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Principle and Simulation of Fixture Configuration Design for Sheet Metal Assembly with Laser Welding. Part 2: Optimal Configuration Design with the Genetic Algorithm

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A fixture plays a key role in ensuring proper metal fit-up in sheet metal assembly with laser welding. In this paper, an optimal fixture configuration design model is proposed based on a new locating scheme which includes both total locating and direct locating on welds. In this model, both the number and the location of the concerned locators are taken as objectives. The proposed prediction and correction method is used for determining the number of direct locators, and a pattern sorting method is developed for determining the number of total locators. Considering the degree of metal fit-up (DMF) to be a fuzzy quantity, a feasible evaluation criterion of DMF is also developed using a fuzzy synthesis evaluation method. A powerful optimisation technique – genetic algorithm – is employed as the optimisation procedure. A case study is presented which demonstrates the effectiveness of the optimal model, and the degree of metal fit-up can be improved compared to the initial locating scheme.

Keywords: Fixture configuration design; Genetic algorithm; Laser welding; Sheet metal assembly

1. Introduction

The automotive industry is concerned with the joining techniques and automation of automotive body manufacturing. Welding processes play an important role in the assembly process. Laser welding, which has economic advantages and small thermal distortion, has gained acceptance for meeting the increasing demand for high welding quality in sheet metal assembly. In particular, laser welding can provide deep and narrow welds at high speed with minimum thermal distortion, and this makes it a strong candidate for high-volume production with increased requirements concerning precision and degree of automation [1]. These advantages over traditional welding techniques give it great potential for replacing other welding techniques. However, some problems related to laser welding limit its applications in industry. Metal fit-up is an important factor that affects the implementation of laser welding and weld quality [2]. A fixture in sheet metal assembly will play a critical role in improving metal fit-up since the quality of current stamping processes cannot satisfy the metal fit-up tolerances that laser welding requires.

Much work has focused on automating the fixture design process considered as a rigid workpiece [3–6]. Asada and By [3] were the first to conduct the kinematic analysis for automatically reconfigurable fixtures systematically. They modelled the rigid workpieces as simple connected piecewise smooth surfaces. The conditions for deterministic locating and total restraining are derived. Menassa and DeVries [6] were the first to apply optimisation to fixture design. They developed a method to select the locations of the machining fixture that can minimise workpiece deflection normal to the primary reference plane using finite-element modelling techniques.

Work has since been carried out on fixture design of deformable sheet metal assembly. Rearick et al. [7] proposed a method to combine nonlinear programming and finite-element analysis (FEA) to design and evaluate fixtures for deformable sheet metal workpieces in resistance spot welding (RSW); Cai et al. [8,9] set up the principle and algorithm of a fixture scheme for deformable sheet metal with RSW in which only the location of the fixtures is considered in their optimisation of fixture configuration for the sheet metal assembly. Based on the RSW, the influence of sheet metal assembly springback was considered. A brief review on optimal fixturing design for RSW is given below.

The fixturing design of deformable sheet metal assembly with RSW aims to control the total deformation of key process control (KPC) points when weld force is applied, regardless of the source variation, and nonlinear programming is employed for optimisation. As stated by Rearick et al. [7] sequential quadratic programming (SQP) is one of the most efficient numerical optimisation algorithms; however, the gradient information of design variables is important for this method. Because it is difficult to obtain the derivatives of the objective function, a finite-difference method is employed to calculate

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approximately the gradient of the objective function by the definition of a perturbation vector of the design variables [7,8], as shown in Eq. (1).

$$\mathbf{g}_i = \frac{\mathbf{F}(\mathbf{X} + \Delta\mathbf{X}_i) - \mathbf{F}(\mathbf{X})}{\Delta\mathbf{X}_i} \tag{1}$$

where $\mathbf{g} = [g_1 \cdots g_i \cdots g_N]^T$ and $\Delta\mathbf{X}_i = [0 \ldots \delta_i \ldots 0]^T$.

A problem arises in that locators must be applied on the mesh nodes of the FE model. Because the design space is continuous, if a locator is located between two neighbouring nodes, a localised remeshing procedure must be adopted. Each remeshing will correspond to a new run of the FEA. Moreover, remeshing also leads to another problem, namely, objective function discontinuity. For example, the original mesh shape is shown in Fig. 1(*a*). A dot represents a fixture location. Figure 1(*b*) shows that the dot is closest to node 2. In this case, the remeshing procedure will move node 2 to the fixture location. If the fixture location is close to the midpoint between two nodal points, the remeshing shape is also similar to Fig. 1(*b*). After perturbation, the fixture location will be closest to nodal point 3. Thus, a new remeshing shape like Fig. 1(*c*) appears. This abrupt change of mesh shape will lead to objective function discontinuity. This will lead directly to a requirement for more iteration, or even to divergence.

The problem of objective function discontinuity has been stated by Rearick et al. [7] but no solution was given. Cai et al. [8] adopted a multipoint constraint (MPC) feature which is an option in MSC/NASTRAN to substitute the remeshing technique. The MPC employs linear interpolation between the two neighbouring nodes. This approximate method is feasible, based on the assumption of small linear deformation.

