The International Journal of Advanced Manufacturing Technology

Optimal Shape Design of an Extrusion–Forging Die Using a Polynomial Network and a Genetic Algorithm

C.-Y. Wu and Y.-C. Hsu

Department of Mechanical Engineering, Tatung University, Chung-Sang North Road, 3rd Sec., Taipei, Taiwan

The objective of this paper is to obtain an optimal die shape in consideration of the influence of metal flow deformation in the extrusion-forging processes. The finite-element method was used to analyse comprehensively the effects of different die shapes on extrusion-forging deformation. A polynomial network was used to construct the relationship model and a genetic algorithm was used to obtain the optimal shape of the extrusion-forging die. Through repeated cross-verifications of modelling and experiment, the errors between the results of the finite-element analysis and the prediction of the polynomial network are very small. The result of the deformation analysis and the optimal design approach to die shape in this study could be extended to the design of a more complicated forging die. This would be beneficial to forging industries.

Keywords: Extrusion-forging; Genetic algorithm; Optimisation; Polynomial network

1. Introduction

Closed-die forging is the main metal forming process for the mass-production of middle-size or small forging parts. In this process, there are two modes of flow deformation of the metal, upsetting and extrusion, which are controlled by the shape of the forging die cavity. In the upsetting mode, the height of a billet is compressed and the diameter is expanded subject to the die pressure. Part of a billet is increased in height when extruded when the billet is locally compressed. In general, extrusion and forging always happened simultaneously in the forging of a complicated shape.

The simplest form of the extrusion-forging process is when a cylindrical billet is compressed between two flat parallel plate dies with a circular hole in the upper die. When the upper die is moving down, the material of the billet in the die gap will be forced simultaneously into the upper die orifice and the lateral gap. In other words, in the extrusion-forging process, the material of billet, close to the outer diameter, flows radially outwards and the material near the centre of the top surface is extruded into the orifice. This backward extrusion and lateral spread forging occurs simultaneously in the cylindrical billet. Generally, there are three stages in billet deformation when extrusion-forging. First, the height of the central boss is reduced because the compressive deformation of the billet is caused by the shearing force acting on it. Secondly, the height of the central boss is increased by the extrusion process and the billet material flows outward to the orifice [1,2].

The modes of flow deformation affect not only the quality of the forging, but also the life of the die. The main factors affecting the material flow deformation are die shape, material properties, height/diameter ratio of billet, frictional condition at the billet/die interface, and the diameter of the upper die hole, etc. [3]. The extrusion–forging process is often used to manufacture high strength fasteners and complicated parts for the automobile and aerospace industries. The deformation process of extrusion–forging and initial preforging of closed die forging are similar. Understanding and design of the extrusion– forging process is important for designing a forging die.

The finite-element method is a robust and effective numerical analysis method used in metal forming applications. Although the data derived from the finite-element analysis or other plasticity analysis methods are reliable as a reference in the design process, they are not sufficient to become a quick reference tool for accurate die design. Therefore, it is important to find a reliable and effective model that can predict the result quickly and accurately. Using a neural network to assess the data from the finite-element analysis and then to construct a prediction model for design could be an excellent method, because the accuracy is fairly high and the computational times can be relatively low [4-6]. A polynomial network is a selforganising adaptive modelling tool with the ability to construct the relationships between input variables and output results [7]. Compared with a back-propagation network, a polynomial network had high prediction accuracy and deficient internal network linkages [8-10].

Correspondence and offprint requests to: Dr Y.-C. Hsu, Department of Mechanical Engineering, National Huwei Institute of Technology, 64 Wun-Hua Road, Huwei, Yunlin, Taiwan. E-mail: jasonych@sunws.nhit.edu.tw

After the relationship model between the input and output parameters of the forging process was built, it was convenient for the industry to design forging dies by applying the model for optimising the die shape. The global optimisation method is the process of searching for the optimal design of a complex problem in practical applications. A genetic algorithm (GA) is an evolutionary computation, simulating the rules of the nature, and is one of the global optimisation methods widely used in many fields of application. The GAs possess very good features for optimal design [11].

In order to obtain the optimal cavity shapes for extrusionforging dies, the finite-element method, a polynomial network, and a genetic algorithm can be combined into an optimisation design system. The finite-element method was used to analyse the effects of geometric parameters of the die shape on the extrusion-forging deformation. Using those analysis data, the relationship model between the die shape and the deformation mode can be constructed by applying a polynomial network. Finally, an improved genetic algorithm was used for finding the optimal extrusion-forging die shape. The cross-verification between finite-element analysis and prediction by polynomial network model was checked repeatedly to ensure the constructed model is accurate and reliable. An extrusion-forging experiment using dies made by us was also carried out to confirm that the developed system can work in practice.

