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A Fuzzy TOPSIS Method for Robot Selection

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A fuzzy TOPSIS method for robot selection is proposed, where the ratings of various alternatives versus various subjective criteria and the weights of all criteria are assessed in linguistic terms represented by fuzzy numbers. The values of objective criteria are converted into dimensionless indices to ensure compatibility between the values of objective criteria and the linguistic ratings of subjective criteria. The membership function of each weighted rating is developed by interval arithmetic of fuzzy numbers. To avoid complicated aggregation of fuzzy numbers, these weighted ratings are defuzzified into crisp values by the ranking method of mean of removals. A closeness coefficient is defined to determine the ranking order of alternatives by calculating the distances to both the ideal and negative-ideal solutions. A numerical example demonstrates the computational process of the proposed method.

Keywords: Fuzzy numbers; Fuzzy TOPSIS; Interval arithmetic; Robot selection

1. Introduction

To improve product quality and increase productivity, robot selection has always been an important issue for manufacturing companies. Many potential robot selection attributes (or criteria), e.g. cost, load capacity, man–machine interface, availability of diagnostic software, etc. must be considered for the selection of a particular robot [1–5]. In general, these attributes can be classified into two categories [5]:

1. Objective attributes – these attributes are defined in numerical terms, e.g. cost, reliability, load capacity, repeatability, and positioning accuracy.
2. Subjective attributes – these attributes have qualitative definitions, e.g. vendor's service contract, training, man-machine interface, and programming flexibility.

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Many precision-based methods for robot selection have been developed [1–4,6,7]. All the above methods are developed based on the concepts of accurate measurement and crisp evaluation, i.e. the measuring values must be exact. However, in real life, measures of subjective attributes, e.g. man–machine interface and programming flexibility, may not be precisely defined by decision-makers. Moreover, the evaluation of robot suitability versus subjective criteria and the weights of the criteria are usually expressed in linguistic terms [5,8]. Thus, Liang and Wang [5] proposed a fuzzy multi-criteria decision-making (MCDM) approach for robot selection. Despite the merits, the Liang and Wang [5] method has the following limitations:

1. The equation for converting objective criteria cannot ensure compatibility between the values of objective criteria and the linguistic ratings of subjective criteria. For example, assume that the evaluation of three alternatives under a benefit criterion are $A_1 = (20,40,65)$, $A_2 = (30,55,75)$, and $A_3 = (35,75,95)$. By Liang and Wang's direct relationship Eq. [5], the conversion of A_3 is (0.15,0.44,1.12). This does not fall between [0,1] and results in incompatibility between the converted A_3 and the fuzzy numbers defined in [0,1]. This same problem also exists in their inverse relationship equation.
2. The multiplication of two positive triangular fuzzy numbers is treated as a triangular fuzzy number. This may not be correct. The membership function of the multiplication of two positive triangular fuzzy numbers can be clearly developed.
3. The ranking method, i.e. maximising set and minimising set [9] used in Liang and Wang method was shown to be illogical by Liou and Wang [10] in 1992.

To solve these limitations, this work proposes selecting a robot via a fuzzy TOPSIS method. The technique for order preference by similarity to an ideal solution (TOPSIS) was initiated by Hwang and Yoon [11]. This technique is based on the concept that the ideal alternative has the best level for all attributes considered, whereas the negative-ideal is the one with all the worst attribute values. A solution from TOPSIS is defined as the alternative which is simultaneously the farthest from the negative-ideal and the closest to the ideal alternative. In fuzzy TOPSIS, some (or all) attributes are represented by fuzzy numbers [12].

In the proposed fuzzy TOPSIS method, the conversion of objective criteria is performed by applying the Hsu and Chen method [13], which ensures compatibility between the values of objective criteria and the linguistic ratings of subjective criteria. The membership function of each weighted rating of each alternative versus each criterion, is developed by using interval arithmetic of fuzzy numbers. To avoid complicated aggregation of irregular fuzzy numbers, these weighted ratings are defuzzified into crisp values by the ranking method of mean of removals [14], and then, a closeness coefficient is defined to determine the ranking order of alternatives by calculating the distances of alternatives to both the ideal and negative-ideal solutions. Finally, a numerical example demonstrates the computational process of the proposed method.

