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Comparative measures of multidimensional deprivation in the European Union

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Abstract This paper proposes new fuzzy measures of monetary poverty and also non-monetary deprivation, providing an economic interpretation of the parameters involved. For non-monetary deprivation, the paper provides a step-by-step procedure: dimensions or groupings of initial items of deprivation are identified using explanatory and confirmatory factor analyses, and a weighting system is applied for the aggregation of individual items into the dimension they represent. The methodology is applied to European Union countries using European Union-Statistics on Income and Living Conditions (EU-SILC) data for the 2011 survey wave.

Keywords Multidimensional poverty · Deprivation · Fuzzy sets · EU-SILC

JEL Classification I32 · C43 · C81

1 Introduction

In order to understand poverty, deprivation and social exclusion, it is necessary to consider deprivation simultaneously in its multiple dimensions—low income as well as different non-monetary aspects of deprivation. Using fuzzy set representation of individual risks to poverty, this paper proposes a multidimensional methodology for poverty and deprivation. The emphasis of the paper is on empirical application in a multi-country comparative context.

The need to adopt a multidimensional approach has been noted, among others, by Atkinson and Bourguignon (1982), Tsui (1985), Maasoumi (1986) and Sen (1999). The multidimensional nature of poverty has been widely recognised, not only by

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the scientific community, but also by many official statistical agencies (e.g. Eurostat, Istat) and by international institutions (United Nations, World Bank). In the literature, diverse approaches to the study of poverty under a multidimensional perspective appear to fall into two classes: non-axiomatic versus axiomatic approaches. A non-axiomatic approach implicitly considers indicators in different dimensions of deprivation to be perfectly substitutable. In this approach, different indicators are combined in order to obtain a multidimensional index. This combination can be done at macro-level or at individual level.[1] By contrast, Bourguignon and Chakravarty (2003) and Chakravarty et al. (1998) have contributed to the development of an axiomatic approach.

In the present work, we go beyond the conventional study of poverty based simply on the poor/non-poor dichotomy defined in relation to some chosen poverty line. Instead, poverty and multidimensional deprivation are treated as matters of degree determined in terms of the individual's position in the distribution of income and other aspects of living conditions. The state of deprivation is thus seen in the form of 'fuzzy sets' to which all members of the population belong but to varying degrees. In this way, we are able to clarify and propose a solution for one of the problems raised in the literature, e.g. in Atkinson et al. (2002), Duclos et al. (2001) and especially Atkinson (2003), who notes: 'How can different attributes be aggregated? … A distinction may be drown between those who adopt a union approach and those who use an intersection measure…'.

A number of authors have evoked the concepts of fuzzy sets in the analysis of poverty and living conditions; the present contribution represents a continuation and further development of the work of Cerioli and Zani (1990), Cheli and Lemmi (1995) and Betti and Verma (2008).[2]

The remainder of the present paper is organised as follows. Section 2 describes a set of fuzzy measures of monetary and non-monetary deprivation. The fundamental choices in developing fuzzy measures are (1) the specification of the 'membership function' concerning the degree to which individual units in the population belong to a particular set and (2) rules for the manipulation of fuzzy sets, in particular the various forms of their union, intersection and aggregation. We specify our choices clearly, but only briefly, referring to the literature for details.

Section 3 contains proposals for a step-by-step procedure for calculating non-monetary deprivation measures; this represents the original theoretical contribution of the paper. The empirical contribution of the paper is in Sect. 4, which presents in considerable detail the main results of the methodology applied to the 2011 wave of EU-SILC data. We compare conventional and fuzzy deprivation measures, emphasising the multi-country comparative aspects as well as the overall EU-wide patterns. Section 5 concludes the paper.

[1] At macro-level see Anand and Sen (1997), while at individual level see Townsend (1979).

[2] See also Cheli and Betti (1999) and Betti et al. (2002, 2004) for a longitudinal approach to poverty analysis using fuzzy sets, the book of Lemmi and Betti (2006) for further contributions on philosophy, mathematics, economics of the fuzzy set approach to poverty measurement, and the recent contributions of Belhadj (2011, 2012), Alkire and Foster (2011), Belhadj and Limam (2012) and Betti et al. (2013).

2 Fuzzy measures of monetary and non-monetary deprivation

In the conventional approach, poverty is characterised by a dichotomisation of the population into the poor and the non-poor, defined in relation to some chosen poverty line which may, for instance, represent a certain percentage (50, 60 or 70 %) of the mean or the median of the equivalised income[3] or consumption distribution.

This approach presents two main limitations: firstly, it is one-dimensional, i.e. it refers to only one proxy of poverty, namely low income or low consumption expenditure, and secondly it divides the population into a simple dichotomy. However, poverty is a complex phenomenon that cannot be reduced solely to the monetary dimension and must also take account of non-monetary indicators of living conditions. Moreover, it is not an attribute that characterises an individual in terms of its presence or absence, but is rather a predicate that manifests itself in different shades and degrees.

The fuzzy approach considers poverty as a matter of degree rather than an attribute that is simply present or absent for individuals in the population.

An early attempt to incorporate the concept of poverty as a matter of degree at methodological level was made by Cerioli and Zani (1990) who drew inspiration from the theory of *Fuzzy Sets* initiated by Zadeh (1965). Subsequently, Cheli and Lemmi (1995) proposed the so-called *Totally Fuzzy and Relative* (TFR) approach in which the membership function[4] is defined as the distribution function of income, normalised (linearly transformed) so as to equal 1 for the poorest and 0 for the richest person in the population.

In this paper, we use throughout the more generalised form of the membership function of monetary and non-monetary deprivation from Betti et al. (2006), define for any individual i as:

$$\mu_{i,K} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma | X_\gamma > X_i}{\sum_{\gamma=2}^{n} w_\gamma | X_\gamma > X_1} \right)^{\alpha_K - 1} \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma X_\gamma | X_\gamma > X_i}{\sum_{\gamma=2}^{n} w_\gamma X_\gamma | X_\gamma > X_1} \right) \quad (1)$$

where X is the equivalised income in the monetary deprivation, or the overall score s in the non-monetary deprivation [see Eq. (4) in Sect. 3 below]; w_γ is the sample weight of individual of rank $\gamma (\gamma = 1, \ldots, n)$ in the ascending income distribution, and $\alpha_K (K = 1, 2)$ are two parameters corresponding, respectively, to monetary and non-monetary aspects of deprivation. Each parameter α_K is estimated so that the mean of the corresponding membership function is equal to the at-risk-of-poverty rate (ARPR) computed on the basis the official poverty line (60 % of the median national equivalised income). All measures refer to the income distribution at the level of individual country.

[3] The equivalised income of a household is obtained by dividing its total disposable income by the household's equivalent size computed by using an equivalence scale which takes into account the actual size and composition of the household.

[4] *Membership function* (m.f.) is a quantitative specification of an individual's or household's degree of poverty or deprivation.

Both the parameters α_K have an economic interpretation: the mean of the membership functions is expressible in terms of the generalised Gini measures G_{α_K}, which is a generalisation of the standard Gini coefficient, $\frac{\alpha_K + G_{\alpha_K}}{\alpha_K(\alpha_K+1)} = \text{ARPR}$.

The monetary-based indicator has been defined as Fuzzy Monetary (FM), while the non-monetary indicator has been defined as Fuzzy Supplementary (FS).

Some benchmarking such as the above is necessary for fixing the scale of the fuzzy measures. In addition to FM and FS as defined above, one may construct fuzzy measures bench-marked for all EU countries to a single at-risk-of-poverty rate computed using a poverty line based on the whole EU equivalised income distribution. The choice of a common EU benchmark makes the resulting measures more 'absolute' in the sense that national differences in them reflect not only differences in relative disparities, but also differences among countries in the average level of living. Some indicative results using a common EU threshold will be presented in Sect. 4.1. Generally, however, we have not chosen this alternative since it loses the direct correspondence between conventional and fuzzy measures of poverty levels at the country level.