2. Review of Related Work

Many studies about the extrusion-forging process have been published. The objects of the studies were to analyse the process considering different factors including material properties, the geometry of the billet, friction conditions and the geometry profile of the die. Much work has focused on the deformation behaviour of the process including studies on the flow mode, deformation energy, external load, and cavity filling. The tools used were numerical methods or an experimental approach [1-3,12-17].

The finite-element method is a robust and effective numerical method for metal forming applications. Maccarini et al. [2] combined the finite-element method with experiments to explore the external force and the friction problem at the die/billet interface in the extrusion-forging and backward extrusion process. In order to understand the flow behaviour of the material, Rao et al. [14] used the finite-element method and physical modelling to analyse the grid distortion and strain distribution on spike forging between a tapered top die and a flat bottom die. Maccarini et al. [15] also used the finiteelement method to study the influence of the die geometry profile on the problem of filling the die cavity during cold forging of copper. The effects of profile variations and the external load during the extrusion-forging stage of a rectangular lead billet has been studied using the finite-element method and experiments by Hu & Hashmi [16]. Giardini et al. [17] also applied the finite-element method to study the influence of die geometry and the friction condition in the extrusionforging operation based on the ductile fracture criterion and material flow. Hu et al. [18] presented a finite-element simulation of the automotive tyre nut forming process. They analysed strain, strain rate, and velocity distribution in the workpiece during the forming process. The stress distribution in the punch during pulling it out and material plastic forming law during extrusion–upsetting forming were also discussed. Wang et al. [19] have used the rigid-plastic finite-element method to simulate the cold upsetting–extruding of tube flanges. The material flow and the cause of forming defects were analysed using the finite-element method.

The die shape crucially affects the forging processes and it is important in the design stage of the forging die. Pioneering work on the theoretical consideration of an optimal die shape in metal extrusion and drawing should perhaps be attributed to Richmond and Devenpeck (see [3]). The slip-line method was used for simulation of the mathematical modelling of the metal flow to achieve the ideal die profile with minimum forming energy. Subsequently, this method was expanded to other metal forming processes such as extrusion or forging. Ulysse [20] applied the finite-element method in combination with optimisation techniques to determine the optimal bearing length of an extrusion die. Jo et al. [21] also used the finiteelement method and optimisation technique to design the optimal tool shape based on uniform microstructure in both hot extrusion and hot forging.

In design of a simple extrusion-forging die, the hole diameter, draft angle and fillet radius are the important parameters for the shape of die cavity. Although Maccarini et al. [15] and Giardini et al. [17] have analysed the influence of die shape on the extrusion-forging process, the former just considered the hole diameter and the fillet radius, whereas the latter used different friction conditions with the fillet radius as the main objective. There are only few studies on prediction models and optimal design related to the shape of the cavity of an extrusion-forging die.

3. Extrusion–Forging Model

The finite-element method used for modelling metal flow deformation has now become the most used analysis tool in the die industry. When the finite-element method is applied to solve the metal forming problem of rigid–plastic material, some equations are required, such as the force equilibrium equation, yield criterion, constitutive equation, geometrical compatibility equation, and boundary conditions.

From the variational principle, the function of the finiteelement method for a rigid–plastic material can be expressed as follows [22]:

$$\pi = \int_{v}^{-} \sigma \,\dot{\epsilon} \, \mathrm{d}V - \int_{s} F_{i} u_{i} \mathrm{d}s \tag{1}$$

where σ is effective stress, $\dot{\epsilon}$ is effective strain-rate, F_i represents surface traction, and u_i is the velocity component. The solution of the original boundary-value problem is then obtained from the solution of the dual variational problem, where the first-order variation of the functional vanishes, namely,

$$\delta \pi = \int_{v}^{-} \delta \dot{\epsilon} \, \mathrm{d}v + k \int_{v}^{+} \dot{\epsilon}_{v} \, \delta \dot{\epsilon}_{v} \, \mathrm{d}V - \int_{s_{F}} F_{i} \delta u_{i} \mathrm{d}s = 0 \qquad (2)$$

where σ is effective stress, $\dot{\epsilon}$ is effective strain-rate, $\dot{\epsilon}_v = \dot{\epsilon}_x + \dot{\epsilon}_y + \dot{\epsilon}_z$ is volume strain-rate, k is a penalty constant, F_i represents surface traction, and u_i is the velocity component. In the process of finite-element analysis, the variational functional function is converted into a system of nonlinear algebraic equations. In order to solve those equations, the direct iteration method was used first to obtain the initial guess, then the iteration was implemented using the modified Newton–Raphson method to obtain the converged value.

In this paper, the commercial finite-element program DEFORM-2D is used for the analysis of the extrusion-forging process. The configuration of the billet and die used in the simulation is illustrated in Fig. 1. From the bottom of the die to a height of 30 mm along the centreline of the upper die, the different through hole diameters (*d*), draft angles (α) and fillet radii (*r*) are given in Table 1. The bottom die is a flat plate. The billets for the extrusion-forging are made from 6061 aluminium alloy of 40 mm diameter and 30 mm height. The constant shear frictional factor is 0.3 at the billet/die interface. For simulation, the upper die is compressed downward for 25 mm, then the die gap finally becomes 5 mm. In this condition, a portion of billet materials is extruded up the orifice of centre hole while some other portions are spread laterally. This phenomenon is just the same as reported elsewhere [1–3,12].

