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Modelling of Five-Axis Machine Tool Metrology Models Using the Matrix Summation Approach

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Five-axis machining has been used widely in manufacturing freeform surfaces. The traditional approach of using a homogeneous transformation matrix (HTM) relies on heavy symbolic manipulation of the matrix multiplication. In this work, a new approach – the matrix summation approach – is developed and implemented for modelling the geometric errors of five-axis machine tools. This approach breaks down the kinematic equation into six components, each of which has clear physical meaning. It reduces the computations substantially and makes the five-axis kinematic model manageable and understandable.

Keywords: Five-axis machining; Kinematic error modelling; Matrix summation

1. Introduction

Five-axis machining has attracted much attention in the manufacturing community because parts with complex surfaces (sculptured surfaces) are becoming increasingly common [1]. Although a three-axis NC milling machine is usually sufficient for machining most sculptured surfaces, a five-axis machine is particularly powerful in that more geometrically complicated parts can be machined in a single set-up with versatile tools such as fillet end mills or flat mills [2].

However, five-axis machining has some drawbacks. Owing to the complexity of the machine configuration, it often encounters more accuracy problems (dimensional deviation) than three-axis machining, because the simultaneous five-axis motions often increase the machine volumetric errors [3], and also the modelling of these volumetric errors for five-axis machines is far more complicated than the modelling for three-axis machines. Therefore, unlike the commonly acknowledged 21 parametric errors in three-axis machines, more error components must be included in a five-axis model, such as the squareness/parallelism errors among axes and the constant offsets of a rotary axis. As pointed out by Suh et al. [2], error modelling and measurement for rotary tables is still not well developed. In particular, five-axis systems encounter more computational problems than their three-axis counterparts. The error equation is very large and many calculations are required. Some model derivations can only be performed by a computer and often result in long equation with over 100 terms. Without a good modelling approach, it is difficult to understand the physical meaning of the error terms in these equation or check possible modelling mistakes.

Compared with three-axis machines, there has been relatively little work on the kinematic modelling for five-axis systems. Soons et al. [4] presented a general methodology to obtain error models for multi-axis machines, including rotation axes. However, the procedures to derive specific error models from the general methodology were not well delineated. Lin and Ehmann [5] presented a direct volumetric error analysis method for the evaluation of the position and orientation errors in the workpiece of a multi-axis machine. Their work provides a basis for the automatic derivation of error synthesis models for arbitrary machine configurations, although their approach is very complicated. It is difficult to decode the error model obtained by their direct analysis approach to obtain physical meanings. Kiridena and Ferreira [6] classified five-axis machines into TTTRR, RTTTR, and RRTTT systems and used the Denavit–Hartenberg convention to develop kinematic models for each of these three machine types. However, their model considered only five parametric errors (one positioning error for each axis). Srivastava et al. [7] developed a geometric error model for a five-axis machining centre. However, their work focused on one specific machine type (TTTRR), which is not comprehensive. Hai [8] developed a generalised model formulation technique for the error synthesis on machine tools, but his work required much symbolic manipulation, which is very complicated and can only be done by a computer. It is difficult to understand the physical meaning of the error terms in these equation or to check possible modelling errors. Yang dedicated one chapter of his dissertation to the formulation of a generalised 5D error synthesis model [9]. However, he defined only 27 geometric error components, which is incomplete. Unlike the commonly acknowledged 21 parametric errors in three-axis machines, more constant error components should

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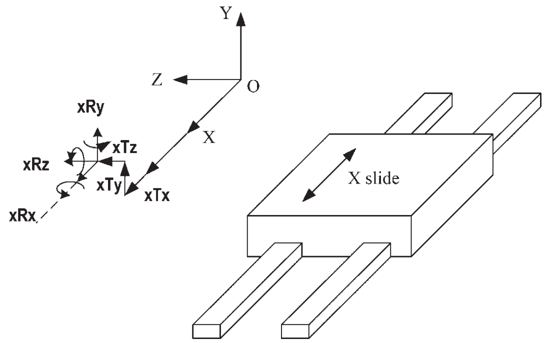

Fig. 1. Parametric errors associated with a prismatic joint.

be included in a five-axis model, such as the squareness/parallelism and constant offset of a rotary axis, etc.

Therefore, the main challenge in this work is the development of a robust five-axis error model, which must be generic enough to handle most of the common five-axis machine types.

