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A Surface Based Approach for Constant Scallop Height Tool-Path Generation

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The machining of sculptured surfaces such as moulds and dies in 3-axis milling relies on the chordal deviation, the scallop height parameter and the planning strategy. The choice of these parameters must ensure that manufacturing surfaces respect the geometrical specifications. The current strategies for machining, consist primarily in driving the tool in parallel planes which generates a tightening of the tool paths. A constant scallop height planning strategy has been developed to avoid this tightening. In this paper, we present a new method of constant scallop height tool-path generation based on the concept of the machining surface. The concept of the machining surface is developed and its use to generate constant scallop height tool paths is described. The approach is compared with existing methods in terms of precision and in particular its aptitude to treat curvature discontinuities.

Keywords: Ball endmill; Freeform surfaces; Machining surface; Scallop height; 3-axis milling; Tool path

1. Introduction

The machining of sculptured surfaces such as moulds and dies in 3- or 5-axis end milling requires the construction of successive tool paths and their juxtaposition according to a machining strategy. The machining strategy relies on the choice of a tool driving direction and two discretisation steps, the step length along the path or chordal deviation (longitudinal step) and the cutter path interval or scallop height (transversal step). The tool path is then made of a discrete set of points representing the successive positions of the tool centre which will enable it to cover the surface. If the numerical control unit can read and interpret tool paths expressed in a polynomial, canonical, B-spline or NURBS format, the tool path consists of curves respecting the chordal deviation criterion.

The choice of the chordal deviation and the scallop height parameters must lead to the realisation of manufactured surfaces conforming to the required geometrical specifications of form deviation and surface roughness [1]. For a given part, the use of high-speed milling (HSM) makes it possible to increase the number of tool paths, and therefore to reduce the scallop height, without increasing machining time [2]. However, the current strategies for machining consist primarily in driving the tool in parallel planes, which does not optimise the rate of material removal. Along a tool path, the variations of the normal orientation on the surface in the perpendicular plane of the tool path generate a tightening of the toolpath over the whole surface. This tightening increases machining time and creates over machining in some areas (Fig. 1).

In order to increase the quality and the speed of machining, we propose to exploit the constant scallop height planning strategy. This strategy generates uniform scallops on the surface and ensures a better coverage of the surface by the tool path. The narrowing of successive paths obtained with other strategies can be removed.

Few published papers deal with the generation of constant scallop height tool paths for 3-axis milling with a ball endmill [3–5]. Furthermore, these methods are quite similar. Tool paths are planned in parametric space and the first two fundamental forms are used to evaluate the surface properties at the point considered.

In this paper, we use a new method of constant scallop height tool-path generation based on the concept of the machining surface [6,7]. The machining surface enables us to consider the tool path as a surface and not only as a set of points (linear interpolation) or a set of curves (polynomial interpolation). Every curve onto the machining surface is a potential tool path which machines the surface without collision. Given a machining surface, the tool-path generation consists in choosing a set of curves belonging to the surface. The knowledge of this surface gives us more elements, to plan more precisely, the relative position of these curves.

In this paper, the concept of the machining surface is presented, and its use to generate constant scallop height tool paths. The approach is then compared with the existing methods and in particular with its aptitude to treat curvature discontinuities on the surface.

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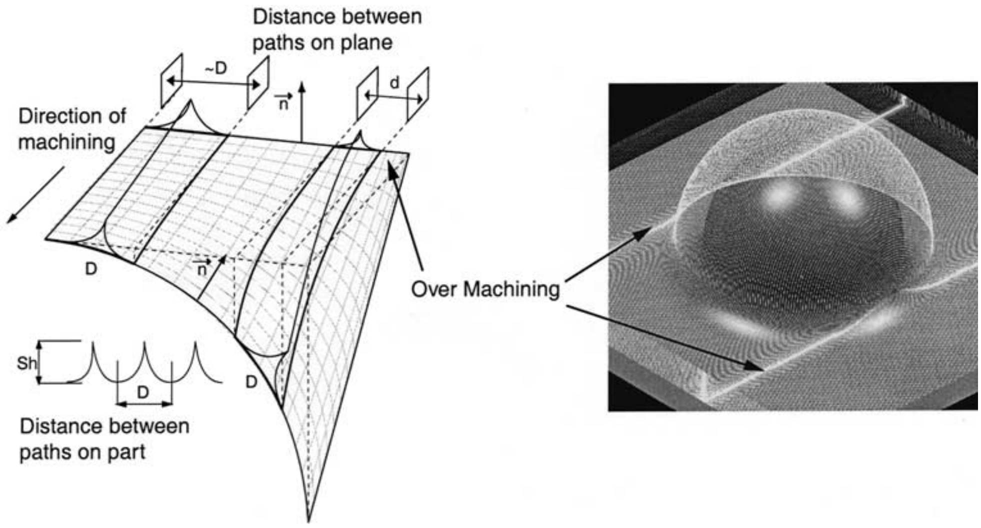

