Int J Adv Manuf Technol (2001) 17:257–262

The International Journal of
Advanced Manufacturing Technology

Theoretical Tolerance Stackup Analysis Based on Tolerance Zone Analysis

E. E. Lin and H.-C. Zhang

Department of Industrial Engineering, Texas Tech University, Lubbock, Texas, USA

In this paper, both dimensional tolerance stackup and geometrical tolerance stackup in one-, two-, and three-dimensions are theoretically analysed. The tolerance analysis in this study is based on the analysis of tolerance zones. The manufacturing errors are classified into two general types, locating errors and machining errors. Generative formulation of tolerance stackup is explored. A simulation example of 3D geometrical tolerance stackup is illustrated.

Keywords: Dimensional; Formulation; Geometrical; Tolerance stackup; Tolerance zone

1. Introduction

1.1 The Motivation for this Study

The purpose of this work is as follows:

1. Tolerance stackup analysis is used to deal with dimensional tolerances in one-dimension, the resultant tolerance is always the sum of the component tolerances [1]. Analysis and control of dimensional tolerances are relatively well developed compared to those for geometric tolerances [2]. The stackup of geometrical tolerances was usually ignored or replaced by the stackup of component tolerances. In this paper, both dimensional tolerances and geometrical tolerances will be considered in one, two, and three-dimensions.

2. Mathematical presentation is a feature of dimensioning and tolerancing [3]. HB Voelcker predicted that one of the most important advances in geometrical tolerancing would be made in the next decade: "One or more generative formulations of geometrical tolerancing will be produced. A generative formulation will be more general than current practice but should contain the current GD&T facilities as special cases. A generative formulation should be teachable in the engineering colleges because it will be based on a small set of underlying mathematical principles" [4]. This paper is a contribution to the generative formulation of geometrical tolerancing.

1.2 Tolerance Stackup Versus Error Stackup

Tolerance is the total amount that a specific feature is permitted to vary, it is the difference between the maximum and minimum limits [5]. *Error* (variation) is the deviation of a feature (geometrical element, surface, or line) from its nominal size or shape [6]. Hence, tolerance stackup deals with the variation limits in machining, whereas error stackup deals with virtual variation. In this paper, tolerance stackup analysis is based on error stackup analysis. The mathematical formulae for tolerance stackup and those for error stackup coincide by substituting error variables with tolerance variables.

1.3 Principle of Tolerance Independency

It is complicated to consider dimensional tolerance and geometrical tolerance simultaneously, in error and tolerance analysis. The International Standard Committee ISO/TC10/SC5 "Technical drawings, dimensioning and tolerancing" and ISO/TC3 "Limits and fits", in ISO 8015 stated that the principle of independency is the fundamental tolerancing principle. It states that:

> "Each requirement for dimensional or geometrical tolerancing specified on a drawing shall be met independently, unless a particular relationship is specified, i.e. maximum material requirement, least material requirement, or envelope requirement."

This study conforms to the principle of tolerance independency.

1.4 Tolerance Zone

Chase et al. considered geometric feature variations in the tolerance analysis of mechanical assemblies [7]. The tolerance zone can be regarded as limits of feature variation. The tolerance analysis in this study is based on the analysis of tolerance zones. Henzold [6] discussed all kinds of tolerance zones. Those tolerance zones can be summarised as typical types, as

Correspondence and offprint requests to: E. E. Lin, 2407 13th Street, Apartment No 1, Lubbock, TX 79401, USA. E-mail: elin@ttacs.ttu.edu

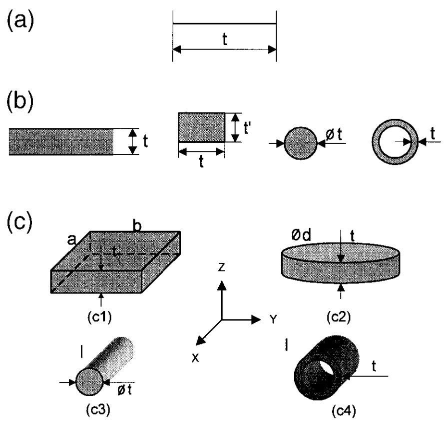

Fig. 1. Typical tolerance zones. (*a*) 1D, (*b*) 2D, and (*c*) 3D tolerance zones.

