

Real interest rates: nonlinearity and structural breaks

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Abstract Real interest rate is a crucial variable that determines the consumption, investment and saving behavior of individuals and thereby acts as a key policy tool that the central banks use to control the economy. Although many important theoretical models require the real interest rates to be stationary, the empirical evidence accumulated so far has not been able to provide conclusive evidence on the mean reverting dynamics of this variable. To resolve this puzzle we re-investigate the stochastic nature of the real interest rates by developing unit root tests for nonlinear heterogeneous panels where the alternative hypothesis allows for a smooth transition between deterministic linear trends around which stationary asymmetric adjustment may occur. When the newly developed panel unit root tests are applied to the real interest rates of the 17 OECD countries, we were able to uncover overwhelming empirical support in favor of mean reversion in the short-run and long-run real interest rates. Therefore, these results show that the conclusions drawn from a misspecified test that ignores the presence of either nonlinearity, structural breaks or cross sectional

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dependence can give quite misleading results about the stochastic behavior of the real interest rates.

Keywords Smooth break · Panel unit root · Cross-sectional dependence · Nonlinearity · Real interest rate

JEL Classifications C2 · C5 · E4

1 Introduction

One of the central issues in macroeconomics and finance is the stochastic properties of the real interest rates. The question of whether the real interest rate is constant and follows a mean reverting stochastic process has attracted a great deal of attention among economists since the seminal work of Fisher (1930). At stake, among other things, is the validity of the intertemporal Euler equation implied by the consumption-based asset pricing models (CAPM) (Lucas 1978; Breeden 1979; Hansen and Singleton 1982), the Black-Scholes model used to determine the price of options (Black and Scholes 1973), the Fisher hypothesis and efficiency of capital markets, the long-run super neutrality hypothesis, the real interest rate parity hypothesis (RIPH) and the implications of many models of monetary transmission mechanism. Thus, many important theoretical models in the macroeconomics literature imply or assume stationary real interest rates. However, the empirical studies after decades of investigation still could not provide a definite answer regarding the stationary properties of the real interest rates.

After a comprehensive overview, the following conclusions can be drawn from the existing econometric analysis¹. First, when the conventional univariate unit roots are applied to the data, the null hypothesis of a unit root in general could not be rejected (e.g., Rose 1988; Moosa and Bahatti 1996; Kapetanios et al. 2003; Choi and Moh 2007; Tsong and Lee 2012). Second, there is strong evidence that the real interest rate reverts to a long-run mean, but that the adjustment to this mean follows a nonlinear process (e.g., Garcia and Perron 1996; Kapetanios et al. 2003; Million 2004; Lanne 2006; Christopoulos and Leon-Ledesma 2007). In particular, the real interest rates are shown to exhibit threshold dynamics, which coincides quite well with the central bank's "opportunistic disinflation" policy strategy. Third, many empirical studies report the existence of structural breaks in the mean of the real interest rate series, which if exists greatly reduces the power of the conventional unit root tests (e.g., Clemente et al. 1998; Bai and Perron 1998; Caporale and Grier 2000; Bai and Perron 2003; Lai 2008). Fourth, studies that utilize a much longer span of data and/or allow for the use cross-sectional information in the panel data tend to find strong evidence in favor of mean reverting behavior in the real interest rates (e.g., Costantini and Lupi 2007; Sekioua and Zakane 2007; Westerlund 2008; Tsong and Lee 2013; Pesaran et al. 2013). Thus, it seems that the reported empirical results that drive a wedge between the theoretical models and the data are due to the econometric challenges involved in identifying stationary real interest rate series. To the best of our knowledge, the

¹ See Neely and Rapach (2008) for a comprehensive survey of this literature.

empirical literature has not considered testing for the long run mean reversion of the real interest rates using panel data allowing for structural breaks and regime-wise nonlinearity together with cross-sectional dependence. However, as mentioned above, existing evidence clearly points to the presence of both structural breaks and nonlinear dynamics in the real interest rate at the international level, which implies that these have to be accounted for when analyzing the stochastic properties of the series in a panel setting. Hence, in this paper to analyze the stochastic properties of the real interest rate series, we accomplish this task and develop unit root tests that allow simultaneously for the existence of structural breaks and regime-wise nonlinearity in heterogeneous panels (see Lundbergh et al. 2000; Skalin and Teresvirta 2002; Lütkepohl et al. 1998; Sollis et al. 2002 and Sollis 2004 for the univariate case).

To this end, the present study extends the nonlinear unit root testing framework of Sollis (2004) to heterogeneous panels by drawing upon the work of Im et al. (2003) (hereafter, IPS). This allows us to combine smooth transition (ST) and threshold autoregressive (TAR) models within a panel framework². Furthermore, to correct for the size distortion that is caused by cross-sectional dependence (CSD), we implement Chang's (2004) sieve bootstrap methodology³. Small sample properties of the newly developed tests are compared to that of the conventional IPS panel unit root test, which ignores the presence of both structural breaks and nonlinearity. We have also compared our test with the panel Leybourne et al. (1998) (PLNV) test proposed by Omay et al. (2013) that only considers the presence of smooth structural breaks in a heterogeneous panel setting (henceforth, OHS). We show that our newly proposed test has significant power gains over both the IPS and PLNV test. This makes our proposed test quite relevant for analyzing the stationary properties of the real interest rate series, which are argued to exhibit both threshold dynamics and structural breaks.

By implementing our newly proposed panel unit root tests to the real interest rates of 17 OECD countries over the period 1979:Q2–2011:Q2, we find conclusive evidence

² In order to consider smooth breaks in the deterministic components of a series, many studies have alternatively developed unit root tests based on Gallant's (1981) flexible Fourier form. However, as discussed in Omay et al. (2013), the smooth transition methodology has many advantages over the Flexible Fourier form (see also Omay 2015). Thus, in this article we have employed the smooth transition model to handle the structural shifts in the data. Moreover, the dummy variable based structural break unit root testing procedures skip the idea of heterogeneity. The most important conclusion that can be drawn from the simulation studies carried out in this study and the ones in Omay et al. (2013) is that if the panel structural break tests do not estimate the break point heterogeneously, they are insufficient to test the real-life data. Fortunately, the proposed tests in this study have proven to be quite successful in both heterogeneous and homogeneous parameter settings via the Monte Carlo simulation studies.

³ There are other methods to remedy the CSD problem such as the common correlated effect (CCE) estimator proposed by Pesaran (2006, 2007). The CCE estimator augments the panel regressions by cross-sectional averages of lagged first-difference terms, and this approach is valid for panels where N and T are of the same order of magnitudes (Pesaran 2007) unlike the principal component approach that requires $N/T \rightarrow 0$. Although Kapetanios et al. (2011) have shown that the CCE estimators have better small sample properties than the factor-based estimators, using the CCE estimator in our present context is quite troublesome since our testing procedure utilizes an indicator function that may produce problems while taking the averages of the independent variables. Hence, we prefer to use the bootstrap methodology which also solves the CSD problem in a multifactor error structure. Besides, the bootstrap methodology is shown to be a better remedy for smooth structural break panel unit root tests with CSD, as opposed to the CCE methodology (Omay et al. 2013).

that the real interest rates are integrated of order zero (i.e., $I(0)$). Also, the scope of the paper covers different terms of the real interest rate—short-term and long-term—that provides a comprehensive analysis of the topic. Particularly, using the panel ST-TAR unit root tests, we show that the short-term real interests rates of all the 17 OECD countries are stationary around a broken trend. For the long-term interest rates, only the real interest rate series of France is found to contain a unit root.

The remainder of the article is structured as follows. In Sect. 2, the newly proposed unit root tests are developed. In Sect. 3, the critical values of this test under cross sectional independence are presented. Sections 4 and 5 illustrate the methodology used to handle the cross-sectional dependence problem and provide the empirical powers of our test in comparison to that of the IPS and the OHS tests. Section 6 applies the newly proposed unit root tests to analyze the mean reversion of the real interest rates. Section 7 is reserved for concluding remarks.

2 The model and testing framework

Let y_{it} be a panel ST-TAR process with changing trend function on the time domain $t = 1, 2, \dots, T$ for the cross-sectional units $i = 1, 2, \dots, N$. Consider y_{it} , which is generated by one of the following smooth transition (ST) processes:

$$\text{Model A: } y_{it} = \alpha_{i1} + \alpha_{i2}S_{it}(\gamma_i, \tau_i) + \varepsilon_{it} \tag{2.1}$$

$$\text{Model B: } y_{it} = \alpha_{i1} + \beta_{i1}t + \alpha_{i2}S_{it}(\gamma_i, \tau_i) + \varepsilon_{it} \tag{2.2}$$

$$\text{Model C: } y_{it} = \alpha_{i1} + \beta_{i1}t + \alpha_{i2}S_{it}(\gamma_i, \tau_i) + \beta_{i2}tS_{it}(\gamma_i, \tau_i) + \varepsilon_{it} \tag{2.3}$$

where $S_{it}(\gamma_i, \tau_i)$ is the logistic smooth transition (LST) function based on a sample of size T and N , $S_{it}(\gamma_i, \tau_i) = [1 + \exp\{-\gamma_i(t - \tau_i T)\}]^{-1}$ where $\gamma_i > 0$ and τ_i determine the mid-point of transformation for each $i = 1, 2, \dots, N$, and ε_{it} is generated by the following TAR model;

$$\Delta\varepsilon_{it} = \rho_{i1}I_{it}\varepsilon_{i,t-1} + \rho_{i2}(1 - I_{it})\varepsilon_{i,t-1} + \eta_{it} \tag{2.4}$$

where I_{it} is the Heaviside indicator function such that $I_{it} = 1$ if $\varepsilon_{i,t-1} \geq 0$, $I_{it} = 0$ if $\varepsilon_{i,t-1} < 0$, and η_{it} is a zero-mean stationary process.