4. Constructing the Relationship Model of The Die Shape

Polynomial networks proposed by Ivakhnenko [23] are a selforganising adaptive modelling tool with the ability to construct the relationships between input variables and output data [10]. Polynomial networks are a special class of biologically inspired networks with machine intelligence and can be used effectively as predictors for estimating the outputs of complex systems. In a polynomial network, the complex system is decomposed



Fig. 1. Configuration of billet and die (*a*) before extrusion forging, (*b*) after extrusion forging.

into smaller, simpler subsystems and grouped into several layers by using polynomial functional nodes. Inputs of the network are subdivided into groups, then transmitted into individual functional nodes. The general polynomial function in a polynomial functional node can be expressed as:

$$y_0 = A_0 + \sum_{i=1}^n A_i x_i + \sum_{i=1}^n \sum_{j=1}^n A_{ij} x_i x_j + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n A_{ijk} x_i x_j x_k + \dots$$
(3)

where x_i , x_j , x_k are the inputs, y_0 is the output, and A_0 , A_i , A_{ij} , A_{ijk} are the coefficients of the polynomial functional node.

In this paper, several specific types of polynomial functional nodes are used in a polynomial network to predict boss height, flange width, and external load, for different kinds of extrusion– forging die shapes. These polynomial functional nodes are: normaliser, unitiser, single node, double node, and triple node, as shown in Figs 2 to 4.

For the case of constructing a polynomial network, a training database with the information of inputs and outputs is first required. Then, a criterion for the synthesis of the polynomial network, called the predicted square error (*PSE*) criterion, is used to determine an optimal network structure. The principle of the *PSE* criterion is to select the network which is as accurate and simple as possible. To accomplish this, the *PSE* criterion is composed of two terms, *FSE* and K_p , where *FSE* is the average squared error of the network for fitting the training data, and K_p is the complex penalty of the network. The average squared error of the network can be expressed as

$$FSE = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$
(3)

where *N* is the number of training data, \hat{y}_i is the desired value in the training set, and y_i is the predicted value of the network. The complex penalty of the network K_p can be expressed as $K_p = CPM (2\sigma_p^2 Q/N)$ where *CPM* is the complex penalty multiplier, *Q* is the number of coefficients in the network, and σ_p^2 is a prior estimate of the model error variance.

In this study, there are 27 data sets, as shown in Table 1, used as training data for the polynomial network to construct the relational models of "die shape and boss height", "die shape and flange width" and "die shape and load", as shown in Figs 2 to 4. In Fig. 2, X_i (i = 1, 2, 3) represents the hole diameter, the draft angle, and the fillet radius, respectively, of the extrusion-forging die for the original input of the network. They can also be used as the notation for the input, determined from the previous layer of a specific layer, for subsequent calculation. N is the normalised input transformed from the original input. The values of D_1 and W_1 represent the output of nodes in the first layer calculated using different types of polynomial equation, and they also will be used as the input for the next layer. Similarly, T_1 is the nodal output of the second layer. T_2 is the final result of the network system which can be converted into the real output (U) such as the boss height is subjected to the original inputs relating to the extrusion-forging die parameters. The real outputs, such as flange width and load, can also be obtained using the same extrusion-forging die parameters and algorithm as shown in Figs 3 and 4.

Table 1. The deformation result of finite-element analysis of the extrusion-forging with different die shapes.

Number	Hole diameter (<i>d</i>) (mm)	Draft angle (α) (degree)	Fillet radius (r) (mm)	Boss height (mm)	Flange width (mm)	Load (mm) (× 10 ⁵ N)
1	8	2	2	19.653	95.984	22.193
2	8	2	4	20.174	95.770	22.361
3	8	2	6	20.758	95.540	20.791
4	8	7	2	26.353	93.708	19.659
5	8	7	4	25.528	93.624	19.495
6	8	7	6	24.735	91.962	19.978
7	8	12	2	33.566	90.450	19.133
8	8	12	4	31.873	90.576	18.470
9	8	12	6	30.045	90.526	19.107
10	10	2	2	21.788	95.854	24.952
11	10	2	4	22.066	95.022	20.258
12	10	2	6	22.475	94.580	20.144
13	10	7	2	23.504	92.792	20.598
14	10	7	4	22.308	92.208	18.709
15	10	7	6	21.810	92.024	18.692
16	10	12	2	30.796	90.946	18.148
17	10	12	4	29.409	90.754	18.639
18	10	12	6	26.924	88.656	17.448
19	12	2	2	23.613	95.258	21.032
20	12	2	4	23.592	93.774	20.093
21	12	2	6	23.998	93.408	19.642
22	12	7	2	30.511	90.262	18.347
23	12	7	4	29.338	90.258	17.820
24	12	7	6	28.504	90.162	18.719
25	12	12	2	37.551	86.712	16.544
26	12	12	4	35.864	86.636	17.477
27	12	12	6	34.761	88.182	17.175





Fig. 2. The model identified by the polynomial network for prediction of boss height.