Fig. 1. Tool paths tightening.

2. The Concept of the Machining Surface

The concept of the machining surface has been developed to improve the quality of the machined surfaces by associating a surface representation to the tool paths. First of all, qualitative advantages come from the integration of the functional constraints of design in the construction of the machining surface so that the machined surface conforms with the design intent. The tool-path generation is improved by the continuous representation of the tool path, instead of the discrete representation of the conventional approaches.

A general definition of the machining surface (MS) is a surface including all the information necessary for the driving of the tool, so that the envelope surface of the tool movement sweeping the MS gives the expected free form [6].

Considering each tool geometry and for each type for machining (3- or 5-axis) in end milling or in flank milling, a specific definition of the machining surface is proposed [7]. In 3-axis end milling, the definition of the machining surface corresponds to the definition of a general offset surface. Since we use a ball-end mill in this work, the machining surface is a traditional offset surface.

Tool-path generation using offset surfaces has been the subject of numerous papers [8,9]. Among the problems encountered in tool-path generation using offset surfaces, the most constraining are the problems of loops or self-intersections and of precision [10,11] The problem of loops arises when using a tool of radius larger than the smallest concave curvature radius of the surface. In order to be free from loop problems, we will consider tools whose radius is smaller than the smallest concave radius of curvature of the surface to be machined. This appears consistent within the concept of finishing milling used to generate constant scallop height tool paths. The problem of precision comes from the model for representing offset surfaces. For most cases, it is not possible to model these surfaces by a parameterised NURBS surface without approximations. Therefore, we will adopt an implicit representation of the machining surface:

$$S_M(u,v) = S_D(u,v) + RN(u,v)$$

The advantage of using the machining surface to generate tool paths with a constant scallop height, is that the distance between the driven point (the tool centre) and the associated point on the scallop curve is constant and equal to the radius of the tool. In the methods where the point of contact between the tool and the surface (C_C point) is driven, the distances between the C_C point and the associated points of the scallops vary along the tool path.

3. The Geometry of a Constant Scallop Height Tool Path

Let us consider two adjacent tool paths C_i and C_{i+1} located on the machining surface S_M, offset surface of distance R from the design surface, and a surface of constant scallop height S_{Sh}, offset by a distance equal to the scallop height Sh. For each path, the envelope surface of the tool movement is a pipe surface the radius of which is equal to the tool radius and whose spine is the curve followed by the tool centre. The scallop curve generated by the two paths is thus the intersection of the two pipe surfaces. In the case of a constant scallop height machining, this curve belongs to the surface of constant scallop height S_{Sh}.

In practice, the previous geometrical problem can be reduced to two successive problems. The first problem consists of finding the scallop curve T_i which is generated by the first path C_i, where T_i is the intersection of the envelope surface associated to C_i, with the surface of constant scallop height S_{Sh} (Fig. 2). In the second problem, we build the adjacent tool path C_{i+1} which belongs to the machining surface S_M starting from the scallop curve, so that the scallop curve T_i is the intersection of the two pipe surfaces associated with C_i and C_{i+1} (Fig. 3).