2. Five-Axis Parametric Errors

Five-axis machines usually have both translational axes and rotational axes. For each translational axis, six parametric errors are identified: three translational errors (*xTx*, *xTy*, and *xTz*), and three angular errors (*xRx*, *xRy*, and *xRz*). Figure 1 shows these six parametric errors associated with a prismatic joint (*X*-axis). In the notations used to depict parametric errors, *R* means rotation, and *T* means translation. The left hand lowercase letter means the moving slide and the right hand lowercase letter means the error direction. For example, *xTy* means the straightness error (in the *y*-direction) of the *X* slide; *zRx* means the pitch/yaw error (rotated on the *x*-axis) for the *Z* slide.

For a rotary joint, the nominal motion is the rotation about its axis. There are also six-degrees-of-freedom error motions associated with this type of joint, i.e. three translational errors and three rotational errors. As shown in Fig. 2, the nominal axis rotation is the Z-axis (depending on different naming conventions on the shop floor, it may be called the *A*-axis, *B*-axis, or *C*-axis) where *XYZ* is the original (error-free) rotary axis coordinate frame and *X′Y′Z′* is the rotated coordinate frame with error motions.

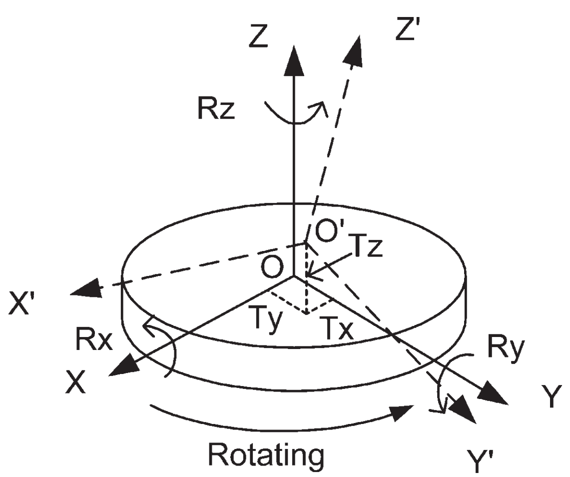

Fig. 2. Six parametric errors associated with a rotary joint.

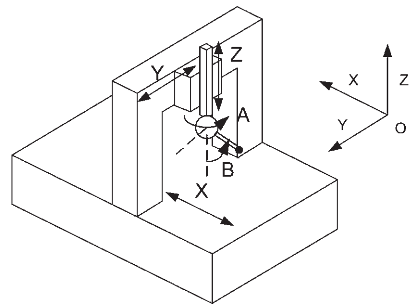

Fig. 3. The FTTTRR system.

Furthermore, there is the misalignment (error) between the axes of two nominally perpendicular carriages owing to mechanical and kinematic imperfections. In a generic five-axis machine tool, seven squareness errors are defined as the constant parametric errors. They can be defined as *Sxy*, *Syz*, *Szx*, *Sax*, *Say*, *Sbx*, and *Sbz* (assuming the *A*-axis is parallel to the *Z*-axis and the *B*-axis is parallel to the *Y*-axis), where *S* means squareness, and the two following letters indicate that the error is between these two reference axes.

3. Kinematic Models for Five-Axis Machine Tools

Most of the common types of five-axis machines have three translational axes and two rotational axes [6]. The 3T2R configurations can be classified into smaller groups based on the location of the rotational axes with respect to the translational axes. The common configurations are TTTRR, RTTTR, and RRTTT machine types. In other words, the machine configurations we study are based on FTTTRR (Fig. 3), RFTTTR (Fig. 4), RRFTTT (Fig. 5) systems, where F means the machine fixed base. To begin with, we study FXYZAB, AFXYZB, and ABFXYZ systems where the *A*-axis is parallel to the *Z*-axis and the *B*-axis is parallel to the *Y*-axis.

For an FTTTRR machine system, the schematic drawing of the kinematic chain from the workpiece to the cutting tool is given in Fig. 6. According to Fig. 6, the tool tip position can be described in the workpiece coordinate frame *OXYZ* (in this

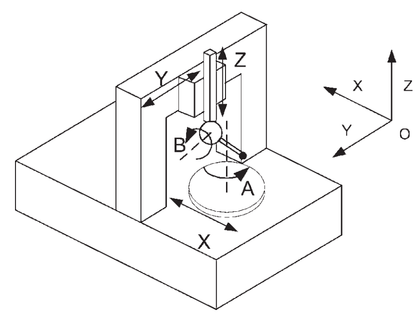

Fig. 4. The RFTTTR system.