In Eq. (2.4) if $H_0: \rho_{i1} = \rho_{i2} = 0$ for all i , then ε_{it} and therefore y_{it} contains a unit root, while if $\rho_{i1} = \rho_{i2} < 0$ for some i , y_{it} is a stationary panel ST-TAR process with symmetric adjustment. On the other hand, if $\rho_{i1} < 0$, $\rho_{i2} < 0$ and $\rho_{i1} \neq \rho_{i2}$ for some i , then y_{it} is a stationary panel ST-TAR process this time displaying asymmetric adjustment. These hypotheses are valid irrespective of whether Model A, B or C is used to describe the deterministic components of y_{it} .

In this modeling strategy, likewise [Leybourne et al. \(1998\)](#) (hereafter, LNV), the structural change is modeled as a smooth transition between different regimes rather than as an instantaneous structural break. The transition function $S_{it}(\gamma_i, \tau_i)$ is a continuous function bounded between 0 and 1. Thus, the ST model can be interpreted as a regime-switching model that allows for two regimes associated with the extreme values of the transition function, $S_{it}(\gamma_i, \tau_i) = 0$ and $S_{it}(\gamma_i, \tau_i) = 1$, where the transition

from one regime to the other is gradual. The parameter γ_i determines the smoothness of the transition, and thus, the smoothness of transition from one regime to the other. The two regimes are associated with the small and large values of the transition variable $S_{it} = t$ relative to the threshold τ_i . For the large values of γ_i , $S_{it}(\gamma_i, \tau_i)$ passes through the interval $(0,1)$ very rapidly, and as γ_i approaches $+\infty$ this function changes value from 0 to 1 instantaneously at time $t = \tau_i T$. Therefore, if we assume that ε_{it} is a zero mean $I(0)$ process and use model (1), then y_{it} becomes a stationary process around a nonlinear mean which changes from the initial value α_{i1} to the final value $\alpha_{i1} + \alpha_{i2}$. However, assuming that ε_{it} is generated by the TAR model given in Eq. (2.4) allows asymmetric adjustment around this nonlinear mean with differing degrees of autoregressive persistence parameters given by ρ_{i1} and ρ_{i2} . Similar conditions follow for models given by Eqs. (2.2) and (2.3)⁴.

The models given in (2.1), (2.2) and (2.3) can be used to test the following hypothesis:

$$H_0: y_{it} = \mu_{it}, \mu_{it} = \mu_{i,t-1} + \varepsilon_{it}, \mu_{i0} = \psi_i \tag{2.5a}$$

$$H_1: \text{Model A, Model B or Model C} \tag{2.5b}$$

or

$$H_0: y_{it} = \mu_{it}, \mu_{it} = \kappa_i + \mu_{i,t-1} + \varepsilon_{it}, \mu_{i0} = \psi_i \tag{2.6a}$$

$$H_1: \text{Model B or Model C} \tag{2.6b}$$

where ε_{it} is assumed to be a stationary processes with zero mean.

Following Sollis (2004), LNV and OHS the test statistics proposed here are calculated using a two-step procedure:

Step 1 Using a nonlinear least squares (NLS) algorithm, we estimate only the deterministic components of the preferred model for each of the cross-sectional units and collect the NLS residuals by

$$\text{Model A} \quad \hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i)$$

$$\text{Model B} \quad \hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\beta}_{i1}t - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i)$$

$$\text{Model C} \quad \hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\beta}_{i1}t - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i) - \hat{\beta}_{i2}tS_{it}(\hat{\gamma}_i, \hat{\tau}_i)$$

Step 2 We test for the presence of a unit root in the residuals obtained from the first step regression using the following TAR model,

$$\Delta \hat{\varepsilon}_{it} = \rho_{i1}I_{it}\hat{\varepsilon}_{i,t-1} + \rho_{i2}(1 - I_{it})\hat{\varepsilon}_{i,t-1} + \sum_{j=1}^{k_i} \delta_{ij} \Delta \hat{\varepsilon}_{i,t-j} + \eta_{it} \tag{2.7}$$

Sollis t -test for the i th individual is the most significant of the t -statistics used for testing $\rho_{i1} = 0$ and $\rho_{i2} = 0$ in Eq. (2.7) and is defined by:

⁴ For further discussions see LNV and Sollis (2004).

$$t_i = \max(t_{i1}, t_{i2}) \tag{2.8}$$

where t_{i1} and t_{i2} are individual t -statistics for $\rho_{i1} = 0$ and $\rho_{i2} = 0$, respectively. The t -statistics are defined as follows:

$$t_{ij} = \frac{\sqrt{T - k_i - 2} \left(\hat{\varepsilon}'_{ij,-1} M_{Q_{ij}} \Delta \hat{\varepsilon}_i \right)}{\left(\hat{\varepsilon}'_{ij,-1} M_{Q_{ij}} \hat{\varepsilon}_{ij,-1} \right)^{1/2} \left(\Delta \hat{\varepsilon}'_i M_{X_{ij}} \Delta \hat{\varepsilon}_i \right)^{1/2}} \quad j = 1, 2 \tag{2.9}$$

where $\Delta \hat{\varepsilon}_i = (\Delta \hat{\varepsilon}_{i1}, \Delta \hat{\varepsilon}_{i2}, \dots, \Delta \hat{\varepsilon}_{iT})'$, $\hat{\varepsilon}_{i1,-1} = (I_i \hat{\varepsilon}_{i0}, \dots, I_i \hat{\varepsilon}_{i,T-1})'$, $\hat{\varepsilon}_{i2,-1} = ((1 - I_i) \hat{\varepsilon}_{i0}, \dots, (1 - I_i) \hat{\varepsilon}_{i,T-1})'$, $Q_{i1} = ((1 - I_i) \hat{\varepsilon}_{i2,-1}, \Delta \hat{\varepsilon}_{i,-1}, \dots, \Delta \hat{\varepsilon}_{i,-k_i})$, $Q_{i2} = (I_i \hat{\varepsilon}_{i1,-1}, \Delta \hat{\varepsilon}_{i,-1}, \dots, \Delta \hat{\varepsilon}_{i,-k_i})$, $M_{Q_{ij}} = I_T - Q_{ij} (Q'_{ij} Q_{ij})^{-1} Q'_{ij}$, $M_{X_{ij}} = I_T - X_{ij} (X'_{ij} X_{ij})^{-1} X'_{ij}$, $X_{ij} = (\hat{\varepsilon}_{ij,-1}, Q_{ij})$ and I_T is an identity matrix of order T .

For testing the unit root null $H_0: \rho_{i1} = \rho_{i2} = 0$ in Eq. (2.7), non-standard individual F -test statistics is given by:

$$F_i = (R \hat{P}_i)' \left[\hat{\sigma}_i^2 R (C'_i C_i)^{-1} R' \right]^{-1} (R \hat{P}_i) / 2 \tag{2.10}$$

where $C_i = [I_i \hat{\varepsilon}_{i,-1}, (1 - I_i) \hat{\varepsilon}_{i,-1}, \Delta \hat{\varepsilon}_{i,-1}, \dots, \Delta \hat{\varepsilon}_{i,-k_i}]$, $R = [I_2, 0_{2 \times k_i}]$, $\hat{P}_i = [\hat{\rho}_{i1}, \hat{\rho}_{i2}]'$, which $\hat{\rho}_{i1}$ and $\hat{\rho}_{i2}$ are the Ordinary Least Square (OLS) estimators of ρ_{i1} and ρ_{i2} , and $\hat{\sigma}_i^2$ is the OLS estimator of σ_i^2 .

We propose testing for whether y_{it} contains a unit root using the F -statistic for testing $\rho_{i1} = \rho_{i2} = 0$ in (2.7), and/or the most significant of the t -statistics from those for testing $\rho_{i1} = 0$ and $\rho_{i2} = 0$. When the full model is given by Eqs. (2.1) and (2.4), the relevant F and t -statistics will be referred to as $\bar{F}_{1p,\alpha}$ and $\bar{t}_{1p,\alpha}$. When the full model is given by Eqs. (2.2) and (2.4), the relevant F and t -statistics will be referred to as $\bar{F}_{2p,\alpha}$ and $\bar{t}_{2p,\alpha}$, and when the full model is given by Eqs. (2.3) and (2.4), the relevant F and t -statistics will be referred to as $\bar{F}_{3p,\alpha}$ and $\bar{t}_{3p,\alpha}$. For each of the models, we obtain the mean group statistics as follows:

$$\bar{F}_{jp,\alpha} = N^{-1} \sum_{i=1}^N F_i \quad j = 1, 2, 3 \tag{2.11}$$

$$\bar{t}_{jp,\alpha} = N^{-1} \sum_{i=1}^N t_i \quad j = 1, 2, 3 \tag{2.12}$$

As the NLS estimation of the parameters γ_i and τ_i does not allow for closed-form solutions, it would be extremely hard to find any analytical relationship between the $\hat{\varepsilon}_{it}$ and y_{it} . This, of course, makes the determination of the null asymptotic distribution of the test statistics by analytical means more or less intractable (Leybourne et al. 1998). Leybourne et al. (1998) stated that the linearity property in the intercept and

trend terms ensures that the residual $\hat{\varepsilon}_i$ from all the models are invariant to the choice of the starting values (i.e., $\mu_{i0} = \psi_i$) and that Model A, B and C are invariant to both starting values and the drift term κ_i . Therefore, the obtained critical values are invariant to the parameters γ and τ , and as a result invariant to α and β . To further prove this statement, we employ a new Monte Carlo design to see the invariance of the critical values with respect to the parameters α , β , γ and τ . For this purpose, we use $\{\gamma, \tau\}$: $\{1.0, 0.5\}$, $\{1.5, 0.6\}$ and $\{0.5, 0.4\}$ for generating the critical values. The simulated density functions are obtained for $N = 1, T = 100$ for 1000 and 50000 trials and show us that the critical values are invariant to the parameters α , β , γ and τ . The simulated density functions for different parameter values of $\{\gamma, \tau\}$ are given in ‘‘Appendix’’.

