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Estimating Europe's natural rates

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Abstract This article estimates potential output, the natural rate of unemployment, and the core inflation rate using aggregated euro area data. The empirical model consists of a Phillips curve linking inflation to unemployment. An Okun-type relationship is used to link the output gap to cyclical unemployment. The model further accounts for new developments in unobserved component models by allowing (i) for correlation between shocks to the natural rates and the corresponding gaps and (ii) structural breaks in the drift of potential output and the natural rate of unemployment.

Keywords Natural rates · Correlated unobserved components · Kalman filter · Structural breaks

1 Introduction

Measuring equilibrium rates of key macroeconomic variables and the resulting gaps has a long-standing history in the economic literature. Since the seminal contribution of Burns and Mitchell (1946) several studies estimated business cycles using various different concepts of equilibrium rates and different statistical techniques to extract them from the data. The deviation of actual output from its long-run level, referred to as potential output, provides an estimate about the cyclical position of the economy. A positive output gap, i.e. actual output exceeds potential output, implies inflationary pressure as demand exceeds supply. A closely related concept is that of the natural rate of unemployment (NRU) and has been pioneered by Friedman (1968) and Phelps (1968) who claim that unemployment is at its natural level when neither

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inflationary nor deflationary pressure emanates from the labour market. This is called the non-accelerating-inflation-rate of unemployment (NAIRU).

Various methodologies have been suggested for estimating natural rates and the corresponding gaps. They can be divided into two groups, purely statistical and economic based. The latter approach estimates a production function and obtains a measure of potential output from various factor inputs multiplied by total factor productivity. The production function approach, used for instance by the European Commission, has a closer link to economic theory compared to the statistical approaches. However, it also requires assumptions of the functional forms of the production technology, return to scale etc. Recent contributions in this literature are Proietti et al. (2007) and Roeger (2006). Contrary to this, the statistical approach used in this article contains less economic theory but has the advantage of specifying a dynamic process for the trends and thus allows to compare the results over different trend specifications.

A popular method to estimate natural rates is the Unobserved Component (UC) model in which trend and cycle are treated as latent state variables, the former modelled as a non-stationary process while the cycle is mean reverting. After casting the model in state-space form, it can be estimated using the Kalman filter. The UC model, pioneered by Harvey (1985); Watson (1986) and Clark (1987), typically assumes zero correlation between shocks to the trend and the cycle. However, Morley et al. (2003) showed that this restriction can strongly influence the resulting decomposition in a univariate model. They further showed that, under certain conditions, this correlation is identified and can be estimated. Moreover, by focusing on the decomposition of US postwar real GDP, Perron and Wada (2005) showed the importance of properly specifying a process for potential output that is capable to account for structural breaks. Specifically they argue that modelling trend growth as either deterministic or a simple random walk yields an estimate in which the trend is very close to actual output, leaving little to the cycle. Hence, it does not accord with what is commonly viewed as a business cycle. It is important to note that even the latter approach can not adequately account for infrequent shifts in the drift of the trend function. As Perron and Wada point out the variance of an estimated random walk drift is very small implying a drift that changes only little but every period though. As a consequence, if there are infrequent but large shifts in the slope of the trend function, the random walk drift is inadequate to capture them.

The result of a small and noisy cycle seems to be more important in univariate models. Studies that estimate multivariate models typically find the cycle to be large and persistent. Intuitively this might be due to the additional information contained in, e.g., the inflation rate. Nevertheless, potential output remains misspecified if there are infrequent but large shifts which are not accounted for.

The existing literature is dominated by studies that focus on the US economy. Kuttner (1994) is the first to estimate the US output gap from a bivariate UC model including output and inflation. More recent contributions include Basistha and Nelson (2007) who estimate the US output gap from a bivariate system combining inflation and output through a forward-looking Phillips curve. Additionally they allow for correlated trend and cycle shocks. Domenech and Gomez (2006) estimate potential output, core inflation, and the NRU from a model that combines information contained in real GDP, inflation, unemployment, and investment. Their model includes a forward-looking Phillips curve and allows for some volatility breaks. The literature that focuses on the euro area is much scarcer. As an exception, Ruenstler (2002) estimates the real-time output gap for the euro area and studies its reliability over different model specification. Fabiani and Mestre (2004) focus on estimating the euro area NRU using a backward-looking Phillips curve and do not allow for correlated trend cycle shocks.

This article estimates potential output, the NRU, and the core inflation rate using aggregate euro area data.[1] It contributes to the existing literature in two ways. First, it provides euro area natural rates of key macroeconomic variables. To the best of our knowledge this is the first attempt to simultaneously estimating euro area potential output, the NRU and a time-varying core inflation rate. Second, the model further accounts for the new developments in UC models by allowing (i) for correlation between shocks to the natural rates and the corresponding gaps and (ii) structural breaks in the drift of potential output and the NRU. In order to investigate the importance of these points an alternative model are estimated and compared to the baseline specification.

