What is an oil shock? Panel data evidence

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Abstract This article characterizes the nonlinear relation between oil price change and GDP growth, focusing on the panel data of various industrialized countries. Toward this end, the article extends a flexible nonlinear inference to the panel data analysis where the random error components are incorporated into the flexible approach. The article reports clear evidence of nonlinearity in the panel and confirms earlier claims in the literature—oil price increases are statistically and economically significant while oil price decreases are not and previous upheaval in oil prices causes the marginal effect of any given oil price change to be reduced. Our result suggests that the nonlinear oil–macroeconomy relation is generally observable over different industrialized countries and it is desirable for one to use the nonlinear function of oil price change for GDP forecast.

Keywords Oil shock · Nonlinear flexible inference · Panel data · Error components model · Economic fluctuation

JEL Classification E32 · C33

1 Introduction

Quite a few studies have reported that changes in the price of oil appear to have a significant effect on economic activity. Examples include Rasche and Tatom (1977, 1981), Hamilton (1983, 1996, 2003, 2008, 2009a), Burbidge and Harrison (1984), Gisser and Goodwin (1986), Mork (1989), Dotsey and Reid (1992), Lee et al. (1995),

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Rotemberg and Woodford (1996), Carruth et al. (1998), Davis and Haltiwanger (2001), Cuñado and de Gracia (2003), Lee and Ni (2002); Leduc and Sill (2004); Hamilton and Herrera (2004), among others. However, Hooker (1996) argues that since the mid-1980s, the linear relation between oil prices and economic activity appears to be either unstable or misspecified. Blanchard and Gali (2008) show that the effects of a given change in the price of oil have changed substantially over time and conclude that three hypotheses—(i) more flexible labor market over time, (ii) changes in the way of conducting monetary policy, and (iii) a decline in the share of oil in the economy—seem to have played an important role in explaining the different effects of oil prices during the 1970s and during the last decade.

In the recent studies about the oil–macroeconomy relation, particularly two issues were debated. The first issue is what causes the oil price increases. Hamilton (1996, 2003, 2009a) claims that physical disruptions of supply of oil caused significant impact of oil price changes on macroeconomy whereas Barsky and Kilian (2002, 2004) and Kilian (2008a, b, 2009a, b) argue that expectations of future oil supply interruptions induce shocks to the precautionary demand for oil that reflect fears about future oil supplies while exogenous oil supply shocks account for only a comparatively small part of oil price movement and thus the oil price increases were driven by strong global demand for oil in conjunction with capacity constraints in crude oil production.¹ The second issue is the functional form of the oil–macroeconomy relation. Lee et al. (1995), Hamilton (1996, 2003, 2009a), Cuñado and de Gracia (2003), Jimenez-Rodriguez and Sanchez (2005), and Herrera et al. (2010) show that the relation is nonlinear while Kilian and Vigfusson (2009) find little evidence on the nonlinear relation.

This article focuses on the latter issue. Many authors have concluded that the nonlinearity of the relation between oil prices and economic activity is responsible for the instability of the empirical relation or misspecification of the functional form.² Hamilton (2003) applies a flexible approach to nonlinear inference developed by Hamilton (2001) and tries to isolate an exogenous component of oil price movements by measuring the oil supply curtailed by five separate military conflicts during the postwar period to address a statistically significant nonlinear relation. Hamilton (2003) finds that the nonlinear relation of oil prices suggested by the functional form of the conditional expectation function supports the lines suggested in the literature: oil price increases are statistically and economically significant while oil price decreases are not and increases have significantly less predictive content if they simply correct previous decreases. He also finds that the nonlinear transformation of oil prices based on the functional form is in fact quite similar to the first-stage least-squares fit from a regression of oil price changes on these exogenous supply disturbances, and that the dynamic multipliers from the nonlinear relation are similar to those coming from a linear relation estimated by instrumental variables. Mork et al. (1994) conclude from

¹ Hamilton (2009a) claims that the price run-up of 2007–2008 was caused by strong demand confronting stagnating world production whereas previous oil price shocks were primarily caused by physical disruptions of supply but the consequences of the 2007–2008 oil price increases for the economy appear to have been similar to those observed in earlier episodes about the U.S. recessions caused oil price shocks.

² Examples include Loungani (1986), Davis (1987a, b), Mork (1989), Lee et al. (1995), Hamilton (1996, 2003), Davis and Haltiwanger (2001), Cuñado and de Gracia (2003), Jimenez-Rodriguez and Sanchez (2005), among others.

the study of seven OECD countries that the correlation patterns between oil-price change and real GDP growth are not the same for price increases and decreases, and asymmetry in the effects of oil price fluctuation is a reasonably robust empirical finding. Cuñado and de Gracia (2003) found from the use of different transformation of oil price data that similar evidence of nonlinearity is observed for European countries as well as U.S.A. while there are significant differences among some of the countries. Jimenez-Rodriguez and Sanchez (2005) carried out multivariate VAR analysis using both linear and non-linear models for main industrialized countries and found that oil price increases have an impact on GDP growth of a larger magnitude than that of oil price declines, supporting a non-linear impact of oil prices on real GDP. Herrera et al. (2010) found a strong nonlinear response of U.S. industrial production to oil prices.

Kilian and Vigfusson (2009), however, show that the regression models and estimation methods which use measures that censor energy price changes to exclude all energy price decreases and are typically used in the existing literature, produce inconsistent estimates of the true effects of unanticipated energy prices increases and lead to overestimating the impact of energy price shocks on macroeconomic aggregates. They find little evidence against the null hypothesis of symmetry in the responses to energy price shocks. Nevertheless, Hamilton (2009b) notes that it must be differences in the specification and data set between Hamilton (2003) and Kilian and Vigfusson (2009), rather than differences in the testing methodology, that accounts for the different findings, and provides a number of detail differences that could explain Kilian and Vigfusson (2009) weaker evidence of nonlinearity such as different data sets, different measure of oil prices, different price adjustment, the inclusion of contemporaneous regressors, and number of lags.

This article considers two insights to examine the functional form of the oilmacroeconomy relationship. First of all, some studies suggest that the nonlinear relationship between oil-price changes and real GDP growth is present not only within a cross-section unit (a country) but also over the cross-section units (multi-countries). To the best of my knowledge, however, there is no systematic investigation of whether the relation between oil price change and real GDP is nonlinear in terms of panel data analysis. This article extends Hamilton (2001) methodology to the panel framework to characterize the nonlinear relation. Specifically, we consider the error components model of Wallace and Hussain (1969); Baltagi et al. (2002), among others, in the context of a flexible approach to nonlinear inference of Hamilton (2001).³ The methodology developed in this article is useful for analyzing nonlinear relation between economic variables in the panel framework because the benefits of nonlinear flexible inferences claimed in Hamilton (2001) can be strengthened with several advantages from using panel data.⁴ This framework may be applied to re-examine the structural stability of the Phillips Curve as the example of Hamilton (2001) and to investigate

³ The error component model has been considered by Wallace and Hussain (1969), Nerlove (1971), Maddala (1971), Amemiya (1971), Swamy and Arora (1972), Fuller and Battese (1973), Baltagi (1981), Baltagi and Griffin (1983), Breusch (1987), Boehmer and Megginson (1990), and Baltagi and Pinnoi (1995), among others. For further discussion and references see Baltagi (2008).

⁴ For detail discussion of the benefits from using panel data, see Klevmarken (1989), Hsiao (2003), and Baltagi (2008).

the nature of nonlinearity in the monetary policy rule as the example of Kim et al. (2005). Secondly, our parametric approach does not have to use the censored oil price changes to investigate asymmetric oil–macroeconomy relation and thus one would avoid potential problems from using the censored energy price changes as pointed out in Kilian and Vigfusson (2009).

In our model, the nonlinear functional relation is common across countries and over time and the regression error is assumed to be composed of three independent components—one component associated with the cross-sectional units, another with an aggregate shock, and the third being an idiosyncratic shock. The results support the claim of a nonlinear relation along the lines suggested in the literature: oil price increases affect the economy whereas decreases do not and previous upheaval in oil prices causes a reduction in the marginal effect of any given oil price change. The alternative specifications of the panel data model with nonlinear flexible inference as a robustness analysis confirm the asymmetric effect of oil price change on macroeconomy.