3 Critical values of the proposed tests under cross-sectional independence

In this section the critical values of the unit root tests proposed above will be given under the assumption of cross-sectional independence using Monte Carlo simulation. Although the panel unit root tests developed in this study are especially designed for analyzing unit root behavior in the real interest rates, they are also general enough to be used in testing for the presence of unit root in any macroeconomic variable that exhibits both nonlinearity and structural breaks. In studying group of countries that are heavily integrated like the group of OECD countries studied here and for macroeconomic variables with close cross country links like the real interest rates analyzed here, accounting for cross-sectional dependence is crucial. However, for other cases which involve the study of macroeconomic variables in which cross-sectional dependence does not constitute a serious problem the critical values tabulated in this section should be used.

The exact critical values of $\bar{I}_{1p,\alpha}$ and $\bar{F}_{1p,\alpha}$ statistics under the assumption of cross sectional independence are generated by Monte Carlo simulations with 5000 replications and reported for Model A in Table 1⁵.

4 Sieve bootstrap algorithm under cross-sectional dependence

In this paper, we have followed Chang’s (2004) Sieve bootstrap methodology to overcome the cross-sectional dependence problem arising from investigating the real interest rates of the OECD countries that are heavily integrated. The Sieve bootstrap scheme used in this paper is summarized via a five-step algorithm in the following manner. *Step 1* Following Basawa et al. (1991), under the null of a unit root ($\hat{\varepsilon}_{it} = \Delta y_{it}$) we estimate the errors as:

$$\hat{v}_{it} = \hat{\varepsilon}_{it} - \sum_{j=1}^{p_i} \hat{\delta}_{ij} \hat{\varepsilon}_{i,t-j} \quad (4.1)$$

where lag orders p_i are determined via information criteria (e.g. AIC, SBC) by starting with p_{\max} .

⁵ The critical values of $\bar{I}_{2p,\alpha}$, $\bar{F}_{2p,\alpha}$, $\bar{I}_{3p,\alpha}$, and $\bar{F}_{3p,\alpha}$ for Models B and C are available upon request.

Table 1 The exact critical values of test statistics for Model A

<i>T/N</i>	$\bar{t}_{1p,\alpha}$					$\bar{F}_{1p,\alpha}$				
	5	10	15	25	50	5	10	15	25	50
<i>1 %</i>										
30	-3.307	-3.053	-2.903	-2.791	-2.693	8.998	7.652	7.083	6.556	6.086
50	-3.115	-2.905	-2.792	-2.719	-2.616	7.925	6.953	6.499	6.133	5.711
70	-3.074	-2.866	-2.756	-2.674	-2.598	7.825	6.783	6.340	5.958	5.617
100	-3.046	-2.832	-2.736	-2.657	-2.564	7.596	6.610	6.235	5.876	5.488
150	-2.980	-2.799	-2.724	-2.638	-2.556	7.328	6.411	6.194	5.788	5.414
200	-2.978	-2.801	-2.721	-2.642	-2.543	7.335	6.501	6.111	5.833	5.376
<i>5 %</i>										
30	-3.010	-2.855	-2.771	-2.697	-2.625	7.483	6.744	6.379	6.058	5.767
50	-2.897	-2.749	-2.688	-2.623	-2.558	6.777	6.250	5.958	5.722	5.433
70	-2.866	-2.717	-2.652	-2.582	-2.534	6.702	6.090	5.832	5.558	5.333
100	-2.822	-2.681	-2.633	-2.566	-2.511	6.626	5.949	5.737	5.454	5.243
150	-2.785	-2.667	-2.612	-2.556	-2.499	6.388	5.905	5.658	5.422	5.197
200	-2.787	-2.665	-2.606	-2.553	-2.486	6.435	5.860	5.650	5.384	5.155
<i>10 %</i>										
30	-2.883	-2.752	-2.702	-2.643	-2.585	6.829	6.329	6.058	5.834	5.578
50	-2.779	-2.673	-2.620	-2.576	-2.523	6.319	5.899	5.701	5.513	5.293
70	-2.746	-2.638	-2.594	-2.542	-2.499	6.186	5.737	5.582	5.364	5.195
100	-2.728	-2.615	-2.576	-2.523	-2.475	6.111	5.639	5.478	5.270	5.096
150	-2.691	-2.599	-2.559	-2.506	-2.468	5.948	5.589	5.429	5.226	5.069
200	-2.692	-2.592	-2.545	-2.505	-2.458	5.978	5.564	5.398	5.213	5.028

Step 2 **Stine (1987)** states that the residuals have to be centered with

$$\tilde{v}_t = \hat{v}_t - (T - p)^{-1} \sum_{t=p+1}^T \hat{v}_t \tag{4.2}$$

where $\hat{v}_t = (\hat{v}_{1t}, \hat{v}_{2t}, \dots, \hat{v}_{Nt})'$ and $p = \max(p_i)$. Moreover, we construct $N \times T[\tilde{v}_{it}]$ matrix from these residuals. We select the residuals column randomly with replacement at a time to preserve the cross-sectional structure of the errors. The bootstrap residuals are denoted by \tilde{v}_{it}^* where $T = 1, 2, \dots, T^*$ and $T^* = T + M$.

Step 3 We construct bootstrap samples ε_{it}^* recursively from

$$\varepsilon_{it}^* = \sum_{j=1}^{p_i} \hat{\delta}_{ij} \varepsilon_{i,t-j}^* + \tilde{v}_{it}^* \tag{4.3}$$

where $\hat{\delta}_{ij}$ are the estimates obtained from Step 1. Notice that we generate $T + M$ values of Δy_{it}^* and discard the first M values to ensure stationarity.

Step 4 We produce bootstrap samples y_{it}^* from

$$y_{it}^* = y_{i,t-1}^* + \varepsilon_{it}^* \quad (4.4)$$

Step 5 Using bootstrap samples y_{it}^* , the bootstrap test statistics (e.g. $\bar{F}_{1p,\alpha}^*$ and $\bar{t}_{1p,\alpha}^*$) are computed for each relevant model by employing a two-step procedure, which is presented in the previous section.

We repeat steps (2) to (5) many times to find bootstrap empirical distribution of these panel test statistics and obtain the bootstrap critical values by selecting the appropriate percentiles of the sampling distributions.

5 Finite sample performance

In this section, we investigate the Monte Carlo performance of the proposed panel test statistics under the assumption of cross sectional dependence. Before giving the Monte Carlo results for the proposed test statistics, it is important to demonstrate that the presence of cross-sectional dependence will distort the size of the tests. In this case, the data-generating process (DGP) with one factor is given as follows:

$$\begin{aligned} y_{it} &= y_{i,t-1} + \varepsilon_{it} & y_{i0} &= 0 \\ \varepsilon_{it} &= \lambda_i f_t + u_{it} \end{aligned} \quad (5.1)$$

where $f_t \sim i.i.d.N(0, 1)$ and $u_{it} \sim i.i.d.N(0, \sigma_i^2)$ with $\sigma_i^2 \sim i.i.d.U[0.5, 1.5]$. f_t and u_{it} are independent of each other. We distinguish between two degrees of cross-sectional dependence, so that we generate $\lambda_i \sim i.i.d.U[0, 0.2]$ for “low cross-sectional dependence”, and $\lambda_i \sim i.i.d.U[1, 4]$ for “high cross-sectional dependence”⁶. We use 2000 replications to compute the size distortions of the tests at the 5% nominal level⁷.

Tables 2 and 3 present the Monte Carlo results in the presence of high and low cross-sectional dependence, respectively. As to be expected, the presence of high cross sectional dependence given Table 2 leads to size distortions for both test statistics when N and T are large. Even if T is small for all values of N , both test statistics show size distortions. In general, their size distortions rise monotonically with N and T for all models. Otherwise, in case of low cross sectional dependence (Table 3) for all values of N and T , and for all models both test statistics have the correct empirical size.

After illustrating the size distortions problem, next we investigate the small sample properties of our tests under cross-sectional dependence using Monte Carlo simula-

⁶ For two different degrees of cross-sectional dependence, the average pair-wise correlations are computed as 0.01 and 0.816 for $N = 10$, respectively.

⁷ For all Monte Carlo simulations, since random parameter values are used, the whole experiment is simulated 5 times and the mean of the empirical sizes and powers obtained from these simulations are reported for different values of N and T . All programs are available upon request in both MATLAB and RATS formats.