2. Fuzzy Numbers

Definition 1. A real fuzzy number A is described as any fuzzy subset of the real line R with membership function $f_A$which possesses the following properties [15] where a, b, c, and d are real numbers:

1. f_A is a continuous mapping from R to the closed interval [0,1].
2. $f_A(x) = 0$, for all $x \in (-\infty, a]$.
3. f_A is strictly increasing on $[a, b]$.
4. $f_A(x) = 1$, for all $x \in [b, c]$.
5. f_A is strictly decreasing on $[c, d]$.
6. $f_A(x) = 0$, for all $x \in [d, \infty)$.

We may let $a = -\infty$, or $a = b$, or $b = c$, or $c = d$, or $d = +\infty$. Unless elsewhere specified, it is assumed that A is convex, normal and bounded, i.e. $-\infty < a$, $d < \infty$.

The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x) & (a \le x \le b) \\ 1 & (b \le x \le c) \\ f_A^R(x) & (c \le x \le d) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $f_A^L(x)$ and $f_A^R(x)$ are the left and right membership functions of fuzzy number A, respectively.

The fuzzy number A is a triangular fuzzy number if its membership function f_A is given by [16]:

$$f_A(x) = \begin{cases} (x-a)/(b-a) & (a \le x \le b) \\ (x-c)/(b-c) & (b \le x \le c) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where a, b and c are real numbers.

Definition 2. The α-cut of fuzzy number A can be defined as [16]

$$A^\alpha = \{x | f_A(x) \ge \alpha,\}, \text{ where } x \in R, \alpha \in [0,1]$$

A^α is a non-empty bounded closed interval contained in R, and it can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are the lower and upper bounds of the closed interval, respectively. For example, if triangular fuzzy number $A = (a, b, c)$, then the α-cut of A can be expressed as:

$$A^\alpha = [A_l^\alpha, A_u^\alpha] = [(b-a)\alpha + a, (b-c)\alpha + c] \tag{3}$$

Given fuzzy numbers A and B, $A, B \in R^+$, the α-cuts of A and B are $A^\alpha = [A_l^\alpha, A_u^\alpha]$ and $B^\alpha = [B_l^\alpha, B_u^\alpha]$, respectively. By interval arithmetic, some main operations of A and B can be expressed as follows [16]:

$$(A \otimes B)^\alpha = [A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha] \tag{4}$$

$$(A \ominus B)^\alpha = [A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha] \tag{5}$$

$$(A \otimes B)^\alpha = [A_l^\alpha B_l^\alpha, A_u^\alpha B_u^\alpha] \tag{6}$$

$$(A \oslash B)^\alpha = \left[\frac{A_l^\alpha}{B_u^\alpha}, \frac{A_u^\alpha}{B_l^\alpha}\right] \tag{7}$$

$$(A \otimes r)^\alpha = [A_l^\alpha r, A_u^\alpha r], r \in R^+ \tag{8}$$

3. Ranking Fuzzy Numbers with Mean of Removals

A review of many fuzzy number ranking methods can be seen in [12,17–20]. However, no one can rank fuzzy numbers satisfactorily in all cases and situations [20]. In this paper, the mean of removals by Kaufmann and Gupta [14] is applied to help complete the proposed method. Consider a fuzzy number $A = [a, b, c, d]$, $A \in R$, as illustrated in Fig. 1. The left removal of A, denoted by A_L, and the right removal of A, denoted by A_R, are defined as follows:

$$A_L = b - \int_a^b f_A^L(x)\,dx \tag{9}$$

$$A_R = c + \int_c^d f_A^R(x)\,dx \tag{10}$$

The left and right removals stretch from the vertical axis at 0 on the x-axis to the left and right membership functions of A, respectively. The meanings of A_L and A_R can also be seen from Fig. 1. Clearly, the fuzzy number A becomes larger if A_L and/or A_R are larger. Thus, both A_L and A_R must be considered when ranking fuzzy numbers. The mean of the A_L and A_R is then defined as:

$$M(A) = \tfrac{1}{2}(A_L + A_R) \tag{11}$$

In this paper, $M(A)$ is used to rank fuzzy numbers. The larger the $M(A)$, the larger is the fuzzy number A. Therefore,

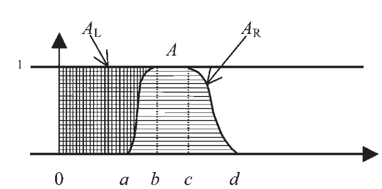

Fig. 1. The left and right removals of A.

for any two fuzzy numbers A_i and A_j, if $M(A_i) > M(A_j)$, then $A_i > A_j$. If $M(A_i) = M(A_j)$, then $A_i = A_j$. Finally, if $M(A_i) < M(A_j)$, then $A_i < A_j$.