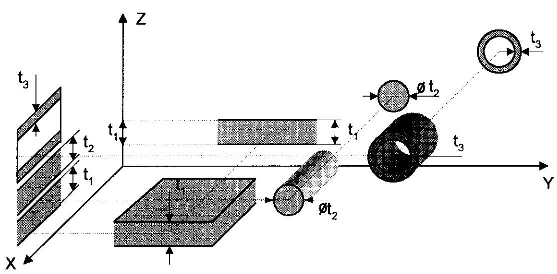

Fig. 2. The projecting relation of tolerance zones.

shown in Fig. 1. The size of the tolerance zone is usually 10^{-3} to 10^{-5} of the feature size. In the following figures, the tolerance zone is exaggerated for illustration. *t* represents tolerance value. There are three typical tolerance zones:

1. 1D tolerance zones.
2. 2D tolerance zones.
3. 3D tolerance zones.

Dimensional tolerance zones belong to type 1. Types 2 and 3 refer to geometrical tolerance zones. In the Cartesian coordinate system, 3D tolerance zones can be projected onto 2D tolerance zones, and 2D zones onto 1D zones, as shown in Fig. 2. Most tolerance zones are 3D; however, tolerance chain and tolerance analysis are usually carried out in two-dimensions or in one-dimension.

1.5 Manufacturing Errors Classification

K. Whybrew and G. A. Britton have summarised 27 sources of errors in a machining process for the following 8 items in machining [4]:

1. Machine.
2. Cutting tool.
3. Fixture.
4. Workpiece.
5. Coolant.
6. Operator.
7. Environmental conditions.
8. Process variable.

Each aspect of the above sources deserves specific study in precision manufacturing. The errors can be classified into two groups: those that are *random*, unpredictable, and cannot be controlled, and those that are *constant*, time dependent or capable of being controlled. Constant errors are added algebraically, while random errors are added arithmetically. A resultant error can be calculated by the following formula:

$$\Delta = \sum_{i=1}^{m} \alpha_i \Phi_i + \sqrt{\left(\sum_{j=1}^{n} (\beta_j \Theta_j)^2 \right)} \qquad (1)$$

where
Δ: resultant error
α_i ($i = 1,2, 3,\ldots,m$): weights of constant error components, with signs
Φ_i (i = 1,2,3,. . .,m): constant error components
β_i (i = 1,2,3,. . .,n): weights of random error components
Φ_i ($i = 1,2,3,\ldots,n$): random error components

The value of β_i depends on the distribution status of the random error component and its geometrical relationship with the resultant error. Much work is required to establish the weights and error components in Eq. (1). However, the exploration of specific sources of the locating error and machining error is unnecessary in this study.

In this study, all types of error source are classified according to their influence on the geometrical positions of the locating features and machining features of the on-line part. Hence, there are two types of error that are directly related to the accuracy of a part:

1. *Locating error*. The variation between the position of a practical datum feature and the position of an ideal datum. After a workpiece has been located and clamped, the set-up error remains constant unless the workpiece is removed from the fixture. Therefore, a locating error is a deterministic error within each set-up.
2. *Machining error*. The variation between the position of a practical machining feature and the position of an ideal machining feature. A machining error is a random error.

Both locating error and machining error are the result of a number of constant and random errors.

