Int J Adv Manuf Technol (2001) 17:330–338


The International Journal of Advanced Manufacturing Technology

# Genetic Algorithm (GA) for Multivariable Surface Grinding Process Optimisation Using a Multi-objective Function Model

R. Saravanan[1] and M. Sachithanandam[2]

[1]Department of Mechanical Engineering, Shanmugha College of Engineering, Tamilnadu, India; and [2]Department of Production Engineering and Management Studies, Regional Engineering College, Tamilnadu, India

*A genetic algorithm (GA) based optimisation procedure has been developed to optimise the surface grinding process using a multi-objective function model. The following ten process variables are considered in this work: wheel speed, workpiece speed, depth of dressing, lead of dressing, cross-feedrate, wheel diameter, wheel width, grinding ratio, wheel bond percentage, and grain size. The procedure evaluates the production cost and production rate for the optimum grinding conditions, subject to constraints such as thermal damage, wheel-wear parameters, machine-tool stiffness and surface finish. A worked example is used to illustrate how this procedure can be used to produce optimum production rate, low production cost, and fine surface quality for the surface grinding process.*

**Keywords:** Genetic algorithm; Multi-objective; Multivariable; Optimisation; Surface grinding

## 1. Introduction

Optimisation analysis of machining processes is usually based on minimising production cost, maximising production rate, or obtaining the finest possible surface quality by using empirical relationships between the tool life and the operating parameters. Optimisation analysis is also applicable to grinding processes, provided suitable tool-life equations are available. Fortunately, many such equation for practical grinding processes have been published in which numerous process variables are involved. The development of comprehensive grinding process models and computer-aided manufacturing provides a basis for realising grinding parameter optimisation.

Previous work on the optimisation of grinding parameters has concentrated on the possible approaches for optimising constraints during grinding [1]. The technique of optimising both grinding and dressing conditions for the maximum workpiece removal rate subject to constraints on workpiece burn and surface finish, in an adaptive-control grinding system, can be found in [2]. The use of quadratic programming for the optimisation of grinding parameters subject to a multi-objective function has been reported in [3]. In our previous work, a GA-based optimisation procedure has been successfully implemented for solving the surface grinding process problem considering four process variables using single [4] and multi-objective functions [5].

This paper describes a genetic algorithm (GA)-based optimisation procedure to optimise grinding conditions using a multi-objective function model with a weighted approach for surface grinding. The procedure evaluates the optimum grinding conditions subject to constraints such as thermal damage, wheel-wear parameters and machine-tool stiffness. In this work ten process variables have been considered which have not been considered previously (only four variables were considered) owing to computational difficulties. Initially, a detailed description of the mathematical model of the grinding process is given. Then, the optimisation procedure is described. Finally, a worked example is used to illustrate this new approach.

## 2. Mathematical Model of the Surface Grinding Process

The mathematical model proposed by Wen et al. [3] is adopted in this work.

### 2.1 Determination of Subobjectives

The aim of carrying out grinding operations is to obtain the finished product with minimum production cost, maximum production rate, and the finest possible surface finish. The authors, therefore, chose the production cost and production rate as subobjectives for the surface grinding process. The resultant objective function of the process is a weighted combination of the two objectives.

*Correspondence and offprint requests to*: R. Saravanan, Department of Mechanical Engineering, Shanmugha College of Engineering, Thanjavur-613402, Tamilnadu, India. E-mail: saradharani@hotmail.com

## 2.2 Optimisation Variables

Since numerous process variables are involved in grinding, especially where changes are extremely influential on the final performance of the parts, it is required to optimise every variable. Unfortunately, among the numerous process variables, only four variables have been considered up to now [3–5], because of the complexity of solving the equations. In this paper, in addition, six more variables have been included (in total, ten variables):

1. Wheel speed.
2. Workpiece speed.
3. Depth of dressing.
4. Lead of dressing.
5. Cross-feed rate.
6. Wheel diameter.
7. Wheel width.
8. Grain size.
9. Wheel bond percentage.
10. Grinding ratio.

## 2.3 Relationships Between the Two Subobjectives and the Ten Optimisation Variables

### 2.3.1 Production Cost

In the surface grinding process, the production costs comprise three elements: the cost directly related to the grinding of the part, the cost of non-productive time, and the cost of material consumption. The total production cost during the grinding process CT, considering the various elements mentioned above is shown in Eq. (1):

$$\mathrm{CT} = \frac{M_c}{60p}\left(\frac{L_w + L_e}{V_w 1000}\right)\left(\frac{b_w + b_e}{f_b}\right)\left(\frac{a_w}{a_p} + S_p + \frac{a_w b_w L_w}{\pi D_e b_s a_p G}\right) + \frac{M_c}{60p}\frac{S_d}{V_t} + t_1 + \frac{M_c t_{ch}}{60N_t} + \frac{M_c \times 1 \times \pi b_s D_e}{60pN_d L V_s 1000} + C_s\left(\frac{a_w b_w L_w}{pG} + \frac{\pi \mathrm{doc} b_s D_e}{pN_d}\right) + \frac{C_d}{pN_{td}} \tag{1}$$

### 2.3.2 Production Rate

The production rate is represented by the workpiece removal parameter WRP. The WRP is directly related to the grinding conditions and details of wheel dressing preceding the grinding operations.

$$\mathrm{WRP} = 94.4\,\frac{\left(1 + \dfrac{2\mathrm{doc}}{3L}\right) L^{11/19}\left(\dfrac{V_w}{V_s}\right)^{3/19} V_s}{D_e^{43/304}\,\mathrm{VOL}^{0.47} d_g^{5/38} R_c^{27/19}} \tag{2}$$

Where VOL $= 1.33X + 2.2S - 8$, and where the values of $X$ is 0, 1, 2, 3, 4, etc., for wheel hardness of $H$, $I$, $J$, $K$, $L$, $M$, etc., respectively, and $S$ is wheel structure number, 4, 5, 6, etc.