5. Optimal Design of the Extrusion–Forging Die Shape

Genetic algorithms (GAs) [11] are based on the mechanism of natural selection and emulate the evolution process using the three basic operators of reproduction, crossover, and mutation. The fittest individuals among population surviving in the evolution process are combined with a structured and randomised information exchange mechanism to form a search algorithm. Genetic algorithms have been proved theoretically and empiri-

Fig. 3. The model identified by the polynomial network for prediction of flange width.

cally to provide robust search in many fields of applications, it will be used as the search algorithm for global optimum shape design in this study.

The value of the fitness function for each individual is an index of design improvement and the probability of survival for reproduction in a genetic algorithm. The individual with higher fitness has a higher probability of being chosen for the next generation. The most important attributes of the problem dealt with here are selected and combined carefully to construct a fitness function.

	Polynomial network
Hole $\underbrace{N_1}_{Fillet}$ $\underbrace{N_2}_{Hole}$ $\underbrace{N_3}_{Draft}$ $\underbrace{N_4}_{Draft}$ $\underbrace{N_4}_{Draft}$ $\underbrace{N_5}_{Table}$	D ₁ T ₁ T ₂ U Load
Hole N ₆	Polynomial equation
$ \begin{split} N_1 &= -6.01 + 0.601 x_1 \\ N_2 &= -2.4 + 0.601 x_1 \\ N_3 &= -6.01 + 0.601 x_1 \\ N_4 &= -1.68 + 0.24 x_1 \\ N_5 &= -1.68 + 0.24 x_1 \\ N_6 &= -6.01 + 0.601 x_1 \end{split} $	$ \begin{split} & D_1 \!\!= \! 0.0305 \!\!- \! 0.368 x_1 \!\!- \! 0.229 x_2 \!\!- \! 0.152 x_1^2 \!\!+ \! 0.121 x_2^2 \!\!+ \! 0.0231 x_1 x_2 \\ & S_1 \!\!= \! 0.197 \!\!- \! 0.752 x_1 \!\!+ \! 0.204 x_5 \\ & T_1 \!\!= \! - \! 0.429 \!\!+ \! 2.04 x_1 \!\!+ \! 0.0239 x_2 \!\!- \! 0.792 x_3 \!\!+ \! 2.5 x_1^2 \!\!+ \! 0.334 x_2^2 \!\!+ \! 0.20 \\ & 4 x_3^2 \!\!+ \! 1.58 x_1 x_2 \!\!- \! 0.958 x_1 x_3 \!\!- \! 0.391 x_2 x_3 \!\!- \! 0.112 x_1 x_2 x_3 \!\!- \! 2.66 x_1^3 \\ & T_2 \!\!= \!\!- \! 0.963 \!\!+ \! 0.564 x_1 \!\!- \! 3.93 x_2 \!\!- \! 0.032 x_3 \!\!+ \! 0.495 x_1^2 \!\!+ \! 0.06 x_2^2 \!\!+ \! 0.276 \\ & x_3^2 \!\!- \! 0.558 x_1 x_2 \!\!+ \! 0.673 x_1 x_3 \!\!- \! 0.417 x_2 x_3 \!\!- \! 0.262 x_1 x_2 x_3 \!\!- \! 0.0265 x_1 \\ & {}^3 \!\!+ \! x_2^3 \\ & U \!\!= \! 1.95 \!\!+ \! 1.8 x_1 \end{split} $

Fig. 4. The model identified by the polynomial network for prediction of the extrusion-forging load.

The forging load, the die filling capacity, the amount of flash and the quality of the forged component are important criteria for evaluating the forging process and die design. With a low extrusion-forging load, the machine capacity required is low and the die life is long because the die is subjected to a small die pressure. This means that the die filling capacity is high when the material flow and deformation occur easily in the cavity. The boss height formed the in extrusion-forging process is analogous to the die filling capacity. The material required in forging is minimised and the trimming in postprocessing is reduced when the boss height is formed slowly with a small amount of flash. The flange of the extrusionforging is analogous to the flash formed. Consequently, the criterion for optimal design of die shape in the current study is to maximise the boss height and minimise the flange width and the extrusion-forging load simultaneously. The fitness function is then defined as

$$Fit = \left(1 + \frac{B - B_{\min}}{B_{\max} - B_{\min}}\right) \left(2 - \frac{F - F_{\min}}{F_{\max} - F_{\min}}\right) \left(2 - \frac{L - L_{\min}}{L_{\max} - L_{\min}}\right)$$
(4)

where *B* represents the boss height of the extrusion-forging workpiece, B_{\min} is the minimum boss height, F_{\min} is the maximum boss height, *F* is flange width, F_{\min} is the minimum flange width, F_{\max} is the maximum flange width, *L* represents the extrusion-forging load, F_{\min} is minimum extrusion-forging load, and L_{\max} is the maximum extrusion-forging load.