For the fixture design of a sheet metal assembly with laser welding, it is rather different. A prediction and correction method with a finite-element model under the specified "3-2-1" locating scheme has been proposed in Part 1 [10]. The proposed method is convenient for fixture configuration design, but not optimal. The objective of this paper is to propose an optimal fixture configuration design model and carry out optimal fixturing.

This paper is organised as follows: in Section 2 a brief introduction of the genetic algorithm and the application in optimal fixturing are given, then an optimal fixture configuration model for laser welding is proposed in Section 3, and a case study and summary are given in Sections 4 and 5.

2. Genetic Algorithm and its Application in Optimal Fixturing

2.1 Problem Statement

As mentioned before, in optimal fixturing with RSW, the employment of sequential quadratic programming will lead to remeshing and discontinuity of the objective function. The same will apply in laser welding. Thus, sequential quadratic programming cannot be applied directly to fixture design for laser welding. When fixture configuration design is carried out, the locator is assumed to be a rigid-point contact. However, this is impossible. One reason is that, in the structure, elements of the locator errors exist from manufacturing and assembly. Moreover, since not all continuous values are effective in industrial application, it is possible to regard the design variables of locator configuration as discrete variables, thus, the mesh pattern will be kept unchanged. In the discrete design space, the designed locators will move on the nodes of the locating area. In this way, the corresponding remeshing procedure can be avoided. In order to improve the solution accuracy of the optimisation process, a local high-meshing procedure is desired during geometric modelling, and the mesh size should be small enough (say, L_0 = 1 mm). Briefly, when the design space is limited to discrete space, the geometric model will be fixed, the changes in the optimal design are only changes of different load sets and constraint sets corresponding to different locating schemes. Thus, data file updating will be easier than changing the geometric model.

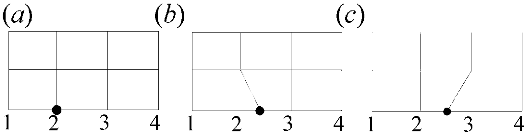

Fig. 1. Remeshing scheme.

However, another problem arises. Since the design space is discrete, the gradient calculation by the perturbation method will not work. A simplified treatment for gradient calculation can be used instead of the perturbation method. A Taylor series expansion is shown in Eq. (2). By neglecting the higher-order terms, the gradient vector can be written as Eqs (3) and (4). Thus, the calculation of the gradient vector is only related to the mesh nodes. In this way, the objective function discontinuity is avoided.

$$y_{i+1} = y_i + \frac{\partial y_{i+1}}{\partial x_{i+1}}(x_{i+1} - x_i) + \frac{\partial^2 y}{2!\partial x^2}(x_{i+1} - x_i)^2 + \cdots \tag{2}$$

$$\frac{\partial y_{i+1}}{\partial x_{i+1}} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \tag{3}$$

$$\mathbf{g}_i = \left[\frac{\partial y_1}{\partial x_1} \cdots \frac{\partial y_i}{\partial x_i} \cdots \frac{\partial y_N}{\partial x_N}\right] \tag{4}$$

The evaluation criterion of metal fit-up is very strict; and the precision requirement for fixture design is high. The derivatives derived from the Taylor series expansion do not have enough precision. After all, sequential quadratic programming is an optimal algorithm for continuous variables, whereas in this paper the design variables of the optimal fixturing for laser welding are regarded as discrete variables. So, a new optimal algorithm with the capability of solving discrete optimisation is desirable.

2.2 Genetic Algorithm (GA)

A genetic algorithm (GA), based on the principles of natural biological evolution, has received considerable and increasing interest over the past decade. Compared with the traditional optimisation method, GA is robust, global and may be applied generally, without recourse to domain-specific heuristics [11]. In engineering design problems, the design variables are often zero-one, discrete or continuous, since most classical optimisation techniques are designed to work with continuous variable, and these methods deal with mixed variables by adding artificial constraints to penalise infeasible values of variables. This combination increases the complexity of the underlying problem, and also spends considerable effort in evaluating infeasible solutions [12]. GA is an ideal optimisation algorithm for not only mixed variables, but also for unconvex or indifferent optimal problems. So GA has been widely used for function optimising, machine learning, etc. Figure 2 is a description of a generic GA.

There are 6 steps in a simple GA. The first step is encoding, the encoded strings can be used with various alphabets based on the need of the actual problem, such as, binary, integer, floating-point, etc. Then, an initial population is produced randomly. The third step is to evaluate fitness. In engineering problems, fitness is generally taken as an objective function, and it is also the basis for selection and evaluation. The higher the fit of individuals, the higher the probability of being selected for reproduction. By applying genetic operators to selected individual pairs with certain probability, a new offspring results. In order to avoid preconvergence, a mutation operator is required. Then, fitness of the new offspring will be evaluated. If the convergence criterion is satisfied, the optimal solution is reached. Otherwise, the same procedure is continued to select and regroup by genetic operators until convergence.