## 2.4 Constraints

It is well recognised that a more complete solution to the grinding problem is one that takes into account several realistic constraints of the actual operations. The constraints can be divided into process constraints and variable constraints. The process constraints considered in the present work are thermal damage, wheel wear parameter, machine tool stiffness and surface finish. The variable constraints are the upper and lower limits of the grinding conditions.

### 2.4.1 Thermal Damage Constraints

Because the grinding process requires an extremely high input of energy per unit volume of material removed, and almost all of the energy is converted into heat that is concentrated within the grinding zone, there may be thermal damage to the workpiece. One of the most common types of thermal damage is workpiece burn, which limits the production rate directly. On the basis of heat transfer analysis and experimental measurements, it has been shown that burning occurs when a critical grinding zone temperature is reached. This temperature is related directly to the specific energy, which consists of chip formation energy, ploughing energy, and sliding energy. Combining the relationships, the specific grinding energy $U$, is given in terms of the operating parameters by the Eq. (3).

$$U = 13.8 + \frac{9.64 \times 10^{-4}\, V_s}{a_p V_w} + \left(6.9 \times 10^{-3} + \frac{2102.4\, V_w}{D_e V_s}\right) \times \left(A_0 + \frac{k_u V_s L_w a_w}{V_w D_e^{1/2} a_p^{1/2}}\right)\frac{V_s D_e^{1/2}}{V_w a_p^{1/2}} \tag{3}$$

The corresponding critical specific grinding energy $U^*$ at which burning starts, can be expressed in terms of the operating parameters as

$$U^* = 6.2 + 1.76\left(\frac{D_e^{1/4}}{a_p^{3/4} V_w^{1/2}}\right) \tag{4}$$

In practice, the specific energy must not exceed the critical specific energy $U^*$, otherwise workpiece burn occurs. According to the relationship between grinding parameters and specific energy (Eq. (4)), the thermal damage constraint can be specified as

$$U \leq U^*$$

### 2.4.2 Wheel Wear Parameter Constraint

Another constraint is the wheel wear parameter WWP, which is related to the grinding conditions and the details of wheel dressing preceding the grinding operations, and can be expressed as follows:

$$\mathrm{WWP} = \left(\frac{\mathrm{k}_p \mathrm{a}_p \mathrm{d}_g^{5/38}\,\mathrm{R}_c^{27/19}}{\mathrm{D}_e^{1.2/\mathrm{VOL}-43/304}\,\mathrm{VOL}^{0.38}}\right)\left(\frac{(1 + d_{oc}/L)L^{27/19}\,(V_s/V_w)^{3/19} V_w}{(1 + 2d_{oc}/3L)}\right) \tag{5}$$

The grinding ratio G is determined by the typical wheel wear behaviour given by a plot of WWP against the accumulated workpiece removal (WRP). The wheel wear constraint can be obtained as follows:

$$\text{WRP/WWP} \geq G$$

#### 2.4.3 Machine Tool Stiffness Constraint

In grinding, chatter results in undulation roughness on the grinding wheel or workpiece surface and is highly undesirable. A reduction of the workpiece removal rate is usually required to eliminate grinding chatter. In addition, wheel surface unevenness necessitates frequent wheel redressing. Thus, chatter results in worsening of surface quality and it lowers the machining production rate. Chatter avoidance is therefore a significant constraint in the selection of the operating parameters.

The relationship between grinding stiffness $K_c$, wheel wear stiffness $K_s$ and operating parameters during grinding is expressed as follows:

$$K_c = \frac{1000\, V_w f_b}{\text{WRP}} \tag{6}$$

$$K_s = \frac{1000 V_s f_b}{\text{WWP}} \tag{7}$$

In this paper, it is proposed that the grinding stiffness and wheel wear stiffness during grinding, as well as the static machine stiffness must satisfy the following constraint in order to avoid excessive chatter during grinding:

$$\text{MSC} \geq |R_{em}|/K_m$$

where

$$\text{MSC} = \frac{1}{2K_c}\left(1 + \frac{V_W}{V_s G}\right)\frac{1}{K_s} \tag{8}$$

#### 2.4.4 Surface Finish Constraint

The surface finish $R_a$, of a workpiece is usually specified to be within a certain $R_a^*$ value. The operating parameters and wheel dressing parameters influence the surface finish strongly.

$$T_{\text{ave}} = 12.5 \times 10^3 \frac{d_g^{16/27} a_p^{19/27}}{D_e^{8/27}} \left(1 + \frac{d_{oc}}{L}\right) L^{16/27} \left(\frac{V_w}{V_s}\right)^{16/27} \tag{9}$$

$$R_a = \begin{cases} 0.4587\, T_{\text{ave}}^{0.30} & \text{for } 0 < T_{\text{ave}} < 0.254 \\ 0.7866\, T_{\text{ave}}^{0.72} & \text{for } 0.254 < T_{\text{ave}} < 2.54 \end{cases} \tag{10}$$

All the aforementioned deterministic constraint equations were empirically developed from experimental data. These constraints were obtained from five independent sources [3]. For the purpose of this paper, these equations are applicable.