The plan of the article is as follows: Sect. 2 considers the error components model of the panel data in the context of a parametric approach to flexible nonlinear inference. Empirical results for the analysis of oil–macroeconomy relation are presented in Sect. 3. Concluding remarks are offered in Sect. 4.

2 A parametric approach to nonlinear flexible inference in the panel

2.1 Model

Consider the general nonlinear regression model of the form based on the panel

$$y_{it} = \mu_i(\mathbf{x}_{it}) + \varepsilon_{it}, \ i = 1, 2, \dots, N, t = 1, 2, \dots, T,$$
 (1)

where, y_{it} is a scalar-dependent variable at time t for country i, \mathbf{x}'_{it} is a k-dimensional vector of explanatory variables, and ε_{it} is Gaussian with dependence structure with mean zero and independent of both $\mu_i(.)$ and $\mathbf{x}_{i\tau}$ for i = 1, ..., N, and $\tau = t, t - 1, ..., 1$. This specification considers the nonlinear relation over the group as well as within the group and thus allows the functional relation to be different over cross-country units. Following Hamilton (2001), the conditional mean function in the panel data, $\mu_i(\mathbf{x}_i)$, is written as

$$\mu_i(\mathbf{x}_{it}) = \alpha_0 + \boldsymbol{\alpha}'_{i1}\mathbf{x}_{it} + \lambda_i m(\mathbf{g}_i \odot \mathbf{x}_{it}), \ i = 1, 2, \dots, N,$$
(2)

where m(.) denotes the realization of a scalar-valued Gaussian random field with mean zero and unit variance, α_0 , α'_{i1} , λ_i , and \mathbf{g}_i are population parameters to be estimated, $\mathbf{g}_i = (g_{i1}, g_{i2}, \ldots, g_{ik})'$ and \odot indicates element-by-element multiplication. λ_i^2 governs the overall importance of the nonlinear component, and \mathbf{g}_i governs the variability of the nonlinear component with respect to each explanatory variable.

As an approach to combining cross section and time series data, we consider the use of an error components model where one component of random error ε_{it} is an

unobserved individual effect which is constant through time, another component is an unobserved time effect which is the same for all individuals at a given time, and the third component is an unobserved remainder which differs among individuals both at a point in time and through time. Thus, we assume that the residual, ε_{it} , is decomposed into the sum of three components:

$$\varepsilon_{it} = \omega_i + a_t + v_{it},\tag{3}$$

where, ω_i is an individual specific variable, a_t a time-specific variable, and v_{it} is the remainder. $\omega'_i s, a'_t s$, and $v'_{it} s$ are random, have zero means, have variances $\sigma_{\omega}^2, \sigma_a^2$, and σ_v^2 , and are independent of each other. That is, it is assumed that $E\omega_i = Ea_t = Ev_{it} = 0$, $E\omega_i\omega_j = 0$ for $i \neq j$, $Ea_t a_s = 0$ for $t \neq s$, $Ev_{it}v_{js} = \sigma_v^2$ for i = j, t = s, and zero otherwise, $E\omega_i a_t = E\omega_i v_{it} = Ea_t v_{it} = 0$. In addition, $\mathbf{x}_{i\tau}$ is strictly exogenous and \mathbf{x}_{it} and ε_{it} are independent of the realization of the random field $m(\cdot)$ in Eq. 2.

For the case of strictly exogenous regressors and no lagged dependent variables, we assume that conditional on the full sample of observations on the exogenous explanatory variables $(X = \{\mathbf{x}_{it}\}_{i=1,\dots,N;t=1,\dots,T})$, the variables ω_i , a_t and v_{it} are all Normal with zero means, variances σ_{ω}^2 , σ_a^2 , and σ_v^2 , respectively, and are mutually independent. That is, for $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1', \dots, \boldsymbol{\varepsilon}_N')'$ and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \dots, \varepsilon_{iT})'$, we assume that

$$\boldsymbol{\varepsilon}|X \sim N\left(\mathbf{0}, \sigma_{\omega}^{2}(\mathbf{I}_{N} \otimes \mathbf{J}_{T}) + \sigma_{a}^{2}(\mathbf{J}_{N} \otimes \mathbf{I}_{T}) + \sigma_{v}^{2}\mathbf{I}_{NT}\right)$$

where \mathbf{J}_T and \mathbf{J}_N denote $(T \times T)$ and $(N \times N)$ matrices of ones, respectively. In our application below we include lagged values of y_{it} along with lagged values of oil prices, with the latter taken to be strictly exogenous. In this case the conditioning set X corresponds to the full sample of observations on oil prices and the pre-sample observations on y_{it} , that is, $X = \{\{o_{it}\}_{i=1,...,N;t=-3,-2,...,T}, \{y_{it}\}_{i=1,...,N;t=-3,-2,-1,0}\}$ for the above conditional distribution.

For simplicity, we further assume that the slopes in the linear component in Eq. 2 are homogenous among different individuals, and λ_i and \mathbf{g}_i are not specific to cross-section units. In general, allowing nonlinear parameters to be country-specific (heterogenous nonlinear components), may be useful for considering the panel heterogeneity issue in the application of our method to various economic application. In our application to the industrialized countries for oil–macroeconomy relation, however, such heterogeneity across the panel may be less likely and the homogenous assumption for nonlinear parameters over different countries would make one focus on common oil–macroeconomy relation across countries.

Under these assumptions, the general specification, (1) and (2), with random-effect and *k*-explanatory variables and the conditional mean function of Eq. 2 in the panel can be rewritten

$$y_{it} = \alpha_0 + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}) + \varepsilon_{it}, \qquad (4)$$

$$\varepsilon_{it} = \omega_i + a_t + v_{it},\tag{5}$$

$$\mu(\mathbf{x}_{it}) = \alpha_0 + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}), \ i = 1, 2, \dots, N, \ t = 1, \dots, T.$$
(6)

Hamilton (2001) chooses a generalized version of the so-called spherical covariance function and this function can be applied to the panel as

$$H_k(h_{is,jt}) = \begin{cases} G_{k-1}(h_{is,jt},1)/G_{k-1}(0,1) & if \quad h_{is,jt} \le 1\\ 0 & if \quad h_{is,jt} > 1, \end{cases}$$
(7)

$$G_k(h_{is,jt},r) = \int_{h_{is,jt}}^{\prime} (r^2 - z^2)^{k/2} dz,$$
(8)

$$h_{is,jt} = (1/2) \{ [\mathbf{g} \odot (\mathbf{x}_{is} - \mathbf{x}_{jt})]' [\mathbf{g} \odot (\mathbf{x}_{is} - \mathbf{x}_{jt})] \}^{1/2}, i, j = 1, \dots, N, t, s = 1, \dots, T$$
(9)

where $H_k(h_{is,jt})$ denotes the $\{is, jt\}$ entry in the $NT \times NT$ covariance function matrix H_k .

2.2 Inference about the conditional expectation function

If each component in ε_{it} (Eq. 5) is random and normally distributed and if the regressor \mathbf{x}'_{it} is strictly exogenous, then the specification of Eqs. 4, 5 and 6 implies a GLS regression model of the form

$$\mathbf{y}|\mathbf{X} \sim N(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}), \tag{10}$$

where

$$\mathbf{y} = (\mathbf{y}'_1, \dots, \mathbf{y}'_N)', \ \mathbf{y}_i = (y_{i1}, \dots, y_{iT})',$$

$$\mathbf{X} = \begin{bmatrix} 1 & \mathbf{x}'_{11} \\ 1 & \mathbf{x}'_{12} \\ \vdots & \vdots \\ 1 & \mathbf{x}'_{1T} \\ \vdots & \vdots \\ 1 & \mathbf{x}'_{NT} \end{bmatrix},$$

$$\mathbf{\Omega} = \mathbf{C}_0 + \sigma_{\omega}^2 (\mathbf{I}_N \otimes \mathbf{J}_T) + \sigma_a^2 (\mathbf{J}_N \otimes \mathbf{I}_T) + \sigma_v^2 \mathbf{I}_{NT}, \tag{11}$$

$$\mathbf{C}_0 = [\lambda^2 H_k(h_{i3}; i)]; \ i=1, \dots, k \in I_{n-1}, T. \tag{12}$$

$$\mathbf{C}_0 = [\lambda^2 H_k(h_{is,jt})]_{i,j=1,...,N,\ \&\ t,s=1,...,T},$$
(12)

 $\boldsymbol{\beta} = (\alpha_0, \alpha'_1)'$, the (1+k)-dimensional vector, \mathbf{J}_T and \mathbf{J}_N are the $T \times T$ and the $N \times N$ ones matrixes, respectively, \mathbf{I}_{NT} is the $NT \times NT$ identity matrix, the function $H_k(.)$ is as specified in Eqs. 4 and 5, and $h_{is,jt}$ is given in Eq. 9. Note that $\sigma^2 = \sigma_{\omega}^2 + \sigma_a^2 + \sigma_v^2$.

