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Combined Robust Parameter and Tolerance Design Using Orthogonal Arrays

A. Jeang and C.-L. Chang
Department of Industrial Engineering, Feng Chia University, Taichung, Taiwan

Conventionally, parameter design is carried out prior to tolerance design for economic considerations. However, this non-combined (two-step) design strategy cannot guarantee an economic and quality product for some quality characteristics. The rapid development of new products, and the planning and early implementation of product development are the important keys to competitiveness. This paper presents an approach based on the techniques of orthogonal arrays, computer simulation, and statistical methods. It also adopts a cost function that represents the combined impact of assigned parameters and tolerance values. This cost function is the sum of the tolerance cost and the quality loss. Computer simulation generates a set of experimental data by following the experimental design suggested by the orthogonal array. The orthogonal array experiment enables an engineer to generate experimental data required for statistical analysis with less experimental effort. Based on the experimental data and cost function, a set of response values are found for statistical analysis, and for detecting the critical assignment of parameter and tolerance values. As a result, a combined robust parameter and tolerance design for quality improvement and cost reduction can be achieved effectively at an early stage of design and planning.

Keywords: Computer simulation; Orthogonal arrays; Parameter; Quality loss; Robust design; Tolerance; Tolerance cost

1. Introduction

In traditional product development, each step is conceived as a unit with clear inputs and outputs. Steps further downstream, such as manufacturing process development, are not supposed to start until the results of previous steps, such as component design, are well defined. Although it is true that downstream work must take upstream decisions into account, the major problem that is ignored by the traditional model is that upstream steps may produce unrealistic, impractical, or non-optimal results for downstream implementation. For these reasons, concurrent engineering (CE) aims at starting all development process steps as early as possible, or even simultaneously [1]. Its success comes from each step influencing the others as the development process moves forward. One of the most important features of CE is the inclusion of manufacturing concerns early in the product design process. Hence, an effective method is essential to ensure that a successful integration of design and manufacturing affairs can be achieved. Statistical modelling, which includes computer simulation and statistical analysis, is one of the methods that may be used to meet this goal.

In the past, statistical models were constructed using approximate analytical methods, and, at best, were estimated during the first few moments of the distribution of the designed product based on the component distributions. This process was long and tedious and often did not include all of the relevant factors; however, using computer simulation techniques, this process takes a fraction of the time. In this study, a Monte Carlo simulation is adopted [2]. Decision-making during design activities is a dynamic and evolutionary process which involves adding or deleting design criteria, changing the associated criteria values, and adjusting preference priorities for the relevant criteria. To maximise the effectiveness of the design process, it is necessary to provide a method which enables designers to have feedback and direction for design improvement. Statistical analysis is adopted as a series of tests during which changes are made to the independent variables (input variables) to observe and identify the reasons for changes in the dependent variables (response values).

To reduce the number of computer simulation runs required for statistical analysis, powerful statistical tools such as the techniques of designed experiments should be adopted [3]. There are various approaches for studying specific experimental design such as build–test–fix, one factor at a time, full factorial experiments, and orthogonal array experiments. Build–test–fix assumes that any results that fall within the specification limits are equally good. This approach is slow and inefficient because it is strongly dependent on the skill of the experimenter. Consequently, there is always a need to rework and improve the performance. The one-factor-at-a-time experiment assumes that one factor-at-a-time is thoroughly studied separately and under fixed conditions. This approach is ineffective because

Correspondence and offprint requests to: Dr A. Jeang, Chairman, Department of Industrial Engineering, Feng Chia University, Taichung, Taiwan.

pairwise comparison is needed for each factor until the factor has been evaluated. Additionally, this approach does not tell the experimenter how the effect of a factor changes when the other factors change owing to an interaction. The full factorial experiment investigates all possible combinations, maximising the possibility of finding a favourable result. The weakness of this approach is that too many experiments are used for the amount of information required to understand the factor effects. In fact, full factorial experiments are only practical for a small number of factors and levels. An orthogonal array experiment is a method of setting up experiments that require only a fraction of the experiments required for a full factorial combination [4,5]. The treatment combinations are chosen to provide sufficient information to determine the factor effects. When a design project is initiated, the knowledge base is relatively small and typically there exists a large number of input factors that the experimenter may want to examine. To identify important input variables effectively, efficient methods such as orthogonal array experiments are used.