### 2.5 Resultant Objective Function Model

Through the analysis discussed above, the optimisation problem for the surface grinding process can be formulated as a multi-objective, multivariable, nonlinear optimisation problem with multiconstraints.

In order to overcome the large differences in numerical values between the subobjectives, normalisation of each sub-objective is introduced. The resultant weighted objective function to be minimised here is:

$$\text{COF} = W_1 \frac{\text{CT}}{\text{CT}^*} - W_2 \frac{\text{WRP}}{\text{WRP}^*} \tag{11}$$

Subject to:

$$U \leq U^*$$

$$\text{WRP/WWP} \geq G$$

$$\text{MSC} \geq |R_{em}|/K_m$$

$$R_a \leq R_a^* \text{ (for rough grinding)}$$

$$\text{WRP} \geq \text{WRP}^* \text{ (for finish grinding)}$$

## 3. Implementation of GA [6, 7, 9–11]

### 3.1 About Genetic Algorithms

GAs form a class of adaptive heuristics based on principles derived from the dynamics of natural population genetics. The searching process simulates the natural evolution of biological creatures and turns out to be an intelligent exploitation of a random search. A candidate solution (chromosomes) is represented by an appropriate sequence of numbers. In many applications the chromosome is simply a binary string of 0 and 1. The quality of its fitness is the function which evaluates a chromosome with respect to the objective function of the optimisation problem. A selected population of the solution (chromosome) initially evolves by employing mechanisms modelled after those currently believed to apply in genetics. Generally, the GA mechanism consists of three fundamental operations: reproduction, crossover, and mutation. Reproduction is the random selection of copies of solutions from the population, according to their fitness value, to create one or more offspring. Crossover defines how the selected chromosomes (parents) are recombined to create new structures (offspring) for possible inclusion in the population. Mutation is a random modification of a randomly selected chromosome. Its function is to guarantee the possibility to explore the space of solutions for any initial population and to permit the freeing from a zone of local minimum. Generally, the decision about the possible inclusion of crossover/mutation offspring is governed by an appropriate filtering system. Both crossover and mutation occur at every cycle, according to an assigned probabilty.The aim of the three operations is to produce a sequence of populations that, on the average, tends to improve.

### 3.2 The Optimisation Procedure Using GA

*Step 1.* Choose a coding to represent problem parameters, a selection operator, a crossover operator, and a mutation operator. Choose a population size $n$, crossover probability $p_c$, and mutation probability $p_m$. Initialise a random population of strings of size l. Choose a maximum allowable generation number $t_{\max}$. Set $t = 0$.

*Step 2.* Evaluate each string in the population.

*Step 3.* If $t > t_{max}$ (or) other termination criteria are satisfied, terminate.

*Step 4.* Perform reproduction on the population.

*Step 5.* Perform crossover on random pairs of strings.

*Step 6.* Perform bitwise mutation.

*Step.* Evaluate srings in the new population. Set $t = t+1$ and go to step 3.

End

### 3.3 GA Parameters [7, 9, 10]

| | |
|---|---|
| Population size | = 20 |
| Number of generations | = 25 |
| Probability of crossover | = 0.8 |
| Probability of mutation | = 0.05 |

### 3.4 Special Coding

In order to solve this problem using GA, a special type of coding system is used to represent the variables $V_s$, $V_w$, $d_{oc}$, $L$, $f_b$, $D_e$, $D_b$, $d_g$, VOL, and $G$. The coding consists of 19 digits. The first 10 digits are binary numbers (0 or 1) and the next 9 digits are numbers ranging from 0 to 9.

(e.g.)

*Coding*

| 1111101000 | 2 | 7 | 3 | 7 | 9 | 2 | 5 | 2 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| $V_s$ | $V_w$ | $d_{oc}$ | $L$ | $f_b$ | $D_e$ | $b_s$ | $G$ | VOL | $d_g$ |

*Decoding*

$V_s$ = 1000 + (decoded value of first 10 digits) (1) = 2000 (m min$^{-1}$)

$V_w$ = 15.5 + (11th digit) (0.5) = 16.5 (m min$^{-1}$)

$d_{oc}$ = 0.03 + (12th digit) (0.005) = 0.065 (mm)

$L$ = 0.03 + (13th digit) (0.005) = 0.045 (mm rev$^{-1}$)

$f_b$ = 1.92 + (14th digit) (0.02) = 2.06 (mm pass$^{-1}$)

$D_e$ = 351 + (15th digit) (1) = 360 (mm)

$b_s$ = 24 + (16th digit) (0.2) = 24.4 (mm)