4. Review of Liang and Wang Method

A quick review of the Liang and Wang method [5] is as follows. Let T_{it}, $i = 1 \sim m$, $t = h+1 \sim k$, denote the objective attribute values assigned to robot R_i by the committee for objective criterion C_t, i.e. M_{it}. The following conversion equation are applied:

For direct relationship:

$$M_{it} = T_{it} \oslash [T_{1t} \otimes \ldots \otimes T_{mt}] \quad (12)$$

For inverse relationship:

$$M_{it} = T_{it}^{-1} \oslash \lfloor T_{1t}^{-1} \oplus \ldots \oplus T_{mt}^{-1} \rfloor \quad (13)$$

Let $X_{itj} = (q_{itj}, o_{itj}, p_{itj})$, $i = 1 \sim m$, $t = 1 \sim h$, $j = 1 \sim n$, be the linguistic rating assigned to robot R_i by decision-maker D_j for subjective criterion C_t. Let $A_{tj} = (c_{tj}, a_{tj}, b_{tj})$, $t = 1 \sim k$, $j = 1 \sim n$, be the linguistic weighting given to criterion C_t by decision-maker D_j. Define

$$M_{it} = \begin{cases} (1/n) \otimes (X_{it1} \oplus \ldots \oplus X_{itn}) & (i = 1 \sim m, t = 1 \sim h) \\ (q_{it}, o_{it}, p_{it}), & (i = 1 \sim m, t = h+1 \sim k) \end{cases} \quad (14)$$

and

$$N_t = (1/n) \otimes (A_{t1} \oplus \ldots \oplus A_{tn}) \quad (t = 1 \sim k) \quad (15)$$

where M_{it} and N_t are the average fuzzy suitability ratings of robot R_i under criterion C_t as well as the importance weight of criterion C_t.

$$q_{it} = \sum_{j=1}^{n} q_{itj} / n, \quad o_{it} = \sum_{j=1}^{n} o_{itj}/n,$$

$$p_{it} = \sum_{j=1}^{n} p_{itj} /n \quad c_t = \sum_{j=1}^{n} c_{tj} / n,$$

$$a_t = \sum_{j=1}^{n} a_{tj} / n, \quad b_t = \sum_{j=1}^{n} b_{tj} / n$$

The fuzzy suitability index G_i and its membership function of the ith robot can be produced as follows:

$$G_i = (1/k) \otimes [(M_{i1} \otimes N_1) \oplus \ldots \oplus (M_{ik} \otimes N_k)] \quad (16)$$

$$f_{G_i}(x) = \quad (17)$$

$$\begin{cases} -H_{i1} + [H_{i1}^2 + (x - Y_i) / T_{i1}]^{\frac{1}{2}} & (Y_i \le x \le Q_i) \\ H_{i2} - [H_{i2}^2 + (x - Z_i) / U_{i1}]^{\frac{1}{2}} & (Q_i \le x \le Z_i) \\ 0 & \text{otherwise} \end{cases}$$

where,

$$T_{i1} = \sum_{t=1}^{k} \frac{(o_{it} - q_{it})(a_t - c_t)}{k},$$

$$T_{i2} = \sum_{t=1}^{k} \frac{[q_{it} (a_t - c_t) + c_t(o_{it} - q_{it})]}{k}$$

$$U_{i1} = \sum_{t=1}^{k} \frac{(p_{it} - o_{it})(b_t - a_t)}{k},$$

$$U_{i2} = \sum_{t=1}^{k} \frac{[b_t(o_{it} - p_{it}) + p_{it}(a_t - b_t)]}{k}$$

$$H_{i1} = \frac{T_{i2}}{2T_{i1}},$$

$$H_{i2} = -\frac{U_{i2}}{2U_{i1}},$$

$$Y_i = \sum_{t=1}^{k} \frac{q_{it}c_t}{k},$$

$$Q_i = \frac{\sum_{t=1}^{k} o_{it}a_t}{k},$$

$$Z_i = \frac{\sum_{t=1}^{k} p_{it}b_t}{k}$$

By applying Chen's maximising set and minimising set [9] with an index of rating attitude $k = 0.5$, the ranking value $U_T(G_i)$ of the fuzzy suitability index G_i is expressed as:

$$U_T(G_i) = [H_{i2} - (H_{i2}^2 + (x_R - Z_i) / U_{i1})^{\frac{1}{2}} + 1 + H_{i1} - (H_{i1}^2 + (x_L - Y_{i1})^{\frac{1}{2}}]/2 \quad (18)$$

where $x_1 = \inf E$, $x_2 = \sup E$, $E = \bigcup_{i=1}^{m} E_i$, $E_i = \{x | f_{G_i}(x) > 0\}$, $i = 1 \sim m$,

$$x_R = \{2x_1 + 2H_{i2}(x_2 - x_1) + (x_2 - x_1)^2 / U_{i1} - (x_2 - x_1) [(2H_{i2} + (x_2 - x_1) / U_{i1})^2 + 4(x_1 - Z_i) / U_{i1}]^{\frac{1}{2}}\}/2$$

$$x_L = \{2x_2 + 2H_{i1}(x_2 - x_1) + (x_2 - x_1)^2 / T_{i1} - (x_2 - x_1) [(2H_{i1} + (x_2 - x_1) / T_{i1})^2 + 4(x_2 - Y_i) / T_{i1}]^{\frac{1}{2}}\}/2$$

Despite its merits, the Liang and Wang method has limitations, which have been stated in Section 1. To resolve these limitations, a fuzzy TOPSIS method is suggested in the next section.

5. A Fuzzy TOPSIS Method

Assume that a committee of n decision-makers (D_j, $j = 1 \sim n$) is responsible for assessing m alternatives (A_i, $i = 1 \sim m$) under each of k criteria (C_t, $t = 1 \sim k$) as well as assessing the importance weights of the criteria, where the suitability ratings of alternatives under subjective criteria as well as the weights of all criteria are assessed in linguistic terms [8] represented by triangular fuzzy numbers. Criteria (C_t, $t = 1 \sim k$) are classified into subjective and objective criterion.

5.1 Aggregate the Importance Weights

Many methods are available to pool the decision-makers' opinions, for example, mean, median, max, min, and mixed operators [21]. Each of the operators has its limitations. Criteria for selecting an appropriate aggregation operator can be found in Zimmermann [22]. Since the average operation is the most commonly used aggregation method, in this paper, the mean operator is used to pool the decision-makers' opinions [5].

Let $W_{tj} = (a_{tj}, b_{tj}, c_{tj})$, $t = 1 \sim k$, $j = 1 \sim n$, be the linguistic weight assigned to criterion C_t by decision-maker D_j. The aggregated linguistic weight, $W_t = (a_t, b_t, c_t)$, $t = 1 \sim k$, for criterion t from n decision-makers' opinions can be calculated by:

$$W_t = (1/n) \otimes (W_{t1} \oplus W_{t2} \oplus \dots \oplus W_{tn}) \tag{19}$$

where,

$$a_t = \frac{\sum_{j=1}^{n} a_{tj}}{n}, \quad b_t = \frac{\sum_{j=1}^{n} b_{tj}}{n} \quad c_t = \frac{\sum_{j=1}^{n} c_{tj}}{n}$$

5.2 Aggregate Ratings of Alternatives under Subjective Criteria

Let $R_{itj} = (o_{itj}, p_{itj}, q_{itj})$ where $i = 1 \sim m$, $t = 1 \sim h$, $j = 1 \sim n$, denote the linguistic rating assigned to alternative A_i by decision-maker D_j for subjective criterion C_t. The mean operator is also used to pool the decision-makers' opinions. The aggregated linguistic rating, $R_{it} = (o_{it}, p_{it}, q_{it})$ where $i = 1 \sim m$, $t = 1 \sim h$, of alternative A_i under subjective criterion C_t from n decision-makers' opinions can be calculated by:

$$R_{it} = (1/n) \otimes (R_{it1} \oplus R_{it2} \oplus \dots \oplus R_{itn}) \tag{20}$$

where,

$$o_{it} = \frac{\sum_{j=1}^{n} o_{itj}}{n}, \quad p_{it} = \frac{\sum_{j=1}^{n} p_{itj}}{n}, \quad q_{it} = \frac{\sum_{j=1}^{n} q_{itj}}{n}$$

