{A.24}$$

Note that a random variable x follows a multivariate skew-normal distribution, $SN(\overline{\Omega}, \alpha)$, with density given by

$$f_{SN}(x; \bar{\Omega}, \alpha) = 2\phi(x; \bar{\Omega})\Phi(\alpha' x), \qquad (A.25)$$

where $\phi(x; \overline{\Omega})$ is the pdf of the multivariate zero mean $N(0, \Omega)$ abd $\Phi(.)$ is the cdf of the univariate N(0, 1) distribution.

In our Bayesian analysis, we use the following stochastic representation of the multivariate skew-normal distribution.

$$\epsilon_t^* = \delta_m Z_m + D(\delta_m) U_m^*, \tag{A.26}$$

$$\eta_t^* = \delta_s Z_s + D(\delta_s) U_s^*, \tag{A.27}$$

where $\delta = [\delta_1, \delta_2, ...]'$ and $D(\delta) = \sqrt{I - \text{diag}(\delta)^2}$. The random variable Z follows a half normal $N^+(0, 1)$ and U a normal $N(0, \bar{\Psi})$. Note that there is a 1-to-1 relation between $(\bar{\Omega}, \alpha)$ and $(\delta, \bar{\Psi})$:

$$\bar{\Omega} = D(\delta)(\bar{\Psi} + \lambda\lambda')D(\delta), \tag{A.28}$$

$$\alpha = (1 + \lambda' \bar{\Psi}^{-1} \lambda)^{-0.5} D(\delta)^{-1} \bar{\Psi}^{-1} \lambda, \tag{A.29}$$

where $\lambda = D(\delta)^{-1}\delta$.

The scaled error terms thus have the following distributions:

$$\epsilon_t \equiv \boldsymbol{\omega} \epsilon_t^* \sim SN(\Omega_m, \alpha_m), \quad \Omega_m = \boldsymbol{\omega} \bar{\Omega}_m \boldsymbol{\omega}, \tag{A.30}$$

$$\eta_t \equiv \boldsymbol{\sigma} \eta_t^* \sim SN(\Omega_s, \alpha_s), \qquad \Omega_s = \boldsymbol{\sigma} \bar{\Omega}_s \boldsymbol{\sigma}. \tag{A.31}$$

The corresponding stochastic representation is as follows:

$$\epsilon_t = \omega \delta_m Z_m + \omega D(\delta_m) U_m^* \equiv \theta_m Z_m + U_m, \quad U_m \sim N(0, \Sigma_m)$$
(A.32)

$$\eta_t = \boldsymbol{\sigma} \,\delta_s Z_s + \boldsymbol{\sigma} \, D(\delta_s) U_s^* \equiv \theta_s Z_s + U_s, \quad U_s \sim N(0, \,\Sigma_s). \tag{A.33}$$

The only difficulty comes from the non-Gaussian term $Z = [Z_m, Z_s]$. Conditional on Z, however, the model reduced to the following standard linear Gaussian state space model:

$$y_t - \theta_m Z_{m,t} = H\beta_t + U_{m,t}, \quad U_{m,t} \sim N(0, \Sigma_m)$$
(A.34)

$$\beta_t = \theta_s Z_{s,t} + F \beta_{t-1} + U_{s,t} \quad U_{s,t} \sim N(0, \Sigma_s).$$
(A.35)

The FFBS can be used to draw samples from conditional distribution of β^T . Therefore, the Gibbs sampler will be completed by drawing sample from conditional distribution of Z given ϵ^T and η^T . Note that $y_t - H\beta_t = \epsilon_t$ and $\beta_t - F\beta_{t-1} = \eta_t$.

Take Z_m as example. Conditional on ϵ^T and other parameters,

$$p(Z_{m,t}|\epsilon_t, \cdot) \propto p(\epsilon_t | Z_{m,t}, \cdot) p(Z_{m,t}| \cdot)$$

$$\propto exp\left(-0.5(\epsilon_t - \theta_m Z_{m,t})' \Sigma_m^{-1}(\epsilon_t - \theta_m Z_{m,t})\right)$$

$$\times exp\left(-0.5Z_{m,t}^2\right) I(Z_{m,t} > 0)$$

$$\propto exp\left(-0.5\left[Z_{m,t}^2\left(\theta_m' \Sigma_m^{-1} \theta_m + 1\right) - 2Z_{m,t}\theta_m' \Sigma_m^{-1} \epsilon_t\right]\right) I(Z_{m,t} > 0)$$

Therefore, completing the square leads to the conditional distribution as follows:

$$Z_{m,t}|\epsilon_t, \cdot \sim N^+(a_t, A), \text{ where}$$

$$a_t = A\theta'_m \Sigma_m^{-1} \epsilon_t, \quad A = \left(\theta'_m \Sigma_m^{-1} \theta_m + 1\right)^{-1}.$$
(A.36)

Conditional on Z and other blocks, we have the following regression system:

$$\begin{bmatrix} \epsilon_{1,t} \\ \vdots \\ \epsilon_{d,t} \end{bmatrix} = \begin{bmatrix} \theta_{m,1} \\ \vdots \\ \theta_{m,d} \end{bmatrix} Z_{m,t} + U_m.$$
(A.37)

Drawing θ_m and Σ_m can be done easily.