The log likelihood function is

$$\ln f(\mathbf{y}; \boldsymbol{\beta}, \sigma_{\omega}^{2}, \sigma_{a}^{2}, \sigma_{v}^{2}, \lambda, \mathbf{g}) = -\frac{NT}{2} \ln 2\pi - \frac{1}{2} \ln |\boldsymbol{\Omega}| - \frac{1}{2} (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta})' \boldsymbol{\Omega}^{-1} (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta}).$$
(13)

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The maximum likelihood estimators of $\boldsymbol{\beta}$, σ_{ω}^2 , σ_a^2 , σ_v^2 , λ , and \mathbf{g} are obtained by maximizing the log likelihood function (13) with respect to those parameters. However, even if the $\boldsymbol{\varepsilon} = \mathbf{y} - \boldsymbol{X}\boldsymbol{\beta}$, were observable, it is very difficult to solve explicitly this maximization problem. Hamilton (2001) shows that one useful way to deal with this problem is to use reparameterization to obtain the concentrated likelihood function. For each pair of observation *is* and *jt*, calculate $\tilde{\mathbf{x}}_{it} = \mathbf{g} \odot \mathbf{x}_{it}$ and $h_{is,jt}(\mathbf{g}) = (1/2)[(\tilde{\mathbf{x}}_{is} - \tilde{\mathbf{x}}_{jt})]^{1/2}$. Let $\mathbf{H}(\mathbf{g})$ denotes the $(NT \times NT)$ matrix whose (is, jt) element is $H_k(h_{is,jt}(\mathbf{g}))$. Let $\mathbf{Y}_{nt} = (\mathbf{y}'_{nt}, \mathbf{x}'_{nt}, \mathbf{y}'_{nt-1}, \mathbf{x}'_{nt-1}, \dots, \mathbf{y}'_{n1}, \mathbf{x}'_{n1})'$, $\mathbf{y}_{nt} = (y_{1t}, y_{2t}, \dots, y_{nt})'$ and $\mathbf{x}_{nt} = (\mathbf{x}'_{1t}, \mathbf{x}'_{2t}, \dots, \mathbf{x}'_{nt})'$, denote information observed through date *t* forn number of individuals. Define $\zeta \equiv \lambda/\sigma_v$ to be the ratio of the standard deviation of the nonlinear component $\lambda m(\mathbf{x})$ to that of the residual *v* and let $\phi_{\omega} = \frac{\sigma_{\omega}}{\sigma_{v}}, \phi_{a} = \frac{\sigma_{a}}{\sigma_{v}} \psi = (\alpha_{0}, \boldsymbol{\alpha}', \sigma_{v}^{2})', \theta = (\mathbf{g}', \zeta, \phi_{\omega}, \phi_{a})'$ and

$$W(X; \boldsymbol{\theta}) \equiv \zeta^2 \mathbf{H}(\mathbf{g}) + \phi_{\omega}^2 (\mathbf{I}_N \otimes \mathbf{J}_T) + \phi_a^2 (\mathbf{J}_N \otimes \mathbf{I}_T) + \mathbf{I}_{NT}.$$
 (14)

Now the log likelihood can be written from (13)

$$\ln f(\mathbf{y}; \boldsymbol{\psi}, \boldsymbol{\theta}) = -\frac{NT}{2} \ln 2\pi - \frac{NT}{2} \ln \sigma_v^2 - \frac{1}{2} \ln |W(\boldsymbol{X}; \boldsymbol{\theta})| -\frac{1}{2\sigma_v^2} (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta})' W(\boldsymbol{X}; \boldsymbol{\theta})^{-1} (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta}).$$
(15)

For given θ , the value of ψ that maximizes (15) can be calculated analytically as

$$\widetilde{\boldsymbol{\beta}}(\boldsymbol{\theta}) = [\mathbf{X}' \boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1} \mathbf{X}]^{-1} [\mathbf{X}' \boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})^{-1} \mathbf{y}],$$
(16)

$$\widetilde{\sigma}_{v}^{2} = (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta})' \boldsymbol{W}(\boldsymbol{X};\boldsymbol{\theta})^{-1} (\mathbf{y} - \boldsymbol{X}\boldsymbol{\beta}) / NT.$$
(17)

Now, these allow us to concentrate the log likelihood (15) as

$$L_{c}(\boldsymbol{\theta}; \boldsymbol{y}, \boldsymbol{X}) = \sum_{i=1}^{N} \sum_{t=1}^{T} \ln f(y_{it} | \mathbf{x}_{it}, \mathbf{Y}_{Nt-1}; \widetilde{\psi}(\boldsymbol{\theta}), \boldsymbol{\theta})$$
$$= -\frac{NT}{2} \ln 2\pi - \frac{NT}{2} \ln \widetilde{\sigma}_{v}^{2}(\boldsymbol{\theta}) - \frac{1}{2} \ln |\boldsymbol{W}(\boldsymbol{X}; \boldsymbol{\theta})| - (NT/2).$$
(18)

Numerically maximizing (18) gives the MLE $\hat{\theta}$, which from (16) and (17) gives $\hat{\psi}$.

2.3 Bayesian analysis

In this subsection, we extend Hamilton (2001) Bayesian analysis to the error components model of the panel data considered above. Let $\psi = (\beta', \sigma_v^{-2})'$, and

 $\theta = (\mathbf{g}', \zeta, \phi_{\omega}, \phi_a)'$. We adopt a standard prior for the linear components as in Hamilton (2001, 2003).⁵ The prior distribution of σ_v^{-2} is $\Gamma(v, \xi)$:

$$p(\sigma_v^{-2}) = \frac{\xi^v}{\Gamma(v)} \sigma_v^{-2(v-1)} \exp[-\xi \sigma_v^{-2}],$$
(19)

where v = 0.25 and $\xi = (vs_y^2/2)$, $s_y^2 = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (y_{it} - \overline{y})^2$, $\overline{y} = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^N y_{it}$. The prior distribution of $\boldsymbol{\beta}$ conditional on σ_v^{-2} is Gaussian:

$$p(\boldsymbol{\beta}|\sigma_v^{-2}) = \frac{1}{(2\pi\sigma_v^2)^{(k+1)/2}} |\mathbf{M}|^{-1/2} \times \exp\left[\left(\frac{-1}{2\sigma_v^2}\right)(\boldsymbol{\beta}-\mathbf{m})\mathbf{M}^{-1}(\boldsymbol{\beta}-\mathbf{m})\right], \quad (20)$$

where we set the first element of **m** to the sample mean of y_{it} and all other components to zero, $\mathbf{m} = (\bar{y}, 0, ..., 0)'$ and we take $\mathbf{M} = NT(\mathbf{X}'\mathbf{X})^{-1}$, so that the prior has the weight of a single observation on $(y_{it}, \mathbf{x}'_{it})$. The prior distribution of each element of $\boldsymbol{\theta}$ is a lognormal distribution:

$$p(\boldsymbol{\theta}) = \prod_{i=1}^{k+3} \frac{1}{\sqrt{2\pi}\tau_i \theta_i} \exp\left[\frac{-[\ln(\theta_i) - \vartheta_i]^2}{2\tau_i^2}\right].$$
 (21)