The empirical model, building on Apel and Jansson (1999a,b), consists of a Phillips curve linking inflation to unemployment. An Okun-type relationship is used to link the output gap to cyclical unemployment. The core inflation rate is time-varying and the transitory component in inflation is linked to the output gap. The rest of the article is organised as follows. Section 2 outlines and estimates the model. In Sect. 3 the results are presented. Section 4 concludes.

2 A multivariate correlated unobserved component model

This section lays out a multivariate correlated UC model that consists of output, unemployment and inflation. Euro area data are taken from the area–wide model of Fagan et al. (2005) and range from 1970 until 2005. The key feature of this dataset is that it treats the euro area as a single economy. The data are constructed by aggregating individual country series. The weights used in the aggregation are calculated from individual countries real GDP.[2] The unemployment rate, u_t, is the quarterly unemployment rate. For inflation, π_t, the first difference of the log of the seasonally adjusted quarterly GDP deflator is used. Output is the log of seasonally adjusted quarterly GDP in constant prices.

2.1 Output decomposition

Following Watson (1986) and Clark (1987) output is modelled as the sum of two components, potential output $\overline{y}_t$ and cyclical output y_t^c

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[1] Estimating the natural rate of interest, although interesting, is beyond the scope of this article. See Mesonnier and Renne (2007) for a recent contribution in this line of the literature.

[2] The aggregated data are calculated from 12 individual countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. For details see Fagan et al. (2005).

$$y_t = \overline{y}_t + y_t^c. \tag{1}$$

Potential output is specified as a random walk with drift

$$\overline{y}_t = \mu_i + \overline{y}_{t-1} + \eta_t^{\bar{y}}, \tag{2}$$

where μ_i with $i = 1, \ldots, m^{\mu}$, is the drift of potential output, often referred to as potential output growth. This specification allows for infrequent but large shifts in the growth rate of potential output. Cyclical output, i.e. the output gap, is modelled as an $AR(2)$ process

$$y_t^c = \phi_1 y_{t-1}^c + \phi_2 y_{t-2}^c + \eta_t^c, \tag{3}$$

where η_t^c is a Gaussian mean zero white noise error term. The $AR(2)$ specification allows the output gap to exhibit the standard hump-shaped pattern.

2.2 Unemployment decomposition

The rate of unemployment is disentangled into two components, a non-stationary trend component u_t^*, and a cyclical component u_t^c

$$u_t = u_t^* + u_t^c. \tag{4}$$

The NRU, i.e. the trend component in the unemployment rate, is specified as a random walk with drift

$$u_t^* = \gamma_t + u_{t-1}^* + \eta_t^{u^*} \tag{5}$$

$$\gamma_t = \delta\gamma_{t-1} + \eta_t^{\gamma}, \tag{6}$$

where the drift, γ_t, evolves according to an $AR(1)$ process. $\eta_t^{u^*}$ and η_t^{γ} are Gaussian mean zero white noise error terms. From an economic perspective it would be sufficient to model long-run unemployment as a simple random walk. Any shock to the NRU, which may reflect changes in its underlying labour market institutions such as the benefit system or employment protection legislation, would have a permanent impact on the NRU and hence on the rate of unemployment. However, modelling the euro area NRU as a simple random walk yields to implausible results. The reason is that, contrary to the US, unemployment in Europe increased from the early 1970s to the mid 1980s and remained persistently high since then. In order to capture this increase, a random walk NRU would need to have a sequence of positive shocks which are not about to reversed within the sample. Thus, the shocks driving the random walk NRU can hardly be described as a standard Gaussian white noise process with mean zero. In NRU estimates for the euro area the trend in long-run unemployment is often modelled as a random walk implying that the NRU is an I(2) process (see e.g. Orlandi and Pichelmann 2000; Laubach 2001; Fabiani and Mestre 2004). The advantage of the $AR(1)$ specification applied here is that the smoothness of the NRU is not imposed but estimated. Moreover, as long as $0 < \delta < 1$ the NRU is I(1). The trend in the rate of unemployment should be seen as a special feature for the time span analysed here rather than as characteristic for the rate of unemployment as such.[3] As argued by Fabiani and Mestre (2004) this upward trend is no longer present and might even about to be reversed. In order to allow for a changing trend in the NRU the following specification is considered as an alternative to (6)

$$\gamma_t = \theta_j, \qquad j = 1, \ldots, m^{\gamma} + 1, \tag{6a}$$

where m^{γ} denotes the number of structural breaks. The trend in the NRU is modelled as a constant which is allowed to break infrequently. The subscript j refers to different values of θ. If the trend in the euro area NRU is indeed a special characteristic of the 1970s and 1980s one would expect θ to be zero in the second half of the sample and thus long-run unemployment to be an I(1) process.