Generally, there are two types of input variable in product or process design: those with tolerance requirements and those without tolerance requirements. Since the values of the second type of input variable do not influence product applications and manufacturing operations, only the nominal values need to be determined; although, parameter design is usually carried out prior to tolerance design for economic reasons, for the first type of input variable. For a comprehensive review of parameter design, see [4,6–11]. If the quality value measured by the output variables (quality characteristics) which results from parameter and tolerance design is in an identical unit, the parameter and tolerance design can be combined and completed in one step [12–14]. The combined design strategy increases the flexibility of the allocation of parameter and tolerance values during the design process. This ensures a further cost reduction, and quality improvement can be achieved. Previous work has formulated the problems mainly in a deterministic model. However, design activities are dynamical and evolutionary decision-making processes. These activities involve possible design changes and value adjustments which depend on appropriate suggestions from design analysis, particularly in uncertain design environments. In addition, it is possible that a great number of design inputs may be considered during product development. When a combined design strategy is required, the number of inputs doubles. It is impractical and difficult to analyse the design problem by representing all the inputs in one deterministic model. Hence, it is essential that the important inputs can be identified in the initial design stage before a detailed design analysis can be performed. Of course, one possible method for detailed design at a later stage could be a deterministic model. For these reasons, a statistical approach which involves less experimental effort is introduced in the present study to achieve our goals.

2. Experimental Design via Orthogonal Array

An experimental design matrix consists of a set of experiments which contain various levels of combinations of the input variables to be studied, from one experiment to another. After conducting a matrix experiment, the data from all of the experiments in the set taken together are analysed to determine the effects of the various combinations of levels. Generally, these initial experiments are purposely limited to relatively simple designs so that the important factors can be determined efficiently. Among the various types of matrices used for planning experiments, robust design makes heavy use of orthogonal arrays [4]. Orthogonal refers to the fact that the effect of each variable can be mathematically assessed independently, without considering the effects of the other variables. The treatment combinations (matrix experiments) are chosen to provide sufficient information to determine the variable effects using statistical analysis. Conducting matrix experiments using these special matrices (orthogonal arrays) allows the effects of several parameters to be determined efficiently. This study uses this type of matrix to analyse several input variables simultaneously to find the most robust parameter and tolerance design.

Before performing computer simulation, an appropriate experimental design matrix over the space of the input variables must be chosen. The matrix consists of a set of trials which depends on the number of input variables and the choice of the experimental design. In this study, the input variables consist of parameters U_1, U_2, U_3, …, U_n, and tolerances t_1, t_2, t_3, …., t_n. That is, the total number of input variables is $2n$. The associated low, middle, and high levels for input variables U_i and t_i should be decided before computer simulation. The combination of low, middle, and high levels for the input variables is determined by following the suggestions from the orthogonal array. Assume that the quality value, X_i, for each input forms a normal distribution $N(U_i,\sigma_i)$ in this study, where $i = 1, 2, 3, \ldots, n$. The standard deviation, σ_i, is estimated as $\sqrt{(S_i^2/9C_{Pm}{}^2 - (U_i - T_i)^2)}$ [15]. C_{Pm} is known the process capability index [16]. Most of the time, a component's parameter value U_i is equal to its T_i, and t_i is equal to its S_i. Hence, σ_i can be further simplified as $\sigma_i = t_i/3C_{Pm}$. Then, a Monte Carlo simulation is performed with a known quality function $Y = f(X_1,X_2,X_3, \ldots, X_n)$, where X_i is a normal distribution, as mentioned above. From the simulation output, the resultant parameter value, U_Y, and the resultant variance, σ_Y^2, are obtained. In a practical exercise, there may be more than one quality function. A subscript, s, is added to Y, that is Y_s, which represents a situation where multiple quality functions exist. Each quality function does not necessarily contain the identical or the same amount of input variables.

3. Response Value for Experimental Design Analysis

Measurement scores are used which are converted from the values, U_{Ys}, σ_{Ys}, found by a numerical simulation method based on an orthogonal array experiment. The measurement scores include the cost items of quality loss and tolerance cost. The optimal strategy is to determine the parameter and tolerance values simultaneously to minimise the measurement scores. By referring to Appendix A, the function in representing this score is:

$$TC = \sum_{s=1}^{q} \frac{(k_{s1} + k_{s2})}{2} [(U_{Ys} - T_{Ys})^2 + \sigma_{Ys}^2] + \sum_{i=1}^{m} C_M(t_i) \quad (1)$$