Note that the prior for θ_i is taken to be independent of that for $\boldsymbol{\beta}$, σ_v^{-2} and θ_j , $j \neq i$. We use $\tau_i = 1$ for i = 1, ..., k + 3, and allow ϑ_j to depend on the standard deviation of variable $j, \vartheta_j = -\ln\left(\sqrt{ks_j^2}\right)$ with $s_j^2 = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (x_{jit} - \overline{x}_j)^2$ and $\vartheta_\omega = -\ln\left(\sqrt{\frac{s_\omega^2}{s_y^2}}\right)$, $\vartheta_a = -\ln\left(\sqrt{\frac{s_a^2}{s_y^2}}\right)$, where $s_\omega^2 = N^{-1} \sum_{i=1}^N (\overline{y}_{i.} - \overline{y})^2$, $s_a^2 = T^{-1} \sum_{t=1}^T (\overline{y}_{.t} - \overline{y})^2$, $\overline{y}_{i.} = T^{-1} \sum_{t=1}^T y_{it}$, $\overline{y}_{.t} = N^{-1} \sum_{i=1}^N y_{it}$. $\vartheta_{k+1} = 0$. Our interest is to infer the posterior expected value of some function $l(\boldsymbol{\theta})$,

$$E[l(\boldsymbol{\theta})|\boldsymbol{Y}_{NT}] = \int l(\boldsymbol{\theta}) f(\boldsymbol{\theta}|\boldsymbol{Y}_{NT}) d\boldsymbol{\theta}, \qquad (22)$$

where $f(\theta|Y_{NT}) = f(\theta, y|X) / \int f(\theta, y|X) d\theta$. Following Geweke (1989) and Hamilton (2001), we evaluate (22) by using importance-sampling algorithm. We consider an arbitrary importance sampling distribution $I(\theta)$ and generate an artificial i.i.d. sample $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(D)}$ drawn from the $I(\theta)$, and calculate

$$\hat{E}[l(\boldsymbol{\theta})|\boldsymbol{Y}_{NT}] = l(\boldsymbol{\theta})^* = \frac{\sum_{j=1}^{D} l(\boldsymbol{\theta}^{(j)}) w(\boldsymbol{\theta}^{(j)}, \boldsymbol{Y}_{NT})}{\sum_{j=1}^{D} w(\boldsymbol{\theta}^{(j)}, \boldsymbol{Y}_{NT})},$$
(23)

⁵ As in Hamilton (2001, 2003) for time-series analysis, the method here requires nondiffuse priors in order for the posterior distribution to be well-defined. For further details about the prior distribution, see Hamilton (2001).

$$w(\boldsymbol{\theta}^{(j)}, \mathbf{Y}_{NT}) = \frac{f(\boldsymbol{\theta}^{(j)}, \mathbf{y} | \mathbf{X})}{I(\boldsymbol{\theta}^{(j)})}.$$
(24)

Following Hamilton (2001), we have an algorithm based on a truncated mixture density. With probability 0.5, we generate θ from a multivariate Student *t* distribution with $\varphi = 2$ degrees of freedom, centered at the MLE with precision matrix given by (-1/2) times the matrix of second derivatives of the log likelihood function. With probability 0.5, the elements of $\ln(\theta)$ are drawn independently from $N(\vartheta_i, 4)$ distributions, so that the logs have the same mean but twice the standard deviation of the prior. The truncation can be achieved by throwing out any draw for which some $\vartheta_i < 0$. Thus, we have

$$I(\boldsymbol{\theta}) \propto (0.5) \frac{\Gamma[(k+3+\varphi)/2]}{\Gamma(\varphi/2)(\varphi\pi)^{(k+3)/2}} |\hat{\boldsymbol{\Omega}}|^{-1/2} \times [1+\varphi^{-1}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})'\hat{\boldsymbol{\Omega}}^{-1}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})]^{-1(k+3+\varphi)/2} + (0.5) \prod_{j=1}^{k+3} \frac{1}{\sqrt{2\pi}(2\tau_j)\theta_j} \exp\left[\frac{-[\ln(\theta_j)-\vartheta_j]^2}{2(2\tau_j)^2}\right],$$
(25)
for $\theta_j \ge 0, j = 1, 2, ..., k+3,$

where the constant of proportionality reflects the truncation, $\hat{\theta}$ is the MLE, $\hat{\Omega}$ is twice its asymptotic variance, $\varphi = 2$, $\tau_j = 1$, $\vartheta_j = 0$, for j = k + 1, k + 2, k + 3 and ϑ_j is given by in the part of the prior distribution $\vartheta_j = -\ln(\sqrt{ks_j^2})$ for j = 1, ..., k.

2.4 Testing for nonlinearity

We consider Hamilton (2001) LM test in the error components model of the panel data. In what follows, we briefly describe the test procedure. Following Hamilton (2001) we fix **g** from the scale of the data, for example, by setting g_i equal to the mean of the prior distribution in (21). Let $\sigma^2 = (\sigma_{\omega}^2, \sigma_a^2, \sigma_v^2)'$ and suppose these variances are observable. Let \mathbf{H}_{NT} be a known ($NT \times NT$) positive semidefinite matrix and let

$$\mathbf{\Omega}_{NT} = \lambda^2 \mathbf{H}_{NT} + \sigma_{\omega}^2 \mathbf{Q}_{\omega} + \sigma_a^2 \mathbf{Q}_a + \sigma_v^2 \mathbf{I}_{NT}, \qquad (26)$$

where $\mathbf{Q}_{\omega} = (\mathbf{I}_N \otimes \mathbf{J}_T), \mathbf{Q}_a = \mathbf{J}_N \otimes \mathbf{I}_T$. Consider the likelihood function under the assumption that $\mathbf{y}|\mathbf{X}, \sigma^2 \sim N(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}_{NT})$:

$$\ln f(\mathbf{y}|\mathbf{X}, \boldsymbol{\sigma}^{2}; \boldsymbol{\zeta}) = -\frac{NT}{2} \ln 2\pi - \frac{1}{2} \ln |\boldsymbol{\Omega}_{NT}| - \frac{1}{2} tr(\boldsymbol{\Omega}_{NT}^{-1} \boldsymbol{\varepsilon} \boldsymbol{\varepsilon}'), \quad (27)$$

for $\boldsymbol{\varepsilon} = \boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}$ and $\boldsymbol{\zeta} = (\lambda^2, \boldsymbol{\beta}')'$. Then, we have a score:

$$\frac{\partial \ln f(\mathbf{y}|\mathbf{X}, \boldsymbol{\sigma}^{2}; \boldsymbol{\zeta})}{\partial \lambda^{2}}|_{\lambda^{2}=0} = -(1/2)tr(\mathbf{Q}_{NT}^{-1}\mathbf{H}_{NT}) + (1/2)tr(\mathbf{Q}_{NT}^{-1}\mathbf{H}_{NT}\mathbf{Q}_{NT}^{-1}\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'],$$
(28)

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where $\mathbf{Q}_{NT} = \sigma_{\omega}^2 \mathbf{Q}_{\omega} + \sigma_a^2 \mathbf{Q}_a + \sigma_v^2 \mathbf{I}_{NT}$. The information matrix is:

$$-E\left\{\frac{\partial^2 \ln f(\mathbf{y}|\mathbf{X}, \sigma^2; \boldsymbol{\zeta})}{\partial \boldsymbol{\zeta} \partial \boldsymbol{\zeta}'}|_{\boldsymbol{\zeta}=\boldsymbol{\zeta}_0}\right\} = \begin{bmatrix} (1/2)tr[(\mathbf{Q}_{NT}^{-1}\mathbf{H}_{NT})^2] & \mathbf{0}\\ \mathbf{0} & \sigma^{-2}\mathbf{X}'\mathbf{X} \end{bmatrix}.$$
(29)

Then, the Lagrange multiplier test of the null hypothesis that $\lambda = 0$ conditional on σ^2 is given by:

$$\boldsymbol{\aleph}_{NT} = \frac{\left[\left(\boldsymbol{\varepsilon}' \mathbf{Q}_{NT}^{-1} \mathbf{H}_{NT} \mathbf{Q}_{NT}^{-1} \boldsymbol{\varepsilon}\right) - tr(\mathbf{Q}_{NT}^{-1} \mathbf{H}_{NT})\right]}{\sqrt{tr[(\mathbf{Q}_{NT}^{-1} \mathbf{H}_{NT})^2]}}.$$
(30)