$G$ = 56 + (17th digit) (1) = 61

VOL = 6.6 + (18th digit) (0.1) = 6.8 (%)

$d_g$ = 0.255 + (19th digit) (0.005) = 0.275 (mm)

With this coding we obtain the following solution accuracy in the given interval:

| | $V_s$ (m min$^{-1}$) | $V_w$ (m min$^{-1}$) | $d_o$c (mm) | $L$ (mm rev$^{-1}$) | $f_b$ (mm pass$^{-1}$) |
|---|---|---|---|---|---|
| Accuracy | 1.0 | 0.5 | 0.005 | 0.005 | 0.02 |
| Interval | (1000, 2023) | (15.5, 20) | (0.03, 0.075) | (0.03, 0.075) | (1.92, 2.10) |

| | $D_e$ (mm) | $b_s$ (mm) | $G$ | VOL (%) | $d_g$ (mm) |
|---|---|---|---|---|---|
| Accuracy | 1.0 | 0.20 | 1.0 | 0.10 | 0.005 |
| Interval | (351, 360) | (24, 25.8) | (56, 65) | (6.6, 7.5) | (0.255, 0.300) |

### 3.5 Objective Function Transformation

GAs are naturally suitable for solving maximisation problems. Since the above problem is a minimisation problem, it is converted into an equivalent maximisation problem by the following transformation.

$$\text{Maximise, NOF} = 1/(1+\text{COF}) \tag{12}$$

Where, COF is the combined objective function and NOF is the new objective function.

It is also a constrained optimisation problem. Penalty terms corresponding to the constraint violation are added to the new objective function and a fitness function is obtained. Penalty terms are added only if the constraints are violated.

### 3.6 Fitness Function (FFN)

$$\text{FFN} = \text{NOF} - \left(\frac{U-U^*}{U}\right) WWP \left(G\ WWP \left(\frac{\text{WRP}}{WWP}\right)\right) - (1 - \text{MSC} \times 10^{-5}) - \left(\frac{R_A - R_a^*}{R_a^*}\right) \tag{13}$$

where, MSC is the machine tool stiffness constraint.

### 3.7 Reproduction [6, 9]

A rank selection method is used for reproduction. The individuals in the population are ranked according to fitness, and the expected value of each individual depends on its rank rather than on its absolute fitness. Ranking avoids giving the largest share of offspring to a small group of highly fit individuals, and thus reduces the selection pressure when the fitness variance is high. It also maintains the selection pressure when the fitness variance is low: the ratio of expected values of individuals ranked $i$ and $i + 1$ will be the same whether their absolute fitness differences are high or low.

The linear ranking method proposed by Baker (see [6]) is as follows: each individual in the population is ranked in increasing order of fitness, from 1 to $N$. The expected value of each individual $i$ in the population at time $t$ is given by

$$\text{Expected value } (i, t) = \text{Min} + (\text{max-min}) \frac{\text{rank}(i, t) - 1}{N - 1}$$

where $N = 20$.

Minimum and maximum values for the above equations are obtained by performing reproduction with the following set of values.

| Number | Max | Min | $P_{s1}$ | $P_{s20}$ |
|---|---|---|---|---|
| 1 | 1.1 | 0.9 | 0.045 | 0.055 |
| 2 | 1.5 | 0.4 | 0.025 | 0.075 |
| 3 | 1.6 | 0.4 | 0.020 | 0.080 |

where,

$P_{s1}$ = probability of selecting the first rank

$P_{s20}$ = probability of selecting the 20th rank

From the above results, in order to have very low selection pressure for the first rank and high selection pressure for the 20th rank, and to avoid quick convergence, maximum and minimum values are selected as 1.6 and 0.4, respectively.

### 3.8 Crossover [11]

The strings in the mating pool formed after reproduction are used in the crossover operation. In a single-point crossover, two strings are selected at random and crossed at a random site. Since the mating pool contains strings at random, we pick pairs of strings from the top of the list. When two strings are chosen for crossover, first a coin is flipped, with a probability $p_c = 0.8$, to check whether a crossover is desired or not. If the outcome of the coin-flipping is true, the crossover is performed, otherwise the strings are placed directly in the intermediate population for subsequent genetic operation. Flipping a coin with a probability 0.8 is simulated as follows: A 3 digit number between 0 to 1 is chosen at random. If the random number is smaller than 0.8, the outcome of coin flipping is true, otherwise the outcome is false.

The next step is to find a crossover site at random. A crossover site is chosen by creating a random number between 1 to 18. For example, if the random number is 11, the strings are crossed at site 11 and two new children strings are created. After crossover, the children strings are placed in the intermediate population.