The values of U_{Ys}, σ_{Ys}, T_{Ys}, K_{s1}, and K_{s2} are the resultant parameter values, resultant variances, target values, and sth quality loss coefficients. The former two values are found from simulation outputs; s ranges from 1 to q; q is the total number of quality characteristics in a designed product. The tolerance costs, $C_M(t_i)$, are based on the tolerance levels, t_i, established in the experiments. For a comprehensive review on tolerance design, see [17–20]. The value, m, is the total number of components with q quality characteristics. As mentioned above, the component standard deviation, σ_i, is estimated as $t_i/3$. Then, based on component parameter values, U_i, and standard deviation, σ_i, the values of U_{Ys} and σ_{Ys} can be found through simulation. The standard deviation, σ_i, is estimated from tolerance t_i, and is used together with U_i to perform a simulation for finding the outputs, U_{Ys} and σ_{Ys}. These values are substituted into Eq. (1) to obtain the measurement scores TC. It is evident that there is an interaction between quality loss and tolerance cost in Eq. (1). This interaction ensures that a simultaneous parameter and tolerance design can be realised using this approach. The TC value is non-negative and continuous. The desired value of TC is zero for the smaller-the-better type of problem. Minimisation of the TC value is the goal of this problem. Minimising TC is equivalent to maximising the η as defined in the following equation:

$$\eta = -10\log_{10}TC \quad (2)$$

Then, the values found from Eq. 2 are considered as the response values in the statistical analysis.

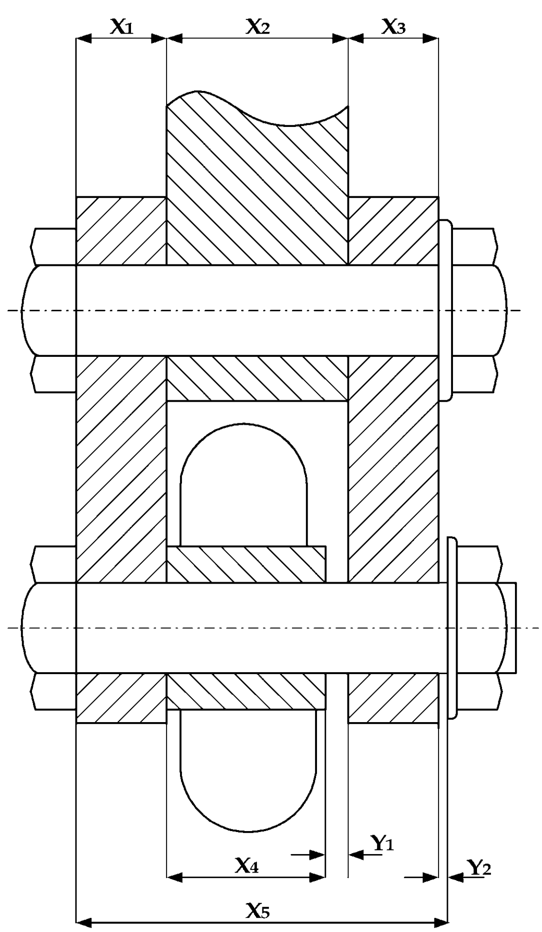

Fig. 1. The wheel mounting assembly drawing.

4. An Application

Assembly is the process by which the various parts and subassemblies are brought together to form a completed assembly or product which is designed to fulfil a certain mechanical function. Since assembly in the manufacturing process consists of putting together all the component parts and subassemblies of a given product, a proper allocation and analysis of tolerances among the assembly components is important to ensure that the functionality and quality of the design requirement are met. However, from the preceeding discussion, in addition to tolerance design, the element of component dimensions (parameter values) should also be considered in an assembly design.

Figure 1 is a wheel mounting assembly which consists of components X_1, X_2, X_3, X_4, and X_5 [21]. They are linked with two interrelated tolerance and dimension chains. The assembly functions (or quality functions) for representing these two tolerance and dimension chains are:

$$Y_1 = X_2 - X_4 \quad (3)$$
$$Y_2 = -X_1 - X_2 - X_3 + X_5$$

Because there are two assembly functions in this example, the possible values for s are 1 and 2. That is, q is 2. The associated component dimensions and tolerances, U_1, U_2, U_3, U_4, and U_5, t_1, t_2, t_3, t_4, and t_5, must be determined simultaneously so that the distances between Y_1 and T_{Y1}, and between Y_2 and T_{Y2} are minimised, where the target values T_{Y1} and T_{Y2} are 0.14 and 0.20 mm, respectively. The two quality function coefficients, k_{11}, k_{12}, k_{21}, and k_{22} are 5000, 7000, 12000, and 8000, respectively. The associated low, middle, and high levels for input factors t_i and U_i are decided as shown in Tables 1 and 2. Various levels of t_i and U_i are selected to provide appropriate combinations according to the experimental design matrix. Because the number of columns of an array represents the

Table 1. Tolerance levels t_i and tolerance costs $C_M(t_i)$ for each component.