Following Wallace and Hussain (1969) and Amemiya (1971) for the best quadratic unbiased estimators of the variance components, we turn to estimates of ε_{it} , say $\hat{\varepsilon}_{it}$ which are observed residuals obtained by least squares. Amemiya (1971) points out that the estimate of variance components based on the ordinary least squares are less efficient and provides following process: ε is obtained by first estimating β by $\hat{\beta} = (\mathbf{X}'\mathbf{G}\mathbf{X})^{-1}\mathbf{X}'\mathbf{G}\mathbf{y}$, $\mathbf{G} = \mathbf{I}_{NT} - (1/T)\mathbf{Q}_{\omega} - (1/N)\mathbf{Q}_{a} + (1/NT)\mathbf{J}_{NT}$, \mathbf{J}_{NT} is the $NT \times NT$ matrix consisting only of ones, and α_0 by $\hat{\alpha}_0 = (1/NT)\mathbf{e}'_{NT}(\mathbf{y} - \mathbf{X}\hat{\beta})$ and then predicting ε by $\mathbf{y} - \hat{\alpha}_0\mathbf{e}_{NT} - \mathbf{X}\hat{\beta} = \mathbf{y} - (\mathbf{J}_{NT}/NT)\mathbf{y} - (\mathbf{1}/NT)\mathbf{J}_{NT}\mathbf{X}(\mathbf{X}'\mathbf{G}\mathbf{X})^{-1}\mathbf{X}'\mathbf{G}\mathbf{y} - \mathbf{X}(\mathbf{X}'\mathbf{G}\mathbf{X})^{-1}\mathbf{X}'\mathbf{G}\mathbf{y}$. Then, the analysis of variance estimates are

$$\hat{\sigma}_{v}^{2} = \frac{1}{(N-1)(T-1)} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(\hat{\varepsilon}_{it} - \frac{1}{T} \hat{\varepsilon}_{i.} - \frac{1}{N} \hat{\varepsilon}_{.t} + \frac{1}{NT} \hat{\varepsilon}_{..} \right)^{2} = \frac{1}{(N-1)(T-1)} \hat{\varepsilon}' G \hat{\varepsilon},$$
(31)

$$\hat{\sigma}_{\omega}^{2} = \frac{1}{T(N-1)} \sum_{i=1}^{N} \left(\frac{1}{T} \hat{\varepsilon}_{i.} - \frac{1}{NT} \hat{\varepsilon}_{...} \right)^{2} - \frac{1}{T} \hat{\sigma}_{v}^{2}$$
$$= \frac{1}{T(N-1)(T-1)} \hat{\boldsymbol{\varepsilon}}' \left[\frac{T-1}{T} \mathbf{Q}_{\omega} - \frac{T-1}{NT} \mathbf{J}_{NT} - \mathbf{G} \right] \hat{\boldsymbol{\varepsilon}}, \qquad (32)$$

$$\hat{\sigma}_{a}^{2} = \frac{1}{N(T-1)} \sum_{t=1}^{T} \left(\frac{1}{N} \hat{\varepsilon}_{.t} - \frac{1}{NT} \hat{\varepsilon}_{...} \right)^{2} - \frac{1}{N} \hat{\sigma}_{v}^{2}$$
$$= \frac{1}{N(N-1)(T-1)} \hat{\varepsilon}' \left[\frac{N-1}{N} \mathbf{Q}_{a} - \frac{N-1}{NT} \mathbf{J}_{NT} - \mathbf{G} \right] \hat{\varepsilon}, \qquad (33)$$

where $\hat{\varepsilon}_{i.} = \sum_{t=1}^{T} \hat{\varepsilon}_{it}$, $\hat{\varepsilon}_{.t} = \sum_{i=1}^{N} \hat{\varepsilon}_{it}$, $\hat{\varepsilon}_{..} = \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{\varepsilon}_{it}$. The Lagrange multiplier test of (30) with the estimate of variance components (31), (32) and (33) is given

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$$\hat{\aleph}_{NT} = \frac{\left[\left(\hat{\boldsymbol{\varepsilon}}'\,\hat{\boldsymbol{\mathcal{Q}}}_{NT}^{-1}\mathbf{H}_{NT}\,\hat{\boldsymbol{\mathcal{Q}}}_{NT}^{-1}\hat{\boldsymbol{\varepsilon}}\right) - tr\left(\hat{\boldsymbol{\mathcal{Q}}}_{NT}^{-1}\mathbf{H}_{NT}\right)\right]}{\sqrt{tr\left[\left(\hat{\boldsymbol{\mathcal{Q}}}_{NT}^{-1}\mathbf{H}_{NT}\right)^{2}\right]}},$$
(34)

where $\hat{\boldsymbol{\varepsilon}} = \boldsymbol{y} - \hat{\alpha}_0 \mathbf{e}_{NT} - \boldsymbol{X}\hat{\boldsymbol{\beta}}, \ \hat{\alpha}_0 = (1/NT)\mathbf{e}'_{NT}(\mathbf{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}), \ \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{G}\mathbf{X})^{-1}\mathbf{X}'\mathbf{G}\mathbf{y},$ and $\hat{\boldsymbol{Q}}_{NT} = \hat{\sigma}_{\omega}^2 \mathbf{Q}_{\omega} + \hat{\sigma}_a^2 \mathbf{Q}_a + \hat{\sigma}_{\upsilon}^2 \mathbf{I}_{NT}.$

3 Empirical results

3.1 Data

The countries included in this study are Australia, Canada, Germany, Netherlands, U.K., and U.S.A.⁶ The series used for real output, y_{it} is the quarterly growth rate of real GDP and we use the world produce price index for crude oil for all countries in dollars and convert it into each country's currency by means of the market exchange rate. The sample used is from 1960:I to 2003:IV. All these data have been downloaded from the International Financial Statistics (IFS) in IMF except U.S.A. The data for U.S. real GDP are from the Bureau of Economic Analysis (BEA) and the produce price index for crude oil for U.S.A. are from the Bureau of Labor Statistics. A total of $T \times N = 1026$ observations is available.

3.2 Linear error components model

When $\lambda = 0$, the model of Eqs. 4, 5, and 6 is a two-way error component model as follows:

$$y_{it} = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_{it} + \varepsilon_{it}, \tag{35}$$

$$\varepsilon_{it} = \omega_i + a_t + v_{it}, i = 1, 2, \dots, N, t = 1, \dots, T.$$
 (36)

Following Amemiya (1971), we consider the interactive MLE for Eqs. 35 and 36 and the estimation results are as follows:

⁶ We initially considered 10 countries (G7 + Australia, Netherlands, Sweden) and excluded four countries, French, Italy, Japan and Sweden. The reason was that as the result of the estimation of the linear model for individual country, these countries have shown quite different dynamics over lagged GDP growth and lagged oil price change, implying that the slopes in the linear component in Eq. 2.2 are not homogenous among different individuals and thus indicating that these countries are far away from homogenous assumption. Japan, in particular, exhibits significantly different dynamics over lagged GDP growth during the sample. An anonymous referee notes that this empirical section may have potential issues arising due to the limited country numbers. This issue would be serious if we estimated the unrestricted model (1–2) with small samples. However, it is unlikely that the estimation of the restricted model (4–6) representing a behavioral equation with the same parameters over time and across countries raises the issue.

$$y_{it} = 2.531 - 0.044 y_{it-1} + 0.068 y_{it-2} + 0.123 y_{it-3} + 0.068 y_{it-4} + 0.011 o_{it-1} - 0.005 o_{it-2} - 0.023 o_{it-3} - 0.039 o_{it-4} (0.014) (0.014) (0.015) (0.014) (0.$$

The coefficient on o_{it-4} is statistically significant at the 1% level. Even though the results seem to support the linear relationship between oil price change and real GDP growth, the test statistic of the null hypothesis of linearity has a value of 30.93, which for a $\chi^2(1)$ variable implies overwhelming rejection of the null hypothesis that the relation is linear in the panel. There seems little question that the relation between oil prices and GDP is nonlinear.