In a multidimensional analysis, it is of interest to know the extent to which deprivation in different dimensions overlaps for individuals. Similarly, in a longitudinal analysis of deprivation, the interest is to determine how far the state of deprivation of individuals persists over time. For these purposes, the fuzzy measures for an individual's deprivation as defined above need to be put together over different dimensions and different time periods. Such analysis requires the specification of rules for the manipulation of fuzzy sets, such as defining aggregation, union and intersection of sets. Any such rules for fuzzy sets must of course reproduce the corresponding rules for crisp sets of the conventional conceptualisation with the membership function reduced to a simple {0,1} dichotomy. However, this requirement of consistency is not sufficient to define the fuzzy rules uniquely: different sets of rules can meet this requirement (Klir and Yuan 1995). We have discussed this issue elsewhere (Betti et al. 2006) and will merely note the most important results here.

The union and intersection of two *similar states* (say, with deprivation membership functions a_i and b_i for individual i) are given by the so-called standard rules for fuzzy manipulation:

$$\begin{aligned} U_i &= \max(a_i, b_i) \\ I_i &= \min(a_i, b_i). \end{aligned}$$

Quantities a_i and b_i may refer to, for instance, deprivation in two different dimensions, or to deprivation at two different times. Union (U_i) means deprivation in any of the two dimensions or at any of the two times under consideration. Interaction (I_i) means the simultaneous presence of deprivation in both dimensions or at both times.

Intersection of two *diverse states* (deprivation in one dimension but non-deprivation in the other; similarly, movement between deprivation and non-deprivation over time) is given by the 'bounded' rule of fuzzy intersection:

$I_i^{(a)} = \max(0, a_i - b_i) = \max(0, a_i + \bar{b}_i - 1)$ for deprivation a_i, non-deprivation $\bar{b}_i$; or
$I_i^{(b)} = \max(0, b_i - a_i) = \max(0, \bar{a}_i + b_i - 1)$ for deprivation b_i, non-deprivation $\bar{a}_i$.

3 Construction of fuzzy supplementary indicator

In this paper, we propose a step-by-step procedure for calculating non-monetary deprivation measures. To quantify and put together diverse indicators of deprivation, several steps are necessary:

1. Identification of items of deprivation to be included in the analysis;
2. Transformation of the items into the [0, 1] interval;
3. Exploratory and confirmatory factor analysis to identify dimensions of deprivation;
4. Calculation of weights within each dimension (each group);
5. Calculation of scores for each dimension;
6. Calculation of an overall score and the parameter α of Eq. (1);
7. Construction of the fuzzy deprivation measures, both overall and separately in each dimension.

3.1 Identification of items

In the present paper, we use data from the European Union-Statistics on Income and Living Conditions (EU-SILC), distributed by Eurostat. The EU-SILC survey was designed to collect detailed information on the income of each household member, and on various aspects of the material and demographic situation of the household. Firstly, from a large set of EU-SILC variables, a selection has been made of items which are substantively meaningful and useful for the construction of non-monetary indicators. All these items are considered at the household level, even if some of them are taken from the individual dataset and then aggregated to household level.

The first set of items regards the lack of possession of a widely desired item. These are telephone including mobile phone ($a1$); colour TV ($a2$); computer ($a3$); washing machine ($a4$); and car ($a5$). In all these cases, we consider a household to be deprived only if the lack of the item is enforced, in the sense that the household would like to have the item but cannot afford it. The second set of items relates to the lack of ability to afford items that are considered as basic: keeping home adequately warm ($b1$); paying for one week annual holiday away from home ($b2$); eating a meal with meat, chicken, fish (or vegetarian equivalent) every second day ($b3$); and being able to meet unexpected financial expenses ($b4$). The third set relates to absence of housing facilities considered so basic that one can presume that all households wish to have them: bath or shower in dwelling ($c1$); and indoor flushing toilet for sole use of the household ($c2$). The fourth set of items relates to problems with accommodation and the environment, with the implicit assumption that the households wish to avoid such problems: leaking roof, damp walls/floors/foundation, or rot in window frames or floor ($d1$); too dark, not enough light in dwelling ($d2$); noise from neighbours or from the street ($d3$); pollution, grime or other environmental problems ($d4$); and crime violence or vandalism in the area ($d5$).

The fifth set relates to financial problems in terms of arrears in paying bills that the household has experienced in the last 12 months: arrears on mortgage or rent payments ($e1$); arrears on utility bills ($e2$); and arrears on hire purchase instalments or other loan

payments ($e3$). The sixth set is just one, subjective but powerful, item related to the household's perception of its ability to make ends meet ($f1$).

The seventh set relates to the health condition of the household members. These items are from individual variables that have been aggregated to household level. We include this dimension because we believe that, in dealing with lifestyle deprivation, a lack of good health is also important. The items considered are general health ($g1$); suffering from a chronic (long-standing) illness or condition ($g2$); limitation in activities because of health problems ($g3$); unmet need for medical examination or treatment ($g4$); and considered separately, unmet need for dental examination or treatment ($g5$).

The eighth set relates to the education. For this set, we have constructed two composite indicators: households with early school leavers not in education or training ($h1$) and households with persons with low educational attainment ($h2$). Indicator ($h1$) relates to household members aged 18–24. The household 'deprivation' is indicated by the presence of a member who has obtained at most lower secondary education (ISCED level currently attended: value 2 or less), and who at the same time is not in education or training leading to a qualification at least to upper secondary level. Indicator ($h2$) concerns persons aged 25–64. Deprivation is indicated by the presence in the household at least one such person who has attained no more than lower secondary education.

The last dimension concerns the labour market. For this set, we have constructed two composite indicators: jobless households ($i1$) and intensity or duration of unemployment at household level ($i2$).

It may be noted that some of the above items have been officially adopted by the European Commission since 2001 in the Open Method of Coordination (OMC)—social inclusion area. This firstly led to the Second Social Report on Income, Poverty and Social Exclusion (Eurostat 2002), where 24 items were utilised to form 5 dimensions (based on EU-15 ECHP data).

It is also of interest to note some other composite indicators of deprivation, which have been proposed. Two new composite indicators on material deprivation were constructed on the basis of the following 9 EU-SILC items (European Commission 2010):

1. to face unexpected expenses;
2. 1 week annual holiday away from home;
3. to pay for arrears (mortgage or rent, utility bills or hire purchase instalments);
4. a meal with meat, chicken or fish every second day;
5. to keep home adequately warm;
6. to have a washing machine;
7. to have a colour TV;
8. to have a telephone;
9. to have a personal car.

The first of these EU indicators measures the *diffusion of deprivation*, defined as the proportion of people living in households who lack at least 3 of these 9 items because they cannot afford them. The second indicator measures the *intensity of deprivation*, that is, the mean number of items (from 0 to 9) lacked by the household. These two

composite measures, following the so-called counting approach, did not aim to identify latent dimensions hidden in the original set of items.

Another recent contribution utilising confirmatory analysis to identifying hidden dimensions (*à-la* Whelan et al. 2001) is the work of Guio (2009); here 3 dimensions have been identified, based on a smaller set of original EU-SILC items, as economic strain; enforced lack of durables; and housing-related problems.

The results of this work are quite consistent with our proposed set of dimensions (see Table 1), since 'economic strain' is the union of our dimensions 1 and 4 (Basic lifestyle and Financial situation); 'enforced lack of durables' represents our dimension 2 (Consumer durables); and 'housing' represents our dimension 3 (Housing amenities).

3.2 Transformation of the items into the [0, 1] interval

When the item is constituted by a fixed number of categories, then it is transformed using the following procedure. For each item, we determine a deprivation score as follows:

$$d_{j,i} = \frac{1 - F(c_{j,i})}{1 - F(1)}; \quad j = 1, 2, \ldots, k; \quad i = 1, 2, \ldots, n \tag{2}$$

where $c_{j,i}$ is the value of the category of the j-th item for the i-th individual and $F(c_{j,i})$ is the value of the j-th item cumulation function for the i-th individual.

We transform the deprivation score to a positive score as follows:

$$s_{j,i} = 1 - \frac{1 - F(c_{j,i})}{1 - F(1)} = \frac{F(c_{j,i}) - F(1)}{1 - F(1)}; \quad j = 1, 2, \ldots, k; \quad i = 1, 2, \ldots, n. \tag{3}$$

In the special, but also common case, where the variable is a dichotomy, the deprivation index d is 1 for deprivation and 0 otherwise, while the positive score s is 0 for deprivation and 1 otherwise. For a polychotomous item, instead of the real value of the category, we assign to each household a value corresponding to the proportion of households that are 'better off' than that household.