Table 2 The exact size of $\bar{F}_{1p,\alpha}$ and $\bar{t}_{1p,\alpha}$ in case of DGP with factor (High cross section dependence)

T/N	$\bar{F}_{1p,\alpha}$			$\bar{t}_{1p,\alpha}$		
	5	10	25	5	10	25
<i>Model A</i>						
50	15.69	21.10	28.15	15.63	22.70	30.94
100	13.10	21.89	29.59	14.30	22.78	31.34
150	17.03	21.33	29.24	18.06	22.65	30.83
200	14.46	22.08	29.81	15.69	22.80	30.86

Table 3 The exact size of $\bar{F}_{1p,\alpha}$ and $\bar{t}_{1p,\alpha}$ in case of DGP with factor (Low cross section dependence)

T/N	$\bar{F}_{1p,\alpha}$			$\bar{t}_{1p,\alpha}$		
	5	10	25	5	10	25
<i>Model A</i>						
50	5.98	5.04	4.35	5.14	4.96	4.57
100	4.70	5.68	5.67	5.32	5.79	5.36
150	6.11	5.24	4.90	6.02	5.16	4.58
200	4.88	4.93	5.07	5.08	4.26	4.28

tions with Sieve bootstrap. We apply the Sieve bootstrap method to obtain critical values of the test statistics. To this end, we report experiments based on 1000 replications with 199 bootstrap replications. The experiments were carried out for all the pair of $N = 5, 10,$ and 25 and $T = 50, 100, 150$ and 200 . First we report the empirical size of the tests for all models. To this end, we employ the following DGP for the empirical size under the null (2.5a)

$$\begin{aligned}
 y_{it} &= y_{i,t-1} + \varepsilon_{it} & y_{io} &= 0 \\
 \varepsilon_{it} &= \lambda_i f_t + u_{it} \\
 u_{it} &= \rho_i u_{i,t-1} + v_{it}
 \end{aligned}
 \tag{5.2}$$

where $f_t \sim i.i.d.N(0, 1)$ and $v_{it} \sim i.i.d.N(0, \sigma_v^2)$ with $\sigma_v^2 \sim i.i.d.U[0.5, 1.5]$. Also, we set as $\lambda_i \sim i.i.d.U[1, 4]$ ⁸. To examine the impact of residual serial correlation, we generate $\rho_i \sim i.i.d.U[-0.4, 0.4]$ which will then be used in the generation of the errors, v_{it} .

Table 4 presents the empirical size of our tests for all models. We see that the empirical size of both test statistics is close to the nominal size for almost all T and N .

For brevity, we consider the empirical size-adjusted powers of the $\bar{t}_{1p,\alpha}$ and $\bar{F}_{1p,\alpha}$. By way of comparison, we also report size-adjusted powers of IPS test (\bar{t}_{IPS}) which

⁸ As an anonymous referee has, Chang’s (2004, page 277–278) DGP causes there to be weak cross-sectional dependence. Therefore, using Pesaran (2007)’s one factor DGP, the size and power analysis of the tests are conducted only for the values of $\lambda_i \sim i.i.d.U[1, 4]$ that generate high cross-sectional dependence. In this situation, the average pair-wise correlation is computed as 0.816 for $N = 10$. All size analysis are available upon request for Model B and C for Tables 2, 3 and 4.

Table 4 The exact size of $\bar{F}_{1p,\alpha}$ and $\bar{t}_{1p,\alpha}$ in case of cross-sectional dependence

T/N	$\bar{F}_{1p,\alpha}$			$\bar{t}_{1p,\alpha}$		
	5	10	25	5	10	25
Model A						
50	5.58	4.92	4.72	5.34	4.92	4.66
100	5.20	4.68	4.76	5.24	4.68	4.60
150	4.82	5.24	4.42	4.88	5.16	4.58
200	5.00	4.40	4.80	4.90	4.62	5.20

includes intercept plus trend term⁹ and the PNLV test (\bar{t}_{LNV}). To evaluate the size-adjusted power of the all test statistics, we employ the following panel ST-TAR(1) DGP under heterogeneous alternative (2.5b).

$$\begin{aligned}
 y_{it} &= \alpha_{i1} + \alpha_{i2}S_{it}(\gamma_i, \tau_i) + \varepsilon_{it} \\
 \Delta\varepsilon_{it} &= \rho_{i1}I_{it}\varepsilon_{i,t-1} + \rho_{i2}(1 - I_{it})\varepsilon_{i,t-1} + u_{it} \\
 u_{it} &= \lambda_i f_t + v_{it} \\
 v_{it} &= \rho_i v_{i,t-1} + \eta_{it}
 \end{aligned}
 \tag{5.3}$$

where $\eta_{it} \sim i.i.d.N(0, \sigma_i^2)$ with $\sigma_i^2 \sim i.i.d.U[0.5, 1.5]$. I_{it} is defined as before. Factor loadings (λ_i) and f_t are generated as Eq. (5.2). We consider two different range of the smooth transition parameters, namely $\gamma_i \sim i.i.d.U[0.5, 1.5]$ and $\gamma_i \sim i.i.d.U[3, 5]$ to represent the slow and fast transitions, respectively. All other parameters are assigned as follows: $\rho_{i1} \sim i.i.d.U[-0.9, -0.7]$, $\rho_{i2} \sim i.i.d.U[-0.2, 0]$, $\alpha_{i1} = 1$, $\alpha_{i2} \in \{2, 10\}$, $\tau_i = 0.5$. In size-adjusted power experiments, under the alternative hypothesis, we use the DGP given in Eq. (5.2) for $i = 1, \dots, [N/2]$ and the one given in Eq. (5.3) for $i = [N/2] + 1, \dots, N$ with $[N/2]$ being the nearest integer value of $N/2$.

The results from Table 5 clearly indicate that the size-adjusted power of the new test statistics $\bar{t}_{1p,\alpha}$ and $\bar{F}_{1p,\alpha}$ increase quite significantly as the number of observations (T) increase for a given panel size (N). The same result holds as the panel size increases (N) for a given number of observations (T). In addition, the increase in the value of γ_i from slow to fast, other things being equal causes the power of the newly proposed tests to decrease. This outcome is not unexpected because as the value of γ_i increases the transition resembles more an abrupt structural change rather than a smooth one, and this leads to power losses in the LNV based tests like ours. The same pattern is observed in the empirical powers of LNV test itself. However, an increase in the time series dimension (T) helps to offset this power loss. As shown in the first two panels of Table 7 for a small break ($\alpha_{i2} = 2$), irrespective of the value of the parameter γ_i , the power of IPS test \bar{t}_{IPS} exceeds that of the newly proposed tests $\bar{t}_{1p,\alpha}$, $\bar{F}_{1p,\alpha}$ and the OHS test \bar{t}_{LNV} . This may stem from the fact that the calculation of $\bar{t}_{1p,\alpha}$, $\bar{F}_{1p,\alpha}$ and \bar{t}_{LNV} requires the estimation of more parameters than does the IPS test. Thus,

⁹ Leybourne et al. (1998) pointed that Model A is capable to proxy a function containing a linear trend with certain combinations of parameters. In this context, the IPS test with intercept and linear trend is a natural competitor to our proposed $\bar{t}_{1p,\alpha}$ statistic.

Table 5 The empirical size-adjusted power comparison of alternative tests under cross-sectional dependence

<i>T/N</i>	5				10				25			
	$\bar{I}_{1p,\alpha}$	$\bar{F}_{1p,\alpha}$	\bar{I}_{LNV}	\bar{I}_{IPS}	$\bar{I}_{1p,\alpha}$	$\bar{F}_{1p,\alpha}$	\bar{I}_{LNV}	\bar{I}_{IPS}	$\bar{I}_{1p,\alpha}$	$\bar{F}_{1p,\alpha}$	\bar{I}_{LNV}	\bar{I}_{IPS}
$\gamma_i \sim U(0.5, 1.5) \quad \alpha_{2i} = 2$												
50	13.50	12.50	10.80	19.80	14.20	12.10	8.50	19.90	15.50	14.50	13.20	20.30
100	35.30	32.50	27.60	40.70	40.40	39.00	32.60	47.40	43.20	38.60	33.80	46.90
150	58.60	57.40	50.80	65.10	71.00	70.00	60.80	71.40	75.10	73.10	60.20	73.60
200	81.60	82.50	69.60	82.90	95.70	97.30	71.50	86.20	93.20	93.10	68.40	86.00
$\gamma_i \sim U(3, 5) \quad \alpha_{2i} = 2$												
50	12.80	12.30	10.50	20.10	14.10	12.40	11.70	19.60	14.10	12.70	16.80	23.50
100	35.10	32.30	27.40	41.40	41.00	37.90	34.20	48.40	42.10	40.50	36.90	48.40
150	59.80	58.20	54.20	64.10	70.30	70.20	60.40	72.90	67.70	69.50	52.00	76.40
200	81.20	83.10	70.80	82.20	86.80	89.10	76.50	86.60	86.80	86.90	69.20	91.00
$\gamma_i \sim U(0.5, 1.5) \quad \alpha_{2i} = 10$												
50	14.40	12.30	9.50	3.20	13.50	11.80	10.90	1.60	16.90	14.90	11.80	2.50
100	33.30	30.30	24.80	8.40	37.90	34.30	33.20	7.30	41.00	36.50	33.60	10.60
150	56.70	55.50	46.40	19.40	68.30	67.50	61.40	13.90	72.70	71.20	54.70	27.00
200	80.20	80.10	60.20	33.70	85.90	87.70	76.00	23.40	91.40	92.40	65.20	37.90
$\gamma_i \sim U(3, 5) \quad \alpha_{2i} = 10$												
50	12.20	10.80	9.80	5.00	12.20	10.90	10.80	2.40	15.70	14.20	13.00	3.50
100	33.50	28.60	25.20	10.50	38.40	34.30	29.30	8.00	41.00	35.80	32.40	12.40
150	57.10	55.30	42.70	25.30	68.90	67.90	57.20	16.20	74.60	72.40	52.80	24.10
200	80.90	80.40	65.80	36.60	86.60	87.80	66.70	25.20	92.70	92.90	64.20	41.40

the loss of power from having to do so seems to be offset by the gain in power from using the correctly specified data-generating process. Although the empirical power of the new tests exceed that of the OHS test for the small break case, the power of the OHS test still remains close to power of the new tests with the difference being approximately 11 % at most. On the other hand, for a large break ($\alpha_{i2} = 10$), not only the power of the IPS test collapses but also the power of the newly proposed tests $\bar{I}_{1p,\alpha}$ and $\bar{F}_{1p,\alpha}$ still exceeds that of the OHS test. Thus, ignoring large breaks and nonlinear adjustment when both are present in the panel data series produces large power losses when testing for a unit root¹⁰.