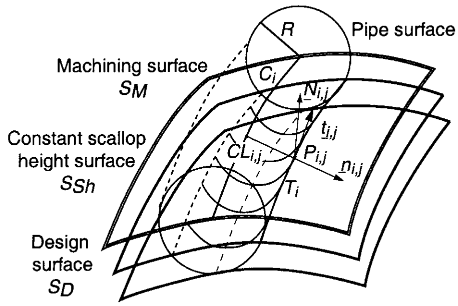

Fig. 2. The geometry of constant scallop height tool path.

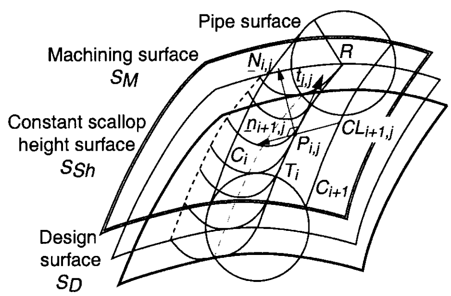

Fig. 3. The geometry of constant scallop height tool path.

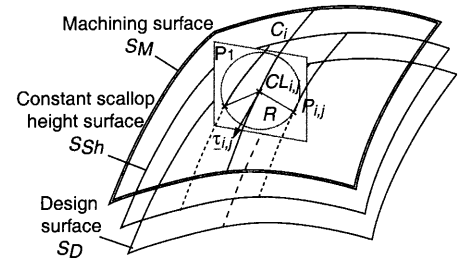

Fig. 4. Discrete construction of the tool path.

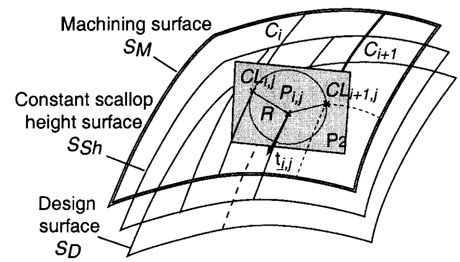

Fig. 5. Discrete construction of the tool path.

We show now that the curve C_{i+1} is the intersection of the machining surface with the pipe surface generated by the scallop curve T_i.

For each point $P_{i,j}$ on the scallop curve T_i, the tangent $\mathbf{t}_{i,j}$ to the curve T_i is given by the cross-product:

$$\mathbf{t}_{i,j} = \mathbf{n}_{i,j} \wedge \mathbf{n}_{i+1,j} \quad (1)$$

with $\mathbf{n}_{i,j}$ and $\mathbf{n}_{i+1,j}$ the unit normals to the pipe surfaces at the point considered:

$$\mathbf{n}_{i,j} = \overrightarrow{CL_{i,j}P_{i,j}}, \quad \mathbf{n}_{i+1,j} = \overrightarrow{CL_{i+1,j}P_{i,j}}, \quad (2)$$

$CL_{i,j}$ and $CL_{i+1,j}$ are the tool locations which generate the point $P_{i,j}$ onto the scallop curve.

At a scallop point $P_{i,j}$, we can associate a tool centre-point $CL_{i+1,j}$ on the path C_{i+1} with:

$$dist(CL_{i+1,j}, P_{ij}) = R, \quad \mathbf{n}_{i+1,j} \cdot \mathbf{t}_{i,j} = 0 \quad (3)$$

The searched path C_{i+1}, and the locus of the points $CL_{i+1,j}$, thus belong to a pipe surface of radius R whose spine is the scallop curve T_i. Finally, the tool path C_{i+1} is the intersection of the previous pipe surface and the machining surface S_M. The intersection of the pipe surface associated to T_i, with the machining surface S_M produces two curves of which one is C_{i+1} and the other is C_i, which is in agreement with Eqs (1) and (2). The construction of constant scallop height tool paths can thus be carried out by successive intersections between pipe surfaces and the machining and constant scallop height surfaces.

The methods developed by Suresh and Yang [3], Sarma and Dutta [4], and Lin and Koren [5] have in common the planning of tool paths in parametric space while using fundamental forms to define differential characteristics of the surface at the considered point. The initial path is sampled to calculate the points of the following path. This path is then built by interpolation of the calculated points. To be able to compare the performance of our approach with existing methods and so as not to leave ambiguities at the surface intersections, we suggest generating successive tool paths point by point in the parametric space. The initial path is sampled and at every point we compute the associated scallop point as well as the corresponding point of the next tool path.