In the few cases where the indicator is defined at the individual level (as a set of dichotomies indicating the presence or absence of an experience by each household member), the score s is taken as the proportion of household members experiencing it.

3.3 Factor analysis

In order to investigate lifestyle deprivation, we have followed the procedure from the Economic and Social Research Ireland (ESRI), as described in Whelan et al. (2001).

In proceeding to construct a summary index of deprivation employing different items, we begin by identifying and investigating the latent dimension of deprivation. By 'dimension', we mean a distinct group of individual items of deprivation. Exploratory and confirmatory factor analyses allow us to achieve this objective. In the literature of multidimensional poverty analysis, these approaches usually deal with the problems of measurement errors, of the redundancy (and double counting attributes) and of the arbitrariness of assigning weights to indicators. We adopt factor analysis for reaching

Table 1 Confirmatory factor analysis results

Goodness of fit (GFI)[a]	0.927
Adjusted GFI[b]	0.914
Parsimonious GFI[c]	0.845
Root Mean Square Residual[d]	0.065
RMSEA[e]	0.052

[a] It is based on the ratio of the sum of squared discrepancies to the observed variances; it ranges from 0 to 1 with higher values indicating a good fit
[b] It is the GFI adjusted for degrees of freedom of the model, that is, the number of the fixed parameters. It can be interpreted in the same manner
[c]. It adjusts GFI for the number of estimated parameters in the model and the number of data points
[d]. The fit is considered really good if RMR is equal or below 0.06
[e]. The Root Mean Squared Error of Approximation (RMSEA) is based on the analysis of residuals, with small values indicating a good fit

the first two goals, while we prefer to construct an *ad hoc* weighting system as described and justified in the next section.

The procedure begins in an exploratory factor analysis so as to give a preliminary framework of the dimensions.[5] The analysis has been conducted using the whole EU-SILC data set at European level, so as to get a unique set of dimensions which would permit comparison of results among countries. We then proceed to rearrange some factors in the dimensions identified in order to create more meaningful groups.

This resulted in identification of the 7 dimensions listed in Table 2.

Finally, we conduct a confirmatory factor analysis to test the goodness of the 7-dimension model hypothesised. The results of the analysis, reported in Table 1, are very good; in fact, all the indicators of goodness of the model are significant.

3.4 Calculation of weights within each dimension

The weights to be given to individual items are determined within each dimension separately, and the set of weights are taken to be item specific, i.e. for a given item, they are common to all individuals in the population.

In multidimensional analysis, there are several methods calculating weights for aggregating different items within dimensions. Filippone et al. (2001) reports a very detailed overview of the most adopted approaches, describing advantages and drawbacks. Moreover, recently an additional categorisation has been proposed making a distinction between consensus and prevalence weighting (see, among others, Guio 2009).

In the present paper, we have adopted the approach proposed by Betti and Verma (1999 a sort of 'prevalence-correlation' method) instead of, for instance, the weights

[5] The exploratory factor analysis has identified 9 dimensions as follows: items $b1$, $b2$, $b3$, $b4$ and f1 in dimension 1; items $g1$, $g2$ and $g3$ in dimension 2; items $d3$, $d4$ and $d5$ in dimension 3; items $c1$ and $c2$ in dimension 4, items $a1$, $a2$, $a3$, $a4$ and $a5$ in dimension 5, items $h1$, $h2$, $i2$ and $i2$ in dimension 6, items $e1$, $e2$ and $e3$ in dimension 7, items $g4$ and $g5$ in dimension 8, and finally items $d1$ and $d2$ in dimension 9.

Table 2 Dimensions after rearrangement and confirmatory factor analysis

Dimension	Indicators	Weights
1. Basic lifestyle	Meals with meat, fish or chicken	1.90
	Household adequately warm	2.18
	Holiday away from home	0.68
	Ability to make ends meet	0.39
2. Consumer durables	Car	2.22
	PC	2.87
	Telephone	8.43
	Washing Machine	8.05
	TV	10.68
3. Housing amenities	Bath or Shower	4.85
	Indoor flushing toilet	5.17
	Leaking roof and damp	1.80
	Rooms too dark	2.95
4. Financial situation	Inability to cope with unexpected expenses	0.80
	Arrears on mortgage or rent payments	3.69
	Arrears on utility bills	1.99
	Arrears on hire purchase instalments	4.06
5. Environment	Crime, violence, vandalism	1.57
	Pollution	1.77
	Noise	1.34
6. Work and education	Early school leavers	5.45
	Low education	1.35
	Worklessness	0.51
	Duration of unemployment	1.79
7. Health related	General health	0.97
	Chronic illness	0.46
	Mobility restriction	0.51
	Unmet need for medical exam.	1.79
	Unmet need for dental exam.	1.69

coming out from the confirmatory factor analysis. This is for three main reasons: (1) it controls the problems of arbitrariness and redundancy in the original choice of items (measurement errors); (2) it gives less importance to items not widely diffused in the population; (3) the procedure has been officially adopted by Eurostat (2002) in the Second Social Report on Income, Poverty and Social Exclusion for comparative analysis. In fact, this weighting system has been widely adopted also in other fields; for instance Aassve et al. (2007) have studied the effects of marital disruption on economic well-being, while Betti et al. (2011) have constructed a fuzzy indicator of educational mismatch for university graduates.

In this approach, the weights comprise two factors: the dispersion of a deprivation item and its correlation with other deprivation items in the given dimension: $w_{hj} = w^a_{hj} \cdot w^b_{hj}, h = 1, 2, \ldots, m;\ j = 1, 2, \ldots, k_h$, where h is a particular dimension and j a particular deprivation item. The first factor is the coefficient of variation of $(1 - s)$ for the deprivation scores s (prevalence weights); the second factor is a measure based on correlations among items, which gives less weight to items more highly correlated with others in the same dimension (correlation weights), so as to reduce the effect of redundancy and arbitrariness in the choice of original items.

The last column of Table 2 reports the results from our data for w_{hj}, the weight of the j-th deprivation variable in the h-th dimension.

3.5 Calculation of scores for each dimension and non-monetary measures

For individual i, aggregation over a group of items in a particular dimension $h(h = 1, 2, \ldots, m)$ is given by a weighted mean taken over j items: $s_{hi} = \sum w_{hi}.s_{hj,i} / \sum w_{hi}$ where w_{hj} is the weight of the j-th deprivation variable in the h-th dimension.

An overall deprivation score for the i-th individual is calculated as the unweighted mean over dimensions:

$$s_i = \frac{\sum_{h=1}^{m} s_{hi}}{m} \tag{4}$$

This is the overall (combined over all dimensions) non-monetary indicator FS for individual i. These values are then used in Eq. (1) to determine parameter α_2, such that the mean of the FS values is equal to the conventional at-risk-of-poverty rate (ARPR). Finally, this overall estimate α_2 is used in Eq. (1) to calculate the indicator for each dimension of deprivation separately (FS1–FS7).

4 Empirical analysis

4.1 Fuzzy supplementary measures by dimensions of deprivation

Table 3 shows indicators of deprivation in various dimensions estimated with the methodology described above, using EU-SILC 2011 data; a list of country names and abbreviations is reported in 'Appendix'. The objective of illustrating those results is both substantive and methodological. They show the relative situation of EU countries in terms of levels of overall deprivation and also in terms of different dimensions of deprivation. At the same time, they illustrate the type of numerical values obtained with the above procedure, thus clarifying details of the methodology.