Cyclical unemployment is linked to the output gap via Okun's Law. Okun (1970) showed that there is an empirical relation between output and unemployment. This relationship has been labelled Okun's Law and can be expressed as

$$u_t^c = \omega(L) y_t^c, \tag{7}$$

where ω is the Okun's Law parameter and the lag polynomial is defined as $\omega(L) = \omega_0 + \omega_1 L + \cdots + \omega_q L^q$. Following Clark (1989) cyclical unemployment is linked to the contemporaneous and lagged output gap. This specification allows the labour market to have a lagged response to the product market. It should be stressed that this relation does not come from a fully specified macroeconomic model but accounts for the negative correlation between the output and the unemployment gap.

2.3 Structural break tests

In order to test for structural breaks in the drift of potential output and the NRU the following simple regression model is considered

$$z_t = c + \varepsilon_t, \tag{8}$$

where z_t refers to the change of (log) real GDP, Δy_t, and the change of the rate of unemployment, Δu_t. We are interested in the stability of the regression parameter c. Suppose there is a structural break at period k, so that

$$c = \begin{cases} c_1 & \text{for } t = 1, \ldots, k \\ c_2 & \text{for } t = k+1, \ldots, T. \end{cases} \tag{9}$$

If the potential breakpoint k is known a priori one could use the Chow (1960) breakpoint test with the null hypothesis of no structural break ($H_0 : c_1 = c_2$) against the

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[3] As the rate of unemployment as a bounded process it obviously cannot be an upward trending process in the long-run.

alternative of a break at time k ($H_1 : c_1 \neq c_2$). The test statistic is based on the Wald statistic and given by

$$F_k = \frac{\left(\hat{\varepsilon}'\hat{\varepsilon} - \left(\varepsilon_1'\varepsilon_1 + \varepsilon_2'\varepsilon_2\right)\right)/r}{\left(\varepsilon_1'\varepsilon_1 + \varepsilon_2'\varepsilon_2\right)/(T-2r)}, \tag{10}$$

where $\hat{\varepsilon}'\hat{\varepsilon}$ denote the sum of least-squared residuals from (8) and $\varepsilon_1'\varepsilon_1$, $\varepsilon_2'\varepsilon_2$ are the sum of least-squared residuals from $t = 1, \ldots, k$ and $t = k+1, \ldots, T$ and r is the number of parameters. Given that the residuals are iid, F_k has an exact finite sample F-distribution. In most practical settings, however, k is not known a priori and has to be determined by the data. Based on Quandt (1960), Andrews (1993) modifies the Chow test and allows for unknown breakpoints. Basically, the Quandt–Andrews test performs the Chow breakpoint test for every observation over the interval $[\xi T, (1-\xi)T]$ and calculates the supremum of the F_k statistics

$$\sup F = \sup{}_{k \in [\xi T, (1-\xi)T]} F_k, \tag{11}$$

where ξ is a trimmimg parameter. Andrews and Ploberger (1994) develop two additional test statistics, the average (ave F) and the exponential (exp F) form. The null hypothesis of no break is rejected if these test statistics are too large. Hansen (1997) derives an algorithm to calculate approximate asymptotic p values of these tests. Bai and Perron (1998, 2006, 2003) (BP) extend this approach to test for multiple structural breaks. Briefly, BP suggest to examine first two tests (the so called $UD_{\max}$ and $WD_{\max}$ tests) to check if there are any structural breaks. If these tests reject the null of no breaks use a sequential procedure to determine the number of breaks. According to the BP notation this means computing a sequence of sup $F_T(l+1|l)$ statistics to test the null of l breaks against the alternative of $l+1$ breaks.[4]

In order to identify the number and the timing of structural breaks in Δy_t and Δu_t the Quandt–Andrews test and the BP multiple structural breaks test are used. The BP methodology has also been used by Rapach and Wohar (2005) to test for structural breaks in real interest rates and inflation rates and by Basistha (2007) to test for structural breaks in Canadian GDP growth within a bivariate UC model.

2.4 The Phillips curve

The new Keynesian Phillips curve (NKPC) states that the difference between actual inflation and its expected value is driven by marginal costs which are in most practical settings assumed to be proportional to the output or unemployment gap. An obvious problem in the NKPC is that the expected value of future inflation is an unobservable variable. However, the UC methodology offers a proxy for expected inflation. Following Nelson and Lee (2007) and Domenech and Gomez (2006), we use the inflation trend, resulting from a decomposition of inflation into a non-stationary trend and a

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[4] A detailed description of this test can be found in BP and Rapach and Wohar (2005).

stationary cycle as a proxy for expected inflation, i.e.