6 Empirical results

In this study, to test the null hypothesis of a unit root in the real interest rate series (RIRs) we have employed quarterly data that include observations on the nominal interest rate and the consumer price index over the period 1979:Q2—2011:Q2 (i.e. 129

¹⁰ A detailed power analysis has been conducted by OHS. For further details refer to Omay et al. (2013).

data points) for seventeen OECD member countries.¹¹ The seventeen OECD countries include Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom (UK), and United States (US).

Since the seminal work of [Rose \(1988\)](#), many studies in the literature have resorted to the *ex post* rate for investigating the stationarity properties of the RIRs. While the *ex-ante* real interest rate is the nominal interest rate minus the expected inflation rate, the *ex post* real interest rate is found by subtracting the realized inflation from the nominal rate. Economic decisions by nature depend on the *ex-ante* real interest rate; however, the *ex-ante* real rates are in fact unobservable in contrast to the *ex post* rates. To prevent the shortcomings involved in specifying the correct way, the agents form their expectations or using survey data that are not available for many countries over a long time span, we will stick with this approach and use the *ex post* real rate. An alternative solution would be to construct inflation forecasts using econometric models. However, as argued in [Neely and Rapach \(2008\)](#), such models fail to include all the necessary variables that agents use to forecast inflation and do not change as the structure of the economy changes¹². Moreover, under the assumption that expectations are formed rationally across countries, the *ex post* (actual) real interest rate will differ from the *ex-ante* real interest rate by a white noise random error which is orthogonal to past information. Therefore, as argued in [Moosa and Bahatti \(1996\)](#), under rational expectations if the *ex post* real interest rate is found to be stationary then this may be interpreted as the *ex-ante* real interest rate being constant over time. Even without the assumption of rational expectations, the difference between the two rates is a stationary nonpersistent process ([Peláez 1995](#)) that is believed to not have an intense effect on the results ([Kapetanios et al. 2003](#)).

Following [Pesaran et al. \(2013\)](#) the *ex-post* real interest rate for each country j is calculated as $r_{jt} = 0.25 \ln(1 + R_{jt}/100) - (p_{jt} - p_{j,t-1})$, where R_{jt} is the nominal rate of interest per annum in per cent in country j at time t , $p_{jt} = \ln(CPI_{jt})$ and CPI_{jt} is the consumer price index of country j at time t .

A vast majority of the studies in the literature have utilized the nominal short-term interest rate (i.e., the three month treasury-bill rate, money market rate, deposit rate or the discount rate) to calculate and test for the stationarity of the RIRs ([Kapetanios et al. 2003](#); [Neely and Rapach 2008](#); [Pesaran et al. 2013](#); [Kapetanios and Shin 2011](#)). However, the nominal long-term interest rates not only are more relevant to saving and investment decisions than the short-term rates ([Tsong and Lee 2012](#)) but also more directly reflect the financing costs for capital investments ([Obstfeld and Taylor 2002](#)). Therefore, to provide a more comprehensive investigation of the stochastic behavior of the RIRs, we utilized both the short-term and long-term nominal interest rates of the countries included in the study. The long-term government bond yield (typically the yield on ten year government bonds) is used as a proxy for the long-term nominal

¹¹ The data used in this study are retrieved from the Global VAR modeling (GVAR) database, which is publicly available at: <https://sites.google.com/site/gvarmodelling/data>. A detailed description of the data and sources are also included.

¹² See [Neely and Rapach \(2008\)](#) and the references therein for a discussion of the further challenges involved in econometrically forecasting inflation.

interest rate. The short-term (three month) nominal interest rates are the treasury-bill rates for Canada, Sweden, UK and US; money market rates for Australia, Austria, Belgium, France, Germany, Italy, Japan, Korea, the Netherlands, Norway, Spain and Switzerland; and the discount rate for New Zealand.

We have carried out a preliminary analysis to determine the stochastic and deterministic features of our real interest rate series before demonstrating the results of our panel unit root tests. The results of this analysis are given in panel A of Table 6 for both the short-term and long-term RIRs. In panel A y_{it} , γ_i and τ_i denote the original real interest rate series (i.e., the r_{jt} 's), the estimate of the transition speed parameter, and the estimate of threshold parameter, respectively. To be comparable we used the same notation given in Eq. (2.1). In the third and seventh columns of panel A, we present the estimates of the threshold parameter c_i obtained from the estimation of the TAR model with de-trended series $\hat{\varepsilon}_{it}$ given in (2.1) and (2.4) using Chan's (1993) methodology¹³. Besides, we have also tested the symmetry of the two regime TAR parameters ρ_1 and ρ_2 given in Eq. (2.4). The results of these asymmetry tests are given in the fourth and eight columns of panel A for the short-term and long-term RIRs, respectively.

As it can be seen from the second column of Table 6, the estimates of the threshold parameters τ_i are clustered around 0.5. This shows us that the structural breaks in the short-term RIRs take place at the middle of the samples. Thus, the sample countries short-term RIRs face a structural break at the middle of the 1990s. Moreover, we can also classify the break dates in the short-term RIRs of these countries with respect to the estimates of the γ_i (first column in panel A). Out of 17 countries included in the study, 8 of them have sharp transition (with estimates $\hat{\gamma}_i > 1.0$), 5 of them have smooth transition (with estimates $0.1 < \hat{\gamma}_i < 1.0$) and 2 of them have highly smooth transitions ($\hat{\gamma}_i < 0.1$)¹⁴. Therefore, the parameter values of γ_i and τ_i obtained are consistent with the power simulations.

In addition, we have investigated the de-trended series $\hat{\varepsilon}_{it}$ considering the threshold parameter and the tabulated results obtained in the third column. The results show us that the threshold parameters are located around $\hat{c}_i = 0.00$. This finding particularly justifies the usage of Enders and Granger's (1998) TAR unit root test¹⁵.

We also test the asymmetry of the parameters in the fourth column of Table 6 for the short-term RIRs. The test results exhibit a symmetric pattern in the TAR parameters. However, we can still order the TAR parameters of these countries from being the most symmetric to most asymmetric. Accordingly Spain, Japan, and Switzerland seem

¹³ The TAR model estimates are available upon request.

¹⁴ As it can be seen from the second column, we have also found out that 2 out of 17 countries have time varying mean structure, but do not have a significant threshold parameter. Therefore, we neither classify them as having sharp nor smooth structural breaks. It is better to indicate this type of estimates as nonlinear trend. Besides, these two countries, namely Austria and Germany, have significant transition speed parameters. This corroborates the fact that these countries have nonlinear trends.

¹⁵ Also Enders and Granger (1998) have proposed two alternative M-TAR unit root tests. One explicitly estimates the threshold parameter, while the other assumes it to be equal to zero. On the other hand, Sollis (2004) combined LNV structural de-trending with the EG-TAR type of unit root tests, which set the threshold parameter exogenously to zero. This type of structural de-trending (nonlinear attractor) also has a secondary effect. It automatically locates the threshold parameter of the TAR model to zero. Therefore, the threshold parameters obtained in the third column of Table 8 show us that extending Sollis (2004) directly to a panel setting is a true strategy to follow rather than explicitly finding the threshold level as in Chan (1993).

Table 6 Pre-test of deterministic and stochastic features of samples

Countries	Short-term RIRs				Long-term RIRs			
	y_{it}		$\hat{\varepsilon}_{it}$		y_{it}		$\hat{\varepsilon}_{it}$	
	$\hat{\gamma}_i$	$\hat{\tau}_i$	\hat{c}_i	Asy-test	$\hat{\gamma}_i$	$\hat{\tau}_i$	\hat{c}_i	Asy-test
<i>Panel A: time series test</i>								
Australia	13.136	0.427*	-0.005	2.721	0.719**	0.625*	0.004	2.070
Austria	0.035*	2.401	0.003	0.073	0.113*	0.647*	0.003	0.396
Belgium	1.300***	0.517*	0.005	0.516	0.114*	0.635*	-0.001	0.278
Canada	1.270	0.533*	-0.002	0.136	0.194***	0.646*	0.001	0.004
France	0.138*	0.647*	0.003	0.247	0.222***	0.684*	0.001	2.118
Germany	0.027*	2.015	0.004	0.382	0.060*	0.745*	0.000	0.437
Italy	0.791**	0.610*	0.005	2.799***	0.651***	0.605*	0.003	2.297
Japan	0.314*	0.466*	-0.002	0.014	0.939**	0.514*	-0.001	0.821
Korea	0.690***	0.648*	-0.003	0.636	8.402*	0.663*	-0.007	8.469*
Netherlands	0.391*	0.495*	0.003	1.558	0.224*	0.568*	0.002	1.223
New Zealand	8.876*	0.628*	-0.008	0.038	23.214*	0.245*	-0.006	8.227*
Norway	11.836	0.473*	-0.005	0.262	10.450**	0.534*	0.005	0.747
Spain	0.395***	0.602*	0.007	0.005	0.579*	0.594*	0.005	0.530
Sweden	0.124*	0.738*	0.006	0.155	0.136***	0.800*	0.000	0.460
Switzerland	7.803*	0.713*	0.002	0.032	14.272**	0.393*	0.000	0.893
UK	12.660***	0.442*	0.005	0.557	0.058*	1.494	0.005	1.510
USA	1.543***	0.392*	-0.002	0.670	0.095***	0.672*	0.005	0.313
	Short-term RIRs			Long-term RIRs				
	CD_{LM1}	CD_{LM2}	CD	CD_{LM1}	CD_{LM2}	CD		