4.1.1 Model parameters

The left hand panel of Table 3 shows FS0, conventional poverty rate (ARPR) for each country. Also shown are the estimates of parameter α_{FM} and α_{FS} determined so as to *make the FM* (*fuzzy monetary*) *and FS* (*fuzzy supplementary*) *deprivation indices both*

Table 3 Fuzzy measures at country level

Country	FS0	ALFA_FM	ALFA_FS	FS_eu	FS_eu/FS0	Ratio to FS0							
						FS1	FS2	FS3	FS4	FS5	FS6	FS7	Mean S1–7
SE	0.141	6.223	6.203	0.040	29	0.71	0.23	0.51	0.55	0.73	0.81	*	0.59
FI	0.137	6.836	6.434	0.064	47	0.75	0.44	0.47	0.68	0.74	0.85	*	0.66
LU	0.136	6.413	6.490	0.076	56	0.72	0.19	0.51	0.45	0.83	0.84	0.77	0.62
ES	0.218	3.857	3.728	0.128	59	0.71	0.29	0.42	0.47	0.66	0.82	0.78	0.59
DK	0.130	6.803	6.857	0.079	61	0.77	0.40	0.59	0.55	0.82	0.82	*	0.66
IE	0.161	5.790	5.374	0.111	69	0.76	0.47	0.51	0.69	0.65	0.83	0.71	0.66
UK	0.162	5.443	5.328	0.113	70	0.80	0.35	0.55	0.52	0.84	0.81	0.77	0.67
DE	0.158	5.357	5.469	0.121	76	0.82	0.42	0.50	0.41	0.96	0.80	0.78	0.67
NL	0.110	8.224	8.246	0.084	77	0.81	0.35	0.52	0.52	0.97	0.86	*	0.67
AT	0.126	7.506	7.061	0.098	77	0.82	0.42	0.57	0.60	0.83	0.88	0.72	0.69
SI	0.136	6.682	6.483	0.112	82	0.87	0.35	0.54	0.67	0.85	0.85	*	0.69
BE	0.153	5.784	5.722	0.136	89	0.85	0.44	0.58	0.56	0.89	0.83	0.76	0.70
PT	0.180	5.197	4.729	0.165	92	0.75	0.53	0.54	0.60	0.82	0.84	0.71	0.69
EE	0.174	5.193	4.916	0.168	96	0.75	0.51	0.71	0.58	0.74	0.84	0.85	0.71
EU_27	0.169			0.169	100	0.86	0.41	0.55	0.56	0.80	0.80	0.87	0.69
FR	0.141	6.520	6.297	0.142	101	0.81	0.33	0.58	0.62	0.87	0.84	0.84	0.70
SK	0.130	7.299	6.886	0.142	109	0.84	0.58	0.51	0.56	0.99	0.88	0.84	0.74
EL	0.214	4.254	3.857	0.234	109	0.87	0.44	0.51	0.80	0.99	0.78	0.84	0.75
IT	0.196	4.424	4.294	0.217	111	0.99	0.23	0.52	0.59	0.87	0.80	0.88	0.70
MT	0.154	5.826	5.637	0.176	114	0.94	0.21	0.52	0.54	0.93	0.82	0.74	0.67

Table 3 continued

Country	FS0	ALFA_FM	ALFA_FS	FS_eu	FS_eu/FS0	Ratio to FS0							
						FS1	FS2	FS3	FS4	FS5	FS6	FS7	Mean S1–7
LT	0.199	4.531	4.191	0.230	115	0.97	0.55	0.76	0.45	0.63	0.83	0.74	0.70
PL	0.177	5.123	4.840	0.212	120	0.95	0.49	0.60	0.54	0.69	0.81	0.99	0.73
CZ	0.098	9.748	9.411	0.138	141	0.95	0.57	0.57	0.49	1.15	0.89	0.89	0.79
CY	0.145	6.458	6.064	0.215	148	0.86	0.25	0.47	0.79	0.95	0.85	0.92	0.73
RO	0.222	4.107	3.733	0.336	151	0.95	0.68	0.70	0.60	0.89	0.78	1.06	0.81
HU	0.138	6.654	6.479	0.239	172	1.10	0.60	0.71	0.77	0.82	0.87	0.97	0.83
BG	0.223	4.139	3.758	0.392	176	1.36	0.68	0.81	0.56	0.82	0.83	0.93	0.85
LV	0.193	4.561	4.439	0.400	208	1.10	0.63	0.80	0.62	0.88	0.84	1.14	0.86

*Data not available

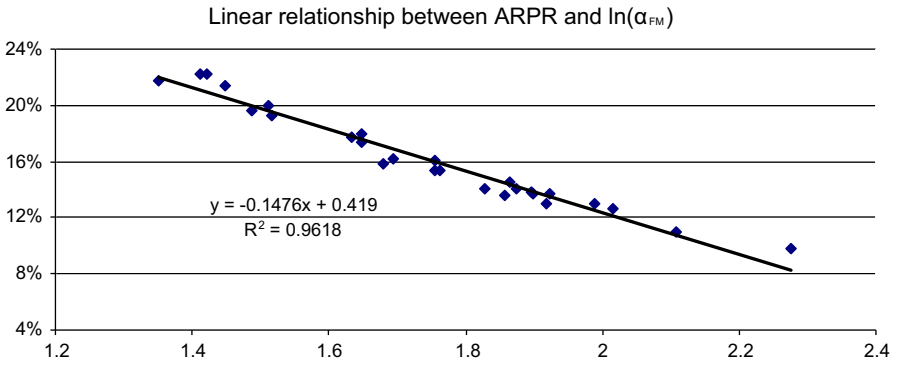

Fig. 1 Relationship between ARPR = FM and ln (α_{FM})

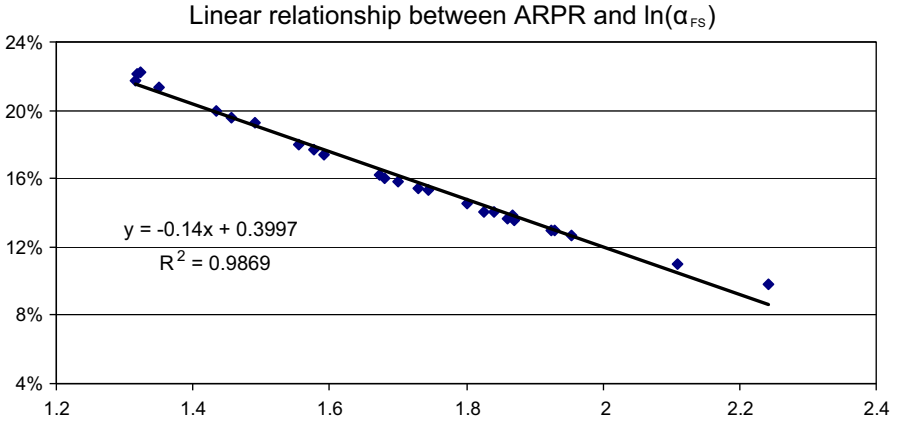

Fig. 2 Relationship between ARPR = FS and ln (α_{FS})

equal to ARPR for each country, as in Eq. (1). For both ln (α_{FM}) and ln (α_{FS}), there is a clear linear relationship with ARPR [$R^2 = 0.96$; $R^2 = 0.99$], as seen in Figs. 1 and 2. This reflects the economic interpretation of the parameters α noted in Sect. 2.

The table also shows the FS measure computed by using a single value of $\alpha_{\mathrm{FS(EU)}}$ estimated using the pooled income distribution for the EU as a whole and pegging the $\mathrm{FS}_{(\mathrm{EU})}$ measure to conventional poverty rate $\mathrm{ARPR}_{(\mathrm{EU})}$ at the EU level.[6] At the European level, there have been concerns on the best approach to be adopted to capture the situation of New Member States and to provide meaningful comparisons with the Old Member States (see, among others, Fusco et al. 2011 and Guio 2009). Measure $\mathrm{FS}_{(\mathrm{EU})}$, incorporating the major part of the effect of national differences in the *average levels* of living, helps to illustrate this issue. The ratio $\left(\mathrm{FS}_{(\mathrm{EU})}/\mathrm{FS0}\right)$ generally tends to

[6] National income distributions have been pooled with weights in proportion to the population size for constructing the EU-level distribution.

be much higher in (often with lower relative poverty rates FS0) New Member States, compared with that in richer Old Member States.[7]

Henceforth, our results will be based on FM and FS measures pegged to FS0 at the national level.

Right hand panel of Table 3 shows dimension-specific fuzzy supplementary deprivation rates (FS1–FS7). As noted, these have been pegged to the country-specific value of parameter α_{FS}, the same for all dimensions in a given country. Hence, numerically, these deprivation rates for individual dimensions are not scaled to automatically equal the overall poverty or deprivation rate FS0. Several interesting results may be noted.