5.3 Convert the Objective Criteria

The objective criteria (fuzzy or non-fuzzy) can be classified into two categories: benefit (B) and cost (C). Objective criteria may have incommensurable units. To ensure compatibility between the fuzzy (or non-fuzzy) evaluation values of objective criteria and the linguistic ratings of subjective criteria, the fuzzy (or non-fuzzy) evaluation values of objective criteria must be converted into a compatible scale (into dimensionless indices) [5]. In this paper, the conversion is performed by applying the Hsu and Chen method [13] since it preserves the property that the ranges of converted triangular fuzzy numbers belong to [0,1]. If $T_{it} = (g_{it}, u_{it}, v_{it})$ where $i = 1 \sim m$, $t = h + 1 \sim k$, represents the fuzzy (or non-fuzzy) total cost/benefit assigned to alternative A_i versus objective criterion C_t, then the converted objective criteria, $R_{it} = (o_{it}, p_{it}, q_{it})$ where $i = 1 \sim m$, $t = h + 1 \sim k$, can be calculated by:

For benefit criteria

$$R_{it} = (g_{it} / v^*_t, u_{it} / v^*_t, v_{it} / v^*_t) \text{ for } t \in B \tag{21}$$

where $v^*_t = \max_i v_{it}$, $o_{it} = g_{it} / v^*_t$, $p_{it} = u_{it} / v^*_t$, $q_{it} = v_{it}/v^*_t$, i = 1 ~ m, t = h + 1 ~ k.

For cost criteria

$$R_{it} = (g_t^- / v_{it}, g_t^- / u_{it}, g_t^- / g_{it}) \text{ for } t \in C \tag{22}$$

where $g_t^- = \min_i g_{it}$, $o_{it} = g_t^- / v_{it}$, $p_{it} = g_t^- / u_{it}$, $q_{it} = g_t^- / g_{it}$, $i = 1 \sim m$, $t = h + 1 \sim k$.

5.4 Construct the Weighted Decision Matrix

$$S_{it} = W_t \otimes R_{it} \tag{23}$$

where S_{it}, $i = 1 \sim m$, $t = 1 \sim k$, denotes the elements of the weighted suitability decision matrix S, i.e. $S = [S_{it}]_{m \times k}$.

The membership function of S_{it}, i.e. the weighted rating, can be developed by Eqs [3] and [6] as follows [19]:

$$\begin{aligned} S_{it}^{\alpha} = W_t^{\alpha} \otimes R_{it}^{\alpha} &= [(b_t - a_t)\alpha + a_t, (b_t - c_t)\alpha + c_t] \\ &\otimes [(p_{it} - o_{it})\alpha + o_{it}, (p_{it} - q_{it})\alpha + q_{it}] = [(b_t - a_t)(p_{it} \\ &- o_{it})\alpha^2 + [a_t(p_{it} - o_{it}) + o_{it}(b_t - a_t)]\alpha + a_t o_{it}, \\ &(b_t - c_t)(p_{it} - q_{it})\alpha^2 + [c_t(p_{it} - q_{it}) + q_{it} \\ &(b_t - c_t)]\alpha + c_t q_{it}] \end{aligned}$$

We now have two Eq. to solve, namely:

$$(b_t - a_t)(p_{it} - o_{it})\alpha^2 + [a_t(p_{it} - o_{it}) + o_{it}(b_t - a_t)]\alpha + a_t o_{it} - x = 0 \tag{24}$$

$$(b_t - c_t)(p_{it} - q_{it})\alpha^2 + [c_t(p_{it} - q_{it}) + q_{it}(b_t - c_t)]\alpha + c_t q_{it} - x = 0 \tag{25}$$

Let $E_{it1} = (b_t - a_t)(p_{it} - o_{it})$, $F_{it1} = a_t(p_{it} - o_{it}) + o_{it}(b_t - a_t)$, $E_{it2} = (b_t - c_t)(p_{it} - q_{it})$, $F_{it2} = c_t(p_{it} - q_{it}) + q_{it}(b_t - c_t)$, $V_{it} = a_t o_{it}$, $Y_{it} = b_t p_{it}$, $Z_{it} = c_t q_{it}$.