2. Dimensional Tolerance Stackup

As shown in Fig. 1, the tolerance zone of a dimension is strictly 1D, hence the formulation of dimensional tolerance stackup is relatively straightforward. Suppose that in a space, the relation of a resultant dimension *d* with its component dimensions is as follows:

$$d = f(x_1 x_2,\ldots,x_l, y_1 y_2,\ldots,y_m, z_1, z_2\ldots,z_n) \qquad (2)$$

where,
d: resultant dimension
x_i, ($i = 1,2,3,\ldots,l$) component dimensions in the X-coordinate
y_j, ($j = 1,2,3,\ldots,m$): component dimensions in the Y-coordinate

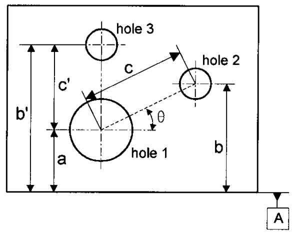

Fig. 3. Dimensional relation of 3 holes in a plane.

z_k ($k = 1,2,3,...,n$): component dimensions in the Z-coordinate

Theoretically, in the worst case:

$$\Delta d = \sum_{i=1}^{l} \left| \frac{\partial f}{\partial x_i} \Delta x_i \right| + \sum_{j=1}^{m} \left| \frac{\partial f}{\partial y_j} \Delta y_j \right| + \sum_{k=1}^{n} \left| \frac{\partial f}{\partial z_k} \Delta z_k \right| \tag{3}$$

where,
Δd: variation of resultant dimension
$\Delta x_i, \Delta y_j, \Delta x_k$: variations of component dimensions

In the statistical case:

$$\Delta d = \left[\sum_{i=1}^{l} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2 + \sum_{j=1}^{m} \left(\frac{\partial f}{\partial y_j} \Delta y_j \right)^2 + \sum_{k=1}^{n} \left(\frac{\partial f}{\partial z_k} \Delta z_k \right)^2 \right]^{\frac{1}{2}} \tag{4}$$

In the following text, only the worst case is dealt with. The statistical case and worst case can be used to deduce similar conclusions in qualitative analysis.

For example, 3 holes are to be drilled in a plane with their dimensional relation shown in Fig. 3. The horizontal dimensions are omitted to simplify the analysis.

The machining procedure and machining requirements are:

Step 1. Use face A as the machining datum and drill hole 1. The vertical dimension from hole 1 to face A is a.

Step 2. Use face A and hole 1 as the machining datum and drill hole 2. The vertical dimension from hole 2 to face A is b, the angle from the horizontal line to the connecting line of hole 1 and hole 2 is θ.

Step 3. Use face A as the machining datum and drill hole 3. The vertical dimension from face A to hole 3 is b'.

Dimensions c and c' are the resultant dimensions.

For c', there is a dimension chain as shown in Fig. 4.

$$c' = b' - a \tag{5}$$

In the worst case,

$$\Delta c' = \left| \frac{\partial c'}{\partial a} \right| \Delta a + \left| \frac{\partial c'}{\partial b'} \right| \Delta b' \tag{6}$$
$$= \Delta a + \Delta b'$$

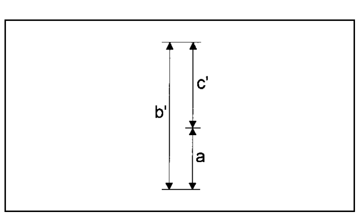

Fig. 4. Dimension chain of c'.

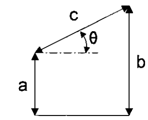

Fig. 5. Dimension chain of c.

The dimension chain of c' is 1D, which is what "dimension chain" used to mean. In a 1D case, the variation stackup is independent of the values of the component dimensions.

For c, there is a dimension chain as shown in Fig. 5.

$$c = \frac{b - a}{\sin\theta} \tag{7}$$

In the worst case,

$$\Delta c = \left| \frac{\partial c}{\partial a} \right| \Delta a + \left| \frac{\partial c}{\partial b} \right| \Delta b + \left| \frac{\partial c}{\partial \theta} \right| \Delta\theta \tag{8}$$
$$= \frac{1}{\sin\theta} \Delta a + \frac{1}{\sin\theta} \Delta b + \frac{(b - a)\cos\theta}{\sin^2 \theta} \Delta\theta$$

The dimension chain of c is 2D. From Eq. (6), 2D error stackup is dependent not only on component errors but also on the basic values of the component dimensions.