1. On the average, dimension-specific deprivation rates are much lower than the overall deprivation rate FS for the country concerned. In order to highlight this point, we have shown in the table the ratios of dimension-specific to the overall deprivation rate for each country, rather than the actual values of these rates. Note that averaged over dimensions and over EU countries, this ratio is around 0.7. This may reflect a tendency for deprivation in different dimensions to be concentrate on the same individuals, increasing the intensity of overall deprivation.
2. The various dimensions appear to fall into two groups with rather different levels of the ratio of dimension-specific to overall deprivation rate. For weighted average over EU countries, the ratio is 0.80–0.87 for four dimensions
   FS1—basic lifestyle
   FS5—environment
   FS6—work and education
   FS7—health related
   but the ratio is much lower (0.41–0.56) for the other three dimensions
   FS2—consumer durables
   FS3—housing amenities
   FS4—financial situation.
3. The ratio tends to be lower for the richer Old Member States, and higher for the New Member States. This may be reflective of higher levels of concentration of relative deprivation in different dimensions on the same individuals in Old Member States. We can infer this conclusion from the higher values of the α parameters.
4. We note that there is a fairly strong correlation between the ranking of countries according to the overall and dimension-specific indices of deprivation (Table 4, top panel). The differences in correlations among dimensions are much more evident when observing individuals in EU. The correlation with FS is particularly high for dimensions FS1, FS3 and FS6.

The lower panel of Table 4 shows correlations among deprivation scores in different dimensions. In the interest of compactness, the top right triangle shows correlation for rankings of countries, and the bottom left triangle shows for individual persons. Quite large differences in the rankings of countries in different dimensions are also present (low values of Spearman correlations), while ranking of individuals is much more consistent in different dimensions (high values of Spearman correlations).

[7] The table has been ordered according to this ratio in order to illustrate the point being made.

Table 4 Correlations among FS measures

	FS1	FS2	FS3	FS4	FS5	FS6
Correlation with overall FS						
Ranking of countries	0.66	0.38	0.42	0.52	0.51	0.47
Individuals in EU	0.90	0.62	0.88	0.77	0.76	0.99
Correlation among dimensional deprivation scores						
Ranking of countries						
FS1	1	0.34	0.23	0.37	0.11	0.29
FS2	0.68	1	0.20	0.22	0.07	0.22
FS3	0.91	0.74	1	0.15	0.15	0.16
FS4	0.77	0.57	0.73	1	0.11	0.24
FS5	0.78	0.48	0.65	0.58	1	0.04
FS6	0.92	0.68	0.89	0.81	0.76	1
Individuals in EU						

Results are not included for FS7 since the data are missing for a number of countries

4.2 Variation in deprivation levels across national regions

There are major differences within countries in the levels of deprivation, both monetary and non-monetary. In Table 5, we provide empirical analysis for a selection of national regions (NUTS1 or NUTS2, where available). Available EU-SILC data permit analysis at the regional level, but not in all countries. Often some assumptions and indirect procedures have to be used to compute sampling variance of the regional indicators. A major reason for this difficulty is the lack of necessary information on survey design in the EU-SILC data sets. In any case, direct estimates for small regions are not reliable, or even not possible, because of limited sample sizes in EU-SILC. We have developed and illustrated the procedures in detail elsewhere (Betti et al. 2012).

Table 5 shows the ratios of FM and FS measures to the conventional poverty rate (ARPR) at the regional level.[8] Note that these ratios equal 1.0 at the national level as a consequence of the chosen scaling of the fuzzy measures. We have shown results for six countries involving 5 or more regions where computations were possible. Since the identity of individual regions is not relevant for the present purpose, we have simply ranked the region within each country according to increasing levels of poverty (ARPR). In three of the countries involving a large number of regions, results have been shown only for a 1-in-3 systematic sample of regions. The following may be observed from the figures.

1. Generally, the ratios (FM/ARPR) and (FS/ARPR) decline with increasing regional ARPR. This means that, compared with the conventional measure ARPR, the fuzzy measures give *less marked differentials among regions*—the regions appear more similar.

[8] Regional poverty rates have been computed from regional income distributions, but always using the national poverty line.

Table 5 Fuzzy measures at regional level, selected countries

Country	Number of regions	Region (ranked according to conventional poverty rate ARPR)												
		1	2	3	4	5	6	7	8					
IT	5													
	ARPR (%)	9.6	10.6	15.0	32.6	38.9								
	FM/ARPR	1.22	1.18	1.07	0.92	0.88								
	FS/ARPR	1.28	1.45	1.20	0.88	0.67								
PL	6													
	ARPR (%)	13.0	15.6	16.0	17.1	18.4	24.4							
	FM/ARPR	1.11	1.03	1.02	1.01	0.96	0.95							
	FS/ARPR	1.48	1.12	1.12	1.05	0.91	0.70							
CZ	8													
	ARPR (%)	4.6	6.1	7.3	7.9	10.0	11.3	15.1	17.1					
	FM/ARPR	1.10	1.04	1.15	1.14	0.97	1.00	0.94	0.89					
	FS/ARPR	2.28	1.45	1.33	1.00	0.85	0.75	0.74	0.84					
		1	4	7	10	13	16	19	22	25	28	31	34	37
ES	19													
	ARPR (%)	8.8	15.9	18.5	20.3	26.4	31.7	33.8						
	FM/ARPR	1.19	1.02	1.08	1.01	0.99	0.95	0.86						
	FS/ARPR	1.54	1.36	1.53	1.04	0.80	0.97	0.84						

Table 5 continued

Country	Number of regions	Region (ranked according to conventional poverty rate ARPR)												
		1	4	7	10	13	16	19	22	25	28	31	34	37
FR	22													
	ARPR (%)	8.5	10.1	12.3	14.1	14.6	16.5	18.5	28.7					
	FM/ARPR	1.31	1.12	1.08	1.06	1.04	0.91	0.90	0.86					
	FS/ARPR	1.01	1.19	0.99	1.19	0.99	1.03	0.55	0.28					
UK	37													
	ARPR (%)	8.7	11.5	12.4	13.7	14.3	15.1	15.7	17.0	17.7	18.7	19.5	20.3	23.6
	FM/ARPR	1.09	1.24	1.14	1.12	1.14	0.94	0.92	0.98	1.03	0.97	0.99	0.98	0.92
	FS/ARPR	1.47	1.43	1.21	1.11	1.12	1.25	0.80	1.03	0.89	1.54	0.83	0.80	0.54

2. The above effect is particularly marked in the case of the FS measure. For instance, in Poland, with regional ARPR varying from 13 to 24 %, ratio (FM/ARPR) varies from 1.1 to 0.95, while ratio (FS/ARPR) varies much more markedly, from 1.5 to 0.7. Indeed, in some countries (Poland and Czech Republic), FS values show little regional variation despite large regional differences in the conventional poverty rates. The effect, however, is less marked in other countries, and the fuzzy measures are higher in regions with higher poverty rates according to the conventional measure. For example in the case of Italy with regional ARPR varying from 10 to 39 %, FM varies from 12 to 34 %, and FS from 12 to 26 %.
3. For countries where computations can be performed for many small regions (ES, FR, UK), the results are of course subject to substantial sampling variability because of the small sample sizes involved.

The empirical observation that the fuzzy measures tend to give less marked differentials among regions is interesting, and needs further exploration.

It is common to base poverty-related indicators on country poverty lines. Income distribution is considered separately at the level of each country, in relation to which a poverty line is defined and the proportion of poor computed. It is also useful to consider poverty lines at other levels (Berthoud 2004; Betti et al. 2012).[9] Especially useful for constructing regional indicators is the use of regional poverty lines, i.e. a poverty line defined for each region based only on the income distribution within that region. So defined, the poverty measures are not affected by disparities in mean levels of income among the regions. The measures are more purely relative. In other words, the use of the regional poverty line provides a relative measure of poverty determined only by the income distribution within the region, independently of the degree of regional disparities in the country. Differentials among regions in the conventional poverty rates based on a national poverty line are generally reduced when computed using regional poverty lines (see for instance, Betti et al. 2012, for many numerical illustrations). Moving from conventional to fuzzy measures (with model parameters estimated by pegging the fuzzy deprivation rates to the conventional poverty rate at the country level), seems to have a similar effect. This is more markedly so for FS compared with FM. This may also imply that fuzzy measures are more 'relative' than the corresponding conventional measures, as already noted.