Equations (24) and (25) can be expressed as:

$$E_{it1}\alpha^2 + F_{it1}\alpha + V_{it} - x = 0 \tag{26}$$

$$E_{it2}\alpha^2 + F_{it2}\alpha + Z_{it} - x = 0 \tag{27}$$

Only roots in [0,1] will be retained in (26) and (27). The left membership function, i.e. $f^L_{S_{it}}(x)$, and the right membership function, i.e. $f^R_{S_{it}}(x)$, of S_{it} can be developed as follows:

$$f^L_{S_{it}}(x) = \{- F_{it1} + [F^2_{it1} + 4E_{it1}\,(x - V_{it})]^{\frac{1}{2}}\}/2E_{it1} \quad (V_{it} \le x \le Y_{it}) \tag{28}$$

$$f^R_{S_{it}}(x) = \{-F_{it2} - [F^2_{it2} + 4E_{it2}\,(x - Z_{it})]^{\frac{1}{2}}\}/2E_{it2} \quad (Y_{it} \le x \le Z_{it}) \tag{29}$$

The membership function of S_{it} may not yield a triangular shape. When $E_{it1} = 0$, $f^L_{S_{it}}(x) = (x - V_{it}) / F_{it1}$. Similarly, when $E_{ij2} = 0$, $f^R_{S_{it}}(x) = (x - Z_{it}) / F_{it2}$ Furthermore, if $E_{it1} = F_{it1} =$

0, there is no left membership function; and if $E_{it2} = F_{it2} = 0$, there is no right membership function. For convenience, S_{it} can be expressed as:

$$S_{it} = (V_{it}, Y_{it}, Z_{it}; E_{it1}, F_{it1}, E_{it2}, F_{it2}) \quad (i = 1 \sim m, \quad t = 1 \sim k) \tag{30}$$

5.5 Determine the Ideal and Negative-Ideal Solutions

To avoid a complicated calculation of irregular fuzzy numbers, all S_{it} terms ($i = 1 \sim m$, $t = 1 \sim k$) are defuzzified into crisp values S_{it} terms by Eq. (11) [18]. Then, we define the ideal (I^+) and negative-ideal (I^-) solutions as:

$$I^+ = (s_1^+, \ldots, s_t^+, \ldots, s_k^+) \tag{31}$$

$$I^- = (s_1^-, \ldots, s_t^-, \ldots, s_k^-) \tag{32}$$

where $s_t^+ = \max_i \{s_{it}\}$ and $s_t^- = \min_i \{s_{it}\}$.

5.6 Calculate the Distance of Each Alternative from I^+ and I^-

The following Eq. are applied to calculate the distance of each alternative from I^+ and I^-.

$$d_i^+ = \left[\sum_{t=1}^{k} (s_{it} - s_t^+)^2 \right]^{\frac{1}{2}} \quad (i = 1 \sim m) \tag{33}$$

$$d_i^- = \left[\sum_{t=1}^{k} (s_{it} - s_t^-)^2 \right]^{\frac{1}{2}} \quad (i = 1 \sim m) \tag{34}$$

where d_i^+ denotes the distance between each alternative and the ideal solution, d_i^- denotes the distance between each alternative and the negative-ideal solution.

5.7 Calculate Closeness Coefficient

The closeness coefficient of alternative A_i with respect to ideal solution A^+ can be defined as:

$$C_i = \frac{d_t^-}{d_t^+ + d_t^-} \quad (0 < C_i < 1, \quad i = 1 \sim m) \tag{35}$$

Clearly, an alternative A_i is closer to I^+ than to I^- as C_i approaches 1, suggesting that the evaluation grade of A_i increases with C_i. The closeness coefficient C_t, can be regarded as the evaluation value of alternative A_t. Thus, the larger C_i, the higher priority the alternative A_i.