Component i	Lower level (mm ($))	Middle level (mm ($))	Upper level (mm ($))
t_1	0.03 (30.0002)	0.06 (12.0250)	0.09 (6.1201)
t_2	0.02 (23.6103)	0.04 (13.6812)	0.06 (9.5133)
t_3	0.03 (30.0002)	0.06 (12.0250)	0.09 (6.1201)
t_4	0.04 (9.0850)	0.06 (6.3041)	0.08 (5.0924)
t_5	0.03 (11.9430)	0.05 (7.4464)	0.07 (5.5926)

Table 2. Parameter levels U_i for each component.

Component i	Lower level (mm)	Middle level (mm)	Upper level (mm)
U_1	4.9654	5.0000	5.0346
U_2	8.4740	8.5000	8.5260
U_3	3.9654	4.0000	4.0346
U_4	8.3311	8.3600	8.3889
U_5	17.6740	17.7000	17.7260

maximum number of variables that can be studied using this particular array, $L_{27}(3^{13})$ is a good choice for this experiment which is shown in Appendix B [4]. C_{Pm} is assumed to be 1, the component standard deviation, σ_i, is estimated as $t_i/3$. By following the appropriate level combinations, we carry out a Monte Carlo simulation S+ using computer software programs under the assumption that $X_i \sim N(U_i,\sigma_i)$ and using Eq. (3). Then the outputs, U_{Y1}, σ_{Y1}, U_{Y2} and σ_{Y2}, for two assembly functions are obtained in the eth experimental run, where e is 1, 2, 3, …, 27. Based on Eqs (1) and (2), the response values, η, can also be found for statistical analysis. The analysis can be completed successfully with statistical software SAS. The effect on the ten input variables is significant at a 5% confidence level. Test statistic F from Table 3 will be used to rank the order of importance of the input variables. These values indicate that input variables t_3, t_1, t_5, t_4, U_4, and U_2 should be closely controlled. When a design improvement is required, these factors must be focused on.

Another goal in carrying out the experiment is to determine the optimum level for each factor (variable). The approximated optimum level for a factor is the level that gives the highest value of factor effect average in the experimental region. The average of factor effects, $(\overline{\eta}_{factor})_{level}$, is found by calculating the average η for each control factor level (variable level). They are summarised in Table 3. The factor effect averages, $(\overline{\eta}_{t1})_3$, $(\overline{\eta}_{t3})_3$, $(\overline{\eta}_{t4})_2$, $(\overline{\eta}_{t5})_2$, $(\overline{\eta}_{U2})_2$, and $(\overline{\eta}_{U4})_1$ with values of -20.624, -20.637, -20.528, -20.482, -20.143, and -39.302 considered as optimum settings which would give the highest $\overline{\eta}_{opt}$. In other words, the optimum factor levels $(t_1)_3$, $(t_3)_3$, $(t_4)_2$, $(t_5)_2$, $(U_2)_2$, and $(U_4)_1$ of 0.09, 0.09, 0.06, 0.05, 8.50, and 8.33, respectively, would result in the least cost TC. This is because, ignoring the factors of t_2, U_1, U_3, and U_5, the corresponding sum of the squares is small. If they are included, the predicted $\overline{\eta}_{opt}$ would be biased on the higher side. An additive model can be used to predict the value of $\overline{\eta}_{opt}$ for the optimum settings. The corresponding equation is as follows:

$$\overline{\eta}_{opt} = \overline{\eta} + ((\overline{\eta}_{t1})_3 - \overline{\eta}) + ((\overline{\eta}_{t3})_3 - \overline{\eta} + ((\overline{\eta}_{t4})_2 - \overline{\eta}) + ((\overline{\eta}_{t5})_2 - \overline{\eta}) + ((\overline{\eta}_{U2})_2 - \overline{\eta}) + ((\overline{\eta}_{U4})_1 - \overline{\eta}) \quad (4)$$

where the overall mean is $\overline{\eta}$ is -20.914.
$\overline{\eta}_{opt}$ is found to be -18.456. Based on the value of $\overline{\eta}_{opt}$, TC^* is 70.086.