2.3 The Application of GA on Optimal Fixturing

Kumar et al. [13] were the first to use the combination of neutral networks and GA for the conceptual design of fixtures. GA is an objective function-dependent algorithm. This is one of the advantages of using GA in optimal fixturing since gradient information will not be used in the algorithm. FEA is time-consuming for optimal fixture design; thus, the optimisation algorithm must be highly efficient. In order to improve the search efficiency of the GA, in this paper, the case control feature of MSC/NASTRAN is employed. By setting subcases of the case control, different priorities can be set on load sets and constraint sets. This will greatly enhance the search efficiency when the GA is adopted.

In the GA, one evolution generation corresponds to one run of FEA, which means evaluation for all individuals of the population within one generation will be obtained by only one run of FEA. If we let the evolution generation number of the GA be N_g, the evaluation of the initial population will also consume one run of FEA, thus, the total run number of FEA is $N_g + 1$. When too many evolution generations are used, it is time-consuming. Three ways are used here to reduce the number of evolution generations. First, reasonable control parameter configurations for the GA are adopted. In particular, a relatively large population number which has a direct effect on the generation number is specified; Secondly, since the locators are on certain locating areas, the feasible design space includes only these locating areas, instead of covering the entire sheet metal area. Thirdly, in order to avoid the degrading of the optimal efficiency, the best locating scheme obtained from each evolution generation is appended directly to the scheme candidates of the next generation.

As stated by Deb and Goyal [12], binary encoding with length L represents exactly 2^L solutions, and binary strings may not be efficient in representing a discrete variable having arbitrary search space because a penalty method with extra constraints is used. In this way, it is difficult to improve the optimal efficiency. So, in this paper, a direct integer-coded procedure is adopted, and the crossover and mutation operations are carried out by taking integer values in a verified convergence of the real-coded space. An interface for the GA and FEA (MSC/NASTRAN) is given in Fig. 3. The modifications of load sets and constraint sets are included in the module for updating the analysis file.

```
Procedure Genetic Algorithm
Begin
    k: =0;
    Generate Initial Population $p(k)$ randomly;
    Compute Fitness of population $p(k)$;
    While (convergence criterion=0) do
        Begin
        k: =k+1;
        Select $p(k)$ from $p(k-1)$ by sorting
            and Reproduction Operator;
        Regroup $p(k)$ by
            Crossover Operator
            Mutation Operator
        Compute Fitness of population $p(k)$;
        End
End
```

Fig. 2. A description of the generic GA.

3. Optimal Fixture Configuration Model for Laser Welding

3.1 Basic Strategy for Optimal Fixturing

A new locating scheme with both total locating and direct locating on welds has been proposed in Part I [10]. The total

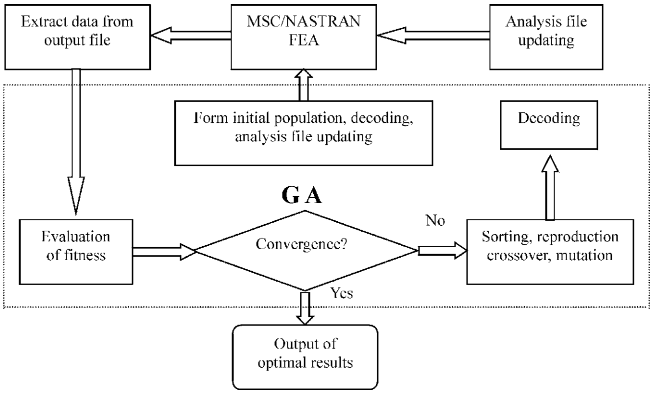

Fig. 3. Interface structure of GA and FEA.

locating scheme is to locate the entire sheet metal assembly, and the direct locating scheme is to locate the weld joints to meet the metal fit-up requirement. A conceptual design model for the fixture design of this paper is shown in Fig. 4. It can be seen that both total locating and direct locating are taken into account. This model is different from those of some other workers on this topic. Not only the location, but also the number of designed locators are taken as the objective function for the optimal design.

Based on the prediction and correction method, the direct locating area and the minimum number of direct locators required for the metal fit-up can be determined. This number will be used directly in the optimal configuration design of this paper. For the total locating scheme, the determination of the number and the locating area of the total locators are given below.

3.1.1 Total Locating Scheme

The total locating scheme can be described as an "*N*-2-1" scheme. The elements of the fixture are pins and locators, the "2-1" locating of the total locating scheme is realised by a 4-way pin and a 2-way pin, while "*N*" is realised by planar locators. The "2-1" locating scheme is to restrain the in-plane motion which is very small since the orientation of variation in sheet metal assembly is primarily normal to the panel surface. Thus, the in-plane variation can be neglected. In order to reduce the "2-1" locating error, two pins should be further apart [8]. The purpose of the optimal configuration design for total locating is to design optimally the number and the location of *N* total locators.

First, a pattern-sorting method for determining the number of total locators is proposed.