Panel B: panel test of cross section dependence

4288.34*	251.77*	63.11*	3747.91*	219.00*	58.25*
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*, **, *** denote significance levels at 1, 5 % and 10 %, respectively

$\hat{\gamma}_i$ and $\hat{\tau}_i$ denotes the transition speed parameter and the threshold parameter of the LNV model. \hat{c}_i is the threshold parameter estimates obtained from the results of the TAR model. In this case the threshold value ids obtained using the Chan's (1993) method. Asy-test is obtained under the null $H_0 : \rho_1 = \rho_2$

to have the most symmetric TAR estimation results, whereas Italy, Australia and the Netherlands are the most asymmetric ones. Therefore, we expect to find the short-term RIRs of Italy, Australia and the Netherlands' to be stationary around a nonlinear trend with our proposed panel unit root test in SPSM procedure. On the other hand, these countries are not expected to be found stationary using the PLNV test¹⁶. Fortunately, our proposed test has power against symmetric cases as well. Therefore, the countries which are found symmetric in the fourth column should be found as stationary with the newly proposed tests as well, since the newly proposed test nests the PLNV test. The same results are obtained in case of the long-term RIRs.

¹⁶ These 3 countries are indeed found to be nonstationary with the implementation of the PLNV test (Table 7).

In panel B of Table 6, cross-sectional dependence of the short-term and long-term RIRs are tested using the CD tests proposed by Pesaran (2004). All the CD test results indicate high cross sectional dependence between the RIRs of the countries in the sample. Therefore, when testing for the integration properties of the RIRs, the cross-sectionally dependent versions of our proposed tests (bootstrap) should be used.

To test whether the short-term and long-term RIRs exhibit mean reversion, in addition to our newly proposed combined tests $\bar{t}_{1p,\alpha}$ and $\bar{F}_{1p,\alpha}$, we have also applied two alternative panel unit root tests to the RIRs of the OECD countries for comparison. The first of these tests is the conventional panel unit root test of IPS. However, an aforementioned weakness of the standard IPS test is that it ignores the possibility of breaks. Since the literature has documented that the RIRs may display structural breaks, the panel LNV test was applied as a second test to the RIRs that allows for both smooth breaks and cross sectional dependence. However, both the IPS and the panel LNV tests are based on the assumption that the RIRs follow a linear adjustment process, which may not be true according to the existing empirical evidence. Thus, to consider the presence of structural breaks and nonlinearity simultaneously, we further apply our proposed tests to the RIRs that also uses additional information from the cross sectional units to achieve a power gain.

The results of the three aforementioned tests are given in Tables 7 and 8 for the short-term and long-term RIRs, respectively. All of the three tests are applied on a reducing dataset using the SPSM procedure¹⁷.

Evidence from panel A of Table 7 shows that the IPS test rejects the null hypothesis of a unit root in the short-term RIRs for the entire panel and in only 6 countries out of 17. The 11 short-term RIRs that are found nonstationary according to the linear panel unit root test belongs to USA, Austria, Japan, Germany, France, Spain, Korea, UK, Belgium, the Netherlands, and Italy. On the other hand, when potential breaks in the RIRs are considered and the panel unit root test with breaks (the panel LNV test) is applied to the panel, the results again point to the stationarity of the panel around a broken trend, but this time with 12 out of 17 countries behaving like an I(0) process (panel B). Thus, accounting for structural breaks within a panel context has considerably increased the number of countries for which the unit root null is rejected for the short-term RIRs, namely the RIRs of only 5 countries (Austria, Spain, Australia, the Netherlands, and Italy) are found to be nonstationary.

At this point, two main conclusions can be made with regard to our findings in Table 7. First, the existence of unit root in the short-term RIRs cannot be conclusively refuted for more than 50%—about 65%—of our sample when the linear panel unit root test without breaks is applied. Second, by considering the presence of structural

¹⁷ Applying the panel unit root tests with the sequential panel selection method (SPSM) proposed by Chortareas and Kapetanios (2009) has allowed us to distinguish the panel into its stationary and nonstationary components. For further technical details on this methodology refer to Chortareas and Kapetanios (2009). On the other hand, recently Costantini and Lupi (2014) have investigated the performance of the SPSM proposed by Chortareas and Kapetanios (2009). The authors suggest that there are some classes of panel unit root tests that are better suited than others to be applied jointly with the SPSM such as the IPS test. Besides, they also recommend that for the Chortareas and Kapetanios (2009) methodology to be used the sample in the investigation must be fairly large and most of the series in the sample should be I(0). Fortunately, our sample fulfills these criteria. For further discussion see Costantini and Lupi (2014).

Table 7 Panel unit root test results for short-term real interest rates

Sequence	Series	\bar{t}_{IPS}	Min. ADF	Sequence	Series	\bar{t}_{LNV}	Min. LNV
<i>Panel A: linear unit root tests without structural break—IPS test</i>				<i>Panel B: linear unit root tests with smooth structural break—PLNV test</i>			
1	Switzerland	-2.611*	-4.666	1	Japan	-4.788*	-7.102
2	N. Zealand	-2.482*	-4.209	2	Germany	-4.643*	-6.827
3	Canada	-2.367**	-3.929	3	Canada	-4.498*	-6.770
4	Sweden	-2.256**	-3.830	4	Switzerland	-4.335*	-5.812
5	Australia	-2.135***	-3.682	5	Sweden	-4.222*	-5.479
6	Norway	-2.006***	-2.953	6	UK	-4.117*	-5.353
7	USA	-1.920	-2.803	7	Korea	-4.004*	-5.158
8	Austria	-1.831	-2.768	8	Belgium	-3.889**	-5.001
9	Japan	-1.727	-2.526	9	Norway	-3.765**	-4.547
10	Germany	-1.627	-2.311	10	N. Zealand	-3.668**	-4.473
11	France	-1.530	-2.210	11	France	-3.553***	-4.191
12	Spain	-1.416	-1.959	12	USA	-3.446***	-4.165
13	Korea	-1.308	-1.578	13	Austria	-3.304	-4.161
14	UK	-1.240	-1.519	14	Spain	-3.088	-3.869
15	Belgium	-1.147	-1.206	15	Australia	-2.828	-3.363
16	Netherlands	-1.118	-1.201	16	Netherlands	-2.560	-2.893
17	Italy	-1.034	-1.034	17	Italy	-2.227	-2.227
Sequence	Series	$\bar{F}_{1p,\alpha}$	Min. $\bar{F}_{1p,\alpha}$	Sequence	Series	Max. $\bar{t}_{1p,\alpha}$	Min. Max. $\bar{t}_{1p,\alpha}$
<i>Panel C: nonlinear unit root tests with smooth structural break—OEE-F and OEE-t tests</i>							
1	Japan	14.970*	25.182	1	UK	-4.589*	-6.672
2	UK	14.331*	24.804	2	Japan	-4.459*	-6.238
3	Germany	13.633*	23.580	3	Germany	-4.340*	-5.611
4	Canada	12.923*	22.895	4	Canada	-4.250*	-5.501
5	USA	12.156*	17.263	5	N. Zealand	-4.153*	-4.889
6	Switzerland	11.730*	17.148	6	Switzerland	-4.092*	-4.889
7	Sweden	11.238*	13.649	7	USA	-4.020*	-4.635
8	Belgium	10.996*	12.724	8	Sweden	-3.958*	-4.465
9	N. Zealand	10.804*	12.416	9	Korea	-3.902*	-4.446
10	Australia	10.603*	12.027	10	Australia	-3.834*	-4.401
11	Italy	10.400*	11.826	11	Belgium	-3.753*	-4.226
12	Korea	10.162*	11.525	12	Netherlands	-2.755*	-4.179
13	Spain	9.889*	11.507	13	Italy	-3.573*	-3.901
14	Norway	9.485*	10.255	14	Spain	-3.491*	-3.784
15	Netherlands	9.228*	10.058	15	Norway	-3.393*	-3.666

Table 7 continued

Sequence	Series	$\bar{F}_{1p,\alpha}$	Min. $\bar{F}_{1p,\alpha}$	Sequence	Series	Max. $\bar{t}_{1p,\alpha}$	Min. Max. $\bar{t}_{1p,\alpha}$
<i>Panel C: nonlinear unit root tests with smooth structural break—OEE-F and OEE-t tests</i>							
16	France	8.813*	8.871	16	Austria	-3.257**	-3.372
17	Austria	8.756**	8.756	17	France	-3.141***	-3.141