The first part of the problem is the search for the point $P_{i,j}$ element of the scallop curve when the tool is located on the point $CL_{i,j}$ on the initial path. The point $P_{i,j}$ is given by (Fig. 4):

$$P_{ij} = \{S_{Sh}\} \cap \{Plane\ P_1\} \cap \{Sphere\ S_1\}$$

where sphere S_1 is the active part of the tool and plane P_1 the plane normal to C_i passing through $CL_{i,j}$.

The second part of the problem consists of determining the point CL_{i+1} of the following path from the scallop point $P_{i,j}$. The point $CL_{i+1,j}$ is given by (Fig. 5):

$$CL_{i+1} = \{S_{Sh}\} \cap \{Plane\ P_2\} \cap \{Sphere\ S_2\}$$

where sphere S_2 is the active part of the tool and plane P_2 the plane normal to T_i passing through $P_{i,j}$.

Contrary to [3] and [5] where the assumption is made that the problem is on a plane, i.e. that the point $CL_{i+1,j}$ is located in the plane perpendicular to C_i passing through $CL_{i,j}$, the construction is now carried out in two different planes. The point $P_{i,j}$ is in the P_1 plane and the point $CL_{i+1,j}$ in the P_2 plane. The problem is indeed on a plane because the three points $P_{i,j}$, $CL_{i,j}$, and $CL_{i+1,j}$ are in the P_2 plane of normal $\mathbf{t}_{i,j}$ (1)(2) but this plane is not known at the beginning of the construction.

4. Algorithms

In existing methods, it is necessary to associate a curve with all of the calculated points C_L to generate the following path. This is necessary to calculate the tangent vector to the tool path or the scallop curve in order to define the study plane. This is not the case when using the method of the machining surface. The tangent vector is given by the cross-product of the normals of both surfaces considered for intersection calculation (Eq. (1)). In order to compare the methods, we thus propose to consider two versions of the method of the machining surface. We use the association of curves to compare the performances of the various methods (algorithm 1), but not to study the impact of curve association on the behaviour of calculated paths (algorithm 2).

Algorithm 1. Computation of the tool positions $CL_{i,j}$ on successive path C_i with association of curve.

Initial conditions:
Design surface S_D: $S(u,v)$
Constant scallop height surface S_{Sh}: $S(u,v) + Sh.N(u,v)$
Machining surface S_M: $S(u,v) + R.N(u,v)$
For $i = 1, n$
  For $j = 1, m$
    Compute cutting plane P_{1j}, perpendicular to C_i at $CL_{i,j}$
    Compute the intersection point $P_{i,j}$ of S_{Sh}, P_{1j} and Sphere S_1
  End
  Associate the scallop curve T_i to $\{P_{i,j}\}$
  For $j = 1, m$
    Compute cutting plane P_{2j}, normal to T_i at $P_{i,j}$
    Compute the intersection point $CL_{i+1,j}$ of S_M, P_{2j} and Sphere S_2
  End
  Associate the tool path C_{i+1} to $\{CL_{i+1,j}\}$
End

Algorithm 2. Computation of the tool positions $CL_{i,j}$ on successive path C_i without association of curve.

Initial conditions:
Design surface S_D: $S(u,v)$
Constant scallop height surface S_{Sh}: $S(u,v) + Sh.N(u,v)$
Machining surface S_M: $S(u,v) + R.N(u,v)$
For $i = 1, n$
  For $j = 1, m$
    Compute the tangent to the intersection curve between the pipe surface (C_i) and S_{Sh}
    Compute the intersection point P_{ij} of S_{Sh}, P_j and sphere S_1
  End
  For $j = 1, m$
    Compute the tangent to the intersection curve between the pipe surface (T_i) and S_M
    Compute the intersection point $CL_{i+1,j}$ of S_M, P_{2j} and sphere S_2
  End
End