To investigate the performance of the test statistics (34), instead of deriving the asymptotic approximation in Eq. 34, we try to approximate the exact small-sample distribution of $\hat{\aleph}_{NT}$ by Monte Carlo methods. Given the data generating process of the linear error components model, we calculated the test statistic $\hat{\aleph}_{NT}$ for 1,000 simulations and did not find any case where $\hat{\aleph}_{NT}$ is greater than that of our original data. The highest $\hat{\aleph}_{NT}$ among 1,000 simulations was 4.46 ($\chi^2(1)$ form of the test = 19.89) while the $\hat{\aleph}_{NT}$ of the original data was 5.56 ($\chi^2(1)$ form of the test = 30.93), implying that the original data for the relation between oil price and GDP growth is far from the linear relation. Furthermore, at the nominal 5% level of significance, the rejection rate of the linearity null hypothesis was 1.6%, implying that overall performance of the developed test statistic (34) has good small-sample property. The Appendix provides detail Monte Carlo methods for the small-sample distribution of the LM test statistic of Eq. 34.

3.3 Nonlinear flexible model with random effect error components

Bayesian posterior estimates and their standard errors for the flexible nonlinear alternative with error components as in the model of Eqs. 4, 5, and 6 are as follows:

$$y_{it} = 2.114 - 0.056 y_{it-1} + 0.050 y_{it-2} + 0.109 y_{it-3} + 0.056 y_{it-4} + 0.010 o_{it-1} - 0.003 o_{it-2} - 0.020 o_{it-3} - 0.038 o_{it-4} + 4.567 [0.336m (0.08 o_{it-1}, 0.07 o_{it-2}, 0.07 o_{it-3}, 0.07 o_{it-4}) + \tilde{\omega}_i + \tilde{a}_t + \tilde{v}_{it}] (0.118) (0.054) (0.10) (0.10) (0.08) (0$$

$$\hat{\sigma}_{\omega}^2 = 0.982, \ \hat{\sigma}_a^2 = 0.476, \ \hat{\sigma}_v^2 = 20.86,$$

where $\tilde{\omega}_i \sim N(0, 1)$, $\tilde{a}_t \sim N(0, 1)$, $\tilde{v}_{it} \sim N(0, 1)$, and m(.) denotes an unobserved realization from a Gaussian random field with mean zero, unit variance and correlations given by Eqs. 7–9.⁷ The innovation ω_i , a_t , and v_{it} in (5) are written as $\sigma_v = 4.567$ times $\tilde{\omega}_i$, σ_v times \tilde{a}_t and σ_v times \tilde{v}_{it} , respectively. The parameter λ in (4) is written

⁷ The Bayesian analysis is based on 100,000 draws from the importance sampling density described in Sect. 2.3.

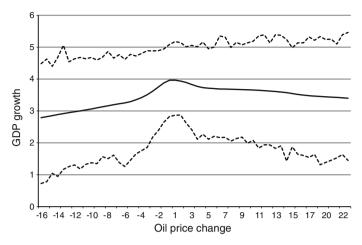


Fig. 1 Effect of oil prices on GDP growth one quarter later. Solid line plots the posterior expectation of the function $\alpha_0 + \alpha' \mathbf{x}_{it} + \delta' \mathbf{z}_{it} + \lambda m(\mathbf{x}_{it})$ evaluated at $\mathbf{x}_{it} = (x_{it-1}, \bar{o}_{it-2}, \bar{o}_{it-3}, \bar{o}_{it-4})'$ and $\mathbf{z}_{it} = (\bar{y}_{it-1}, \bar{y}_{it-2}, \bar{y}_{it-3}, \bar{y}_{it-4})'$ as a function of x_{it-1} where $\bar{z}_{it-j} = T^{-1} \sum_{t=1}^{T} z_{it-j}$ and where the expectation is with respect to the posterior distribution of $\alpha_0, \alpha, \delta, \lambda$, and $m(\mathbf{x}_{it})$ conditional on observation of $\{y_{it}, \mathbf{x}_{it}, \mathbf{z}_{it}\}$, for $t = 1, \ldots, T$; $i = 1, \ldots, N$, with this posterior distribution estimated by Monte Carlo importance sampling with 100,000 simulations. Dashed lines give 95% probability regions

as σ_v times the parameter ζ , whose estimate is 0.336. Each of the four lags of oil price changes exert an overall negative effect on output growth as indicated by the linear coefficients, though only the coefficient on o_{it-4} is statistically significant. Although one would accept a hypothesis of linearity for any one of the lags of oil prices taken individually (as reflected by the insignificant *t*-statistics on the individual coefficients g_i), collectively the nonlinear component makes a highly significant contribution (as evidenced by the *t*-statistic for $\zeta = 0$ or the LM tests).

To take a look at what the nonlinear function $\mu(.)$ looks like and compare the nonlinear function of Hamilton (2003) with that of the panel data, I performed an exercise similar to Hamilton (2003) and fixed the values of o_{it-2} , o_{it-3} , and o_{it-4} equal to their sample means and examined the consequences of changing o_{it-1} alone, that is, I set $\mathbf{x}^* = (x_{i1}, \bar{o}, \bar{o}, \bar{o})$ and evaluated the Bayesian posterior expectation of the optimal inference of the value of the unobserved function $\mu(\mathbf{x}^*)$. Figures 1, 2, 3, and 4 indicate flexible inference on the effect of oil price change in previous specific quarter on current GDP growth along with 95% probability region. For example, Fig. 1 plots the inference on the effect of oil price change in the previous quarter on current GDP growth. The negative figures in the horizontal axis indicate the percentage decrease in the oil price while the positive ones mean the percentage increase in the oil price and the figure in the vertical axis shows GDP growth. The region of dashed lines indicates the degree of confidence about the inference based on the Bayesian posterior estimates.⁸

⁸ The anonymous referee suggests that we plot the deviation of GDP growth from its mean, so that an oil price change of zero would have a zero effect on average GDP growth. However, we use similar format of the figures with Hamilton (2003) to directly compare flexible inference in the panel data with that of U.S. time-series data.

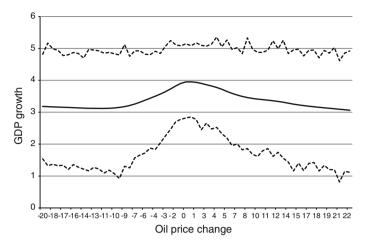


Fig. 2 Effect of oil prices on GDP growth two quarters later. Solid line plots the posterior expectation of the function $\alpha_0 + \alpha' \mathbf{x}_{it} + \delta' \mathbf{z}_{it} + \lambda m(\mathbf{x}_{it})$ evaluated at $\mathbf{x}_{it} = (\bar{o}_{it-1}, x_{it-2}, \bar{o}_{it-3}, \bar{o}_{it-4})'$ and $\mathbf{z}_{it} = (\bar{y}_{it-1}, \bar{y}_{it-2}, \bar{y}_{it-3}, \bar{y}_{it-4})'$ as a function of x_{it-2}

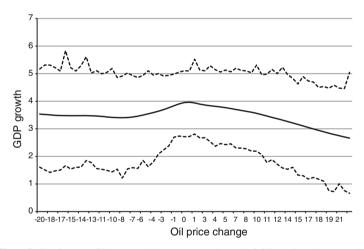


Fig. 3 Effect of oil prices on GDP growth three quarters later. Solid line plots the posterior expectation of the function $\alpha_0 + \alpha' \mathbf{x}_{it} + \delta' \mathbf{z}_{it} + \lambda m(\mathbf{x}_{it})$ evaluated at $\mathbf{x}_{it} = (\bar{o}_{it-1}, \bar{o}_{it-2}, x_{it-3}, \bar{o}_{it-4})'$ and $\mathbf{z}_{it} = (\bar{y}_{it-1}, \bar{y}_{it-2}, \bar{y}_{it-3}, \bar{y}_{it-4})'$ as a function of x_{it-3}

Figure 1 shows the result as a function of x_{i1} along with 95% probability regions. The implied function is nonlinear, suggesting that if oil prices either increase or decrease after three quarters of stability, slightly slower GDP growth is predicted than if oil prices had remained stable, though decreases are a little worse news than increases. Nevertheless, the confidence band of the Fig. 1 indicates that the change in GDP growth does not appear to be statistically significant.