In the search process of genetic algorithms, there is a tradeoff between exploitation (i.e. reproduction) and exploration (i.e. crossover and mutation). The difficulty of applying simple genetic algorithms (SGA) is in seeking a balance between exploitation and exploration which determines the convergence and diversity of the optimal search. GAMAS (Genetic Algorithms based on Migration and Artificial Selection) proposed by Potts et al [24] solves the problem of the balance between exploitation and exploration and alleviates the problem of premature convergence in the search process.

A improved GAMAS scheme [25] is adopted in this study for optimal shape design of an extrusion-forging die. In addition to setting a different crossover rate and mutation rate for each species in the original GAMAS, the improved GAMAS uses different combinations of crossover and mutation operators in each species. The niche mechanism is added to the improved GAMAS system and the individuals of elitist groups are involved in the recycling process. The Ackley function used to test the performance of the algorithm was defined as

$$F(x_1, x_2) = -20 \times \exp\left[-0.2 \sqrt{\left(0.5 \sum_{j=1}^2 x_j^2\right)}\right]$$
(5)
$$-\exp\left[0.5 \sum_{j=1}^2 \cos(2\pi \times x_j)\right] + 20 + e$$

where $-5 \le x_j \le 5$, j = 1, 2 and e = 2.71828.

A three-dimensional graphical display of the function and the local minimums found by improved GAMAS are shown in Fig. 5. Relationship models identified by polynomial networks are used in genetic algorithms to search for the optimal shape of the extrusion–forging die.

6. Verification of the Relative Model of Extrusion–Forging

In order to understand and verify the accuracy of the design system developed combining finite-element analysis with polynomial networks, experiments were carried out on a universal material testing machine, as shown in Fig. 6. The blanks used in this test are made from 6061 aluminium alloy of 40 mm outside diameter and 30 mm height. The blanks were finished by annealing at 420°C for 2 hr and cooling in the furnace. Three sets of dies for the tests are made from SKD11 die steel and have been treated through quenching and tempering. The parameters of die sets (see Fig. 7) used for experiments are:

1.
$$d = 10$$
 mm, $\alpha = 12^{\circ}$, $r = 6$ mm for D1.
2. $d = 10$ mm, $\alpha = 7^{\circ}$, $r = 4$ mm for D2.
3. $d = 12$ mm, $\alpha = 12^{\circ}$, $r = 2$ mm for D_{opt}.

The first two die sets are used to verify the accuracy of the modelling analysis. The third die set, which was the optimal shape of the extrusion–forging die obtained in this study, was used to verify the accuracy of the predicted polynomial model and the optimal result of the genetic algorithm search.

7. Result and Discussion

7.1 The Finite-Element Analysis of Extrusion-Forging

Using finite-element analysis, the deformation modes of the extrusion-forging process including boss height, flange width, and load can be obtained by using dies with different hole diameters, draft angles and fillet radii. The results of the analysis are shown in Table 1. It can be seen that the larger the hole diameter, the higher the boss height and the smaller



Fig. 5. Search results of the Ackley function found by modified GAMAS after 80 generations. (a) 3D display. (b) Local minima.



Fig. 6. Configuration of the universal material test machine for extrusion-forging experiment.



Fig. 7. Photographs of three dies used for extrusion-forging experiments.

the flange width. However, there are still a few sets with different tendencies. For example, the boss height of some extrusion-forging parts made from dies with draft angles of 7° and 12° do not increase with the hole diameter. It is also found that the flange width made from a die with draft angle 12° and hole diameter 10 mm is smaller than others with different hole diameters. For the case of the same draft and fillet radius, the load required for the extrusion-forging die was maximal for a 10 mm diameter hole and minimal for a 12 mm diameter hole. In general, the boss height is higher, the flange width is smaller, and the load is smaller for a larger draft angle, but for a die with a 10 mm diameter hole, a 7° draft angle and a 6 mm fillet radius, the boss height is smaller for a larger draft angle. In addition, for a die with the same hole diameter and draft angle, and almost the same boss height, the flange width and load is decreased with an increase in fillet radius, but there are exceptions. For dies with hole diameter and draft angle combinations of 8 mm/2° and 10

mm/2°, respectively, the boss height increases with the fillet radius. The values of flange width have different variations, but the difference between the maximum and minimum width are all less than 2.5%, when the draft angle is 12° and the hole diameters are 8 mm and 12 mm, respectively. There is a marked change in the extrusion–forging load required when the fillet radius is 4 mm. Therefore, we can see that the optimal die shapes when considering different hole diameters, draft angles, and fillet radii, must have a maximum boss height and the minimum flange width and load.