1. *Determination of the locating pattern.* For a certain subassembly, it is always possible to define four areas to apply fixtures. An illustrative example of the "3-2-1" total locating scheme is shown in Fig. 5. The two pins and locators are distributed on the four specified areas. The labels "12", "23," and "3" refer to the restrained DOF of the fixture. In order to reduce the fixture cost, in sheet metal laser welding owing to the locating effect of direct locators on welds, it is possible to reduce the number of total locators in the "3-2-1" locating scheme. Thus, there are four different candidates for the locating scheme: "3-2-1", "2-2-1", "1-2-1", and "2-1" schemes. The locations of the two pins are relatively fixed, the 4-way pin locates on the bottom-left node of the panel and the 2-way pin locates on the top-right node of the panel. Only the locations of the locator(s) are considered as being changeable and all four corner locations can be resided in by the locator. In this way, for some total locating schemes, different patterns can be configured on the four locating areas of the assembly based on the different locations of the locator(s). A two-level locating pattern diagram can be obtained and is shown in Fig. 6. Level 1 shows the different alternatives of the total locating schemes; level 2 shows the different patterns corresponding to the above locating schemes. For instance, there are two locating patterns for the "3-2-1" locating scheme, apart from the fixed locations of the two pins, two of the three locators also have fixed locations, one locates on the top-left node and the other locates on the bottom-right node of the panel. There are two options for the location of the third locator, if the locator locates on the top-right node this is pattern A, if the locator locates on the bottom-left location, it is pattern B. The same situation will be met in the "1-2-1" locating scheme. There are a total of six patterns for the total locating scheme, as shown in Fig. 6.

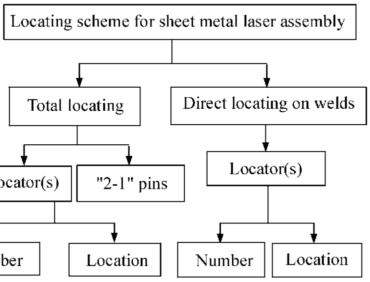

Fig. 4. A conceptual design model for fixture design.

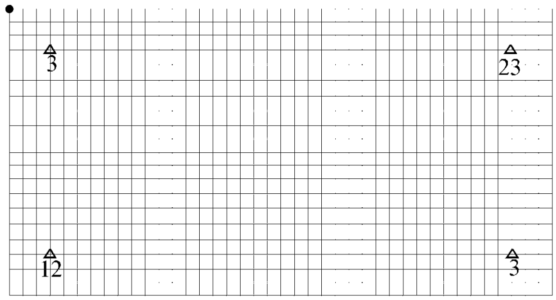

Fig. 5. An illustrative example of the "3-2-1" locating scheme.

2. *Pattern sorting*. A "3-2-1" locating scheme with pattern A is applied for direct locator configuration by the proposed prediction and correction method. By fixing the configuration of the direct locators, the other 5 patterns can be tested by FEA. In this way, we can sort all 6 patterns based on the DMF, then a total locating scheme with a minimum number of fixtures which satisfy the criterion of metal fit-up can be obtained. The number determined is the desired number of total locators for optimal fixturing.

In Fig. 6, if the optimal total locating scheme is a "2-1" configuration, the number "*N*" equals zero, which means the

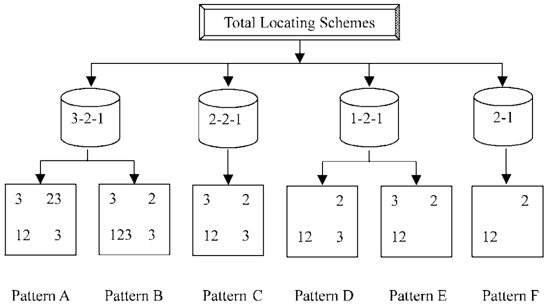

Fig. 6. Locating pattern diagram for total locating schemes.

design variables of the optimal fixture configuration design include only direct locators. Generally speaking, both total locators and direct locators should be considered. Another case should be mentioned here, if the two patterns of one locating scheme are all satisfied with the metal fit-up criterion, the pattern with minimum variation will be selected as the total locating scheme. Taking the three cases from Part 1 [10] of this research as example, a pattern sorting with a single short weld, a single long weld and double short welds is shown in Table 1. From Table 1 we can see, based on the criterion of DMF $\leq$ 0.1 IMF, that the feasible locating schemes for the three cases are Pattern F, Pattern D, and Pattern D (represented by boldface), respectively. This pattern sorting method is used to determine the number of total locating schemes and also the initial total locating scheme for optimisation. The location of the total locators and the location of direct locators will be optimally determined by GA.

3.1.2 Feasible Design Space

Because "2-1" configuration restrains in-plane movement, these locators have little effect on the movement normal to the panel surface which is controlled by total and direct locating. In this paper, the "2-1" locating pins are fixed while allowing the two pins to move further apart. As mentioned before, it is not necessary to set a feasible design space to cover the whole sheet metal since the locating area is locally distributed. The feasible design spaces (FDS) for direct locators and total locators are described as follows:

(*a*) *FDS for direct locators*. A local design space for direct locators can be set near to the initial design scheme obtained from the prediction and correction method. As shown in Fig. 7(*a*), a direct locator is applied at node i and the original mesh size is L_0, and the feasible design space for optimal design is the discrete points on the curve between node $i-1$ and node $i+1$. A high mesh density is required on the locating area and the high mesh is not shown in Fig. 7(*a*). Based on this local design space, a local optimal fixture design scheme can be obtained. Although it may not be a global optimal scheme, in some cases it is a compromise when global optimisation is not available.