$$\pi_t = \pi_t^* + \pi_t^c, \tag{12}$$

where π_t is the inflation rate, π_t^* is the trend of inflation, and π_t^c denotes cyclical inflation with mean zero. Trend inflation is specified as a random walk[5]

$$\pi_t^* = \pi_{t-1}^* + \eta_t^{\pi^*}. \tag{13}$$

As shown by Beveridge and Nelson (1981) π_t^* can be interpreted as the long-run forecast of inflation since the long horizon forecast of transitory inflation is zero

$$\pi_t^* = \lim_{j\to\infty} \mathrm{E}_t\left(\pi_{t+j}\right) = \mathrm{E}_t(\pi_\infty), \tag{14}$$

where $\mathrm{E}(\cdot)$ is the expectation operator based on information up to time t. Assuming that transitory inflation and the output gap are linearly related, i.e. $\pi_t^c = \beta(L)y_t^c + \eta_t^{\pi^c}$, where $\beta(L) = \beta_0 + \beta_1 L + \cdots + \beta_s L^s$, Eq. 12 can be rewritten as

$$\pi_t = \mathrm{E}_t(\pi_\infty) + \beta(L)y_t^c + \eta_t^{\pi^c}. \tag{15}$$

This represents the expectation augmented Phillips-curve. Note that the slope of the Phillips-curve not only depends on $\beta(L)$ but is also affected by the expectation horizon. This point becomes clearer by taking the one-step ahead conditional expectation of Eq. 15[6]

$$\mathrm{E}_t\left(\pi_{t+1}\right) = \mathrm{E}_t\left[\mathrm{E}_{t+1}\left(\pi_\infty\right) + \beta\left(L\right) y_{t+1}^c + \eta_{t+1}^{\pi^c}\right] \tag{16}$$

$$= \mathrm{E}_t\left(\pi_\infty\right) + \beta\left(L\right)\left(\phi_1 y_t^c + \phi_2 y_{t-1}^c\right). \tag{17}$$

Solving (17) for $\mathrm{E}_t\left(\pi_\infty\right)$ and combining it with (15) gives the one-step ahead forward-looking Phillips curve

$$\pi_t = \mathrm{E}_t\left(\pi_{t+1}\right) + \beta\left(L\right)\left(1-\phi_1\right) y_t^c - \beta\left(L\right)\phi_2 y_{t-1}^c + \eta_t^{\pi^c}. \tag{18}$$

It should be noted that Eq. 18 cannot directly be linked to the NKPC which is based on optimising agents and nominal rigidities. The term $\mathrm{E}_t\left(\pi_{t+1}\right)$ in the equation above stems from interpreting trend inflation as the long-run forecast of inflation and thus lacks any structural interpretation in the sense of the New Keynesian literature.

We will refer to the model given by Eqs. 1–5, 6a, and 12–15 as the baseline model. Thus, the baseline specification considers the drift in the NRU as a constant which is allowed to break. Additionally, the variance–covariance matrix of the shocks to the

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[5] In an alternative specification (13) is modelled similar to the NRU, i.e. a unit-root process with an AR(1) trend. However, the variance of the estimated trend and the autoregressive parameter were found close to zero and insignificant. Detailed results are available upon request.

[6] This derivation borrows heavily from Nelson and Lee (2007).

unobserved variables is left unrestricted to allow for non-zero covariances. The baseline model will be compared to an alternative specification in which all covariances are set to zero, the drift in the NRU is modelled according to Eq. 6, and potential output growth is not allowed to break. This model, referred to as model (b), is close to traditional UC models used to estimate natural rates such Apel and Jansson (1999a).

2.5 State-space form

The baseline model can be written in a linear Gaussian state-space representation of the following form. The observation equation

$$y_t = \Gamma \alpha_t, \tag{19}$$

$$\begin{bmatrix} y_t \\ \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \omega_1 & \omega_2 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ \pi_t^* \\ \pi_t^c \\ u_t^* \\ y_t^c \\ y_{t-1}^c \end{bmatrix}. \tag{20}$$

The states equations are given by

$$\alpha_{t+1} = \mu^* + S\alpha_t + R\eta_t, \qquad \eta_t \sim N(0, Q) \tag{21}$$

$$\begin{bmatrix} \bar{y}_{t+1} \\ \pi_{t+1}^* \\ \pi_{t+1}^c \\ u_{t+1}^* \\ y_{t+1}^c \\ y_t^c \end{bmatrix} = \begin{bmatrix} \mu_i \\ 0 \\ 0 \\ \theta_j \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ \pi_t^* \\ \pi_t^c \\ u_t^* \\ y_t^c \\ y_{t-1}^c \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t^{\bar{y}} \\ \eta_t^{\pi^*} \\ \eta_t^{\pi^c} \\ \eta_t^{u^*} \\ \eta_t^{y^c} \end{bmatrix}. \tag{22}$$