In order to understand and verify the accuracy of the simulation of the extrusion-forging process by the finite-element method, experiments using two die sets were carried out to compare the results which are shown in Figs 8 and 9 and Table 2. The curves in the load-stroke diagram and the load required in the extrusion-forging process are very similar for both finite-element simulation and experiments, as shown in Figs 8 and 9. The errors of maximum load between them were less than 4%. The tendencies of those load curves were similar to those reported in the literature, that is, the load curve could be approximately divided into three stages. First, the billet is deformed in the upper die orifice which leads to the stable increase of the initial load. Secondly, the load is increased at a more uniform rate, mainly due to the upsetting of an annular portion of the billet flowing outside a central core. Finally, the formation of the boss takes place due to the backward extrusion of the central portion which is indicated clearly by a sharp



Fig. 8. Comparison of tendency in simulation and experiment of different loads.



Fig. 9. Comparison of load between simulation and experiment for two sets of dies.



Fig. 10. Comparison of boss height in finite-element analysis and polynomial network prediction.

Table 2. Comparison between the profiles of extrusion-forging parts using experimental and modelling approaches.

Item	Die†	Experimental value (mm)	Modelling value (mm)	Error (%)	Mean error (%)
Boss height	D1 D2	21.8 22.2	22.9 23.5	4.8 5.5	5.2
Flange width	D1 D2	68.7 68.6	66.3 67.5	-3.6 -1.6	-2.6

 \dagger D1 represents the die with hole diameter 10 mm, draft angle 12°, and fillet radius 6 mm. D2 represents the die with hole diameter 10 mm, draft angle 7°, and fillet radius 4 mm.

increase in the forming load [3,6]. Table 2 shows the comparison between the experimental approach and the finite-element method for the profile of the deformed billet. It is based on the conditions that the upper die downward stroke is 20 mm and the flange thickness of the deformed billet is 10 mm. From Table 2, we can observe that the difference in the extrusion–forging deformation, using those two methods, is small and the difference is less than 6%.

7.2 Relationship Model of Extrusion–Forging

After analysing data for different die shapes for an extrusionforging die, we constructed a relational model between extrusion-forging deformation and die shape by using the polynomial network, as shown in Figs 2 to 4. From those relational models, the deformations such as boss height, flange width, and load for different hole diameters, draft angle and fillet radius, can be predicted easily and quickly. Figures 10 to 12 are the comparisons of network prediction and finiteelement analysis, respectively, when using 38 different die shapes for the extrusion-forging process. The boss height, flange width, and load of the simulation and experiment are all close, as shown in those figures. The average errors of boss height, flange width, and load are 0.58%, 0.34%, and 1.95%, respectively. These relational models should be useful for the design of extrusion-forging die shapes. Therefore, using models created by polynomial networks can predict the results of extrusion-forging deformation for different die shapes quickly. The identified models are useful for the forging die design and can also be used as a reference for forging process planning.

7.3 Optimal Design of Extrusion–Forging Die Shape

The optimal die shapes can be obtained by using a genetic algorithm based on the model constructed by the polynomial networks in this study. Table 3 displays four different dies for extrusion-forging, two good designs with high applicability and two bad designs with low applicability, obtained by applying multimodel optimisation of genetic algorithms. Using dies with maximum applicability shown in Table 3 as an example, the optimal die shape for extrusion has a 12 mm hole, a 12° draft angle and a 2 mm fillet radius. The extrusion-forging workpiece obtained from this die would simultaneously have maximum boss height, minimum flange width, and require minimum load. Because the boss height is high, the material flow and the deformation in the cavity became easier, and the die filling capacity is also higher. The flash formation will be slower and smaller for a die with smaller flange width. Consequently, the material required in forging will be minimised and the trimming in post processing will be easy. When the extrusion-forging load is low, the machine capacity required can be reduced and the die will last longer at a smaller die pressure.

The relationship between the draft and fillet to the fitness of three dies with different hole diameters is shown in Figs 13 to 15. It is clear that the fitness will increase with the draft



Fig. 11. Comparison of flange width in finite-element analysis and polynomial network prediction.



Fig. 12. Comparison of extrusion forging load in finite-element analysis and polynomial network prediction.



Fig. 13. The relationship between the fitness of GAs and fillet and draft for extrusion-forging die with 8 mm diameter hole.

angle. Although the fitness is also affected by the fillet radius when the draft angle is small, its effect is very small. The fitness has a minimum value when the draft angle and fillet radius are small.

Four sets of die shapes are selected for comparison of the results of deformation of extrusion-forging, between finiteelement modelling and network prediction as given in Table 3. The maximum errors in individual items are all less than 8%. In the first set with a 7.9219 fitness, the error of boss height obtained from this die is 0.13%, of flange width is 0.09%, and of extrusion-forging load is only 0.85%. The average error for those four sets is less than 4%.