4.3 Variations in deprivation levels by gender, age, and level of education of individuals

Income distribution data such as from EU-SILC are normally household based: income is aggregated over household members and over income sources to obtain the total household income, which is equivalised to account for differences in household

[9] By the 'level of poverty line' is meant the population level to which the income distribution is pooled for the purpose of defining the poverty line. In fact, different levels for the poverty line can be seen as implying a different mix of 'relative' and 'absolute' measures. By relative measures is meant measures concerning purely the distribution of income, and by absolute measures those concerning income levels.

Table 6 Fuzzy measures by gender

Country	ARPR	Ratio (female/male) deprivation score		
		ARPR	FM	FS
BG	0.223	1.13	1.09	1.02
RO	0.222	1.03	1.02	1.02
ES	0.217	1.06	1.04	1.00
EL	0.214	1.05	1.04	1.04
LT	0.199	1.01	1.06	0.98
IT	0.195	1.14	1.11	1.02
LV	0.193	0.93	0.98	0.97
PT	0.180	1.05	1.05	1.01
PL	0.177	0.99	1.01	1.00
EE	0.175	0.99	1.04	0.94
UK	0.162	1.19	1.13	1.00
IE	0.161	1.02	1.05	0.99
DE	0.158	1.13	1.09	1.05
MT	0.154	1.06	1.08	1.00
BE	0.153	1.09	1.09	1.02
CY	0.145	1.29	1.23	0.99
FR	0.140	1.07	1.07	1.02
SE	0.139	1.28	1.23	1.01
HU	0.139	0.96	1.00	1.00
FI	0.137	1.08	1.08	0.99
SI	0.136	1.23	1.14	1.04
LU	0.136	1.14	1.11	1.01
SK	0.130	1.02	1.03	1.01
DK	0.130	1.00	1.02	1.07
AT	0.126	1.15	1.12	1.02
NL	0.110	1.03	1.07	1.03
CZ	0.098	1.19	1.20	1.05
Simple average	0.161	1.09	1.08	1.01

size and composition, and then ascribed each household member. All members of a given household—irrespective of differences in age, gender and other individual characteristics—are therefore assigned the same value of income; the same applies to non-monetary resources. Nevertheless, differentials in levels of deprivation according to individual characteristics arise because persons with these characteristics are distributed differently among households of different types.

In this section, we examine differential in levels of deprivation according to characteristics of individual persons; some differentials according to household-level characteristics will be presented in Sect. 4.4.

Table 6 compares the three deprivation measures (ARPR, FM and FS) in terms of gender differentials. Overall, the conventional poverty rate (ARPR) for females is

10 % higher than that for males. The overall differential is very similar in the case of fuzzy monetary measure FM, but there is practically no gender differential overall in the case of fuzzy supplementary measure FS.

The overall pattern mentioned above generally holds for individual countries as well, but there are a few exceptions and random variations. In the table, we have identified countries where ARPR for females exceeds that for males by more than 15 %. Surprisingly, these include some of the richer countries (AT, SE, UK); the other being CZ, SI and CY.

The attenuation or complete disappearance in the case of FS, of the female–male differential for fuzzy measures is on the same lines as the attenuation noted earlier in the case of geographical region. One may conjecture that fuzzy measures for population groups, while pegged to the conventional poverty rate for the total (national) population, behave more like 'relative' measures of within-group disparities. By contrast, the conventional measure based on the national poverty line fully incorporates the effect of both within-group and between-group disparities.

The pattern of age differentials for the three deprivation measures are compared in Table 7. The overall pattern is that deprivation rates, averaged over countries, do not vary much by age for persons aged 25 and over (age groups 25–49, 50–64 and 65+ in the table), but are higher for children (0–17) and other young persons (18–24). The overall pattern by age is very similar for the three types of deprivation measures, except that the higher deprivation score for children, seen in the case of ARPR and FM, is much less pronounced in the case of FS (its overall average is around 1.4 for the former two, and under 1.1 for FS).

There are some irregularities in the pattern for individual countries; however, these presumably arise from small sample sizes and/or measurement errors. For example, the ratio (age65+/age25–49) for ARPR is at or below 0.50 in some countries (e.g. HU, LV, SK). By contrast, this ratio is nearly 3.4 in one case (CY). In order to control the effect of the presence of a few extreme values, the table also shows the median value in addition to the simple average over countries.

Finally, concerning differentials by level of education of individuals, as expected, there are very strong differentials in the case of all the three deprivation measures. However, as among subgroups defined in terms of other variables (region, gender), the differentials attenuate as we move from ARPR to FM, and further to FS. Table 8 shows the ratio of deprivation scores of persons with the lowest level of education to those with the highest level (ISCED levels 1–5, respectively, where available). The differentials are very pronounced. Averaged over countries, the ratio is 5.4 for ARPR, declining to 4.3 for FM and to 3.1 for FS (the respective median values are 4.2, 3.8 and 3.0). A handful of exceptions to the general pattern may be noted. The observed ratio for ARPR is very low in DK (0.9), low in NL (1.9), and between 2.0 and 2.5 in three other countries among the 27 countries shown. Extremely high values (16–18) have been reported for BG and RO, with values near 10 in three other countries. (The extreme values have been marked in the table.) Apart from such cases, the overall pattern (and the decline in the ratio from ARPR to FM to FS) is quite regular across countries.

Table 7 Fuzzy measures disaggregated at age group level

	ARPR	Age pattern (ARPR)					Age pattern (FM)					Age pattern (FS)				
		0–17	18–24	25–49	50–64	65+	0–17	18–24	25–49	50–64	65+	0–17	18–24	25–49	50–64	65+
BG	0.223	1.61	1.28	1.00	0.95	1.70	1.46	1.22	1.00	0.93	1.50	1.27	1.23	1.00	1.01	1.11
RO	0.222	1.50	1.32	1.00	0.73	0.65	1.40	1.28	1.00	0.78	0.81	1.25	1.28	1.00	1.09	1.24
ES	0.217	1.33	1.28	1.00	0.91	1.02	1.27	1.24	1.00	0.95	1.09	0.96	1.48	1.00	1.11	0.90
EL	0.214	1.24	1.45	1.00	1.08	1.26	1.18	1.42	1.00	1.03	1.23	0.94	1.23	1.00	1.03	1.11
LT	0.199	1.23	1.35	1.00	0.97	0.64	1.17	1.20	1.00	0.98	0.90	1.02	1.31	1.00	1.18	1.34
IT	0.195	1.35	1.34	1.00	0.78	0.87	1.29	1.27	1.00	0.81	0.91	1.00	1.35	1.00	1.04	1.03
LV	0.193	1.31	1.20	1.00	1.13	0.50	1.25	1.20	1.00	1.13	0.89	1.10	1.26	1.00	1.24	1.05
PT	0.180	1.45	1.53	1.00	1.08	1.32	1.39	1.44	1.00	1.06	1.23	1.08	1.51	1.00	1.18	0.97
PL	0.177	1.34	1.30	1.00	1.03	0.90	1.32	1.28	1.00	1.04	0.99	1.06	1.20	1.00	1.24	1.18
EE	0.175	1.23	1.45	1.00	1.24	0.82	1.16	1.39	1.00	1.28	1.23	1.07	1.25	1.00	1.21	1.03
UK	0.162	1.49	1.58	1.00	1.21	1.80	1.44	1.42	1.00	1.12	1.64	1.07	1.55	1.00	0.88	0.48
IE	0.161	1.45	1.45	1.00	1.25	0.78	1.44	1.44	1.00	1.24	1.10	1.20	1.19	1.00	1.00	0.54
DE	0.158	1.05	1.14	1.00	1.21	0.95	1.12	1.13	1.00	1.11	1.04	0.99	1.13	1.00	1.03	0.70
MT	0.154	1.58	1.16	1.00	1.02	1.37	1.49	1.14	1.00	1.06	1.54	1.11	1.45	1.00	1.23	0.94
BE	0.153	1.41	1.11	1.00	0.91	1.53	1.38	1.23	1.00	0.97	1.66	1.16	1.25	1.00	0.94	0.67
CY	0.145	1.10	0.89	1.00	1.06	3.37	1.14	1.00	1.00	0.99	2.63	1.05	1.49	1.00	1.15	1.00
FR	0.140	1.44	1.70	1.00	0.82	0.74	1.36	1.63	1.00	0.80	0.85	1.11	1.45	1.00	0.97	0.70
SE	0.139	1.18	2.01	1.00	0.60	1.47	1.19	1.86	1.00	0.61	1.57	0.96	1.33	1.00	0.91	0.53
HU	0.139	1.66	1.38	1.00	0.80	0.32	1.54	1.36	1.00	0.87	0.56	1.31	1.33	1.00	1.01	0.78
FI	0.137	1.13	2.58	1.00	1.12	1.83	1.15	2.30	1.00	1.07	1.78	0.97	1.42	1.00	0.97	0.51
SI	0.136	1.30	0.91	1.00	1.14	1.84	1.23	0.98	1.00	1.12	1.61	0.95	1.26	1.00	1.20	1.02