The global design space for direct locators is distributed on the whole direct locating area. As shown in Fig. 8, two finite-element models with local high mesh density will be used as a case study in this paper. The single long weld locates on the centre-line of the two mated panels, as shown in Fig. 8(*a*). The direct locating area refers to the loop area formed by high meshing. In Fig. 8(*b*), the two short welds locate on the centre of the two panels. From the figure, we can see that the direct locating area is shown by the two-loop area formed by high meshing. Each of the two FE models includes two panels, as shown in Fig. 6 of Part 1 [10]. In Fig. 8 of this paper, an *X-Y* view of the sheet metal assembly with a lap joint is shown. The two panels of each figure are projected into one plane, but actually they are two mating panels.

(*b*) *FDS for total locators*. Similar to the design space for direct locating, the locating area for the total locating scheme can be on the whole panel. The total fixture is often applied along the boundary. Considering the dimension of the fixture, the feasible design space is on a curve around the boundary, as shown in Fig. 7(*b*). From the figure, we know that 4 locating areas should be specified, and each area should be located approximately on a quarter of the curve. In fact, if all the four locating areas are applied with the total locators, it is a "4-2-1" total locating scheme. For the proposed locating scheme of this paper a "3-2-1" scheme is applied. Thus, not all

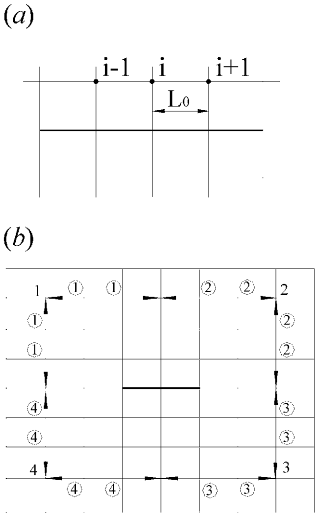

Fig. 7. Illustrative feasible design space for (*a*) direct locators, and (*b*) total locators.

Table 1. Pattern sorting results for two DMF criteria.

DMF criterion	Weld type	Number of direct locators	Maximum variation on weld stitch for six patterns (mm)					
			A	B	C	D	E	F
0.1 IMT	Single short weld	3	−0.08842	−0.0910	0.09290	−0.09135	0.0332	**−0.07306**
	Single long weld	10	−0.06990	−0.07472	−0.07066	**−0.07504**	0.32624	0.32062
	Two short welds	7	0.09243	0.02731	0.1083	**−0.04149**	0.31568	0.35375
0.124 IMT	Single long weld	5	−0.12154	−0.23522	−0.26319	−0.23367	**0.08303**	−0.29242
	Two short welds	6	−0.1228	−0.23543	−0.26181	−0.24298	0.09911	**0.11908**

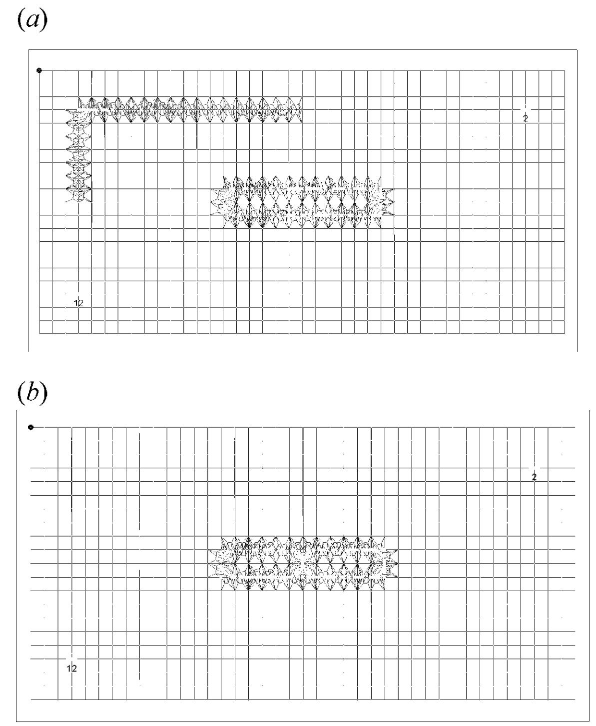

Fig. 8. FE models for the two cases. (*a*) Single long weld. (*b*) Two short welds.

four locating areas are employed in the optimisation. As shown in Fig. 7(*b*), labels 1, 2, 3, and 4 are to show the specified locations for the prediction and correction method, while arrows show the feasible range of different locating areas for optimisation. In Fig. 8(*a*), only one locator need be applied for the total locating. The total locating area is just the "L" shaped high meshing area. In Fig. 8(b) no total locator is applied.