The selected method for curve fitting is the interpolation by cubic B-spline curves. We use a parameter setting proportional to the chord length [12]. Whatever the method, the tool paths are calculated in parametric space. In the case of algorithm 1, it is necessary to calculate the tangent to the current curve (scallop curve or tool path) so that the cutting plane (P_1 or P_2) is defined. The tangent vector $\mathbf{t}$ to a curve $C(t)$ or $C(u(t),v(t))$ lying on the surface $\Phi(u,v) = S(u,v) + d\ \mathbf{N}(u,v)$ where d takes the value R or Sh is given by:

$$\mathbf{t} = u'(t) \cdot \Phi_u + v'(t) \cdot \Phi_v \tag{4}$$

where $u'(t)$ and $v'(t)$ are the parametric tangent vectors and Φ_u and Φ_v the partial derivatives of the surface given by:

$$\Phi_\alpha = S_\alpha + d\,\frac{\partial \mathbf{N}}{\partial \alpha} \tag{5}$$

$\partial\mathbf{N}/\partial\alpha$ is the curvature operator defined by:

$$\frac{\partial \mathbf{N}}{\partial \alpha} = -b_\alpha^\beta S_\beta \tag{6}$$

Finally,

$$\mathbf{t} = u'(t) \cdot (S_u - d \cdot (b_1^1 \cdot S_u + b_1^2 \cdot S_v)) + v'(t) \cdot (S_v - d \cdot (b_2^1 \cdot S_u + b_2^2 \cdot S_v)) \tag{7}$$

b is the tensor of curvature or the second fundamental form.

5. Agorithm Behaviour

At first, the methods are applied on a NURBS surface delimited by two lines and two circular arcs (Fig. 6). It thus has concave and convex areas and does not include any discontinuity of tangency or of curvature. The tool radius R is 10 mm and the specified scallop height is 0.001 mm.

The initial tool path is an isoparametric curve of the surface. Then, the initial tool path is sampled at points, and points of the adjacent path are built one by one to define a first path and so on until the last tool path. Throughout this process we observe the propagation of the initial points, by its traceability. We can thus visualise the tool centre location or the surface contact point calculated before curve associations. Indeed, the interpolation hides the behaviour of each algorithm.

The first test consists of comparing the proposed method with the methods suggested in [3] and [4]. We pointed out that these methods use the association of curves. We thus use the method of the machining surface with curve association

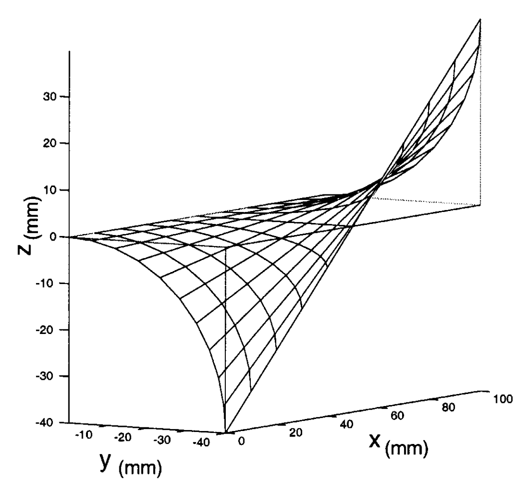

Fig. 6. Test surface 1.

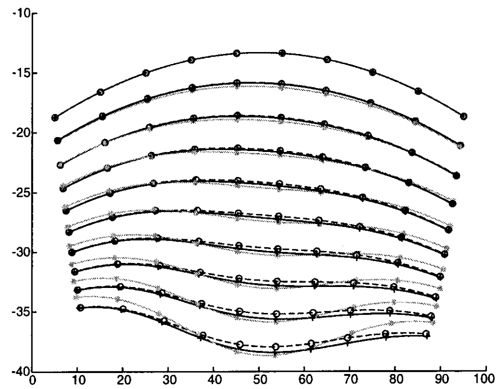

Fig. 7. Traceability of tool paths. *, Suresh & Yang [3]; ×, Sarma and Dutta [4]; ○, MS algorithm 1.