Table 7 continued

	ARPR	Age pattern (ARPR)					Age pattern (FM)					Age pattern (FS)				
		0–17	18–24	25–49	50–64	65+	0–17	18–24	25–49	50–64	65+	0–17	18–24	25–49	50–64	65+
LU	0.136	1.46	1.27	1.00	0.75	0.34	1.38	1.25	1.00	0.78	0.53	1.05	1.53	1.00	1.20	0.74
SK	0.130	1.71	1.24	1.00	0.89	0.50	1.56	1.29	1.00	0.89	0.85	1.20	1.11	1.00	1.12	1.09
DK	0.130	0.92	3.71	1.00	0.46	1.40	0.98	2.96	1.00	0.60	1.69	0.90	1.78	1.00	0.76	0.41
AT	0.126	1.43	1.06	1.00	0.97	1.46	1.37	1.03	1.00	0.96	1.35	1.10	1.15	1.00	1.01	0.70
NL	0.110	1.54	2.04	1.00	0.67	0.63	1.43	1.84	1.00	0.79	1.01	0.88	1.41	1.00	0.99	0.40
CZ	0.098	1.67	1.42	1.00	0.84	0.73	1.59	1.43	1.00	0.89	1.09	1.17	1.32	1.00	1.08	0.90
Simple average	0.161	1.38	1.49	1.00	0.95	1.14	1.32	1.42	1.00	0.96	1.23	1.07	1.34	1.00	1.07	0.85

Table 8 Fuzzy measures disaggregated at education level

Country	ratio (ed1/ed5)			
	ARPR	ARPR	FM	FS
BG	0.223	16.1	7.9	5.9
RO	0.222	17.6	11.0	4.2
ES	0.218	2.7	2.5	2.1
EL	0.214	4.1	3.3	1.9
LT	0.199	2.5	2.5	3.3
IT	0.196	3.0	2.8	2.5
LV	0.193	4.4	3.9	4.7
PT	0.180	9.2	7.0	3.3
PL	0.177	6.2	5.0	3.7
EE	0.174	3.6	2.7	3.3
UK*	0.162	2.8	2.6	2.0
IE	0.161	2.2	2.3	2.3
DE	0.158	4.2	3.3	3.4
MT	0.154	3.8	3.8	2.9
BE	0.153	4.2	3.8	2.5
CY	0.145	6.3	4.8	2.3
FR	0.141	2.5	2.6	2.3
SE	0.141	3.3	2.9	1.2
HU	0.138	8.7	7.7	6.4
FI*	0.137	4.5	3.8	2.0
SI	0.136	11.7	8.2	3.4
LU	0.136	4.2	3.9	3.3
DK	0.130	0.9	1.7	3.0
SK	0.130	4.6	3.8	3.2
AT*	0.126	3.1	2.8	2.9
NL	0.110	1.9	2.4	1.8
CZ	0.098	6.7	5.7	4.6
Simple average	0.161	5.4	4.3	3.1
cv				
Median	0.154	4.2	3.8	3.0

Ratio (ed1/ed5) ARPR ≤ 2.5
Ratio (ed1/ed5) ARPR ≥ 8.0
Table sorted by ARPR
* The ratio shown is (ed2/ed5) because of lack of data on ed1

4.4 Deprivation levels according to household characteristics

Results are presented in Tables 9 and 10 for two basic household-level characteristics, respectively, household size and tenure of accommodation.

Concerning household size, the deprivation scores are generally the lowest for household size 3 (such as households composed of a couple with one child). Table 9 shows the ratio of deprivation score for each household size to the score for household size 3, for ARPR, FM and FS measures, respectively.

Table 9 Fuzzy measures by household size

	Pattern for household size	ARPR				FM				FS			
	ARPR	1	2	3	4+	1	2	3	4+	1	2	3	4+
BG	0.223	4.04	1.53	1.00	1.87	2.83	1.46	1.00	1.66	1.29	1.29	1.00	1.38
RO	0.222	1.49	0.80	1.00	1.74	1.49	0.90	1.00	1.61	1.45	1.45	1.00	1.54
ES	0.218	1.40	1.10	1.00	1.42	1.34	1.04	1.00	1.30	0.72	0.72	1.00	1.10
EL	0.214	1.32	1.07	1.00	1.17	1.38	1.11	1.00	1.15	1.04	1.04	1.00	0.93
LT	0.199	1.52	0.97	1.00	1.14	1.70	1.02	1.00	1.15	1.51	1.51	1.00	1.25
IT	0.196	1.50	0.91	1.00	1.47	1.34	0.96	1.00	1.36	0.92	0.92	1.00	1.11
LV	0.193	1.16	1.03	1.00	1.08	1.59	1.06	1.00	1.14	1.31	1.31	1.00	1.33
PT	0.180	1.95	1.30	1.00	1.35	1.71	1.21	1.00	1.35	1.03	1.03	1.00	1.11
PL	0.177	2.21	1.16	1.00	1.74	1.99	1.13	1.00	1.58	1.31	1.31	1.00	1.16
EE	0.174	1.63	0.85	1.00	1.04	1.80	0.97	1.00	0.99	0.88	0.88	1.00	1.03
UK	0.162	1.76	1.04	1.00	1.02	1.60	1.02	1.00	1.06	0.84	0.84	1.00	1.17
IE	0.161	1.75	1.02	1.00	1.36	1.84	1.03	1.00	1.29	0.98	0.98	1.00	1.05
DE	0.158	3.04	1.25	1.00	1.02	2.46	1.22	1.00	1.18	1.28	1.28	1.00	0.90
MT	0.154	2.22	1.85	1.00	1.52	2.20	1.78	1.00	1.42	1.18	1.18	1.00	1.31
BE	0.153	1.83	1.42	1.00	1.18	1.86	1.37	1.00	1.22	1.26	1.26	1.00	1.13
CY	0.145	2.78	2.17	1.00	0.86	2.40	1.89	1.00	0.98	0.89	0.89	1.00	0.98
FR	0.141	1.58	0.76	1.00	1.40	1.43	0.80	1.00	1.30	0.91	0.91	1.00	1.02
SE	0.141	2.67	0.81	1.00	0.88	2.39	0.88	1.00	0.94	1.06	1.06	1.00	0.84
HU	0.138	1.59	0.81	1.00	1.88	1.62	0.85	1.00	1.64	1.18	1.18	1.00	1.38
FI	0.137	4.21	1.10	1.00	1.36	3.25	1.14	1.00	1.27	1.22	1.22	1.00	0.90
SI	0.136	4.75	1.46	1.00	1.25	3.61	1.46	1.00	1.17	1.28	1.28	1.00	0.88
LU	0.136	1.24	0.58	1.00	1.35	1.25	0.66	1.00	1.29	0.76	0.76	1.00	0.83
DK	0.130	2.60	0.80	1.00	0.74	2.55	0.99	1.00	0.85	1.23	1.23	1.00	0.74
SK	0.130	1.77	0.62	1.00	1.40	1.95	0.83	1.00	1.38	1.25	1.25	1.00	1.09
AT	0.126	3.17	1.43	1.00	1.58	2.44	1.32	1.00	1.51	1.10	1.10	1.00	1.08
NL	0.110	2.09	0.80	1.00	1.74	2.16	0.92	1.00	1.50	1.33	1.33	1.00	0.89
CZ	0.098	2.57	1.03	1.00	1.57	2.78	1.18	1.00	1.58	1.15	1.15	1.00	1.22
Simple average	0.161	2.22	1.10	1.00	1.34	2.04	1.12	1.00	1.29	1.12	1.12	1.00	1.09

With household size = 3 as the base, overall (averaged over countries) ARPR is higher for size = 2 by 10 %, by a third for size = 4+, but by over 2.2 times for single person households. (One person households often consist of single young persons or of older persons—more often women—living alone.)