In order to explain the difference between the combined and the non-combined design strategy, an additional experiment representing the non-combined design strategy (traditional approach or two-step design strategy) is performed. In this experimental run, only the tolerance values with the conditions of $T_{Y1} = U_2 - U_4$ and $T_{Y2} = -U_1 - U_2 - U_3 + U_5$ are considered as factors which are usually assumed in the non-combined design strategy. In this example, $U_1 = 16$, $U_2 = 18$, $U_3 = 29$, $U_4 = 1.8$, and $U_5 = 2.3$. As for the experimental run in the combined design strategy, the associated low, middle, and high levels for input factors t_i must also be pre-defined; they are given in Table 1. The experimental combinations should also follow those suggested from the design matrix shown in Appendix C [4]. The results for statistical analysis are summarised in Table 4. The optimum settings, $(\overline{\eta}_{t1})_3$, $(\overline{\eta}_{t2})_3$, and $(\overline{\eta}_{t3})_3$ are -22.504, -22.514, and -22.494. The optimum factor levels, $(t_1)_3$, $(t_2)_3$, and $(t_3)_3$, are 0.09, 0.06, and 0.09, respectively. $\overline{\eta}_{opt}$ is -21.938 and TC^* is 156.237. Apparently, the combined design strategy has a lower TC than the non-combined design strategy because additional factors such as parameter values are included. This increases the flexibility in the assignment of parameter and tolerance values. Consequently, a further cost reduction and quality improvement can be achieved.

Table 3. Factor effect average and ANOVA analysis.

Factor	Average η by factor level $(\overline{\eta}_{factor})_{level}$			DF	SS	MS	F-ratio
	1	2	3				
U_1	−20.821	−20.704	−21.218	2	2.613	1.306	1.101
U_2	−21.465	−20.615	−20.664	2	0.852	0.426	0.359
U_3	−20.835	−20.661	−21.247	2	2.474	1.237	1.043
U_4	−20.143	−21.084	−21.376	2	2.874	1.437	1.211
U_5	−21.181	−20.793	−20.769	2	2.528	1.264	1.066
t_1	−21.346	−20.774	−20.624	2	1.306	0.653	0.551
t_2	−20.664	−21.060	−21.019	2	4.100	2.050	1.728
t_3	−21.335	−20.771	−20.637	2	1.632	0.816	0.6889
t_4	−21.326	−20.528	−20.889	2	8.135	4.068	3.429
t_5	−21.137	−20.482	−21.125	2	0.965	0.483	0.407
Error	–	–	–	6	7.118	1.186	–
Total	–	–	–	26	34.597	–	–

Table 4. Factor effect average and ANOVA analysis.

Factor	Average η by factor level $(\overline{\eta}_{factor})_{level}$			DF	SS	MS	*F*-ratio
	1	2	3				
t_1	−23.157	−22.700	−22.503	2	1.349	0.675	1018.714
t_2	−23.093	−22.754	−22.514	2	1.015	0.507	766.194
t_3	−23.152	−22.715	−22.494	2	1.344	0.672	1014.506
t_4	−22.872	−22.784	−22.705	2	0.083	0.041	62.866
t_5	−22.902	−22.778	−22.680	2	0.148	0.074	112.450
Error	–	–	–	7	0.004	0.001	–
Total	–	–	–	17	3.945	–	–

5. Conclusion

This study illustrates the efficiency of using Monte Carlo simulation, orthogonal array experiments, and statistical analysis for the quality improvement and cost reduction of a product. To reflect the combined impact of designed parameter and tolerance values, the sum of quality loss and tolerance cost is considered as a measurement score for statistical analysis. An assembly problem demonstrates the approach, and the results reveal that the critical and optimal parameter and tolerance values can be determined effectively with less experimental effort. A comparison between a combined design stragegy and a non-combined design strategy indicates that the former is superior to the latter. This results from the increased flexibility of the allocation of parameter and tolerance values during the design process. The design strategy presented can be applied to find a set of key factors from a large number of input factors to focus on before detailed design, particularly, in the earlier stage of planning and design.