In all cases *, **, and *** denote rejection of the unit root null hypothesis at 1, 5 and 10 % significance levels, respectively. We include only a constant term in the test regressions. Test results are obtained employing 1000 bootstrap replications. In each panel, the countries given in the second column are sorted in a descending order according to the univariate counterparts of each panel unit root test applied on the RIRs of the OECD countries included in the study. Min. ADF denotes the individual minimum ADF (the univariate counterpart of the IPS test) statistics that are used to decide on the individual series to be dropped from the panel when implementing the SPSM with the IPS test. Min. LNV denotes the individual minimum LNV (the univariate counterpart of the panel LNV test) statistics that are used to carry out the reduction of the individual series for the SPSM and PLNV test. Min. $\bar{F}_{1p,\alpha}$ and Max. $\bar{t}_{1p,\alpha}$ denote Sollis's (2004) individual minimum F - and t -statistics (the univariate counterparts of our proposed tests) that are used to carry out the reduction of the individual series for the SPSM and the OEE-F and OEE-t tests

Table 8 Panel unit root test results for long-term real interest rates

Sequence	Series	\bar{I}_{IPS}	Min. ADF	Sequence	Series	\bar{t}_{LNV}	Min. LNV
<i>Panel A: linear unit root tests without structural break—IPS test</i>				<i>Panel B: linear unit root tests with structural break—PLNV test</i>			
1	N. Zealand	-2.498*	-5.048	1	Switzerland	-4.780*	-6.520
2	Australia	-2.339*	-3.768	2	Belgium	-4.672*	-6.467
3	Sweden	-2.243*	-3.766	3	Japan	-4.552*	-6.197
4	Switzerland	-2.135**	-3.166	4	Austria	-4.434*	-5.771
5	UK	-2.055**	-2.822	5	Canada	-4.332*	-5.760
6	Germany	-1.991***	-2.782	6	N. Zealand	-4.213*	-5.717
7	Norway	-1.919	-2.647	7	Germany	-4.076**	-5.646
8	Austria	-1.847	-2.645	8	Spain	-3.919**	-5.461
9	USA	-1.758	-2.500	9	Australia	-3.748**	-5.233
10	Japan	-1.665	-2.161	10	Norway	-3.562**	-5.124
11	Canada	-1.594	-2.066	11	USA	-3.339***	-4.584
12	Italy	-1.516	-1.838	12	Netherlands	-3.131	-4.297
13	Korea	-1.451	-1.781	13	UK	-2.898	-3.777
14	Netherlands	-1.369	-1.765	14	France	-2.678	-3.700
15	France	-1.237	-1.724	15	Sweden	-2.337	-3.174
16	Spain	-0.994	-1.470	16	Korea	-1.919	-2.646
17	Belgium	-0.517	-0.517	17	Italy	-1.193	-1.193
Sequence	Series	$\bar{F}_{1p,\alpha}$	Min. $\bar{F}_{1p,\alpha}$	Sequence	Series	Max. $\bar{t}_{1p,\alpha}$	Min. Max. $\bar{t}_{1p,\alpha}$
<i>Panel C: non-linear unit root tests with structural break—OEE-F and OEE-t tests</i>							
1	Korea	16.626*	22.837	1	Korea	-4.981*	-6.692
2	UK	16.238*	22.460	2	UK	-4.874*	-6.501

Table 8 continued

Sequence	Series	$\bar{F}_{1p,\alpha}$	Min. $\bar{F}_{1p,\alpha}$	Sequence	Series	Max. $\bar{t}_{1p,\alpha}$	Min. Max. $\bar{t}_{1p,\alpha}$
<i>Panel C: non-linear unit root tests with structural break—OEE-F and OEE-t tests</i>							
3	Belgium	15.823*	21.791	3	N. Zealand	-4.766*	-5.958
4	Switzerland	15.397*	21.336	4	Japan	-4.681*	-5.793
5	Japan	14.940*	19.698	5	Australia	-4.595*	-5.745
6	Australia	14.544*	19.135	6	Switzerland	-4.499*	-5.475
7	N. Zealand	14.126*	18.658	7	Belgium	-4.410*	-5.248
8	Austria	13.673*	16.742	8	Germany	-4.327*	-4.881
9	Canada	13.332*	16.530	9	Norway	-4.265*	-4.854
10	Germany	12.932*	16.139	10	Austria	-4.191*	-4.774
11	Italy	12.474*	15.182	11	Canada	-4.108*	-4.745
12	Spain	12.023*	14.790	12	Italy	-4.002*	-4.485
13	USA	11.470*	14.681	13	Netherlands	-3.906*	-4.449
14	Norway	10.667*	13.754	14	Spain	-3.770*	-4.233
15	Netherlands	9.638*	13.160	15	USA	-3.615*	-4.188
16	Sweden	7.877**	8.907	16	Sweden	-3.329**	-3.722
17	France	6.846	6.846	17	France	-2.936	-2.936

In all cases *, **, and *** denote rejection of the unit root null hypothesis at 1, 5 and %10 significance levels, respectively. We include only a constant term in the test regressions. Test results are obtained employing 1000 bootstrap replications. In each panel, the countries given in the second column are sorted in a descending order according to the univariate counterparts of each panel unit root test applied on the RIRs of the OECD countries included in the study. Min. ADF denotes the individual minimum ADF (the univariate counterpart of the IPS test) statistics that are used to decide on the individual series to be dropped from the panel when implementing the SPSM with the IPS test. Min. LNV denotes the individual minimum LNV (the univariate counterpart of the panel LNV test) statistics that are used to carry out the reduction of the individual series for the SPSM and PLNV test. Min. $\bar{F}_{1p,\alpha}$ and Max. $\bar{t}_{1p,\alpha}$ denote Sollis's (2004) individual minimum F - and t -statistics (the univariate counterparts of our proposed tests) that are used to carry out the reduction of the individual series for the SPSM and the OEE- F and OEE- t tests

breaks in the RIRs and further applying the panel LNV test we can uncover mean reverting dynamics in the RIRs for about 70 % of the sample.

Altogether our findings so far highlight the importance of taking structural breaks into account when studying the stochastic properties of the RIRs. However, even if a possible structural break in the short-term RIRs was taken into account, the test did not yield very strong evidence that the short-term RIRs behave like an $I(0)$ process. The panel LNV test still cannot reject the unit root null in 30 % of the countries, though it provides additional 6 rejections of unit root compared to the results of the IPS test. On the whole, neither the linear panel model nor the panel model with smooth breaks can overwhelmingly support the long-run mean reversion of the short-term RIRs for our sample. Hence, the more powerful tests proposed in this study that exploits the possibility of nonlinearities and breaks in the RIRs simultaneously will be implemented.

Panel C of Table 7 contains the results of our proposed tests on the short-term RIRs. By applying the same SPSM outlined above, the null hypothesis of a unit root

in the short-term RIRs was rejected for all the countries included in the study. Thus, we have obtained conclusive evidence in favor of the long-run mean reversion of the short-term RIRs for all the OECD countries included in the study. Ability to obtain overwhelming evidence of mean reversion seems to suggest that nonlinearity and breaks in the short-term RIRs co-exist in all the countries we studied.

Table 8 contains the results on the long-term RIRs. The results are similar to those obtained for the short-term RIRs. We again started by conducting a standard IPS panel unit root test with bootstrap and the SPSM methodology (Panel A in Table 7) and concluded that for a vast majority of the countries the unit root null hypothesis cannot be rejected (only in 6 out of 17 cases, which corresponds to about 35 % of rejections). Afterward, to consider the presence of structural breaks in the long-term RIRs, we applied the panel smooth break unit root test with again bootstrap and the SPSM methodology (Panel B in Table 7), but even in this case we were able to provide only some support for the long-run mean reversion of the long-term RIRs (about 65 % of rejections of the unit root null hypothesis). This finding probably suggests that allowing for structural breaks only offers a partial solution to the extensive non-rejection of the unit root null again in the long-term RIRs.

On the other hand, when both nonlinearity and smooth breaks in the long-term RIRs are simultaneously considered while tackling cross-sectional dependence, the null hypothesis of a unit root in the RIRs is considerably rejected in Panel C in sharp contrast with the results in Panels A and B. Using our proposed tests together with the SPSM methodology outlined above, we find that 16 out of 17 countries exhibit mean-reverting dynamics in their long-term RIRs. Particularly, the unit root null is not rejected only for France. Comparing the results in Panels B and C, we find that an additional five rejections of the unit root dynamics can be found in our sample, including the Netherlands, UK, Sweden, Korea, and Italy. More rejections for the long-term RIRs may again be attributed to the improved power of our proposed tests.

After determining the stochastic properties of the RIR series, we next employed a heterogeneous Panel ST-TAR estimation. As in Sollis (2004), this estimation is carried out in two steps. First the deterministic component is estimated (the smooth trend is removed) and then the panel TAR estimation is carried out using Eq. (2.4) and thus the residuals obtained from the first step. For the short-term RIRs, we have found out that the group mean estimator of the speed of adjustment coefficient is -0.549 ($\hat{\rho}_1$) if $\varepsilon_{i,t-1} \geq 0$, which is also classified as the high regime, and -0.572 ($\hat{\rho}_2$) for the low regime. Both of the regime parameters are significant at 1 % significance level. In case of the long-term RIRs the estimates obtained are -0.549 and -0.647 for the high regime and low regime, respectively. Thus, the RIRs are found to display differential speeds of adjustment with the positive phase of the RIR sequence being more persistent than the negative phase, which implies faster adjustment for negative RIRs than for positive ones.