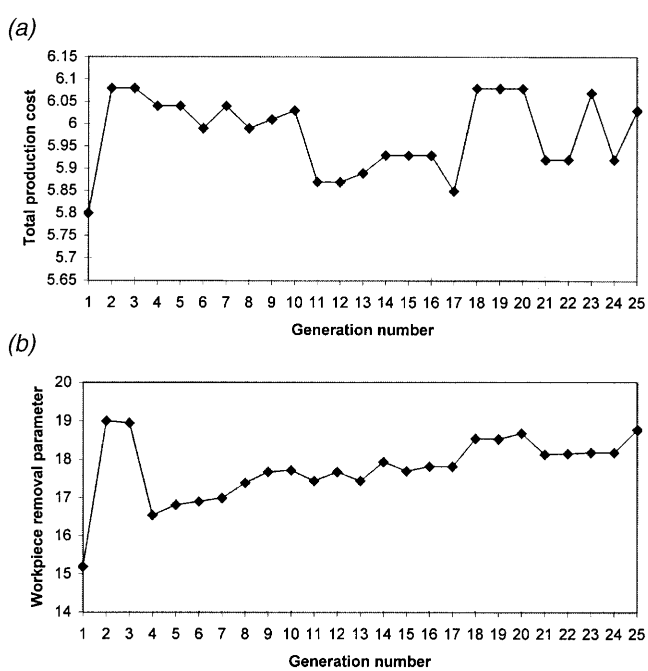


**Fig. 1.** (*a*) Total production cost obtained after each generation. (*b*) Workpiece removal parameter obtained after each generation.

### 3.9 Mutation [11]

For bitwise mutation, a coin is flipped with a probability $p_m = 0.05$ for every bit. If the outcome is true, the bit is altered to 1 or 0 depending on the bit value. If it is a number from 1 to 9, then this value is exchanged with the next one selected for mutation.

## 4. Data of the Problem

| Description | Symbol | Value |
|---|---|---|
| Number of workpiece loaded on table (pc) | $p$ | 1 |
| Length of the workpiece (mm) | $L_w$ | 300 |
| Empty length of grinding (mm) | $L_e$ | 150 |
| Width of workpiece (mm) | $b_w$ | 60 |
| Empty width of grinding (mm) | $b_e$ | 25 |
| Total thickness of cut (mm) | $a_w$ | 0.1 |
| Grinding down feed (mm pass$^{-1}$) | $a_p$ | 0.0505 |
| Number of sparkout grinding passes (pass) | $S_p$ | 2 |
| Distance of wheel idling (mm) | $S_d$ | 100 |
| Speed of wheel idling (mm min$^{-1}$) | $V_r$ | 254 |
| Time for loading and unloading workpiece (min) | $t_l$ | 5 |
| Time for adjusting machine tool (min) | $t_{ch}$ | 30 |
| Total number of workpieces to be ground between two dressings (pc) | $N_d$ | 20 |
| Batch size of workpiece | $N_t$ | 12 |
| Total number of workpieces to be ground during the life of dresser (pc) | $N_{td}$ | 2000 |
| Cost of wheel per mm$^3$ (\$ mm$^{-3}$) | $C_s$ | 0.003 |
| Workpiece hardness (Rockwell hardness) | $R_c$ | 58 |
| Surface finish limitation-rough (μm) | $R_a^*$ | 1.8 |
| Workpiece removal parameter limitation | WRP* | 20 |
| Static machine stiffness (N mm$^{-1}$) | $K_m$ | 100 000 |
| Dynamic machine characteristics | $R_{em}$ | 1 |
| Initial percentage of wear flat area | $A_0$ | 0 |
| Wear constant (mm$^{-1}$) | $k_u$ | $3.937 \times 10^{-7}$ |
| Constant dependent on coolant and grain type | $k_a$ | 0.0869 |

**Table 1.** Initial random population.

| Number | String | | CT | WRP | COF | NOF | FFN |
|---|---|---|---|---|---|---|---|
| 1 | 1110010000 | 174685095 | 6.16 | 17.55 | –0.131 | 1.15 | 1.15 |
| 2 | 0001001101 | 177270801 | 6.21 | 12.07 | 0.009 | 0.99 | –66.70 |
| 3 | 1010100001 | 774027721 | 5.96 | 17.08 | –0.129 | 1.15 | –19.80 |
| 4 | 1001000110 | 662522914 | 5.89 | 15.27 | –0.087 | 1.10 | –3.73 |
| 5 | 1100011000 | 142256851 | 6.16 | 14.88 | –0.064 | 1.07 | 1.07 |
| 6 | 0011100101 | 684792768 | 5.91 | 12.97 | –0.029 | 1.03 | –58.53 |
| 7 | 0101011011 | 269403685 | 6.10 | 13.77 | –0.039 | 1.04 | –61.89 |
| 8 | 1110101000 | 851574795 | 5.80 | 16.33 | –0.119 | 1.13 | 1.13 |
| 9 | 1001111101 | 111000204 | 6.14 | 11.94 | 0.009 | 0.99 | 0.99 |
| 10 | 0010100100 | 165053448 | 6.25 | 11.60 | 0.022 | 0.98 | –50.55 |
| 11 | 1111101001 | 264574777 | 6.09 | 17.82 | –0.141 | 1.16 | 1.30 |
| 12 | 0000111101 | 952707995 | 5.73 | 10.02 | –0.036 | 0.97 | –55.03 |
| 13 | 0000111110 | 678975438 | 5.87 | 11.92 | –0.005 | 1.00 | –90.30 |
| 14 | 1110011011 | 732088983 | 5.88 | 15.10 | –0.084 | 1.09 | 1.09 |
| 15 | 1010111010 | 752463382 | 5.86 | 14.94 | –0.081 | 1.09 | 1.09 |
| 16 | 0100011111 | 640518815 | 5.87 | 11.45 | 0.007 | 0.99 | –5.90 |
| 17 | 0111000010 | 268980935 | 6.01 | 14.86 | –0.071 | 1.08 | –53.02 |
| 18 | 1010010100 | 454272684 | 6.02 | 14.81 | –0.069 | 1.07 | –16.46 |
| 19 | 0011100010 | 013900228 | 6.03 | 9.59 | –0.062 | 0.94 | –22.57 |
| 20 | 1011100011 | 873792524 | 5.81 | 17.35 | –0.143 | 1.17 | –7.40 |