The variance–covariance matrix of the state innovations is given by

$$Q = \begin{bmatrix} \sigma_{\eta^{\bar{y}}}^2 & \sigma_{\eta^{\bar{y}}\eta^{\pi^*}} & \sigma_{\eta^{\bar{y}}\eta^{\pi^c}} & \sigma_{\eta^{\bar{y}}\eta^{u^*}} & \sigma_{\eta^{\bar{y}}\eta^{y^c}} \\ \sigma_{\eta^{\bar{y}}\eta^{\pi^*}} & \sigma_{\eta^{\pi^*}}^2 & \sigma_{\eta^{\pi^*}\eta^{\pi^c}} & \sigma_{\eta^{\pi^*}\eta^{u^*}} & \sigma_{\eta^{\pi^*}\eta^{y^c}} \\ \sigma_{\eta^{\bar{y}}\eta^{\pi^c}} & \sigma_{\eta^{\pi^*}\eta^{\pi^c}} & \sigma_{\eta^{\pi^c}}^2 & \sigma_{\eta^{\pi^c}\eta^{u^*}} & \sigma_{\eta^{\pi^c}\eta^{y^c}} \\ \sigma_{\eta^{\bar{y}}\eta^{u^*}} & \sigma_{\eta^{\pi^*}\eta^{u^*}} & \sigma_{\eta^{\pi^c}\eta^{u^*}} & \sigma_{\eta^{u^*}}^2 & \sigma_{\eta^{u^*}\eta^{y^c}} \\ \sigma_{\eta^{\bar{y}}\eta^{y^c}} & \sigma_{\eta^{\pi^*}\eta^{y^c}} & \sigma_{\eta^{\pi^c}\eta^{y^c}} & \sigma_{\eta^{u^*}\eta^{y^c}} & \sigma_{\eta^{y^c}}^2 \end{bmatrix}. \tag{23}$$

Table 1 Test for structural breaks

Quandt–Andrews breakpoint test[a]

	Sup$F$	Exp$F$	Ave$F$
Δy_t	21.70**	6.90**	5.16**
Δu_t	31.08**	12.46**	13.20**

BP multiple structural break test[b]

	$WD_{\max}$	$UD_{\max}$	Sup$F_T(1\|0)$	Sup$F_T(2\|1)$	Sup$F_T(3\|2)$	Sup$F_T(4\|3)$	Sup$F_T(5\|4)$
Δy_t	24.01**	23.31**	23.31*	8.29	3.99	3.61	3.90
Δu_t	19.92**	15.63*	7.70*	6.77	4.98	6.77	5.00

[a] The ** denotes P values less than 0.05. P values are calculated according to Hansen (1997)

[b] The maximum number of breaks is set to 5. The * and ** denote significance at the 10% and the 5% level, respectively. The 5% critical values are $UD_{\max} = 9.52$, $WD_{\max} = 10.39$, Sup$F_T(1|0) = 9.1$, Sup$F_T(2|1) = 10.55$, Sup$F_T(3|2) = 11.36$, Sup$F_T(4|3) = 12.35$, Sup$F_T(5|4) = 12.97$

The likelihood for the linear Gaussian state space model can be calculated by a routine application of the Kalman filter and maximised with respect to the unknown parameters using an iterative numerical procedure (see e.g. Harvey 1989; Durbin and Koopman 2001). The stationary state variables are initialised by drawing from their stationary distributions while a diffuse initialisation is used for the non-stationary state variables. Standard errors for the estimates are calculated by inverting the Hessian matrix.

3 Estimation results

3.1 Structural breaks

Table 1 presents the results of the Quandt–Andrews and the BP tests on structural breaks in Δy_t and Δu_t.[7] All three test statistics of the Quandt–Andrews breakpoint test clearly reject the null hypothesis of no structural breaks in GDP growth at conventional confidence levels. The $WD_{\max}$ and the $UD_{\max}$ statistics of the BP test also reject the null hypothesis of no structural breaks. The sequential analysis further rejects the null of no breaks in Δy_t against the alternative hypothesis of one structural break. However, more than one structural break cannot be found. The detected break date is 1974:Q1. Although the focus here is not on explaining the source of the break one may recognise that this period is usually associated with a worldwide slowdown in productivity growth. Moreover, using the same methodology Basistha (2007) found a single break in Canadian GDP growth in 1973:Q4 and Perron and Wada (2005) also point to a single break in US GDP growth in 1973:Q1. Consequently, the drift of potential output is modelled as a piecewise linear process taking the value μ_1 before 1974:Q1 and μ_2 thereafter. Regarding a structural break in Δu_t the Quandt–Andrews test rejects the null of no breaks. The BP tests statistics confirm this finding although the $UD_{\max}$ and

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[7] The results of the BP tests have been obtained by using the original GAUSS program from P. Perron available on his webpage.