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References

1. B. Prasad, Concurrent Engineering Fundamentals, Prentice Hall, Englewood Cliffs, NJ, 1996.
2. R. V. Rubinstein, "Simulation and the Monte Carlo Method", John Wiley, 1981.
3. D. C. Montgomery, Design and Analysis of Experiments, 3rd edn, John Wiley, New York, 1991.
4. M. S. Phadke, Quality Engineering Using Robust Design, Prentice Hall, Englewood Cliffs, NJ, 1989.
5. C. R. Rao, "Factorial experiments derivable from combinatorial arrangements of arrays", Journal of Royal Statistical Society, Series B, 9, pp. 128–139, 1947.
6. R. N. Kackar, "Off-line quality control, parameter design, and the Taguchi method", Journal of Quality Technology, 17, pp. 176–209, 1985.
7. R. V. Leon and C. F. J. Wu, "A theory of performance measures in parameter design", Statistica Sinica, 2, pp. 335–358, 1992.
8. V. N. Nair, "Taguchi's parameter design: a panel discussion", Technometrics, 34, pp. 127–161, 1992.
9. J. J. Pignatiello and J. S. Ramberg, "Discussion of off-line quality control, parameter design, and the Taguchi method", Journal of Quality Technology, 17, pp. 198–206, 1985.
10. J. Sacks and W. Welch, "Taguchi's parameter design: a panel discussion", Technometrics, 34(2), pp. 127–161, 1992.
11. G. Taguchi, Introduction to Quality Engineering, Asian Productivity Organization, Unipub, White Plains, New York, pp. 21–22, 1989.
12. A. Jeang and C.-Z. Hun, "Process parameter determination for precision manufacturing", Quality and Reliability Engineering International, 11, pp. 33–44, 2000.
13. A. Jeang, "Optimal process parameter determination for computer-aided manufacturing", Quality and Reliability Engineering International, 15, pp. 3–16, 1999.
14. A. Jeang, "Combined optimization of parameter and tolerance design for quality and cost", International Journal of Production Research, 2001.
15. A. Jeang, "Economic tolerance design for quality", Quality and Reliability Engineering International, 11(2), pp. 113–121, 1995.
16. S. Koltz and N. Johnson, Processes Capability Indices, Chapman & Hall, London, 1993.
17. K. W. Chase, W. H. Greenwood, B. Loosli and L. Haunglund, "Least cost tolerance allocation for mechanical assemblies with automated process selection", Manufacturing Review, 3(1), pp. 49–59, 1990.
18. F. H. Speckhart, "Calculation of tolerance based on a minimum cost approach", Journal of Engineering for Industry, 94, pp. 447–453, 1972.
19. M. F. Spotts, "Allocation of tolerances to minimize cost of assembly", Journal of Engineering for Industry, 95, pp. 762–764, 1973.
20. H. C. Zhang and M. E. Hug, "Tolerancing techniques: the state-of-the-art", International Journal of Production Research, 30(9), pp. 2111–2135, 1992.
21. O. Bjorke, Computer-Aided Tolerancing, 2nd edn, ASME Press, New York, 1989.
22. M. W. Kim and W. M. Liao, "Estimating hidden quality costs with quality loss functions", Accounting Horizons, 8(1), pp. 8–18, 1994.

Appendix A

The expected value of Eq. (A1) is:

Case A1. Assuming that the design target, T, is not far from the process mean, U and $U >> \sigma_m$. The quality loss coefficients and design tolerances are different for two directions [22].

The loss function for this case is:

$$L_A(X) = \begin{cases} K_2(X-T)^2 & (X \geq T) \\ K_1(X-T)^2 & (X < T) \end{cases} \qquad (A1)$$

Let $E(L_A(X))$ be the expected quality loss representing the above function.
Derivation:

$$E(L_A(X)) \approx \int_0^T K_1(X-T)^2 h(U,\sigma_m)\mathrm{d}X + 2\int_T^{\infty} K_2(X-T)^2 h(U,\sigma_m)\mathrm{d}X$$

$$\approx \frac{\int_0^{\infty} K_1(X-T)^2 h(U,\sigma_m)\mathrm{d}X}{2} + \frac{\int_0^{\infty} K_2(X-T)^2 h(U,\sigma_m)\mathrm{d}X}{2}$$

$$= \frac{K_1[(U-T)^2 + \sigma_m^2]}{2} + \frac{K_2[(U-T)^2 + \sigma_m^2]}{2}$$

$$= \frac{(K_1 + K_2)}{2}[(U-T)^2 + \sigma_m^2]$$

Appendix B

$L_{27}(3^{13})$ orthogonal array.

Expt.	Column												
No.	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

Appendix C

$L_{18}(2^1 \times 3^7)$ orthogonal array.

Expt.	Column							
No.	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1