6. Numerical Example

The numerical example from Liang and Wang [5] is used to illustrate the feasibility of the proposed fuzzy TOPSIS method. Assume that a manufacturing company requires a robot to perform a material-handling task. After preliminary screening, three robots R_1, R_2, and R_3 are chosen for further evaluation. A committee of four decision-makers, D_1, D_2, D_3, and D_4 is formed to conduct the evaluation and to select the most suitable robot. The robot selection criteria and the importance weights of the criteria are shown in Tables 1 and 2, respectively. The ratings of three subjective criteria are shown in Table 3. The data of objective criteria is shown in Table 4.

The linguistic terms represented by triangular fuzzy numbers for evaluating the alternative robots under subjective criteria

Table 1. The robot selection criteria.

Subjective criteria	Objective criteria
Man–machine interface (C_1)	Purchase cost (C_4)
Programming flexibility (C_2)	Load capacity (C_5)
Vendor's service contract (C_3)	Positioning accuracy (C_6)

Table 2. The weights of criteria and the average weights.

Criteria	Decision makers				Average weights (W_t)
	D_1	D_2	D_3	D_4	
C_1	H	VH	VH	H	(0.6000, 0.8500, 1.0000)
C_2	M	VH	H	VH	(0.5250, 0.8000, 0.9500)
C_3	L	M	L	M	(0.1000, 0.4000, 0.6500)
C_4	L	M	M	M	(0.1500, 0.4500, 0.7250)
C_5	VH	H	VH	VH	(0.6500, 0.9250, 1.0000)
C_6	H	H	VH	VH	(0.6000, 0.8500, 1.0000)

Table 3. Ratings of robots under subjective criteria and the average ratings.

Criteria	Robots	Decision makers				Average ratings (R_{it})
		D_1	D_2	D_3	D_4	
C_1	R_1	*VG*	*G*	*F*	*F*	*(0.5000, 0.7000, 0.8500)*
	R_2	*G*	*F*	*G*	*F*	*(0.4500, 0.6500, 0.8500)*
	R_3	*G*	*VG*	*F*	*G*	*(0.5750, 0.7750, 0.9250)*
C_2	R_1	*F*	*G*	*P*	*G*	*(0.3750, 0.5750, 0.7750)*
	R_2	*F*	*VG*	*G*	*VG*	*(0.6250, 0.8250, 0.9250)*
	R_3	*G*	*VG*	*F*	*G*	*(0.5750, 0.7750, 0.9250)*
C_3	R_1	*F*	*G*	*F*	*F*	*(0.3750, 0.5750, 0.7750)*
	R_2	*G*	*VG*	*F*	*G*	*(0.5750, 0.7750, 0.9250)*
	R_3	*VG*	*G*	*G*	*G*	*(0.6500, 0.8500, 1.0000)*

Table 4. The values under objective criteria.

Robots	Purchase cost ($\$ \times 1000$), C_4	Load capacity (lb), C_5	Positioning accuracy ($\pm$in), C_6
R_1	(72.5,73,74)	(48.5,50,52)	(0.11,0.12,0.14)
R_2	(69,70,72)	(44,45,46.5)	(0.15,0.16,0.18)
R_3	(67.5,68,70)	(43.5,45,47.5)	(0.16,0.17,0.19)

are: VP = very poor = (0, 0, 0.2), P = poor = (0, 0.2, 0.4), F = fair = (0.3, 0.5, 0.7), G = good = (0.6, 0.8, 1), VG = very good = (0.8, 1, 1). The linguistic terms represented by triangular fuzzy numbers for evaluating the importance weights for criteria are: VL = very low = (0, 0, 0.3), L = low = (0, 0.3, 0.5), M = medium = (0.2, 0.5, 0.8), H = high = (0.5, 0.7, 1), VH = very high = (0.7, 1, 1) [5].