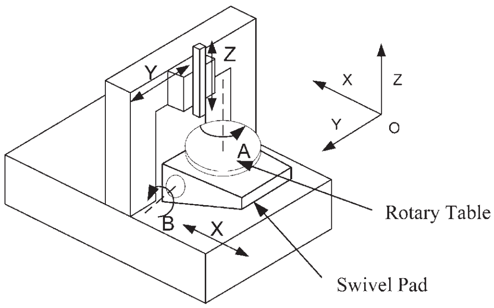

Fig. 5. The RRFTTT system.

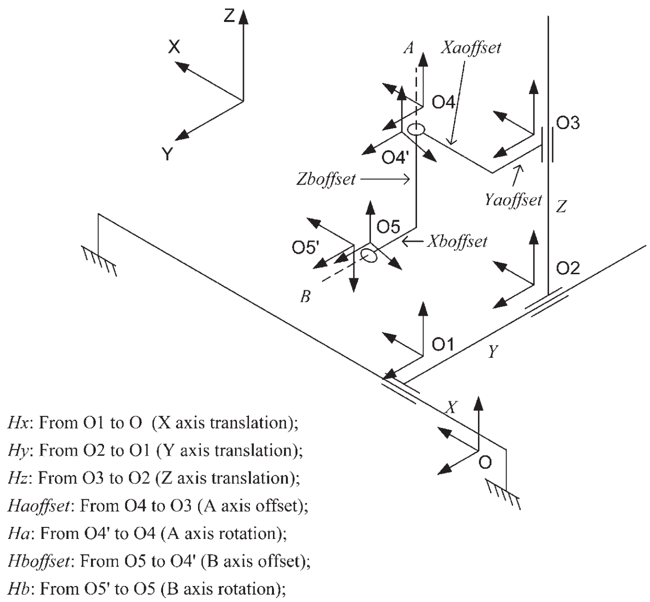

Fig. 6. Diagram of an FTTTRR machine type.

case, the workpiece coordinate frame coincides with the machine coordinate frame). Using the homogenous transformation matrix (HTM) approach,

$$Tsystem = Hx \cdot Hy \cdot Hz \cdot Haoffset \cdot Ha \cdot Hboffset \cdot Hb \cdot T \tag{1}$$

Tsystem is the tool tip position described in the workpiece coordinate frame. In vector form, it can be written as:

$$Tsystem = \begin{bmatrix} Xtrue \\ Ytrue \\ Ztrue \\ 1 \end{bmatrix} \tag{2}$$

T is the tool link offset vector (tool tip position described in tool coordinate frame):

$$T = \begin{bmatrix} Xp \\ Yp \\ Zp \\ 1 \end{bmatrix} \tag{3}$$

Hx, *Hy*, *Hz*, *Ha*, and *Hb* are the transformation matrices for each moving axis. *Haoffset* and *Hboffset* are transformation matrices for the corresponding offsets of the rotary axes. According to [10] and [11], these homogenous transformation matrices can be written as

$$Hx = \begin{bmatrix} 1 & -(xRz + Sxy) & xRy + Sxz & X + xTx \\ xRz + Sxy & 1 & -xRx & xTy \\ -(xRy + Sxz) & xRx & 1 & xTz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$Hy = \begin{bmatrix} 1 & -yRz & yRy & yTx \\ yRz & 1 & -(yRx + Syz) & yTy + Y \\ -yRy & (yRx + Syz) & 1 & yTz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$Hz = \begin{bmatrix} 1 & -zRz & zRy & zTx \\ zRz & 1 & -zRx & zTy \\ -zRy & zRx & 1 & Z + zTz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$Ha = \begin{bmatrix} \cos A - aRz \cdot \sin A & -\sin A - aRz \cdot \cos A & aRy + Sax & aTx \\ aRz \cdot \cos A + \sin A & \cos A - aRz \cdot \sin A & -aRx - Say & aTy \\ -\cos A \cdot aRy - \cos A \cdot Sax + \sin A \cdot aRx + \sin A \cdot aRx & \sin A \cdot aRy + \sin A \cdot Sax + \cos A \cdot aRx + \cos A \cdot Say & 1 & aTz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$Hb = \begin{bmatrix} cosB - bRy \cdot \sin B & -bRz - Sbx & \sin B + bRy \cdot \cos B & bTx \\ \cos B \cdot bRz + coB \cdot Sbx + sinB \cdot bRx + sinB \cdot Sbz & 1 & \sin B \cdot bRz + \sin B \cdot Sbx - \cos B \cdot bRx - \cos B \cdot Sbz & bTy \\ -\cos B \cdot bRy - \sin B & bRx + Sbz & \cos B - bRy \cdot \sin B & bTz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