A comparison of the results of finite-element simulation and experiment for the extrusion–forging die with maximum fitness are shown in Figs 16 to 18 and Table 4. Figure 16

Table 3. The comparisons of extrusion-forging deformation between modelling analysis and network prediction with different geometry and its corresponding GA fitness.

Set	Fitness of GA/ optimisation	Optimal extrusion-forging die				Boss	Flange	Load $(\times 10^5 \text{ N})$
		Hole diameter (mm)	Draft angle (deg.)	Fillet radius (mm)		(mm)	(mm)	(× 10 14)
1	7.9219	12.000	12.0000	2.0000	Modelling analyis value Network predicted value	37.551 37.501 (0.13%)	86.712 86.636 (0.087%)	16.554 16.685 (-0.85%)
2	7.8511	11.9664	11.9995	2.0168	Modelling analyis value Network predicted value	37.568 37.378 (0.51%)	86.870 86.713 (0.18%)	17.215 17.708 (-3.03%)
3	4.5148	8.4060	11.9347	3.4914	Modelling analyis value Network predicted value	33.035 31.738 (3.93%)	91.138 90.760 (0.41%)	18.580 18.788 (-1.12%)
4	4.5106	11.8301	6.0374	5.3720	Modelling analyis value Network predicted value	27.588 27.226 (1.31%)	91.088 89.907 (1.31%)	18.580 17.209 (-7.96%)
					Difference between modelling analysis and Network prediction	1.47%	0.50%	-3.24%

Note: The values in parentheses are the errors compared to finite-element modelling.



Fig. 14. The relationship between the fitness of GAs and fillet and draft for extrusion-forging die with 10 mm diameter hole.



Fig. 15. The relationship between the fitness of GAs and fillet and draft for extrusion-forging die with 12 mm diameter hole.



* The figures of simulation only display the right half view

Fig. 16. The comparison of extrusion-forging deformation between simulation and experiment in different strokes.



Fig. 17. The comparison of boss height and flange width between simulation and experiment in different strokes using the optimal die shape with GA fitness 7.9219.



Fig. 18. The comparison of tendency of load in experiment and simuation using optimal die shape with GA fitness 7.9219.

compares the deformation between simulation and experiment during the extrusion-forging process. Figure 17 compares the boss height and flange width formed at different positions of the stroke when the upper-die is compressed downward. From those two figures, the difference of the deformed results at different positions of the stroke is quite small. The maximum error of the boss height is less than 12% and that of the flange width is less than 6%. It can be seen that the history of change of deformation profile of a workpiece in extrusion-forging, by using those two methods, is similar to that stated in the literature [1,2]. The boss height is shorter than that of the primary billet at the initial stage, then its height remains constant, but the boss height will increase continuously at the final stage. Table 4 compares boss height and flange width at the final stroke of upper die. The errors between modelling and experiment are less than 11%. The shape of the extrusion-forging load for finite-element modelling and experiment was also very close, as shown in Fig. 18. Hence, the new method developed in this study should be reliable enough for die design.

The approach of optimal shape design of extrusion-forging die is accomplished in three steps. First, the finite-element method is used to analyse the effects of the geometric parameters of the die shape on the deformation in the extrusion-forging process. Then a polynomial network is

Table 4. The comparisons of boss height and flange width between modelling and experiment using the optimal die shape with GA fitness 7.9219.

Optimal die	Item to be compared	Modelling value (mm)	Experimental value (mm)	Error
d = 12 mm	Boss height	37.551	33.493	10.81
$\alpha = 12^{\circ}$ r = 2 mm	Flange width	86.712	84.356	2.71

applied to construct a model relating the geometric parameters and deformation of the extrusion-forging. Finally, genetic algorithms are used to obtain the optimal shape of the extrusion-forging die based on the model constructed by the polynomial networks. Repeated cross-verification of simulation, modelling and experiment showed very small errors. Consequently, the result of deformation analysis and the optimal design approach of die shape in this study can be used as a quick reference for extrusion-forging die design and should be extended to the design of a more complicated forging die. The method should be useful in forging die design and forging process planning.

8. Conclusion

In this study, the finite-element method, a polynomial network and a genetic algorithm are combined to develop a system for the design of the optimal shape of an extrusion–forging die. Information from finite-element analysis of different die shapes was used to construct a geometry-deformation model of an extrusion–forging die by applying a polynomial network. The relationship equations of the constructed model could accurately and quickly predict the effects of die shape, with different geometric parameters, on the extrusion–forging deformation. Based on the model constructed, genetic algorithms were used to search for the optimal shape for extrusion–forging which could be done easily and effectively. As a consequence, the simplicity and effectiveness of the design approach developed in this study can be beneficial to the optimal design of the shape of an extrusion–forging die.