Figures 2 and 3 answer the analogous question, fixing o_{it-1} , o_{it-3} , and o_{it-4} at their sample means and varying the value of o_{it-2} in Fig. 2 and fixing o_{it-1} , o_{it-2} , and o_{it-4} at their sample means and varying the value of o_{it-3} in Fig. 3. Both Figs. 2 and 3

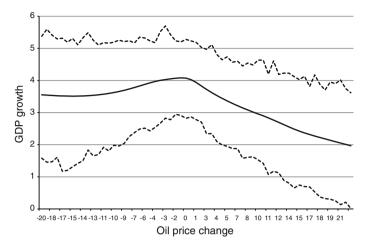


Fig. 4 Effect of oil prices on GDP growth four quarters later. Solid line plots the posterior expectation of the function $\alpha_0 + \alpha' \mathbf{x}_{it} + \delta' \mathbf{z}_{it} + \lambda m(\mathbf{x}_{it})$ evaluated at $\mathbf{x}_{it} = (\bar{o}_{it-1}, \bar{o}_{it-2}, \bar{o}_{it-3}, x_{it-4})'$ and $\mathbf{z}_{it} = (\bar{y}_{it-1}, \bar{y}_{it-2}, \bar{y}_{it-3}, \bar{y}_{it-4})'$ as a function of x_{it-4}

show different impression from that of Fig. 1 though the implied function is nonlinear. Both figures indicate the opposite situation to the Fig. 1 where oil price increases are worse news than decreases. However, in Figs. 2 and 3, the confidence interval is quite broad, implying that such an inference might not be warranted statistically.

Figure 4 describes the effect of o_{it-4} alone and shows more dramatic relation, suggesting that decreases in oil prices four quarters earlier have almost no consequences for current GDP growth, whereas oil price increases significantly reduce expected GDP growth. Furthermore, the confidence interval shows a statistically significant relation. This figure indicates an asymmetric specification as in Mork (1989) and Hamilton (1996, 2003). Even though there is not simply a mechanical relation between oil prices and output, we view the demand-side effect of oil price shock as an explanation of nonlinear oil–macroeconomy relation. As outlined in Hamilton (2003), when oil prices and availability are uncertain, people feel uncertain about the future and tend to postpone their spending on cars, housing, appliances, and investment goods, resulting in allocative disturbances. In this mechanism, an oil price increase results in the postponement of purchases of energy-sensitive big-ticket items that produce the downturn, whereas it seems not to be reasonable to assume that an oil price decrease would produce an economic boom that mirrors the recession induced by an oil price increase.

I calculated how o_{it-3} is affected by different values of o_{it-4} to examine the interactive effects. Figure 5 compares the three functions $\hat{\mu}(\bar{o}, \bar{o}, x_{i3}, 0)$, $\hat{\mu}(\bar{o}, \bar{o}, x_{i3}, 5)$, $\hat{\mu}(\bar{o}, \bar{o}, x_{i3}, -5)$, plotted as a function of x_{i3} . The dotted line represents the relation between various x_{i3} and $o_{it-4} = 0$, which is essentially the same as the mean value plotted in Fig. 3. The solid line represents the relation to show how the effect of an x_{i3} % oil price change three quarters ago would be different if oil prices had also increased 5% the quarter before that. The one point to make is that the solid line is uniformly lower than the dotted line. This implies that regardless of the value of o_{it-3} , one forecasts

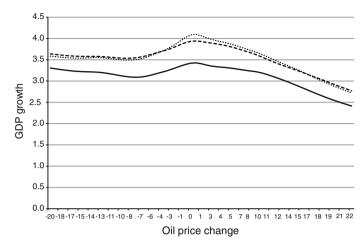


Fig. 5 Effect of oil prices on GDP growth three quarters later different possible values of o_{t-4} . Three *lines* plot the posterior expectation of the function $\alpha_0 + \alpha' \mathbf{x}_{it} + \delta' \mathbf{z}_{it} + \lambda m(\mathbf{x}_{it})$ evaluated at $\mathbf{x}_{it} = (\bar{o}_{it-1}, \bar{o}_{it-2}, x_{it-3}, x_{it-4})'$ and $\mathbf{z}_{it} = (\bar{y}_{it-1}, \bar{y}_{it-2}, \bar{y}_{it-3}, \bar{y}_{it-4})'$ as a function of x_{it-3} . For the *Solid line*, $x_{it-4} = 5$, for the *dashed line*, $x_{it-4} = -5$, and for the *dotted line*, $x_{it-4} = 0$

definitely lower GDP growth when an oil price increase four quarters earlier than when an unchanged oil price four quarters earlier. Furthermore, one can notice that the slope of the dotted line is steeper than that of the solid line, implying that the additional information content of any change in quarter t - 3 is reduced by an oil price increase four quarters earlier. This result appears to be relevant to two views in existing literature. The first view claimed by Lee et al. (1995) and Hamilton (2003) is that oil price changes are less useful for forecasting GDP if they follow a period of earlier uncertain price changes. The second one claimed by Hamilton (1996, 2003) is that if oil price does not exceed the previous 3-year peak, no oil shock is said to have occurred. Given the similar result in the panel data with Hamilton (2003), it is considered such two views as driving the result.

The dashed line plots the predicted GDP growth for quarter t when $o_{it-3} = x_{i3}$ and $o_{it-4} = -5$. The almost same shape of the dotted line and the dashed line indicates that if oil prices decreased four quarters earlier, this has little consequences for forecasting GDP if it was followed by no change in oil price in quarter t - 3. In other words, one should downweight another oil price change in period t - 3 following a 5% decrease in period t - 4. The overall conclusion of Fig. 5 supports the view of Lee et al. (1995) and Hamilton (1996, 2003) that previous upheaval in oil prices tends to reduce the marginal effect of any given oil price change.

In sum, our results of the panel data analysis confirm the findings of Hamilton (2003, 2009b); Cuñado and de Gracia (2003), and Jimenez-Rodriguez and Sanchez (2005). The relation between oil price change and GDP growth is nonlinear and oil price increases are statistically and economically significant while oil price decreases are not, and increases that come after a long period of stable prices have a bigger effect than those that simply correct previous decreases for industrialized countries as well as U.S.A.

3.4 Alternative specifications

To examine whether the results of previous section rely on the specification of flexible nonlinear error component model we consider other two alternative specifications. The first alternative is a flexible nonlinear model with fixed effects instead of random error components and this specification is as follows:

$$y_{it} = \alpha_i + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}) + \varepsilon_{it}, \qquad (39)$$

$$\varepsilon_{it} = a_t + v_{it},\tag{40}$$

$$\mu(\mathbf{x}_{it}) = \alpha_i + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}), \ i = 1, 2, \dots, N, \ t = 1, \dots, T,$$
(41)

where α_i denotes the individual specific effect and it is assumed that α_i is a fixed parameter to be estimated, $a_t \sim i.i.d.N(0, \sigma_a^2)$ and $v_{it} \sim i.i.d.N(0, \sigma_v^2)$. Inference in this case is conditional on the particular N individuals. The other alternative is a flexible nonlinear model without the individual specific effect and we have a following specification:

$$y_{it} = \alpha_0 + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}) + \varepsilon_{it}, \qquad (42)$$

$$\varepsilon_{it} = a_t + v_{it},\tag{43}$$

$$\mu(\mathbf{x}_{it}) = \alpha_0 + \boldsymbol{\alpha}'_1 \mathbf{x}_{it} + \lambda m(\mathbf{g} \odot \mathbf{x}_{it}), \ i = 1, 2, \dots, N, \ t = 1, \dots, T.$$
(44)

Inference in this case is for the case of pooling across countries but not for the case of pooling over time.