A tolerance chart is usually used for dimensional tolerance stackup analysis. For *rotational parts* a single chart per work-piece is sufficient to control tolerances along the axis of the workpiece. There is no possibility of stackups occurring in the radial direction. For *prismatic parts* it is necessary to control tolerance stackups in at least two dimensions and three charts are necessary for each workpiece. These charts will, in general, not be independent, as some surfaces, and hence tolerance, will appear on more than one chart. The charts must be linked together through common surfaces [8].

3. Geometrical Tolerance Stackup

3.1 One-Dimensional Geometrical Tolerance Stackup Analysis

One-dimensional geometrical tolerance stackup applies to the situation in which component tolerance types are the same and the basic dimensions do not affect tolerance stackup. As an example, a part with 5 identical parallel slots is shown in Fig. 6. Faces A, B, C, D, and E are set-up datums for machining

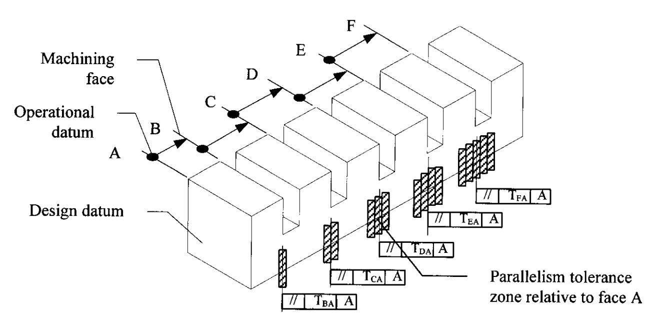

Fig. 6. One-dimensional geometrical tolerance stackup analysis.

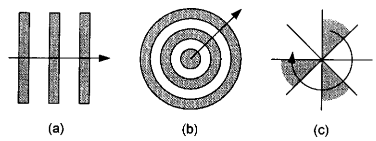

Fig. 7. Tolerance zone distributions for 1D tolerance stackup.

faces *B*, *C*, *D*, *E*, and *F*, respectively, as shown in Fig. 6. The following notations are used for error stack-up analysis:

Γ_M $M = A, B, C, \ldots$ parallelism of locating face relative to ideal vertical face, also called locating error,

λ_M $M = A, B, C, \ldots$ parallelism of machining face relative to ideal vertical face, also called machining error,

T_{MN} $M, N = A, B, C, \ldots$ the parallelism between faces *M* and *N*.

In Fig. 6, the parallelism tolerance stackup can be specified as follows:

$$T_{AB} = \Gamma_A + \lambda_B \tag{9}$$

$$T_{AC} = T_{AB} + \Gamma_B + \lambda_C$$

$$= \Gamma_A + \Gamma_B + \lambda_B + \lambda_C \tag{10}$$

$$T_{AD} = T_{AC} + \Gamma_C + \lambda_D$$

$$= \Gamma_A + \Gamma_B + \Gamma_C + \lambda_B + \lambda_C + \lambda_D \tag{11}$$

$$T_{AE} = T_{AD} + \Gamma_D + \lambda_E$$

$$= \Gamma_A + \Gamma_B + \Gamma_C + \Gamma_D + \lambda_B + \lambda_C + \lambda_D + \lambda_E \tag{12}$$

$$T_{AF} = T_{AE} + \gamma_E + \lambda_F$$

$$= \Gamma_A + \Gamma_B + \Gamma_C + \Gamma_D + \Gamma_E + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F \tag{13}$$

Apparently, in a 1D case, the resultant tolerance is always equal to the sum of the component tolerances. The types of case of ID geometrical tolerance stackup are limited, the tolerance zone distributions of some typical cases for 1D geometrical tolerance stackup are shown in Fig. 7. The case in Fig. 6 belongs to case (*a*) in Fig. 7.

The machining method for the part in Fig. 6 uses face *A* as the set-up datum and machines faces *B*, *C*, *D*, *E*, and *F* in the same set-up. This machining method is common in CNC machine. In this situation, the operational datum, design datum, and set-up datum are the same face – *A*. Hence, there is no error stackup. The error relations are as follows:

$$T_{MN} = (\Gamma_M + \lambda_N),\ M = A,\ N = B,C,D,E,F \tag{14}$$

3.2 Two-Dimensional Geometrical Tolerance Stackup Analysis

Figure 8 shows a 2D view of the tolerance zone of face *B*.