Table 2 Parameter estimates

		Baseline model	Model (b)
Phillips curve	$\sigma_{\eta^{\pi*}}$	0.617 (0.123)	0.496 (0.110)
	$\sigma_{\eta^{\pi^c}}$	1.293 (0.121)	1.192 (0.105)
	β_1	-1.567 (0.237)	0.756 (0.410)
	β_2	1.725 (0.561)	-0.238 (0.413)
Unemployment	$\sigma_{\eta^{u*}}$	0.090 (0.012)	0.079 (0.007)
	$\sigma_{\eta^{\gamma}}$	–	0.010 (0.006)
	ω_1	-0.485 (0.077)	-0.236 (0.092)
	ω_2	-0.103 (0.091)	-0.151 (0.092)
	δ	–	0.979 (0.011)
	θ_1	0.125 (0.016)	–
	θ_2	-0.017 (0.011)	–
Output	$\sigma_{\eta^{\bar{y}}}$	0.453 (0.031)	0.437 (0.031)
	$\sigma_{\eta^{y^c}}$	0.089 (0.022)	0.209 (0.043)
	ϕ_1	1.876 (0.054)	1.809 (0.068)
	ϕ_2	-0.905 (0.041)	-0.828 (0.069)
	μ_1	0.897 (0.121)	0.583 (0.044)*
	μ_2	0.591 (0.038)	–
Covariances	$\sigma_{\eta^{\bar{y}}\eta^{\pi*}}$	0.238 (0.048)	–
	$\sigma_{\eta^{\bar{y}}\eta^{\pi^c}}$	-0.205 (0.071)	–
	$\sigma_{\eta^{\bar{y}}\eta^{u*}}$	-0.005 (0.018)	–
	$\sigma_{\eta^{\bar{y}}\eta^{y^c}}$	0.025 (0.008)	–
	$\sigma_{\eta^{\pi*}\eta^{\pi^c}}$	-0.266 (0.18)	–
	$\sigma_{\eta^{\pi*}\eta^{u*}}$	-0.034 (0.029)	–
	$\sigma_{\eta^{\pi*}\eta^{y^c}}$	0.026 (0.016)	–
	$\sigma_{\eta^{\pi^c}\eta^{u*}}$	0.011 (0.023)	–
	$\sigma_{\eta^{\pi^c}\eta^{y^c}}$	-0.032 (0.031)	–
	$\sigma_{\eta^{u*}\eta^{y^c}}$	0.001 (0.001)	–
log-likelihood		22.699	-2.977

Standard errors are in *parentheses*. Model (b) restricts all covariances to zero, does not allow for a structural break in potential output growth and models the drift in the NRU according to Eq. 6. $*\mu_1$ refers to potential output growth over the full sample

the $SupF_T(1|0)$ test statistics are only significant at the 10% confidence level. The detected break date is 1985:Q2. Thus, the parameter θ_j in Eq. 6a is also piecewise linear. This finding can be seen as evidence of a change in the upward trend in euro area unemployment in the mid 1980s as argued by Fabiani and Mestre (2004).

3.2 UC model parameter estimates

The third column in Table 2 presents the parameter estimates of the baseline UC model. In order to investigate the importance of allowing for non-zero covariances and the structural break in potential output growth an alternative model is estimated. Model (b) sets all covariances to zero and does not allow for a structural break in potential output growth. Additionally the drift in the NRU is an AR(1) process as shown in Eq. 6. In the baseline model, the estimated persistence of cyclical output, as measured by the sum of the autoregressive parameters ϕ_1 and ϕ_2, is found to be very high. The result of strong persistence in the output gap and/or cyclical unemployment is also found in previous studies. Laubach (2001), for instance, estimates the NRU for seven economies and finds that cyclical unemployment is very persistent, particularly in continental European countries.

In the output equation the piecewise linear trend is estimated to be 0.897 before 1974:Q1 and 0.591 thereafter which corresponds to a potential output growth rate of 3.59% and 2.36% p.a., respectively. This sharp decrease highlights once again the need for a proper potential output growth specification.[8] The Okun's Law parameter, ω_1 and ω_2, have the correct sign and a reasonable magnitude.[9] The sum of the ω's is -0.588 and lies within the range of previous estimates. Okun (1970) stated that the relation between cyclical unemployment and the output gap is roughly 1:3 but more recent empirical studies found Okun's Law coefficient somewhat lower (see e.g Orlandi and Pichelmann (2000) and Berger and Everaert (2008) for euro area estimates). Turning to inflation, the slope of the Phillips-curve is found to be 0.158. The estimated covariances not only ensure that the empirical model does not have unnecessary parameter restrictions, it also gives some economic insights. The covariance between the NRU and the output gap can be interpreted as a test for possible hysteresis effects. The term hysteresis, originally stemming from physics, refers to a situation in which transitory unemployment translates into long-run unemployment. This idea has been introduced by Blanchard and Summers (1986) who argued hysteresis effects can arise from insider–outsider effects in wage formation. However, the results here do not show hysteresis effects in the rate of unemployment. Moreover, the covariance between shocks to the NRU and shocks to the core inflation rate is statistically insignificant. Thus, there is no trade-off between inflation and unemployment in the long-run as suggested by theory.

The parameter estimates of model (b) do not deviate substantially from the baseline specification. The point estimate of δ implies strong persistence in the drift of the NRU. Thus, taking the parameter uncertainty into account, an I(2) behaviour for the NRU over the considered sample cannot be ruled out. This result confirms earlier NRU estimates for the euro area which impose an I(2) NRU a priori (see e.g. Orlandi and Pichelmann 2000; Laubach 2001; Fabiani and Mestre 2004). A formal comparison between the baseline model and model (b) is not possible with a standard likelihood-ratio test as model (b) is a different model rather than a restricted model in which parameters are set to zero.[10] However, it should be noted that the log-likelihood of the

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[8] In order to make sure that the estimated break dates do not lead to biased results we do a grid search over different break dates (four quarters before and after the detected break for output growth, eight quarters for the drift break in unemployment) of unemployment and potential output. The highest likelihood value is indeed found at the break dates suggested by the BP procedure.