The test statistic of the null hypothesis of linearity for the case of linear fixed-effect model and for the case of linear pooling model without the country specific effect has a value of 28.41 and 26.99, respectively, which are similar with the case of linear random error component model and both values for a $\chi^2(1)$ variable indicate strong rejection of the null hypothesis of linearity. Following the reparametrization as in Sect. 2.2, we have Bayesian posterior estimates and their standard errors for the model (40–41) and the model (43–44) as follows:

$$y_{it} = \alpha_i - \underbrace{0.055}_{(0.033)} y_{it-1} + \underbrace{0.050}_{(0.030)} y_{it-2} + \underbrace{0.105}_{(0.032)} y_{it-3} + \underbrace{0.049}_{(0.033)} y_{it-4} \\ + \underbrace{0.016}_{(0.014)} o_{it-1} - \underbrace{0.002}_{(0.014)} o_{it-2} - \underbrace{0.023}_{(0.016)} o_{it-3} - \underbrace{0.037}_{(0.016)} o_{it-4} \\ + \underbrace{4.826}_{(0.122)} \underbrace{[0.294m}_{(0.070)} \underbrace{(0.070}_{(0.09)} o_{it-1}, \underbrace{0.070}_{(0.08)} o_{it-2}, \underbrace{0.060}_{(0.07)} o_{it-4}, \underbrace{0.060}_{(0.07)} + \tilde{a}_t + \tilde{v}_{it}] \\ \underbrace{(45)}_{(45)}$$

$$\hat{\sigma}_a^2 = 0.886, \ \hat{\sigma}_v^2 = 23.29,$$

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$$y_{it} = 2.081 - 0.053 y_{it-1} + 0.060 y_{it-2} + 0.107 y_{it-3} + 0.059 y_{it-4} + 0.010 o_{it-1} - 0.003 o_{it-2} - 0.023 o_{it-3} - 0.036 o_{it-4} + 4.587 [0.336 m (0.09 o_{it-1}, 0.08 o_{it-2}, 0.07 o_{it-3}, 0.07 o_{it-4}) + \tilde{a}_t + \tilde{v}_{it}] (0.111) (0.054) (0.10) (0.10) (0.10) (0.10) (0.10) (0.10) (0.08) (0.07) (0.$$

$$\hat{\sigma}_a^2 = 0.964, \ \hat{\sigma}_v^2 = 21.04$$

where α_i is a fixed parameter to be estimated for the country specific effect.⁹ The values of estimated fixed parameter α_i in the nonlinear fixed effect model (45) are between 1.49 and 2.58 over all sample countries and all estimated fixed parameters are statistically significant.

Estimated parameter governing the overall importance of the nonlinear component λ , is written as $\zeta \cdot \sigma_v$ and their estimates in both models are statistically significant and are quite similar to those of the nonlinear random effect model in Eq. 38. Furthermore, in both models, the coefficient on o_{it-4} is statistically significant at the 5% level and the value of the coefficient is close to that of Eq. 38. Figure 6a and b describes the effect of o_{it-4} alone on current economic growth based on the nonlinear fixed effect model (45) and on the nonlinear pooling model (46), respectively. Interestingly, both figures are quite similar with Fig. 4, suggesting that two alternative specifications supports similar nonlinear oil–macroeconomy relation which the nonlinear random error component model of Eq. 38 showed.

Figure 7a and b plots the effect of oil price on GDP growth three quarter later for different possible values of $o_{it-4} - \hat{\mu}(\bar{o}, \bar{o}, x_{i3}, 0)$, $\hat{\mu}(\bar{o}, \bar{o}, x_{i3}, 5)$, and $\hat{\mu}(\bar{o}, \bar{o}, x_{i3}, -5)$ —to examine the view that previous turbulence in oil prices causes the marginal effect of any given oil price change to be reduced. Both figures are also quite similar with Fig. 5 in the case of nonlinear random error component model of Eq. 38. Overall, two alternative specifications supports similar oil–macroeconomy relation as in the nonlinear random error component model and thus the claim of nonlinear relation appears to be robust to the nonlinear flexible inference with various specifications of the panel data model.

4 Concluding remarks

The instability over time in a linear regression of output growth on lagged oil prices has triggered the investigation of functional relation between oil price and real economic activity. Hamilton (2003) shows by employing the methodology of nonlinear flexible inference of Hamilton (2001) that the true relation is nonlinear in the case of U.S. economy. Some literature has found evidence of nonlinear effects of oil price on real economic activity of main industrialized countries from the studies of individual country analysis. Kilian and Vigfusson (2009), however, show that the regression models and estimation methods which use measures that censors energy price

⁹ The Bayesian analysis is based on 20,000 draws from the importance sampling density described in Sect. 2.3.

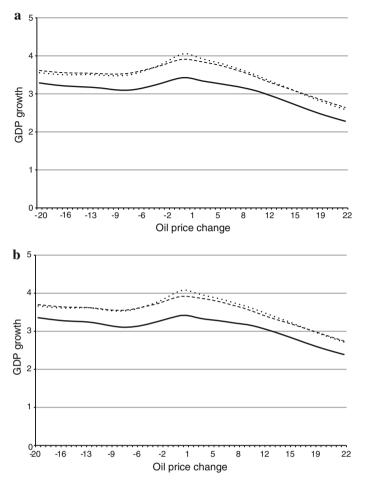


Fig. 6 a Effect of oil prices on GDP growth four quarters later: With fixed effect. b Effect of oil prices on GDP growth four quarters later: Without country specific effect

changes, produce inconsistent estimates of the true effects of unanticipated energy price increases and lead to overestimating the impact of energy price shocks on macroeconomy aggregates.

This article examines the issue of whether the relation between oil price change and the business cycle is nonlinear along with two insights. First of all, this article extends Hamilton (2001) methodology for time-series data to the panel data framework to investigate whether the relation is nonlinear in terms of panel data analysis. Specifically, we consider nonlinear flexible inference with random error components. Secondly, our parametric approach does not have to use the censored oil price changes and thus avoids potential problems from using the censored energy prices changes.

Our results show from the study of the panel data for six industrialized countries that oil price increases are statistically and economically significant while oil price

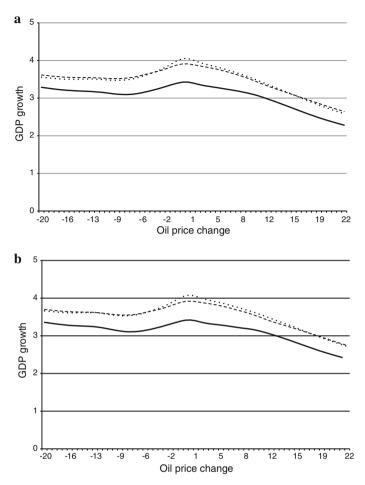


Fig. 7 a Effect of oil prices on GDP growth three quarters later different possible values of o_{t-4} : With fixed effect. **b** Effect of oil prices on GDP growth three quarters later different possible values of o_{t-4} : Without country specific effect

decreases are not, and previous upheaval in oil prices causes the marginal effect of any given oil price change to be reduced and support the claim in the literature. The alternative specifications of the panel data model with nonlinear flexible inference as a robustness analysis support similar nonlinear oil–macroeconomy relation. Therefore, the result of such a panel data analysis suggests that one should use a nonlinear function of oil price changes if the goal is to forecast GDP growth.

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Appendix: Monte Carlo analysis for approximating the small-sample distribution of the LM test statistic

In order to approximate the exact small-sample distribution of $\hat{\aleph}_{NT}$ and to examine how good the test statistic developed in Eq. 34 performs, we undertook a small-scale Monte Carlo analysis. We consider following linear data-generating process based on the linear error-components model:

$$y_{it} = 2.531 - 0.044y_{it-1} + 0.068y_{it-2} + 0.123y_{it-3} + 0.068y_{it-4} + 0.011o_{it-1} - 0.005o_{it-2} - 0.023o_{it-3} - 0.039o_{it-4} + \varepsilon_{it},$$
(A1)
$$\varepsilon_{it} = \omega_i + a_t + v_{it},$$

where $\omega_i \sim N(0, 0.035)$, $a_t \sim N(0, 2.75)$, $v_{it} \sim N(0, 21.39)$ and the disturbances are assumed to be mutually and temporally independent normal variables with variance equal to those estimated in the linear error-component model. Using 1,000 simulations of the data generated for this system corresponding to 6 countries oil price changes and GDP growth, we calculated the test statistic $\hat{\aleph}_{NT}$ conditional on the best quadratic unbiased estimators for each simulation. We did not find any case where $\hat{\aleph}_{NT}$ is greater than that of the original data, implying that the original data shows strong evidence on the nonlinear relation between oil price change and GDP growth. Furthermore, at the nominal 5% level of significance, the rejection rate of the linearity null hypothesis was 1.6%, implying that the developed test statistic leads to some under-rejection of the true linearity null hypothesis but overall performance of the test statistic has good small-sample property in terms of conventional analysis.

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