3.2 Fuzzy Determination of Evaluation Criterion of DMF

In the fixture configuration design by the prediction and correction method, a stringent criterion of DMF is employed: DMF $\le$ 0.1 IMT where IMT is impact metal thickness. The resultant direct locating scheme based on this criterion is used as the initial scheme. After optimisation, the DMF can reach very high accuracy, while DMF with 0.1 IMT is sufficient in engineering applications, improving the DMF will require more direct locators. The tooling cost will increase also. In fact, if DMF is within 0.15 IMT the weld process is acceptable [2]. So a DMF criterion within (0.1–0.15) IMT is reasonable. A question arises as to what percentile of the DMF criterion is much more feasible for prediction and correction.

In this paper, the desired DMF criterion is examined as a fuzzy quantity and can be expressed as Eq. (5). As shown in Fig. 9, the membership function μ_D of the fuzzy allowable section D can be written as Eq. (6).

$$\begin{aligned} &DMF = \delta \cdot IMT \\ &\text{and} \quad \delta \subset D \end{aligned} \tag{5}$$

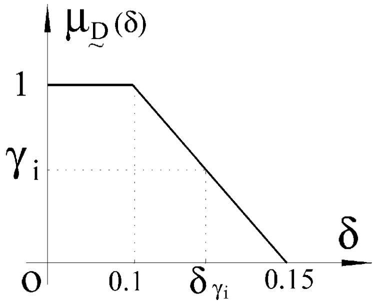

Fig. 9. Membership function.

$$\mu_{D(\delta)} = \begin{cases} 1 & (\delta \le 0.1) \\ 1 - \dfrac{\delta - 0.1}{0.15 - 0.1} & (0.1 \le \delta \le 0.15) \\ 0 & (\delta \ge 0.15) \end{cases} \tag{6}$$

From the figure, we can see that when $\delta \le 0.1$ the degree of satisfaction is 1, and when $\delta \ge 0.15$ the degree of satisfaction is 0. A straight line is used to show the transition between the degree of satisfaction and dissatisfaction. On the straight line different level set γ_i will correspond to different δ_{γ_i}. The key to this issue is to determine an optimal level set γ_0. A fuzzy synthesis evaluation method based on fuzzy set theory [14,15] is employed to obtain an optimal level set γ_0. The adopted method is given as follows:

1. *Determination of factor set U*. There are a lot of influential factors on DMF in the process of laser welding for a lap joint. In this paper, the following four factors are included in the factor set:

   u_1: degree of penetration into the lower layer panel
   u_2: weld bead width requirement
   u_3: material type
   u_4: welding speed

   The four factors form the factor set $U = \{u_1\ u_2\ u_3\ u_4\}$.
2. *Determination of selection set V*. The variation range of δ is (0.1–0.15). If we discretise this range, we can obtain a set {0.1 0.11 0.12 0.13 0.14 0.15}. Corresponding to this set, a selection set that reflects the degree of satisfaction can be written as $\mathbf{V} = \{\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ \mathbf{v}_4\ \mathbf{v}_5\ \mathbf{v}_6\} = \{1\ 0.8\ 0.6\ 0.4\ 0.2\ 0\}$.
3. *Determination of evaluation matrix* **R**. Considering each factor in the factor set, there will be a different degree of satisfaction related to the selection set. Thus, each factor can be specified an evaluation vector with the same size as the selection set. In total, an evaluation matrix can be formed. In order to determine this evaluation matrix, we assume that the levels of the four factors are in the following status:

   (*a*) Full penetration is not allowed.
   (*b*) Weld bead width is moderate.
   (*c*) The material type is mild steel.
   (*d*) The welding speed is slow.

   Thus, based on the above factor status, an evaluation matrix can be written as follows:

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0.5 & 0.8 & 1.0 & 0.5 & 0.0 \\ 0.5 & 0.7 & 1.0 & 0.6 & 0.2 & 0.0 \\ 1 & 0.8 & 0.6 & 0.4 & 0.2 & 0.0 \\ 0 & 0.3 & 0.5 & 0.7 & 0.8 & 1.0 \end{bmatrix}$$

Each row of the matrix shows the degree of satisfaction to the selection set under the specified factor status.

4. *Determination of factor weight set* **W**. Different factors will have different effects on DMF. Factor weight is used to evaluate the degree of importance of a factor. The factor weight set should be specified by experts. Factors u_1, u_2, and u_3 are intervolved and equally important while the importance of material type is no more than the other three, so the following factor weight set is specified: $\mathbf{W} = (0.3\ 0.3\ 0.1\ 0.3)$.
5. *Determination of evaluation set* **B**. Based on the above four steps, a fuzzy synthesis evaluation process is carried out. There are many composition rules [15], but in this paper a "multiply and plus" rule **M** (●,+) is adopted, the operation of this rule is similar to a generalised matrix multiplication. The evaluation set is written as follows:

$$\mathbf{B} = \mathbf{W} \circ \mathbf{R} = (w_1\ w_2\ w_3\ w_4) \circ \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} \end{bmatrix}$$

$$= (b_1\ b_2\ b_3\ b_4\ b_5\ b_6) \qquad (7)$$

By substituting the actual value into the above matrix, an evaluation set can be obtained: $\mathbf{B} = (0.31\ 0.53\ 0.75\ 0.73\ 0.47\ 0.3)$.