B Estimates of the output gap using CBO's method

The CBO estimates the potential output based on a neoclassical growth model, in which the output is determined by the production function as follows:

$$\ln(Y) = 0.7\ln(L) + 0.3\ln(K) + \ln(\text{TFP}) + C, \quad (B.1)$$

where *L* represents hours worked, K is capital input, TFP is total factor productivity, and C is a constant. The potential output, denoted by Y^* , is then computed by

$$\ln(Y^*) = 0.7\ln(L^*) + 0.3\ln(K) + \ln(\text{TFP}^*) + C, \qquad (B.2)$$

where L^* and TFP^{*} are potential level of hour worked and total factor productivity.

The key device used by the CBO to obtain the potential level of a variable *X* is a cyclical adjustment equation as follows:

$$\ln(X) = \alpha(U - U^*) + f_t(\beta) + \epsilon, \qquad (B.3)$$

 $f_t(\beta)$ is a linear function of time which breaks at the peak in the business cycle.¹⁵ *U* is the unemployment rate, and U^* is the CBO's estimates of the natural rate of unemployment. Equation (B.3) can be estimated consistently using the ordinary least squares for all variables needed to construct CBO's potential output estimates.

The new potential output can be written as

$$\hat{y} = \hat{y}_{nf} + \hat{y}_{other}.$$

where y_{nf} is the potential output in the nonfarm business sector, and y_{other} is the real output in other sector. CBO estimates y_{nf} using a neoclassical growth model, but using the cyclical adjustment Eq. (B.3) to obtain y_{other} due to data availability. We take an approximation of the above equation around the CBO's estimate, denoted by superscript *, as follows:

$$\ln(\hat{y}) - \ln(y^*) \approx \frac{y_{\text{nf}}^*}{y^*} \frac{\hat{y}_{\text{nf}} - y_{\text{nf}}^*}{y_{\text{nf}}^*} + \frac{y_{\text{other}}^*}{y^*} \frac{\hat{y}_{\text{other}} - y_{\text{other}}^*}{y_{\text{other}}^*}$$

¹⁵ Thus, there are as many break points as the number of recessions after 1950.

$$\approx w_{\rm nf} \left(\ln(\hat{y}_{\rm nf}) - \ln\left(y_{\rm nf}^*\right) \right) + w_{\rm other} \left(\ln\left(\hat{y}_{\rm other}\right) - \ln\left(y_{\rm other}^*\right) \right).$$
(B.4)

Thus, we only need to compute the two term on the right-hand side; then, we can obtain the potential output $\ln(\hat{y})$. Note that

$$\ln (y_{nf}^*) = 0.7 \ln (L_{nf}^*) + 0.3 \ln(K_{nf}) + \ln (\text{TFP}_{nf}^*) + C$$

$$\Rightarrow (\ln (\hat{y}_{nf}) - \ln (y_{nf}^*)) = 0.7 (\ln(\hat{L}_{nf}) - \ln (L_{nf}^*))$$

$$+ (\ln(\text{TFP}_{nf}) - \ln (\text{TFP}_{nf}^*))$$

The variables with $\hat{}$ are obtained using cyclical adjustment Eq. (B.3) with the shortrun unemployment rate, while those with * are using Eq. (B.3) with total unemployment rate. The total factor productivity of the nonfarm business sector is obtained from Fernald (2012). For the other sector, we obtain the real output in other sector by subtracting real output in the nonfarm business sector from the total real output. The potential output in the other sector is thus constructed using cyclical adjustment Eq. (B.3).

We are confronted with a minor difficulty while using cyclical adjustment Eq. (B.3) with the short-run unemployment rate. The natural rate of the short-run unemployment is not available and needed to be estimated. Let p_s be the sample mean of the proportion of the short-run unemployment in the total unemployment, which is 0.8331 in our sample. We obtain the estimates of the natural rate of the short-run unemployment by $p_s \times U^*$, where U^* is the CBO's estimates of the natural rate of unemployment.

C Measures of the output gaps from 1947:3 to 2013:3

Figure 9 shows the estimated output gap from 1947:3 to 2013:3.



Fig. 9 Measures of the output gap

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