By Eqs (19) and (20), the average weights for criteria and the average ratings for robots versus subjective criteria can be obtained, and they are also presented in Tables 2 and 3, respectively. By Eqs (21) and (22), the converted values of objective criteria of alternative robots can be obtained as:

R_{14} = (0.9122, 0.9247, 0.9310), R_{24} = (0.9375, 0.9643, 0.9783), R_{34} = (0.9643, 0.9926, 1.0000)

R_{15} = (0.9327, 0.9615, 1.0000), R_{25} = (0.8462, 0.8654, 0.8942), R_{35} = (0.8365, 0.8654, 0.9135)

R_{16} = (0.7857, 0.9167, 1.0000), R_{26} = (0.6111, 0.6875, 0.7333), R_{36} = (0.5789, 0.6471, 0.6875)

By Eq. (30), the weighted ratings, S_{it} (i = 1 ~ 3, t = 1 ~ 6), can be produced as:

S_{11} = (0.3000, 0.5950, 0.8500; 0.0500, 0.2450; 0.0225, −0.2775)

S_{12} = (0.1969, 0.4600, 0.7363; 0.0550, 0.2081; 0.0300, −0.3063)

S_{13} = (0.0375, 0.2300, 0.5038; 0.0600, 0.1325; 0.0500, −0.3238)

S_{14} = (0.1368, 0.4161, 0.6750; 0.0037, 0.2755; 0.0018, −0.2607)

S_{15} = (0.6063, 0.8894, 1.0000; 0.0079, 0.2752; 0.0029, −0.1135)

S_{16} = (0.4714, 0.7792, 1.0000; 0.0327, 0.2750; 0.0125, −0.2333)

S_{21} = (0.2700, 0.5525, 0.8500; 0.0500, 0.2325; 0.0300, −0.3275)

S_{22} = (0.3281, 0.6600, 0.8788; 0.0550, 0.2769; 0.0150, −0.2338)

S_{23} = (0.0575, 0.3100, 0.6013; 0.0600, 0.1925; 0.0375, −0.3288)

S_{24} = (0.1406, 0.4339, 0.7092; 0.0080, 0.2853; 0.0038, −0.2792)

S_{25} = (0.5500, 0.8005, 0.8942; 0.0053, 0.2452; 0.0022, −0.0959)

S_{26} = (0.3667, 0.5844, 0.7333; 0.0191, 0.1986; 0.0069, −0.1558)

S_{31} = (0.3450, 0.6588, 0.9250; 0.0500, 0.2638; 0.0225; −0.2888)

S_{32} = (0.3019, 0.6200, 0.8788; 0.0550, 0.2631; 0.0225, −0.2813)

S_{33} = (0.0650, 0.3400, 0.6500, 0.0600, 0.2150; 0.0375, −0.3475)

Table 5. Relative closeness to ideal solution.

Robots	d_i^+	d_i^-	C_i
R_1	0.2108	0.2385	0.5308
R_2	0.2282	0.1875	0.4510
R_3	0.2380	0.1985	0.4547

S_{34} = (0.1446, 0.4467, 0.7250; 0.0085, 0.2935; 0.0020, −0.2803)

S_{35} = (0.5438, 0.8005, 0.9135; 0.0079, 0.2488; 0.0036, −0.1166)

S_{36} = (0.3474, 0.5500, 0.6875; 0.0170, 0.1856; 0.0061, −0.1436)

By Eqs (31)–(34), the distances of alternative robots, R_i from I^+ and I^- can be obtained, and they are displayed in Table 5. By Eq. (35), the closeness coefficient C_i of each alternative robot to I^+ can be obtained, and they are also displayed in Table 5. According to Table 5, the ranking order of the three robots is R_1, R_3 and R_2. Thus, the best selection is robot 1. By the Liang and Wang method [5], the ranking order of the three robots is R_3 (0.5180), R_2 (0.4944), and R_1 (0.4635).

7. Conclusions

Liang and Wang [5] proposed a fuzzy multi-criteria decision-making approach for robot selection. Despite the merits, the Liang and Wang method has several limitations. To resolve the limitations, a fuzzy TOPSIS method for the robot selection problem is suggested, where the importance weights of different criteria and the ratings of various alternatives under different subjective criteria are assessed in linguistic terms represented by triangular fuzzy numbers.

In this work, the Hsu and Chen conversion method [13] is applied to ensure the compatibility between the values of objective criteria and the linguistic ratings of subjective criteria. The membership function of each weighted rating of each alternative versus each criterion is clearly developed. To avoid complicated calculation of fuzzy numbers, these weighted ratings are defuzzified into crisp values to help calculate the distances of each alternative to both the ideal and negative-ideal solutions. A closeness coefficient is then defined to determine the ranking order of alternatives. A numerical example has demonstrated the computational process of the proposed method. The proposed method can also be applied to other fuzzy management problems.

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