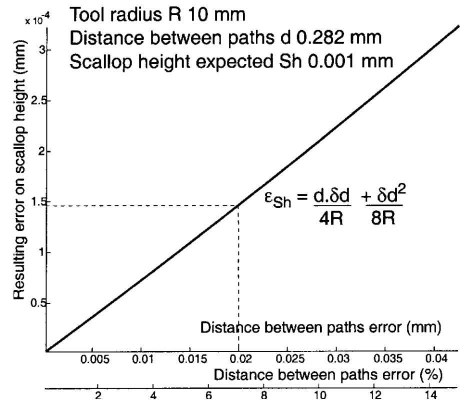

Fig. 9. Scallop height error.

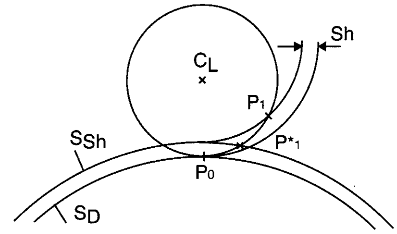

Fig. 10. Curvature approximation.

(algorithm 1). The selected initial tool path is the isoparametric curve $v_0 = 1/3$, and the machining is in the -*y*-axis direction (Fig. 7). The tool paths are represented by curves with the points used for computation. The progression is from top to bottom and, for clarity, only one tool path in ten is represented. We notice first of all that the three curves diverge progressively when machining. Curves generated by Suresh & Yang [3] present the greatest variation in comparison to the others. In their approach, the problem is regarded as on a plane, and we pass from one path to the other without passing by the scallop curve (this makes it the fastest method). The distance between successive tool paths is calculated approximately. The two other methods are relatively close despite a slight divergence.

The second test consists of studying the influence of curve association on the calculated tool paths (Fig. 8). The two versions of our method are compared. It can be seen that the interpolation largely influences the results. Although the tool paths are very similar, the positions of points calculated by both methods varies. This is explained by the fact that the interpolation modifies the direction of the tangent vector to the curves at the calculated points, but does not modify the distance between two adjacent tool positions. Adjacent points are not evaluated in the correct direction. We could have introduced tangency constraints in the interpolation scheme but it would not have been representative of the possibilities offered by the other methods.

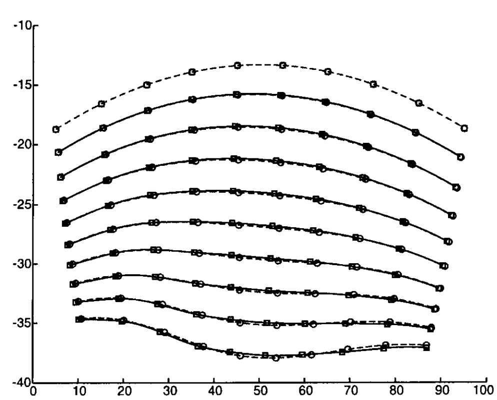

Fig. 8. Traceability of tool paths. ○, MS algorithm 1; □, MS algorithm 2.

Differences between tool paths are found on the resulting scallop height. Let us take the example of the difference between the tool paths generated by the method in [3] and those generated with the method of the machining surface with curve association (Fig. 7). At the end of 100 tool paths, the distance between the methods is approximately 2 mm. If we consider that the drift is constant progressively with the construction of the tool paths, it represents approximately 20 μm (7%) of error between two consecutive tool paths, that is to say an error on the scallop height of 0.15 μm (15%) (Fig. 9).

6. Behaviour on Curvature Discontinuities

In this section, we study the treatment of curvature discontinuities. Existing methods make the assumption that the surface

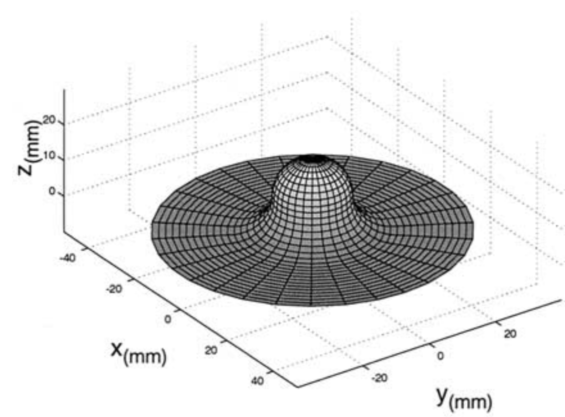

Fig. 11. Test surface 2.