[9] More lags in the Okun's Law equation were found to be statistically insignificant.

[10] This is due to modelling the drift in the NRU as an AR(1) process.

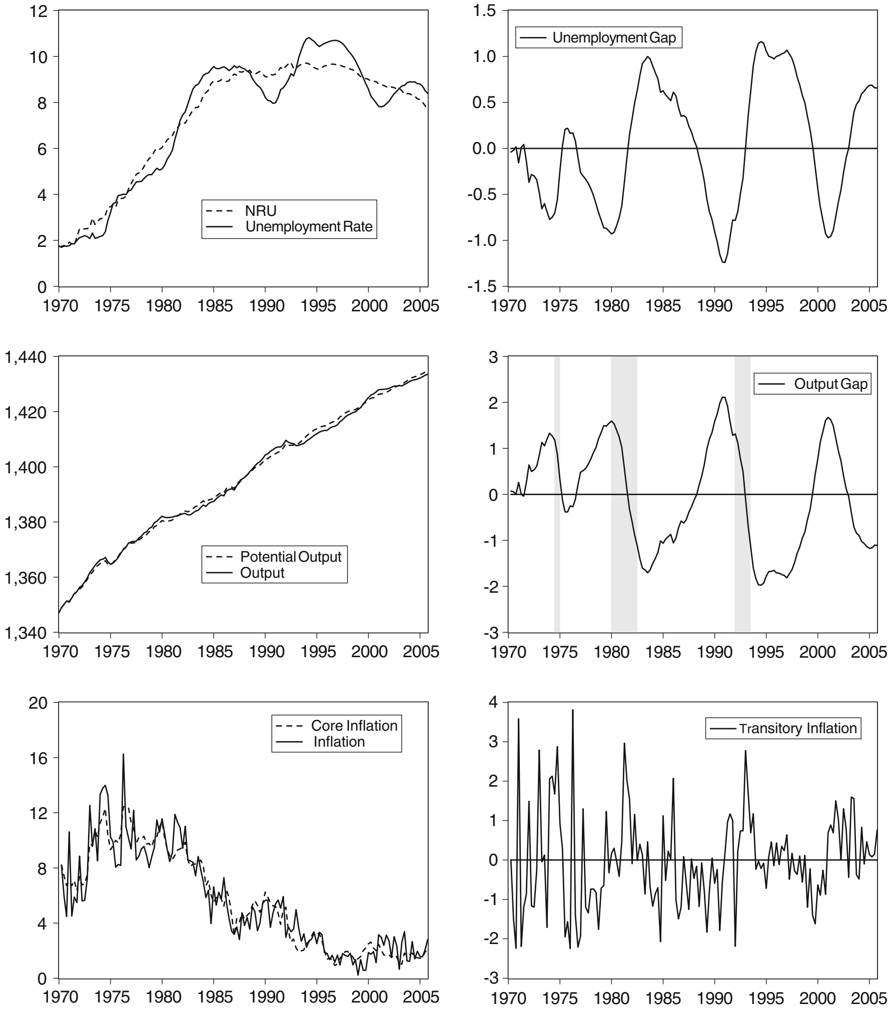

Fig. 1 State estimates: baseline model

baseline model is substantially higher as compared to model (b). This suggests that the restrictions imposed by model (b) are not supported by the data.

3.3 State estimates

Figure 1 shows the estimated states of the baseline model together with the original data. The sharp increase in the rate of unemployment until the mid 1980s is explained by movements of the NRU. Before 1985:Q2 the NRU is upward drifting and increases at a rate of 0.5% p.a. However, after the break the drift is estimated to be close to zero. Thus, from the mid 1980s the NRU evolves according to a random walk implying