For AFXYZB (RFTTTR system), referring to Fig. 4,

$$Hsystem = Ha^{-1} \cdot Haoffset^{-1} \cdot Hx \cdot Hy \cdot Hz \cdot Hb \tag{9}$$

For ABFXYZ (RRFTTT system), referring to Fig. 5,

$$Hsystem = Ha^{-1} \cdot Haoffset^{-1} \cdot Hb^{-1} \cdot Hboffset^{-1} \cdot Hx \cdot Hy \cdot Hz \tag{10}$$

The *Haoffset* and *Hboffset* matrices provide the constant offset transformation from the local rotary joint coordinate system to the machine coordinate system.

$$Haoffset = \begin{bmatrix} 1 & 0 & 0 & Xaoffset \\ 0 & 1 & 0 & Yaoffset \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$Hboffset = \begin{bmatrix} 1 & 0 & 0 & Xboffset \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Zboffset \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Although Eqs (1), (8), and (9) can be used to derive the kinematic error model for five-axis machine tools, they are not ready for implementation in practice. The final derivation of the model relies on heavy symbolic manipulation of the matrix multiplication, which often results in a long equation with over 100 terms. Here, we found that the HTM approach, i.e. the matrix multiplication approach, has several difficulties:

1. Low computational efficiency.
2. Difficult to understand the physical meaning of the error equation.
3. Difficult to evaluate the contribution of error motion of individual axes.
4. Difficult to reuse the currently available model for new machine types.

4. Matrix Summation Approach in Five-Axis Error Modelling

To simplify the model, by applying a small error motion assumption and eliminating second-order or higher-order terms, homogenous transformation matrices for each joint can be rewritten as follows:

$$Hx = Hxideal(I + Hxerror) \tag{13}$$

$$Hy = Hyideal(I + Hyerror) \tag{14}$$

$$Hz = Hzideal(I + Hzerror) \tag{15}$$

$$Ha = (I + Haerror)Haideal \tag{16}$$

$$Hb = (I + Hberror)Hbideal \tag{17}$$

where,

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

$$Hxideal = \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$Hxerror = \begin{bmatrix} 0 & -(xRz + Sxy) & xRy + Sxz & xTx \\ xRz + Sxy & 0 & -xRx & xTy \\ -(xRy + Sxz) & xRx & 0 & xTz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

$$Hyideal = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$$Hyerror = \begin{bmatrix} 0 & -yRz & yRy & yTx \\ yRz & 0 & -(yRx + Syz) & yTy \\ -yRy & yRx + Syz & 0 & yTz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$Hzideal = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

$$Hzerror = \begin{bmatrix} 0 & -zRz & zRy & zTx \\ zRz & 0 & -zRx & zTy \\ -zRy & zRx & 0 & zTz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

$$Haideal = \begin{bmatrix} \cos A & -\sin A & 0 & 0 \\ \sin A & \cos A & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

$$Haerror = \begin{bmatrix} 0 & -aRz & aRy + Sax & aTx \\ aRz & 0 & -(aRx + Say) & aTy \\ -(aRy + Sax) & aRx + Say & 0 & aTz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

$$Hbideal = \begin{bmatrix} \cos B & 0 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ \sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

$$Hberror = \begin{bmatrix} 0 & -(bRz + Sbx) & bRy & bTx \\ bRz + Sbx & 0 & -(bRx + Sbz) & bTy \\ -bRy & bRx + Sbz & 0 & bTz \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}$$

Therefore, by substituting Eqs (13) to (28) into Eq. (1), the kinematic error for the TTTRR machine type can be derived as:

$$
\begin{aligned}
Tsytem = {} & Hxideal \cdot Hyideal \cdot Hzideal \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Hxerror \cdot Hyideal \cdot Hzideal \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Hyerror \cdot Hzideal \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Hzerror \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haerror \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haideal \cdot Hberror \cdot Hbideal \cdot T
\end{aligned} \quad (29)
$$