The differentials by household size are somewhat less marked for FM, but are greatly reduced in the case of FS. For example, while the overall ratio (size = 1/size = 3) of deprivation scores is 2.2 for ARPR and 2.0 for FM, for FS this ratio is only 1.1.

Table 10 Fuzzy measures by house tenure

	Ratio (rent/own) deprivation score			
	ARPR	ARPR	FM	FS
NL	0.110	3.98	3.21	3.03
BE	0.153	3.76	2.98	3.03
SE	0.141	3.72	2.97	2.69
LU	0.136	3.66	2.85	1.94
FR	0.141	3.22	2.62	2.90
FI	0.137	3.14	2.66	2.94
DE	0.158	3.07	2.33	2.33
DK	0.130	2.71	2.56	3.18
AT	0.126	2.55	2.17	2.96
SI	0.136	2.44	2.14	2.93
CZ	0.098	2.28	1.98	2.66
UK	0.162	2.05	1.99	2.50
LT	0.199	2.05	1.79	1.73
IE	0.161	1.97	1.84	2.76
HU	0.138	1.97	1.78	2.02
IT	0.196	1.89	1.75	2.11
LV	0.193	1.85	1.62	2.17
MT	0.154	1.80	1.68	2.17
CY	0.145	1.80	1.78	1.49
ES	0.218	1.79	1.60	1.66
PT	0.180	1.78	1.58	2.29
EE	0.174	1.69	1.60	1.46
SK	0.130	1.52	1.56	1.94
PL	0.177	1.30	1.28	1.72
EL	0.214	1.27	1.29	1.54
RO	0.222	1.19	0.98	2.05
BG	0.223	1.16	1.03	1.53
Simple average	0.161	2.28	1.99	2.29

The other notable feature of the results is the greater stability of the values of the fuzzy measures, compared with the conventional measure ARPR. For instance, ratio (size = 1/size = 3) of deprivation scores for ARPR exceeds 4.0 in 3 countries (SI, FI, BG) and exceeds 3.0 for 2 others (AT, DE). Though raised values for these countries are also present in the case of FM, but to a reduced extent; and they are hardly present in the FS measure.

Generally, the greater stability across countries of fuzzy compared with conventional measures can be noted for differentials by other variables as well. This indicates lower values of sampling error.

As to differentials by tenure of accommodation, the differential between owners and non-owner is very marked according to all the three measures, as seen from Table 10.

Averaged over countries, overall ratio (non-owners/owner) of deprivation scores is 2.3 for ARPR, 2.0 for FM, and 2.3 for FS. (Only a small number (4) of countries contribute to the last-mentioned relatively high average value for FS.)

There is a considerable variation in the ratio (non-owners/owner) of deprivation scores across countries. The table identifies particularly high values as ARPR $\geq$ 3.0 and low values as ARPR $\leq$1.5, compared with overall average ARPR = 2.3. The low values of the ratio mostly appear in countries with above-average levels of deprivation, and high values in countries with above-average levels of deprivation. This applies to the fuzzy measures as well—though the sets of countries involved in the two extreme groups are not identical for the three deprivation measures. Overall, the fuzzy measures again show somewhat more stable results across countries (values of the coefficient of variation being 0.36 for ARPR, and 0.30 and 0.24 for FM and FS, respectively).

5 Concluding remarks and further research

The present contribution represents a continuation and further development of the work drawing on the concepts of fuzzy sets in the analysis of poverty and deprivation. The construction of fuzzy measures requires the specification of a membership function concerning the degree to which individual units in the population belong to a set characterising a particular form of deprivation. We began with describing a set of fuzzy measures of monetary and non-monetary deprivation and developed a step-by-step procedure for calculating non-monetary deprivation measures. We show how the multidimensional nature of poverty can be better captured by a set of items properly aggregated into several dimensions. As an empirical contribution, we presented results of the methodology applied to the 2011 wave of EU-SILC data, emphasising the overall EU-wide patterns comparing conventional and fuzzy deprivation measures.

Some benchmarking is necessary for fixing the scale of the fuzzy measures. In most of the empirical analysis presented, we have estimated the fuzzy measures such that their mean is equal to the at-risk-of-poverty rate (ARPR) computed on the basis the official poverty line (60 % of the median equivalised income). This is applied at the level of each EU country individually, and to both the fuzzy monetary (FM) and the fuzzy supplementary (FS) measures. Non-monetary deprivation is decomposed into dimensions or domain of deprivation. In constructing numerical measures of dimension-specific deprivation, we have considered it appropriate to use the model parameters determined for the overall FS measure for the country concerned, rather than to rescale each dimension separately.

Benchmarking of the above type is particularly suited for the study of differential by population subgroup within countries, as shown by our numerical illustrations. Results have been analysed for different types of variables defining subgroups, in terms of geographical (national regions), household (household size, tenure of accommodation), and personal (age, sex, level of education) characteristics. Two noteworthy conclusions that emerge from the illustrations are the following.

(1) Differentials among subgroups are generally attenuated as we move from the conventional measure ARPR to the fuzzy measures FM and FS. This is more markedly so for FS compared with FM. Conventional poverty rates for subgroups (e.g. regions) constructed on the basis of the poverty line defined at the total population (e.g. country) level are affected not only by disparity within the subgroups, but also by subgroup difference in the absolute levels of average members. The first component may be seen as more purely 'relative' and the last as 'absolute'. In comparison with conventional measures, fuzzy measures appear to reflect within-group disparities most strongly than between-group disparities. In this sense, they can be seen as being more relative than corresponding conventional measures. This is reflected in the attenuation of observed subgroups differentials. This possibility needs to be explored further both theoretically and numerically.

(2) Generally, there is greater stability across countries in fuzzy compared with conventional measures. This applies to differentials by different types of variables. Both these areas are interesting and need further research.
This observation indicates lower values of sampling error of fuzzy compared with the conventional measures. Computation of sampling errors of poverty measure, including the basic measure ARPR, in the presence of complex sampling designs is a complex task. We have developed, and applied using EU-SILC data, variance computation procedures for a diversity of conventional poverty measures (Verma and Betti 2011). This analysis could be extended to fuzzy measures as well.

(3) The third major area requiring further research concerns the specification and empirical application of rules for the manipulation of fuzzy sets, such as defining aggregation, union and intersection of sets. This paper has been concerned with the development of deprivation measures, which are essentially cross-sectional and consider one dimension at a time. In dealing with multiple dimensions, it is necessary to ascertain the overlaps in deprivation in different dimensions. Similarly, in analysing persistence of deprivation, it is necessary to ascertain the overlaps over time. For these purposes, the fuzzy measures for an individual's deprivation need to be put together over different dimensions and different time periods. Initial ideas such as in (Betti et al. 2006) need to be developed, applied, and evaluated more thoroughly.

6 Appendix: List of country names abbreviations

See Table 11.

Table 11 List of countries abbreviations

AT	Austria
BE	Belgium
BG	Bulgaria
CY	Cyprus
CZ	Czech republic
DE	Germany
DK	Denmark
EE	Estonia
EL	Greece
ES	Spain
FI	Finland
FR	France
HU	Hungary
IE	Ireland
IT	Italy
LT	Lithuania
LU	Luxembourg
LV	Latvia
MT	Malta
NL	Netherland
PL	Poland
PT	Portugal
RO	Romania
SE	Sweden
SI	Slovenia
SK	Slovakia
UK	United Kingdom

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