References

- S. C. Jain, A. N. Bramley, C. H. Lee and S. Kobayashi, Theory and Experiment in Extrusion Forging, 11th MTDR, Manchester, UK, 1970.
- G. Maccarini, C. Giardini and A. Bugini, "Extrusion operations: FEM approach and experimental results", Journal of Material Processing Technology, 24, pp. 395–402, 1990.
 V. John and M. John, "An upper-bound analysis of a forging–
- V. John and M. John, "An upper-bound analysis of a forgingextrusion process", Journal of Materials Processing Technology, 55, pp. 103–110, 1995
- K. Osakada, G. B. Yang, T. Nakamura and K. Mori, "Expert system for cold forging process based on FEM simulation", Annals CIRP, 39, pp. 249–252, 1990.

- K. Osakada and G. B. Yang, "Application of neural networks to an expert system for cold forging", International Journal of Machine Tools and Manufacture, 41, pp. 577–587, 1991.
- D. J. Kim, B. M. Kim and J. C. Choi, "Determination of the initial billet geometry for a forged product using neural networks", Journal of Materials Processing Technology, 72, pp. 86–93, 1997.
- Journal of Materials Processing Technology, 72, pp. 86–93, 1997.
 R. L. Barron, A. N. Mucciardi, F. J. Cook, J. N. Craig and A. R. Barron, "Adaptive learning networks: development and application in the United State of algorithms related to GMDH", in S. J. Farlow (ed.), Self-Organizing Methods in Modeling: GMDH Type Algorithms, Marcel Dekker, New York, 1984.
- 8. G. Chryssolouris and M. Guillot, "A comparison of statistical and AIM approaches to the selection of process parameters in intelligent machining", ASME Journal of Engineering for Industry, 112, 1990.
- G. J. Montgomery and K. C. Drake, "Abductive reasoning network", Neurocomputing, 2, pp. 97–104, 1991.
- H. V. Ravindra, M. Raghunandan, Y. G. Srinivasa and R. Krishnamurthy, "Tool wear estimation by group method of data handling in turning", International Journal of Production Research, 32, pp. 1295–1312, 1994.
- D. E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison–Wesley, 1989.
- S. J. Hashmi and F. B. Klemz, "Axisymmetric extrusion forging: effects of material property and product geometry", International Journal of Machine Tool Design and Research, 26, pp. 157– 170, 1986.
- M. Brayden and J. Monaghan, "An analysis of closed-die extrusion/forging", Journal of Materials Processing Technology, 26, pp. 141–157, 1991.
- K. P. Rao, S. M. Doraivelu and K. Sivaram, "Physical modeling studies using spike forging to verify analytical prediction", Journal of Materials Processing Technology, 28, pp. 295–306, 1991.
- G. Maccarini, C. Giardini, G. Pellegrini and A. Bugini, "The influence of die geometry on cold extrusion forging operations: FEM and experimental results", Journal of Materials Processing Technology, 27, pp. 227–238, 1991.
- W. Hu and M. S. J. Hashmi, "Study of metal flow in extrusion forging of rectangular billets", Journal of Materials Processing Technology, 43, pp. 51–59, 1994.
- C. Giardini, E. Ceretti and G. Maccarini, "Formability in extrusion forging: the influence of die geometry and friction conditions", Journal of Materials Processing Technology, 54, pp. 302–308, 1995.
- X. L. Hu, Z. J. Wang and Z. R. Wang, "Numerical simulation of extrusion-upsetting forming of automotive tyre nut", Advanced Technology of Plasticity, vol. 2, Proceeding of the 6th ICTP, 19– 24 September 1999.
- Z. Wang, J. Lu and Z. R. Wang, "Numerical and experimental research of the cold upsetting–extruding of tube flanges", Journal of Materials Processing Technology, 110, pp. 28–35, 2001.
- P. Ulysse, "Optimal extrusion die design to achieve flow balance", International Journal of Machine Tools and Manufacture, 39, pp. 1047–1064, 1999.
- H. H. Jo, S. K. Lee, D. C. Ko and B. M. Kim, "A study on the optimal tool shape design in a hot forming process", Journal of Materials Processing Technology, 111, pp. 127–131, 2001.
- 22. S. Kobayashi, S. Oh and T. Altan, Metal Forming and the Finite-Element Method, Oxford University Press, 1989.
- A. G. Ivakhnenko, "Polynomial theory of complex system", IEEE Transactions on Systems, Man, and Cybernetics, 1(4), pp. 364– 378, 1971.
- 24. J. C. Potts, T. D. Giddens and S. B. Yadav, "The development evaluation of an improved genetic algorithm based on migration and artificial selection", IEEE Transactions on Systems, Man, and Cybernetics, 1(24), pp. 73–86, 1994.
- Cybernetics, 1(24), pp. 73–86, 1994.
 25. G. C. Luh and C. Y. Wu, "Nonlinear system identification using genetic algorithms", Proceedings of the Institution of Mechanical Engineers, Journal of System and Control Engineering, 213(1), pp. 105–118, 1999.