Similarly for RTTTR machine type,

$$
\begin{aligned}
Tsystem = {} & Haideal^{-1} \cdot Haoffset^{-1} \cdot Hxideal \cdot Hyideal \cdot Hzideal \cdot Hboffset \cdot Hbideal \cdot T \\
& - Haideal^{-1} \cdot Haerror \cdot Haoffset^{-1} \cdot Hxideal \cdot Hyideal \cdot Hzideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haideal^{-1} \cdot Hxerror \cdot Hyideal \cdot Hzideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haideal^{-1} \cdot Hyerror \cdot Hzideal \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haideal^{-1} \cdot Hzerror \cdot Hboffset \cdot Hbideal \cdot T \\
& + Haideal^{-1} \cdot Hberror \cdot Hbideal \cdot T
\end{aligned} \quad (30)
$$

For RTTTR machine type,

$$
\begin{aligned}
Tsystem = {} & Haideal^{-1} \cdot Haoffset^{-1} \cdot Hbideal^{-1} \cdot Hboffset^{-1} \cdot Hxideal \cdot Hyideal \cdot Hzideal \cdot T \\
& - Haideal^{-1} \cdot Haerror \cdot Haoffset^{-1} \cdot Hbideal^{-1} \cdot Hboffset^{-1} \cdot Hxideal \cdot Hyideal \cdot Hzideal \cdot T \cdot Hbideal^{-1} \cdot Hberror \cdot Hboffset^{-1} \cdot Hxideal \cdot Hyideal \cdot Hzideal \cdot T \\
& + Haideal^{-1} \cdot Hbideal^{-1} \cdot Hxerror \cdot Hyideal \cdot Hzideal \cdot T \\
& + Haideal^{-1} \cdot Hbideal^{-1} \cdot Hyerror \cdot Hzideal \cdot T \\
& + Haideal^{-1} \cdot Hbideal^{-1} \cdot Hzerror \cdot T
\end{aligned} \quad (31)
$$

5. Generalisation from the Matrix Summation Approach

Finally, by generalising Eqs (29) to (31), the generic form of the kinematic models for the five-axis systems can be written as:

$$
Psystem = Pideal + Paerror + Pberror + Pxerror + Pyerror + Pzerror \quad (32)
$$

The advantage of rewriting the kinematic model in the form of Eq. (32) is that we can now see clearly how the errors in each individual axis will propagate in the final error positions of the tool tip related to the workpiece, and we have the ideal (error-free) tool tip position (*Pideal*) automatically for the commanded *X*, *Y*, *Z* and *A*, *B* axis movement.

Table 1 shows the equation for calculating *Pideal* for three different machine types: RRRTT, RTTTTR, and RRTTT. Also, the contribution of the error motions of the translation axes (*XYZ*) to the volumetric errors of the machine, can be regarded as the corresponding three-axis error model, transformed by the ideal angular motions of the rotary axes. For example, Table 2 shows how the error motions of translational axes (*XYZ*) contribute to the volumetric inaccuracy of five-axis machine tools.

Table 1. The ideal tool tip position for three different machine types.

Machines	*Pideal*
TTTRR (FXYZAB)	$Pideal = H_{3axis} \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T$ (33)
RTTTR (AFXYZB)	$Pideal = Haideal^{-1} \cdot Haoffset^{-1} \cdot H_{3axis} \cdot Hboffset \cdot Hbideal \cdot T$ (34)
RRTTT (ABFXYZ)	$Pideal = Haideal^{-1} \cdot Haoffset^{-1} \cdot Hbideal^{-1} \cdot Hboffset^{-1} \cdot H_{3axis} \cdot T$ (35)

$$
H_{FXYZ} = Hxideal \cdot Hyideal \cdot Hzideal \quad (36)
$$

$$
H_{FXYZ} \cdot T = \begin{bmatrix} X + Xp \\ Y + Yp \\ Z + Zp \\ 1 \end{bmatrix} \quad (37)
$$

For the contribution of the error motions of the rotary axes: *Paerror* and *Pberror*, the error equation will be dependent on whether the rotary axis appears before or after the translational axes in the kinematic chain. For these three different systems, their *Paerror* and *Pberror* can be obtained from Eqs (43) to (48) in Table 3, respectively.