From Fig. 8, the tolerance zone indicates two possible maximum movements of the part: horizontal translation of Δ_B

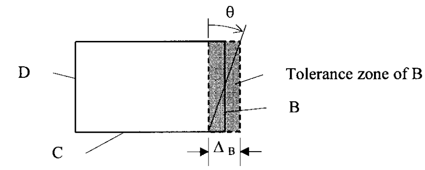

Fig. 8. Two-dimensional tolerance zone of face *B* of the part.

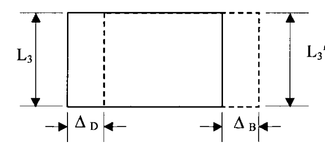

Fig. 9. The effect of the translation of face *B*.

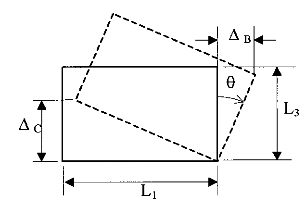

Fig. 10. The effect of the rotation of face *B*.

and rotation of θ. Suppose face *B* is used as a machining datum for machining face *D* and face *C*, and the error zone of face *B* is equal to the tolerance zone of face *B*. The error of face *D* used to be considered as equal to the translation of face *B* (as shown in Fig. 9):

$$\Delta_D = \Delta_B \tag{15}$$

The translation of face *B* has no effect on the error of face *C*. The effect of the error of face *B* on face *C* is through the rotation of θ, as shown in Fig. 10.

From Fig. 10,

$$\tan\theta \cong \frac{\Delta_B}{L_3} \cong \frac{\Delta_C}{L_1} \tag{16}$$

$$\Delta_c \cong \frac{L_1}{L_3}\Delta_B \tag{17}$$

However, there are two problems here:

1. Is θ the maximum rotation angle here?
2. Is Eq. (14) still correct if $L_3 \neq L_3'$?

For the first problem, the answer is no. An actual feature can make the rotation angle either larger or smaller than *θ*. Furthermore, in actual locating and clamping, the other locating and clamping faces may affect the rotation angle. However, for theoretical analysis, θ can adequately represent the average rotation angle.

For the second problem, if $L_3 \neq L_3'$, it seems that Eq. (14) should be changed to:

$$\Delta_c = \frac{L_3'}{L_3}\Delta_B \tag{18}$$

That is, it is also affected by the rotation angle of θ. However, with the restraints of the other locating and clamping faces in addition to the primary datum [9], for two faces in the same coordinate direction, translation rather than rotation is dominant. Hence, Eq. (14) is still approximately correct when $L_3 \neq L_3'$.

3.3 Three-Dimensional Geometrical Tolerance Stackup Analysis

For 3D geometrical tolerance analysis, the kinematic analysis of a rigid body is helpful. Basic transformation of a rigid body includes translation and rotation. Mathematically, the transformation is represented in matrix form, T. Any point P in a 3D Cartesian coordinate frame ($OXYZ$) is defined by its homogenous coordinates [x, y, z, 1]. To transform the point into a new point P' with coordinates [x′, y′, z′, 1] in the $OXYZ$ frame [10],

$$P' = P\,T \tag{19}$$

where T is a 4×4 matrix.

To transform a point by [a, b, c] on the X-, Y-, and Z-axes, denote the translation matrix as T_t,

$$T_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a & b & c & 1 \end{bmatrix} \tag{20}$$

In homogeneous coordinates, the 4×4 transformation matrix for rotating (named rotation matrix) about the X-axis by angle α, about the Y-axis by angle β, and about the Z-axis by angle γ can be written as T_X, T_Y, and T_Z, respectively, as follows:

$$T_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_Y = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_Z = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

In set-up planing we usually select plane faces or cylindrical faces (either convex cylindrical faces or concave cylindrical faces) as locating faces. The tolerance zone of a plane face is usually as (c1) or (c2) in Fig. 1. The tolerance zone of a cylindrical face is usually as (c3) or (c4) in Fig. 1.