7 Conclusion

Real interest rate remains one of the crucial variables in the macroeconomics and finance literature. Although many important theoretical models require the real inter-

est rates to be stationary, the empirical evidence accumulated so far has not been able to provide conclusive evidence on the mean reverting dynamics of this variable. Thus, serious questions have been raised about the validity of many important macroeconomic relationships, models and hypotheses involving stationary real interest rates. In this study we show that a potential solution to this problem is using a test that is specifically designed to fit the stochastic features of the real interest series. A researcher should consider all factors pertinent to a situation when conducting an analysis. Likewise, to investigate the stochastic properties of the real interest rates one should also utilize a test that allows for the possibility of *nonlinearity* and *structural breaks* simultaneously in a panel setting that induces additional power gains with the use of cross-sectional information. Accounting for cross-sectional dependence is especially important in analyzing macroeconomic variables with close cross country links like the real interest rates.

The empirical power of our newly proposed unit root tests are compared with the powers of two alternative panel unit root tests—the IPS test ignoring both nonlinearity and structural breaks and the PLNV test considering only structural breaks—using Monte Carlo simulation exercises. Under the hypothesis that the data are generated by a globally asymmetric stationary ST-TAR process our tests are shown to have better small sample properties than the aforementioned tests.

When the newly proposed panel unit root tests are applied to the short-term and long-term real interest rates of the 17 OECD countries we were able to uncover overwhelming empirical support in favor of the mean reversion in the real interest rates. Particularly, the unit root null is rejected for all OECD countries when the new panel unit root tests are applied to the short-term RIRs. Our findings imply that for all OECD countries included in our study, a one-time shock to the short-term real interest rate will have no permanent effect on the economy. In case of the long-term RIRs, the unit root null is not rejected only in France, which implies that a one-time shock to the long-term real interest rate in France would be permanent and could not be eliminated with the passage of time.

Overall, the empirical results verify mean reversion in the ex post OECD real interest rates. However, the main contribution of this paper is twofold. First we show in this paper that the mean (or equilibrium value) to which the ex post real interest rates revert in the long-run, is not a constant but changes smoothly over time. However; as argued in Kesriyeli et al. (2006) while most studies have allowed for nonlinear dynamics in the interest rates, they have assumed that the interest rate dynamics was constant over time. Researchers have also typically assumed that the interest rate policy was constant in the period of relatively low interest rates since 1984. The authors argue that such assumptions may lead to significant misspecification. We show that this misspecification carries over to our panel of OECD countries. The long-run equilibrium real interest rate may be altered by forces caused by important structural events. In the general equilibrium growth models, these forces include permanent changes in the exogenous rate of time preference, risk aversion, or long-run growth rate of technology. In other more complicated macroeconomic models, permanent changes in public purchases and their financing or changes in the steady-state money growth rate also induce changes in the long-run equilibrium or steady-state real interest rate.

Besides the aforementioned long-run behavior of the real rates, the second contribution of this paper is to show that in the short-run the real interest rate adjusts asymmetrically toward this endogenously changing equilibrium real interest rate. The estimation results demonstrate that asymmetric behavior is detected between the regimes of positive and negative real interest rates. More specifically, the mean reverting process that governs the real interest rate adjustment to its long-run equilibrium value depends on the sign of the disequilibrium or shock. This implies that the real interest rate adjusts differently or asymmetrically to positive and negative deviations of the real interest rate from its long-run equilibrium value¹⁸. Particularly, for the OECD real interest rates, we have demonstrated that the real interest rates tend to adjust faster to equilibrium for negative real interest rates than for positive ones. This observed sign asymmetry can be rationalized with the existence of asymmetric optimal policy rules that arise in case of inflation and/or output targeting¹⁹. According to the nonlinear Phillips curve literature positive aggregate demand shocks manifested in the form of positive inflation and/or output gaps (i.e., occurring in the expansionary phase of the business cycle) are more inflationary than negative shocks (i.e. in case of recessions) which are disinflationary. As argued in Schaling (2004), a positive deviation of inflation from its target level causes the real interest rates to be negative in the short-run. The danger of inflation caused in this case becomes much more severe than the linear case. This causes an inflation targeting central bank to increase the nominal rates by more than if the output inflation trade-off was symmetric²⁰. Similarly, in case of a negative deviation, the real rates will be positive in the short-run and the accompanying disinflationary pressure will be less than in the linear case. Hence, the central bank will only call for a small cut in the nominal rate. In other words, with nonlinear Phillips curve and inflation targeting the optimal monetary policy rule is asymmetric: Positive deviations from the inflation target and or in the output gap imply higher (absolute values of) real interest rates than negative deviations. Thus, inflation targeting central banks react more aggressively to positive aggregate demand shocks that cause negative real interest rates in the short-run and thereby induce a relatively rapid adjustment in the real rates toward equilibrium when they are in the negative regime. The same result carries over to central banks that give equal weights to both output and inflation stabilization (the case of flexible inflation targeting)²¹.

¹⁸ Following Schaling (2004), the long-run equilibrium value in our model is set to zero.

¹⁹ For nonlinear optimal monetary policy rules see Schaling (2004) and Dolado et al. (2005); and the references therein.

²⁰ Inflation targeting here is interpreted in strict terms in the sense that it refers to the case in which the central bank only aims to minimize only deviations of inflation from its assigned target. With no weight given to output stabilization in the central bank's loss function, the real interest rate becomes a nonlinear function of the deviation of the inflation rate from its target (i.e., inflation gap) and the output gap. Thus, a central bank with strict inflation targeting still responds to output gaps. See Schaling (2004) for further details.

²¹ Flexible inflation targeting refers to the case in which the goal of monetary policy is to stabilize not only inflation around the target level (as in the case of strict inflation targeting) but also output around its own target level. This causes again the optimal real interest rate to be a nonlinear function of the deviation of the inflation rate from its target and the output gap. See Schaling (2004) for further details.

8 Appendix

As it can be seen from the generated density functions obtained for different initial values of γ and τ , critical values at convergence are clearly not sensitive to the choices of initial values as stated in [Leybourne et al. \(1998\)](#). Therefore, the test statistics obtained are invariant to parameters α , β , γ and τ . In order to be accurate, we increase the trial number from 1000 to 50000. After carrying out this new simulation experiment, we have obtained the same density functions for the Max t -test and F -Tests as follows, with different initial values of γ and τ (Figs. 1, 2, 3 and 4).

From the figures given above, it can be clearly seen that the critical values are invariant to the parameters α , β , γ and τ in NLS estimation with respect to the choice of different initial values.

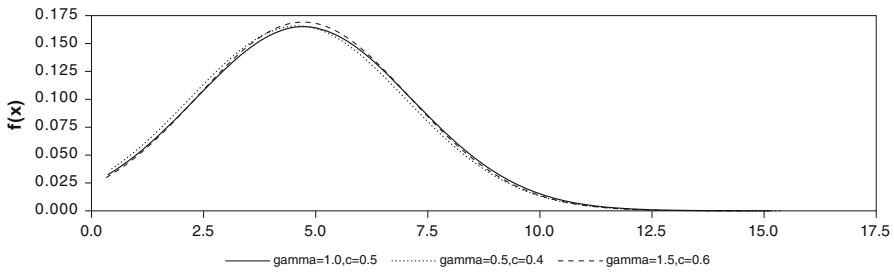


Fig. 1 Density functions comparison of F test

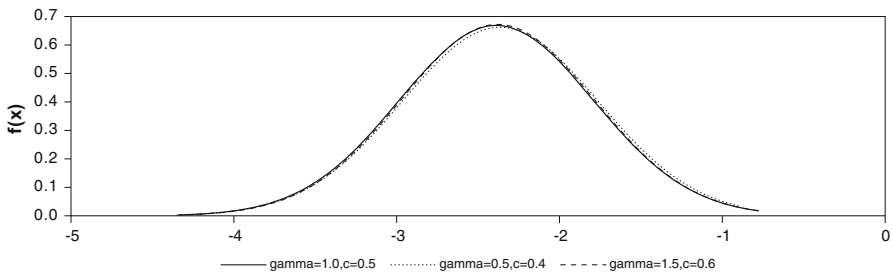


Fig. 2 Density functions comparison of Max- t test

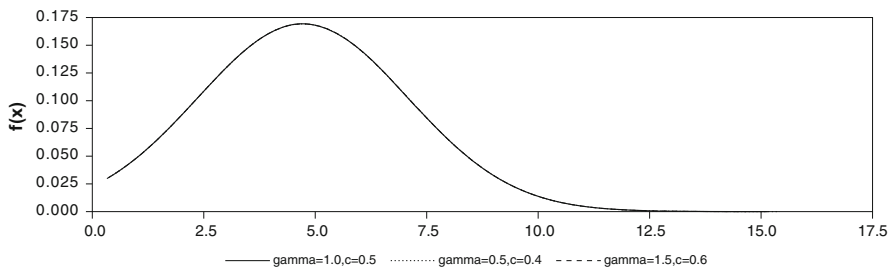


Fig. 3 Density functions comparison of F test

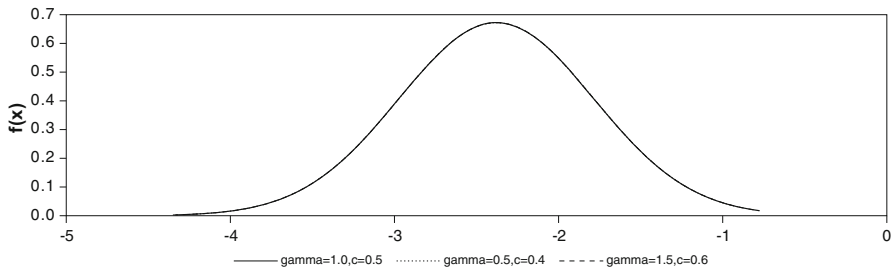


Fig. 4 Density functions comparison of Max- t test

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