6. Conclusions

In this paper, the matrix summation approach is proposed for modelling the geometric errors of five-axis machine tools. This approach breaks down the kinematic equation into six components: the ideal tool tip position under nominal axis motions, and the contribution of the error motions of each axis. It helps to reduce computation in the modelling process and makes the five-axis machine tool metrology model manageable and understandable.

The new approach can be used to deal with different machine configurations and axis definitions such as those derived from the three basic five-axis machine types: TTTRR, RTTTR, and RRTTT. It addresses the versatility issue of various machine types and the naming conventions on the shop floor, without having to build from scratch for each new machine type, and it helps to reduce the modelling effort.

Compared with the homogeneous transformation matrix approach, the matrix summation approach has several advantages:

1. It reduces the computation substantially.
2. It is easy to debug.
3. It is easy to evaluate.
4. It is easy to derive models for new machine types.

The five-axis error models have been implemented and integrated into an enhanced virtual machining research [12]. Simulation of metrology data from a five-axis machine tool and NC

Table 2. Contribution of the error motions of translational axes.

Machines	Pxyz	
TTTRR (FXYZAB)	$Pxyz = Pxerror + Pyerror + Pzerror = E_{3axis} \cdot Haoffset \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T$	(38)
RTTTR (AFXYZB)	$Pxyz = Pxerror + Pyerror + Pzerror = Haideal^{-1} \cdot E_{3axis} \cdot Hboffset \cdot Hbideal \cdot T$	(39)
RRTTT (ABFXYZ)	$Pxyz = Pxerror + Pyerror + Pzerror = Haideal^{-1} \cdot Hbideal^{-1} \cdot E_{3axis} \cdot T$	(40)

$$E_{FXYZ} = Hxerror \cdot Hyideal \cdot Hzideal + Hyerror \cdot Hzideal + Hzerror \quad (41)$$

$$E_{FXYZ} \cdot T = \begin{bmatrix} xTx + yTx + zTx - (xRz+Sxy) \cdot (Y+Yp) + (xRy+Sxz) \cdot (Z+Zp) + yRy \cdot (Z+Zp) - yRz \cdot Yp + zRy \cdot Zp - zRz \cdot Yp \\ xTy + yTy + zTy - xRx \cdot (Z+Zp) + (xRz+Sxy) \cdot Xp - (yRx+Syz) \cdot (Z+Zp) + yRz \cdot Xp - zRx \cdot Zp + zRz \cdot X \\ xTz + yTz + zTz + xRx \cdot (Y+Yp) - (xRy+Sxz) \cdot Xp + (yRx+Syz) \cdot Yp - yRy \cdot Xp + zRx \cdot Yp - zRy \cdot Xp \\ 1 \end{bmatrix} \quad (42)$$

Table 3. Contribution of the error motions of rotary axes.

Machines	Protary	
TTTRR (FXYZAB)	$Paerror = Haerror \cdot Haideal \cdot Hboffset \cdot Hbideal \cdot T$	(43)
	$Pberror = Haideal \cdot Hberror \cdot Hbideal \cdot T$	(44)
RTTTR (AFXYZB)	$Paerror = -Haideal^{-1} \cdot Haerror \cdot Haoffset^{-1} \cdot H_{3axis} \cdot \mathrm{Hboffset} \cdot \mathrm{Hbideal} \cdot \mathrm{T}$	(45)
	$Pberror = Haideal^{-1} \cdot Hberror \cdot Hbideal \cdot T$	(46)
RRTTT (ABFXYZ)	$Paerror = -Haideal^{-1} \cdot Haerror \cdot Haoffset^{-1} \cdot Hbideal^{-1} \cdot Hboffset^{-1} \cdot H_{3axis}$	(47)
	$Pberror = -Haideal^{-1} \cdot Hbideal^{-1} \cdot Hberror \cdot Hboffset^{-1} \cdot H_{3axis}$	(48)

machining of a sculptured surface are used to demonstrate how the five-axis model can be used to predict machine tool volumetric errors, and finally to predict the dimensional and form errors of sculptured-surface parts. The implementation procedures and results are reported in [12].

Five-axis kinematic modelling is at an early stage and the kinematic models developed in this paper can be regarded as the start of work to derive a generic error model for five axis machine tools; however, owing to the diverse configuration of five-axis machine tools, more work should follow to model other machines with various different configurations and axis definitions so that the matrix summation approach can be thoroughly tested and established.

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