6. *Determination of optimal level set* γ_0. In order to determine the optimal level set from the evaluation set **B**, many rules can be employed, i.e. in the simplest way, γ_0 can be either the maximum value or the minimum value of the set **B**. However, the effect of some factors is neglected. In this paper a weighted average method is adopted. The expression is written as:

$$\gamma_0 = \sum_{i=1}^{6} b_i v_i \Big/ \sum_{i=1}^{6} b_i \qquad (8)$$

By substituting the actual value where an optimal level set γ_0 is determined, $\gamma_0 = 1.57/3 = 0.523$, then substituting $\gamma_0 = 0.523$ in Eq. (6), the fuzzy quantity δ is computed, $\delta = 0.124$. Thus, the resultant new criterion of DMF is: DMF = 0.124 IMT.

3.3 Main Flowchart of Optimal Fixturing

Based on the new criterion of DMF determined by fuzzy synthesis evaluation, the number and locating area for direct locators can be determined and the number of total locators can also be determined by the pattern sorting method. Then, an optimal fixture configuration design is carried out. The optimal objective function can be written as:

$$\min F(\mathbf{X}) = \max(x_{ij})\ (i = 1, 2, \ldots, n_w, j = 1, 2, \ldots, n) \qquad (9)$$
$$\text{subject to } G_i(\mathbf{x}) \geq 0$$

where x_{ij} represents the DMF of the ith mating nodes on the jth weld stitch; n_w represents the nodal number on a weld stitch and n represents the total number of weld stitches in the fixture configuration design.

A unequal constraint set is given in this paper. The main constraints include three parts: first, the limits of the search space, this constraint is guaranteed by GA; secondly, the FEA software cannot allow duplicate nodes to be applied by locators. Moreover, on the locating area the mesh density is 1 mm and considering the structural dimension of the locator, the neighbour nodes should be equal to or greater than a specified distance apart (say, 5 mm); thirdly, variation requirement on KPC points specified by user. The KPC points must be located on the nodes of FE mesh, if not, the additional local remeshing procedure will be employed before the run of FEA. The main flowchart of the optimal fixturing is shown in Fig. 10.

4. Case Study

Two of the case studies from Part 1 [10] of this research are used here for fixturing optimisation of laser weld assembly, the procedure of finite-element modelling has already been shown in Part 1. The length × width × thickness of the two panels is 200 mm × 100 mm × 1 mm. All panels are mild steel with Young's modulus $E = 20\,700\ \text{N mm}^{-2}$ and Poisson's ratio $\gamma = 0.3$. As shown in Fig. 8, two different FE models with identical weld location are employed in this study:

(*a*) One single long weld of 50 mm length.

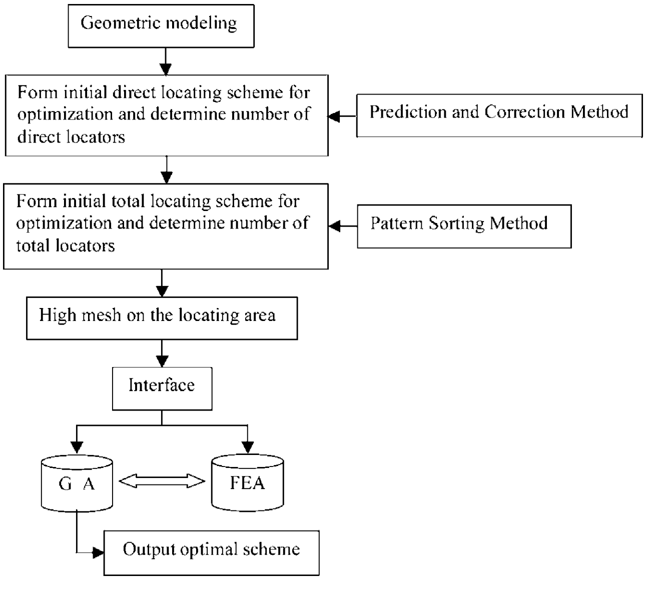

Fig. 10. Main flowchart for optimal fixturing.

(*b*) Two short welds of 20 mm length each.

For illustration purposes, no variation constraints of the KPC points are specified in the optimal models. The origins of the two finite-element models are all located on the top-left corner node.

The initial scheme of direct locating is obtained by the prediction and correction method based on the new DMF criterion of 0.124 IMT. The pattern sorting method is used to determine the initial scheme of total locating, and the pattern sorting results for these two cases are shown in Table 1. From the table, the numbers of direct locators for the two cases are 5 and 6. The total locating patterns corresponding to cases (*a*) and (*b*) are patterns E and F. In case (*a*), the locating scheme is "1-2-1" for total locating, and 5 direct locators are used for direct locating. In case (*b*), the locating scheme is "2-1" for total locating and 6 direct locators are used for direct locating.