The transformation matrices of Fig. 1 (c1) are:

$$T_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & t & 1 \end{bmatrix} \quad T_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\arctan\frac{t}{b}\right) & \sin\left(\arctan\frac{t}{b}\right) & 0 \\ 0 & -\sin\left(\arctan\frac{t}{b}\right) & \cos\left(\arctan\frac{t}{b}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_Y = \begin{bmatrix} \cos\left(\arctan\frac{t}{a}\right) & 0 & -\sin\left(\arctan\frac{t}{a}\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\arctan\frac{t}{a}\right) & 0 & \cos\left(\arctan\frac{t}{a}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

The transformation matrices of Fig. 1 (c2) are:

$$T_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & t & 1 \end{bmatrix} \quad T_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\arctan\frac{t}{d}\right) & \cos\left(\arctan\frac{t}{d}\right) & 0 \\ 0 & -\sin\left(\arctan\frac{t}{d}\right) & \cos\left(\arctan\frac{t}{d}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_Y = \begin{bmatrix} \cos\left(\arctan\frac{t}{d}\right) & 0 & -\sin\left(\arctan\frac{t}{d}\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\arctan\frac{t}{d}\right) & 0 & \cos\left(\arctan\frac{t}{d}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

The transformation matrices of Fig. 1 (c3) and (c4) are:

$$T_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & t & t & 1 \end{bmatrix} \quad T_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_Y = \begin{bmatrix} \cos\left(\arctan\frac{t}{l}\right) & 0 & -\sin\left(\arctan\frac{t}{l}\right) & 0 \\ 0 & 1 & 0 & 0 \\ \sin\left(\arctan\frac{t}{l}\right) & 0 & \cos\left(\arctan\frac{t}{l}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_z = \begin{bmatrix} \cos\left(\arctan\frac{t}{l}\right) & \sin\left(\arctan\frac{t}{l}\right) & 0 & 0 \\ -\sin\left(\arctan\frac{t}{l}\right) & \cos\left(\arctan\frac{t}{l}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

For 3D tolerance stackup analysis, the space positional relationship of the tolerance zones determines which of the matrices are to be used. By Eqs (20) and (21) and the projecting relationship of tolerance zones shown in Fig. 2, 3D tolerance stackup can be decomposed into a series of 2D tolerance stackup analyses based on the coordinate relationship.

4. An Example

For the part shown in Fig. 11, suppose the machining precedence of all the faces is: $D \Rightarrow B \Rightarrow C \Rightarrow E \Rightarrow F \Rightarrow A$. The

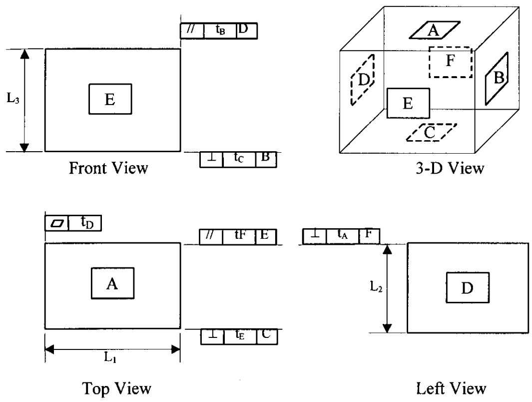

Fig. 11. A part for 3D tolerance stackup analysis.

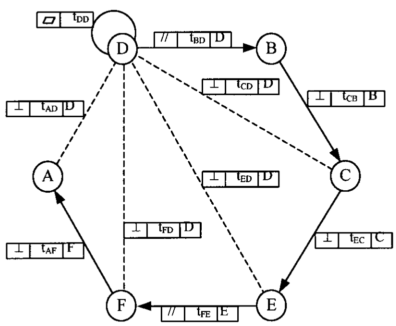

Fig. 12. Tolerance graph of the part.

machining datums of faces *B*, *C*, *E*, *F*, and *A* are *D*, *B*, *C*, *E*, and *F*, respectively. The assumed machining tolerance of each step is specified in Fig. 11, denoted as:

> t_{MN}: the tolerance of face *M* by using face *N* as machining datum

All the specified tolerances in Fig. 11 are component tolerances, suppose the given values are (units: mm):

t_{DD}	t_{BD}	t_{CB}	t_{EC}	t_{FE}	t_{AF}	L_1	L_2	L_3
0.01	0.01	0.02	0.03	0.03	0.02	500	300	400

The geometrical tolerance chain for the part is shown in Fig. 12. Suppose the resultant tolerances illustrated by dotted lines are to be estimated. The component tolerances are illustrated by arrowed lines.