**Table 2.** Population obtained after 2nd generation.

| Number | String | | CT | WRP | COF | NOF | FFN |
|---|---|---|---|---|---|---|---|
| 1 | 1110010000 | 851574795 | 5.80 | 14.42 | –0.071 | 1.08 | –0.33 |
| 2 | 1010100001 | 493792524 | 6.02 | 17.76 | –0.143 | 1.17 | –8.84 |
| 3 | 1011101001 | 873772524 | 5.81 | 17.42 | –0.145 | 1.17 | –7.08 |
| 4 | 0000111111 | 953707995 | 5.73 | 10.17 | 0.032 | 0.97 | –64.21 |
| 5 | 1110001000 | 142792524 | 6.07 | 15.84 | –0.092 | 1.10 | 1.10 |
| 6 | 1010010001 | 473156851 | 6.08 | 16.19 | –0.101 | 1.11 | –2.13 |
| 7 | 1010010101 | 472792524 | 5.98 | 16.27 | –0.108 | 1.12 | –2.24 |
| 8 | 1110011001 | 142463382 | 6.13 | 15.43 | –0.079 | 1.09 | 1.09 |
| 9 | 1110011100 | 454272815 | 6.01 | 17.53 | –0.138 | 1.16 | –10.11 |
| 10 | 1010001000 | 142156858 | 6.18 | 13.64 | –0.032 | 1.03 | 1.03 |
| 11 | 1010011011 | 265053478 | 6.18 | 15.47 | –0.078 | 1.08 | –19.40 |
| 12 | 1111101101 | 454272815 | 6.01 | 18.15 | –0.153 | 1.18 | –7.56 |
| 13 | 1010010001 | 265053478 | 6.08 | 15.54 | –0.085 | 1.09 | –25.70 |
| *14 | 1111101000 | 273792524 | 6.08 | 19.01 | –0.172 | 1.21 | 1.21 |
| 15 | 1011100111 | 154272815 | 6.18 | 15.90 | –0.089 | 1.10 | –10.73 |
| 16 | 1010011100 | 464272684 | 6.04 | 15.44 | –0.084 | 1.09 | –13.74 |
| 17 | 1010001000 | 142156851 | 6.18 | 13.87 | –0.038 | 1.04 | 1.04 |
| 18 | 1111100010 | 142463382 | 6.13 | 15.66 | –0.085 | 1.09 | 1.09 |
| 19 | 1111100010 | 850574795 | 5.80 | 16.74 | –0.129 | 1.15 | 1.15 |
| 20 | 1010101010 | 142156851 | 6.18 | 14.11 | –0.004 | 1.05 | 1.05 |

*Best point in this population.

## 5. Results and Comparison

In this work, 25 generations are used for obtaining a solution to the optimisation of the surface grinding process. The initial random population is given in Table 1 and the population obtained after the second generation is given in Table 2. Note that after the second generation a solution (Table 3) is obtained. The best point obtained after each generation is given in Fig. 1. Results are compared with quadratic programming (4 variables) and genetic algorithms (4 variables) and are given in Table 4.

It is observed from the results that by employing GA, 2.0% reduction in cost and 8.8% increase in workpiece removal

**Table 3.** Optimisation results.

| | | | | |
|---|---|---|---|---|
| $V_s$ | $V_w$ | $d_{oc}$ | $L$ | $f_b$ |
| 2000 | 16.50 | 0.065 | 0.045 | 2.06 |
| $D_e$ | $b_s$ | $G$ | VOL | $d_g$ |
| 360 | 24.40 | 61.0 | 6.80 | 0.275 |
| CT | WRP | COF | NOF | FFN |
| 6.08 | 19.01 | –0.172 | 1.207 | 1.207 |

**Table 4.** Comparison of results.

| Number | Method | Variables | $V_s$ | $V_w$ | $d_{oc}$ | $L$ | CT | WRP | COF |
|---|---|---|---|---|---|---|---|---|---|
| 1 | QP | 4 | 2000 | 19.96 | 0.055 | 0.044 | 6.20 | 17.47 | –0.127 |
| 2 | GA | 4 | 1988 | 18.40 | 0.060 | 0.044 | 6.90 | 18.07 | –0.132 |
| 3 | GA | 10 | 2000 | 16.50 | 0.065 | 0.045 | 6.08 | 19.01 | –0.172 |

parameter is achieved in comparison with quadratic programming (QP) as found in [3]. An overall improvement of 35.46% is obtained in the combined objective function.

In comparison with GA (4 variables), 11.9% reduction in cost and 5.2% increase in WRP is achieved. An overall improvement of 30.3% is obtained in the combined objective function.