The control parameters of the genetic algorithm for this case study are given as follows. The number of the population is 30, the reproduction probability is 0.15, the probabilities for crossover and mutation are 0.9 and 0.03, respectively. The convergence criteria include two parts: one is $\|F_{i+1} - F_i\| \le \epsilon$, F_{i+1} and F_i are objective functions of adjacent generations, the other is the evolution generations reaching a specified number in order to avoid preconvergence of the algorithm.

The optimal results for the two cases are shown in Table 2. We can see the objective function values include initial locating scheme and the optimal locating scheme searching in both local design space and global design space. In the global design space, the most optimal result can be reached. From the table we know by optimal design the degree of metal fit-up (objective function) has obviously been improved, that means the scheme obtained from optimisation is better than the scheme obtained from the prediction and correction method. In this way, a high quality of laser welding for the assembly process can be obtained.

As shown in Fig. 8(*b*) the shape of the locating area is a double-rectangle area, a shared edge of the two rectangles is located in the middle of the locating area. From the optimal result based on the global design space, we can see that there are three direct locators located near the shared edge, which will have a locating effect on both weld stitches. This confirms that it is important for designed weld locations to include a shared locating area to reduce the tooling cost.

A comparison of the maximum evolution generation between cases (*a*) and (*b*) in global design space shows that it requires 7 generations for convergence in case (*a*) and 9 generations in case (*b*). Considering the generation of the initial population, the total number of evolution generations are 8 and 10, respectively, so the amount of time-used is reasonable.

5. Summary

This paper is the first to determine the optimal fixture configuration design for sheet metal assembly with laser welding.

A powerful optimisation technique – genetic algorithm – is used which is not only capable of treating different design variables in engineering problem but is also capable of finding a global or "near global" solution for optimal fixturing. The logistic interface structure between GA and MSC/NASTRAN is set up. An optimisation model for fixture configuration is proposed considering the effects of total locating and direct locating on welds. This is the first design model to consider both the number and the location of the fixtures. The number

Table 2. Design results of case study.

Weld type	"2-1" pins	*X*- and *Y*-coordinate (mm)	(15, −85)	(185, −15)
		Restrained direction	(*X*, *Y*)	(*Y*)
		Designed locators (mm)		
	Locator number	Initial scheme	Optimal scheme	
			local	global
Single long weld	Total 1	(15, −15)	(15, −43)	(62, −15)
	Direct 5	(100, −45)	(107, −45)	(123, −45)
		(105, −45)	(100, −55)	(70, −51)
		(105, −55)	(106, −55)	(127, −55)
		(100, −55)	(94, −55)	(113, −45)
		(95, −55)	(95, −45)	(115, −55)
	Objective function (mm)	0.0830	0.0713	0.0577
Two short welds	Total 0	–	–	–
	Direct 6	(95, −45)	(101, −45)	(127, −55)
		(100, −45)	(107, −45)	(104, −55)
		(105, −45)	(102, −55)	(122, −45)
		(105, −55)	(96, −55)	(108, −45)
		(100, −55)	(94, −45)	(93, −45)
		(95, −55)	(108, −55)	(100, −51)
	Objective function (mm)	0.1191	0.0573	0.0248

of total locator(s) is determined by a pattern sorting method which is developed in this paper. The proposed prediction and correction method is used to determine the number and the locating area of the direct locators. Taking the criterion of degree of metal fit-up as a fuzzy quantity, a new criterion of DMF is developed by the fuzzy synthesis evaluation method.

The case study shows, based on the new locating scheme, that the sheet metal laser assembly does not necessarily take a "3-2-1" total locating scheme because of the influence of direct locating. Moreover, it reveals the importance of weld location design because of the shared locating effect of the shared locators for the case of multiweld stitches. From the case study, we can see by optimisation that the objective functions can be reduced compared to that of the initial locating schemes. With an improved degree of metal fit-up, a high weld quality will be obtained. The case study also shows that the optimal fixturing approach is effective and efficient for fixture configuration design for sheet metal assembly with laser welding.

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Notation

$\mathbf{D}$	fuzzy allowable section
$\mathbf{F(X)}$	objective function with design variable $\mathbf{X}$
$\mathbf{g}_i$	gradient vector with perturbation quantity $\Delta\mathbf{X}_i$
$\mathbf{G}_i(\mathbf{X})$	constraint vector for optimal fixturing
N_g	total number of evolution generation
$\mathbf{R}$	evaluation matrix for fuzzy synthesis evaluation (FSE)
U	factor set for FSE
V	selection set for FSE
$\mathbf{W}$	factor weight set for FSE
x_i	coordinates of the ith locator
y_i	variation of the ith locator
δ	range of evaluation for DMF
δ_i	value of the perturbation quantity for the ith variable
$\mu_D(\delta)$	membership function
γ_0	optimal level set for FSE
$\frac{\partial y_i}{\partial x_i}$	derivative of the function y_i
$\Delta\mathbf{X}_i$	perturbation quantity of the ith variable