The formulae of tolerance stackup in this paper were coded in a program. The running result of the example is:

INPUT:
t_DD t_BD t_CB t_EC t_FE t_AF L1 L2 L3
0.01 0.01 0.02 0.03 0.03 0.02 500 300 400
UNITS: mm
OUTPUT:

Resultant tolerance	Worst case	Statistical case
t_CD	0.0450	0.0320
t_ED	0.0900	0.0522
t_FD	0.1200	0.0602
t_AD	0.1100	0.0634

UNITS: mm

It can be seen that $t_{CD} < t_{ED} < t_{FD} < t_{AD}$ have increasing component tolerances. However, from the output of the worst case, there is one exception, $t_{FD} > t_{AD}$. The exception is because the geometrical tolerance stackup is affected by the basic dimensions of the related features (as discussed in Section 3.2), and so the resultant tolerance is not the straight sum of the component tolerances.

5. Conclusion and Discussion

In this paper, both dimensional tolerance stackup and geometrical tolerance stackup in one, two, and three dimensions are theoretically analysed. The tolerance analysis in this study is based on the analysis of tolerance zones. The principle of tolerance independency is fulfilled, and the dimensional tolerances and geometrical tolerances are discussed separately. The manufacturing errors are classified into two general types, locating errors and machining errors. The basic size of toleranced features used to be considered to be unrelated to 1D tolerance stackup. This paper shows that in two and three dimensions, tolerance stackup is dependent not only on component tolerances but also on the basic sizes of component features.

In this paper, only a primary datum face is considered for tolerance stackup analysis. In practical locating and clamping, a secondary datum face and tertiary datum face should also be considered. The situation will be more complicated and need further study. However, the primary datum face still plays a dominant role, and the analytical methods of this paper can be used as the basis for general tolerance analysis in the set-up planning and fixture planning in future study.

References

1. O. R. Wade, “Tolerance control”, Tool and Manufacturing Engineers Handbook: A Reference Book for Manufacturing Engineers, Managers, and Technicians, 4th edn, vol. 1, Machining, Dearborn, Michigan, Society of Manufacturing Engineers, 1983.
2. Øyvind Bjørke, Computer-Aided Tolerancing, 2nd edn, ASME Press, New York, 1989.
3. ASME, Mathematical Definition of Dimensioning and Tolerancing Principles, ANSI Y14.5.1 M-94, American Society of Mechanical Engineers, 1994.
4. H.-C. Zhang, Advanced Tolerancing Techniques, Wiley series in engineering design and automation, Wiley, New York, 1997.
5. ASME Y14.5M, Dimensioning and Tolerancing (Revision of ANSI Y14.5M-1982 (R1988), American Society of Mechanical Engineers, 1994.
6. G. Henzold, Handbook of Geometrical Tolerancing: Design, Manufacturing and Inspection, Wiley, New York, 1995.
7. K. W. Chase, J. Gao, S. P. Magleby and C. D. Sorensen “Including geometric feature variations in tolerance analysis of mechanical assemblies”, IIE Transactions, 28(10), pp. 795–807, 1996.
8. K. Whybrew, G. A. Britton, D. F. Robinson and Y. Sermsuti-Anuwat, “A graph-theoretic approach to tolerance charting”, International Journal of Advanced Manufacturing Technology, 5, pp. 175–183, 1990.
9. A. Y. C. Nee, K. Whybrew and A. S. Kumar, Advanced Fixture Design for FMS, Springer-Verlag, London, New York, 1995.
10. T.-C. Chang, Expert Process Planning for Manufacturing, Addison-Wesley, 1990.