For solving this problem, using an exhaustive search method, $10 \times 10^{11}$ combinations have to be tried. However, by employing GA, only 520 combinations (25 generations) have been tried, and after evaluating 60 combinations (2 generations) a result is obtained. For the 4 variables problem using QP [3] after 13 iterations, an answer is obtained. Since there is no reference available for solving a 10-variables problem using conventional methods (based on the 4-variables problem using QP) it is assumed that the computational effort will be very high in comparison with GA.

## 6. GA for Other Metal Cutting Applications

With suitable systems, this procedure can be easily modified to suit other metal cutting applications such as turning [5,8], milling, cylindrical grinding and non-conventional machining. An example is given in Appendix A.

## 7. Conclusion

For solving machining optimisation problems, various conventional techniques have been used. It is observed that the conventional methods are not robust, for the following reasons:

The convergence to an optimal solution depends on the chosen initial solution.

Most algorithms tend to get stuck on a suboptimal solution.

An algorithm efficient in solving one machining optimisation problem may not be efficient in solving a different machining optimisation problem.

Algorithms are not efficient in handling multi-objective functions.

Computational difficulties arise in solving multivariable problems (more than four variables).

Also, these methods have problems when applied to the surface grinding process, which involves more variables and constraints. So to overcome the above problems, GA is used in this work for solving the surface grinding problem. It is observed that GA has outperformed the quadratic programming technique [3]. It is also observed that there is a considerable reduction in computational effort. This GA technique has also been successfully implemented for solving the above problem with the single objective function of minimising the production cost [4]. This procedure can be easily modified to suit other metal cutting operations such as turning [5], milling, cylindrical grinding, and non-conventional machining processes.

## References

1. S. Malkin, “Practical approaches to grinding optimization in, Milton C. Shaw Grinding Symposium, ASME Winter Annual Meeting, Miami Beach, pp. 289–299, 1985.
2. G. Amitay, “Adaptive control optimization of grinding”, Journal of Engineering for Industry, ASME, pp. 103–108, 1981.
3. X. M. Wen, A. A. O. Tay and A. Y. C. Nee, “Micro-computer based optimization of the surface grinding process”, Journal of Materials Processing Technology, 29, pp. 75–90, 1992.
4. R. Saravanan, S.Vengadesan and M. Sachithanandam, “Selection of Operating Parameters in Surface Grinding Process Using Genetic Algorithm (GA)”, Proceedings of 18th All India Manufacturing Technology, Design and Research Conference, pp. 167–171, 1998.
5. R. Saravanan, G. Sekar and M. Sachithanandam, “Optimization of CNC machining operations subject to constraints using genetic algorithm (GA)”, International Conference on Intelligent Flexible Autonomous Manufacturing Systems, CIT, Coimbatore, India, 10–12 January 2000.
6. Melanie Mitchell, An Introduction to Genetic Algorithms, Prentice-Hall of India, 1998.
7. Kalyanmoy Deb and Moyank Goyal, “Optimization of engineering designs using a combined genetic search”, Proceedings of the Seventh International Conference on Genetic Algorithms, pp. 521–528, 1997.
8. J. S. Agapiou, “The optimization of machining operations based on a combined criterion, Part 1: The use of combined objectives in single pass operations”, Transactions of ASME, Journal of Engineering for Industry, 114, pp. 500–507, 1992.
9. David E. Goldberg and Kalyanmoy Deb, “A comparative analysis of selection schemes used in genetic algorithms”, Proceedings of the Workshop on the Foundations of Genetic Algorithms and Classifier-Systems, pp. 69–93, 1990.
10. Gregory J. E. Rawlins, Foundations of Genetic Algorithm, Morgan Kaufmann, 1991.
11. Kalyanmoy Deb, Optimization for Engineering Design: Algorithms and Examples, Prentice Hall of India, 1995.

## Appendix A. Turning Optimisation Using GA

### Data of the Problem

| Description | | Value |
|---|---|---|
| $D$, diameter of the workpiece | | 152 mm |
| $L$, length of the workpiece | | 202 mm |
| $V_{min}$, minimum allowable cutting speed | | 30 m $min^{-1}$ |
| $V_{max}$, maximum allowable cutting speed | | 200 m $min^{-1}$ |
| $f_{min}$, minimum allowable feedrate | | 0.254 mm $rev^{-1}$ |
| $f_{max}$, maximum allowable feedrate | | 0.762 mm $rev^{-1}$ |
| $R_{a\max(r)}$, maximum surface roughness of rough cut | | 50 μm |
| $R_{a\max(f)}$, maximum surface roughness of finish cut | | 10 μm |
| $P_{max}$, maximum allowable power of the machine | | 5 kW |
| $F_{max}$, maximum allowable cutting force | | 900 N |
| $\theta_{max}$, maximum allowable temperature of tool–workpiece interface | | 500°C |
| $d_{oc\min(r)}$, minimum allowable depth of cut (rough) | | 2.0 mm |
| $d_{oc\max(r)}$, maximum allowable depth of cut (rough) | | 5.0 mm |
| $d_{oc\min(f)}$, minimum allowable depth of cut (finish) | | 0.6 mm |
| $d_{oc\max(f)}$, maximum allowable depth of cut (finish) | | 1.5 mm |
| constants used in tool life equation | a1 | 0.29 |
| | $a_2$ | 0.35 |
| | $a_3$ | 0.25 |
| | $K$ | 193.3 |
| $t_{cs}$, tool change time | | 0.5 min $edge^{-1}$ |
| $t_R$, quick return time | | 0.13 min $pass^{-1}$ |
| $t_h$, loading and unloading time | | 1.5 min $piece^{-1}$ |
| $C_o$, operating cost | | \$0.08 $min^{-1}$ |
| $C_t$, tool cost per cutting edge | | \$0.4 edge |