curvature is constant around the considered point. This is not a problem during the machining of a B-spline surface made up of only one patch because this type of surface guarantees curvature continuity. However, the majority of industrial parts are modelled with a multitude of patches connected in tangency and eventually with NURBS surfaces presenting curvature discontinuities. For example, this is the case when we introduce blending radii. Let us consider the machining of a cylindrical surface along its generatrices whose profile (Fig. 10) presents a curvature discontinuity at the point P_0. The considered point is in the convex part of the surface before P_0 and with the hypothesis of constant curvature, the adjacent tool location calculated is in P_1^* and not in P_1 as it should be. The resulting scallop height is thus not the expected one.

We thus study the behaviour of our algorithm and those developed in [3] and [4]. We consider the machining of a

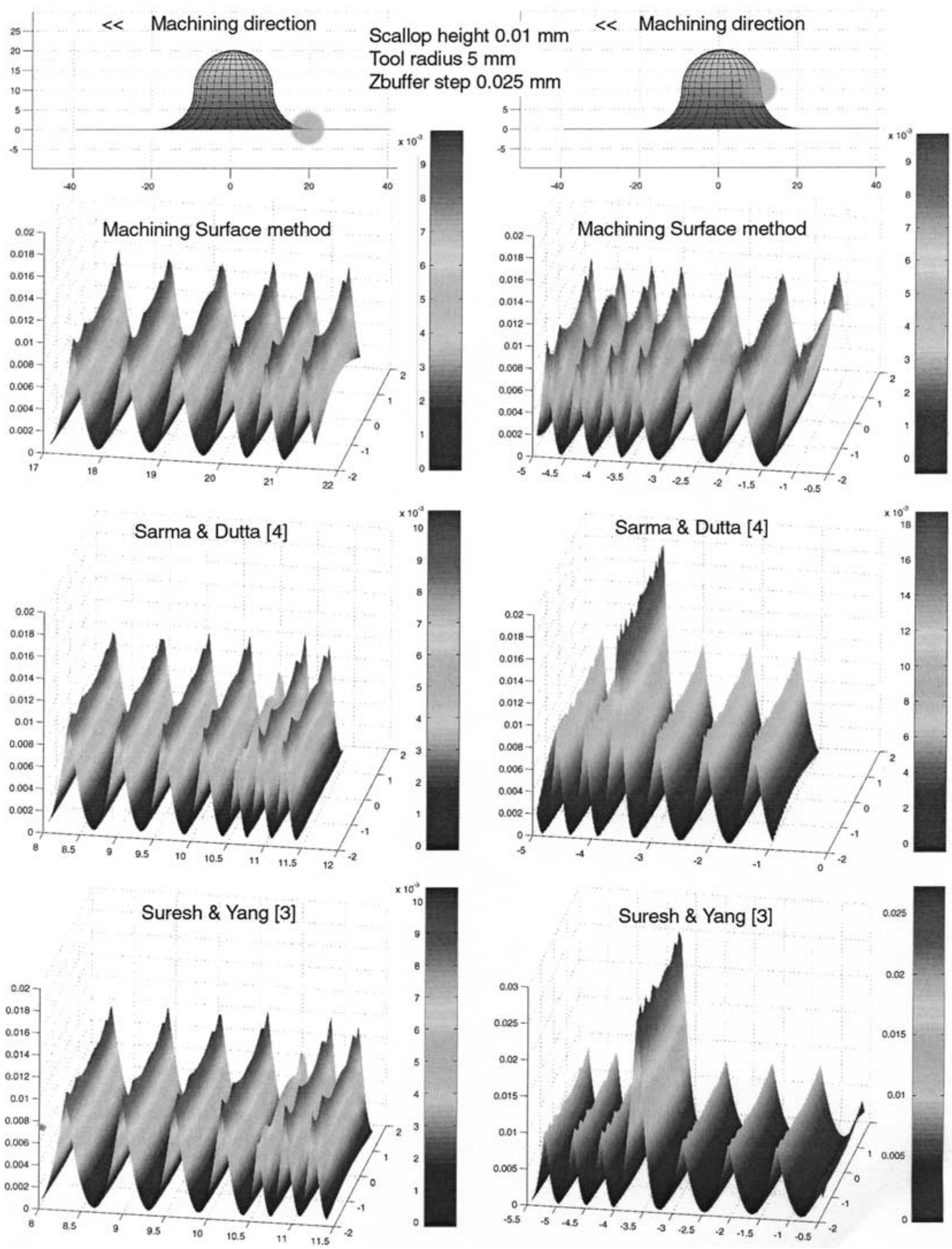

Fig. 12. Performances on curvature discontinuities.

sphere lying on a plane with a connection in tangency (Fig. 11). The surface consists of three surfaces: a half sphere (radius 10 mm), a portion of torus (radii 10 and 20 mm), and a plane. This presents two curvature discontinuities along the profile. The first is located at the linking between the plane and the torus, and the second is between the torus and the sphere.

The adopted machining strategy is a machining according to the circular isoparametric from the outside of the surface towards the top of the sphere. Discontinuities are then well situated between two adjacent tool paths. We observe the scallop left by the tool at the two curvature discontinuities, with the three methods of tool-path calculation. It is also pointed out that for a given scallop height and a given tool radius, the tool paths are more spaced (resp. less) when the curvature is concave (resp. convex).

To compare the methods, the scallops left by the tool are built using the method of the *Z*-buffer. We build, in the zone of interest, a network of lines parallel to the *z*-axis and laid out on a grid whose step size indicates the precision. The step size of the square grid is set to 0.025 mm. Then, we carry out the intersections between this network of lines and the envelope surfaces of the tool movement.