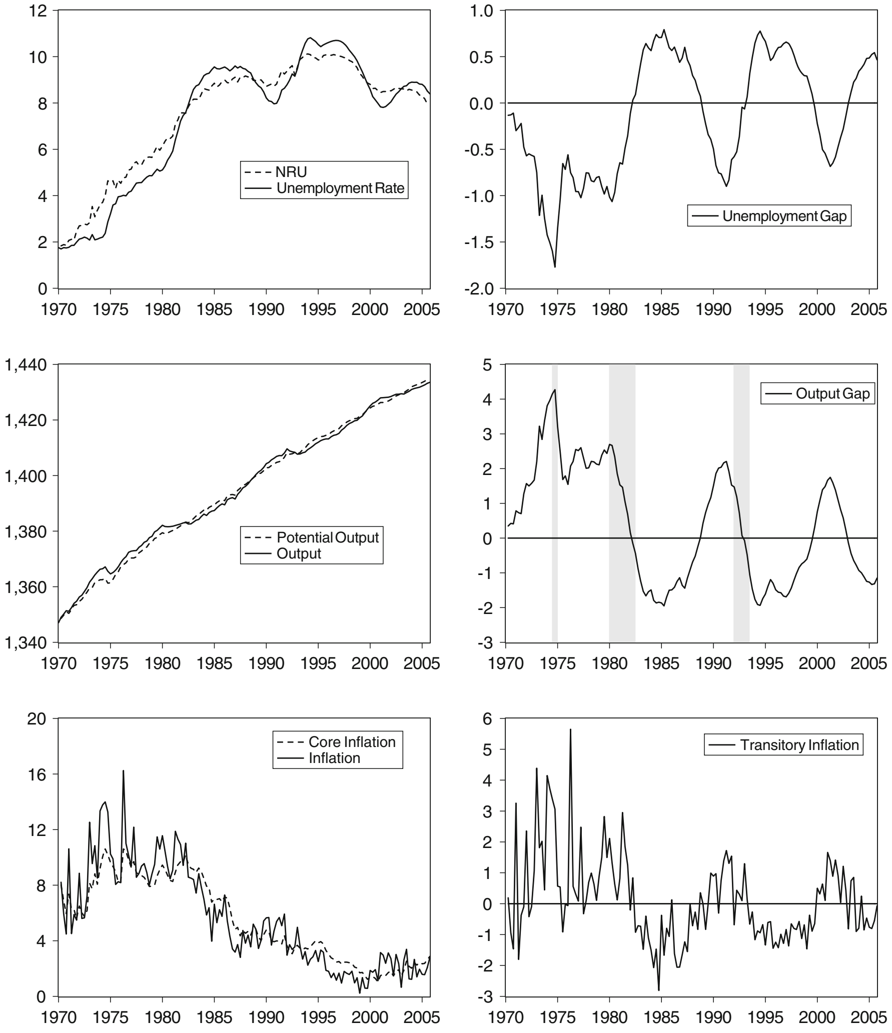

Fig. 2 State estimates: model (b)

that unemployment is I(1). The results imply that modelling the trend in the NRU as a breaking constant provides an alternative to the common unit-root specification in euro area studies.[11] Cyclical unemployment, although very persistent, is rather small in magnitude. The output gap is symmetric to the unemployment gap since they are linked to each other via Okun's Law. The core inflation rate evolves smoothly, and thus transitory inflation appears to be very volatile. Overall, the estimated natural rates and the corresponding gaps are consistent with earlier studies mostly based on a backward-looking Phillips curve (see e.g. Fabiani and Mestre 2004). The shaded area in

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[11] I would like to thank an anonymous referee for suggesting this alternative specification.

the output gap graph indicate recessions as defined by the CEPR.[12] The estimated gap picks up the business cycle turning points quite accurately. Particularly the beginning of recessions is almost similar dated. In terms of magnitude and turning points the output gap obtained here is very similar to recent estimates using the production function approach (see e.g. Roeger 2006, p. 9).

Figure 2 shows the estimated states of model (b). Compared to the baseline model the natural rates and the corresponding gaps differ particularly in the beginning of the sample. Potential output is below actual output even before 1974:Q1. This leads to a positive output gap in the mid 1970s which is twice as big as in the boom of the early 1990s. This difference is due to a constant growth rate of potential output which underestimates its true value before the break in 1974:Q1.

4 Conclusion

This article estimates potential output, the NRU, the core inflation rate, and the corresponding gaps for the euro area over the period 1970–2005. Output, inflation, and the rate of unemployment are decomposed into a non-stationary and a stationary component. An Okun-type relation is used to link the output gap to the unemployment gap. In the Phillips curve, the core inflation rate is allowed to be time-varying. Different from previous studies in the field, the empirical model allows for large but infrequent shifts in the growth rate of potential output and in the drift of the NRU. Moreover, accounting for recent advances in unobserved component models, the shocks to the unobserved states are allowed to be correlated. The results show that there is a one-time large shift in the growth rate of potential output in 1974:Q1. This indicates that the conventional approach of modelling potential output growth as either deterministic or a unit-root process with shocks occurring every period is inadequate of capturing this shift. By comparing the baseline model to an alternative model without a structural break in output growth and covariances that are set to zero, it is shown that these restrictions are not supported by the data. Furthermore, a new way of modelling the trend in the euro area NRU is offered. This breaking constant specification has the advantage of avoiding the I(2) assumption for the rate of unemployment. Finally, the estimated output gap is compared to the business cycle dating of the CEPR and found to accord quite precisely with it. The results presented here are also in line with the production function approach to estimate natural rates.

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[12] The CEPR dating committee labels the decline in GDP from 2001 onwards a *prolonged pause in the growth of economic activity* rather than a recession but notes that this might be reversed as revised GDP statistics appear (see http://www.cepr.org/data/Dating/ for details).

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