### Binary Coding (for *V*)

| Number | Code | Decode | $V$ |
|---|---|---|---|
| 1 | 0000000000 | 0 | 30 |
| 2 | 1111111111 | 1023 | 203.91 |
| 3 | 1001000110 | 582 | 128.94 |

### Binary Coding (for *f* )

| Number | Code | Decode | $f$ |
|---|---|---|---|
| 1 | 000000000 | 0 | 0.254 |
| 2 | 111111111 | 511 | 0.765 |
| 3 | 100100011 | 266 | 0.520 |

### Optimisation Results

| Number | $d_{oc}$ | $V$ | $f$ | $T_U$ |
|---|---|---|---|---|
| 1 | 2.0 | 118.91 | 0.764 | 2.85 |
| 2 | 2.5 | 114.15 | 0.644 | 3.02 |
| 3 | 3.0 | 114.49 | 0.665 | 3.13 |
| 4 | 3.5 | 120.61 | 0.531 | 3.46 |
| 5 | 4.0 | 106.16 | 0.565 | 3.51 |
| 6 | 4.5 | 104.80 | 0.454 | 3.96 |
| 7 | 5.0 | 110.58 | 0.435 | 4.14 |

$T_U$, total production time (min $piece^{-1}$).

## Nomenclature

| | |
|---|---|
| $a_p$ | grinding downfeed (mm $pass^{-1}$) |
| $f_b$ | cross-feedrate (mm $pass^{-1}$) |
| $a_w$ | total thickness of cut (mm) |
| $A_0$ | initial percentage of wear flat area (%) |
| $L_W$ | length of workpiece (mm) |
| $L_e$ | empty length of workpiece (mm) |
| $b_e$ | empty width of grinding (mm) |
| $b_s$ | width of wheel (mm) |
| $b_w$ | width of workpiece (mm) |
| $D_e$ | diameter of wheel (mm) |
| $G$ | grinding ratio |
| $B_k$ | positive definite approximation of the Hessian |
| $L$ | lead of dressing (mm $rev^{-1}$) |
| $d_{oc}$ | depth of dressing (mm) |
| $C_d$ | cost of dresser (\$) |
| $C_s$ | cost of wheel per $mm^3$ (\$ $mm^{-3}$) |
| CT | total production cost (\$ $pc^{-1}$) |
| CT* | expected production cost limitation (\$ $pc^{-1}$) |
| $d_g$ | grain size (mm) |
| $M_c$ | cost per hour labour and administration (\$ $h^{-1}$) |
| $N_d$ | total number of pieces to be ground between two dressings (pc) |
| $N_t$ | batch size of workpieces (pc) |
| $N_{td}$ | total number of workpieces to be ground during the life of dresser(pc) |
| $p$ | number of workpieces loaded on table (pc) |
| $R_a$ | surface finish (μm) |
| $R_a^*$ | surface finish limitation during rough grinding (μm) |
| $R_c$ | workpiece hardness (Rockwell hardness number) |
| $K_c$ | cutting stiffness (N $mm^{-1}$) |
| $K_m$ | static machine stiffness (N $mm^{-1}$) |
| $K_s$ | wheel wear stiffness (N $mm^{-1}$) |
| $k_u$ | wear constant ($mm^{-1}$) |
| $k_a$ | constant dependent on coolant and wheel grain size |
| $R_{em}$ | dynamic machine characteristics |
| $S_d$ | distance of wheel idling (mm) |
| $S_p$ | number of spark out grinding passes (pass) |
| $t_{sh}$ | time for adjusting machine tool (min) |
| $t_i$ | time for loading and unloading workpiece (min) |
| $T_{ave}$ | average chip thickness during grinding (μm) |
| $U$ | specific grinding energy (J $mm^{-3}$) |

| | |
|---|---|
| $U^*$ | critical specific grinding energy (J mm$^{-3}$) |
| $V_r$ | speed of wheel idling (mm min$^{-1}$) |
| $V_s$ | wheel speed (m min$^{-1}$) |
| $V_w$ | workpiece speed (m min$^{-1}$) |
| VOL | wheel bond percentage (%) |
| WRP | workpiece removal parameter (mm$^3$ min$-$N$^{-1}$) |
| WRP* | workpiece removal parameter limitation (mm$^3$ min$-$N$^{-1}$) |
| WWP | wheel wear parameter (mm$^3$ min$-$N$^{-1}$) |
| $W_1$, $W_2$ | weighting factors, $0 \leq W_1, W_2 \leq 1$ ($W_1 + W_2 = 1$) |
| COF | combined objective function |
| NOF | new objective function |
| FFN | fitness function |