Results (Fig. 12) show that methods that rely on constant curvature generate an abnormal scallop on the discontinuity, which is not the case for the method of the machining surface. At the junction between the torus and the sphere, results show a higher scallop height than the others at the discontinuity. The distance between paths is calculated as if the curvature were concave (torus) whereas it is convex (sphere). With a constant distance between paths, the scallop height is larger on the sphere than on the torus. Between the planar zone and the torus, the scallop errors are smaller. The algorithms calculate a distance between paths as if the curvature were null (plane) whereas it is concave (torus). With a constant distance between paths, the scallop height is lower on the torus than on the plane.

The experimental results confirm our assumptions on the influence of the curvature approximations. Approaching a surface by its osculating sphere during the calculation of constant scallop height tool path does not allow the correct treatment of curvature discontinuities. Thus, moulds and dies containing many blending radii cannot be machined with such algorithms; this applies to plastic injected moulds in particular. The method of machining the surface successfully treats curvature discontinuities by leaving a scallop in accordance with the specifications.

7. Conclusion

The concept of the machining surface enables the adoption of a new method to generate constant scallop height tool paths which show benefits. First of all, there is no need to associate an interpolating curve to build consecutive tool paths. Without interpolation, the proposed method is more accurate than the other methods. It does not accumulate error from the beginning of the tool-path calculation. Moreover, the method is characterised by its aptitude to treat curvature discontinuities. However, it should be noticed that these improvements increase computation time.

The concept of the machining surface also offers a framework for generating constant scallop height tool paths with flat-end or filleted end mills in 3-axis milling. However, whatever the method or the tool employed, it becomes necessary to tackle the problem of tool-path planning. Indeed, according to the initial tool path and the topology of the design surface, we have to extrapolate the tool paths close to the borders of the surface. Furthermore, the constant scallop height tool paths might contain loops. These loops could be eliminated, but some tangent discontinuities would appear on the tool path.

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Notation

S_M	machining surface
S_D	design surface
S_{Sh}	constant scallop height surface
R	tool radius (mm)
Sh	scallop height (mm)
C_i	tool path
$CL_{i,j}$	point on the tool path
T_i	scallop curve
$P_{i,j}$	point onto the scallop curve
$\mathbf{N}_{i,j}$	surface normal
$\mathbf{n}_{i,j}$	pipe surface normal
$\mathbf{t}_{i,j}$	tangent vector to T_i
$\tau_{i,j}$	tangent vector to C_i