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Variable Feedrate CNC Interpolation for Planar Implicit Curves

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The machining of planar implicit curves is common in computer numerical control (CNC) machining. It is usually carried out through approximating these curves as segments of straight line/circular arc for CNC interpolation. This paper considers real-time variable feedrate interpolation of planar implicit curves. Geometric entities of planar implicit curves were related to motion entities along the curve. The results were then used to develop real-time variable feedrate schemes for geometry-dependent feedrate and time-varying feedrate interpolation in general. Detailed schemes have been worked out for the cases of varying the feedrate for constant contour error and for initial acceleration/final deceleration. Examples have been provided to demonstrate how these schemes can be used in combination to enhance the efficiency of machining. They demonstrate how the feedrate for curve interpolation can be varied according to feedrate and feed-acceleration constraints on the machine tool and contour error constraint on the machined product.

Keywords: CNC; Implicit curves; Interpolation; Variable feedrate

1. Introduction

Computer numerical control (CNC) machining is an important method for machining complex 3D part surfaces. Surface machining involves four main phases, namely, optimisation of machining process parameters, tool path planning, path interpolation, and servo control.

Path interpolation is concerned with the conversion of tool paths obtained from a tool-path planning system into time-dependent commands for driving the servo control system of a CNC machine. For the determination of feedrate profiles, the interpolation algorithm should take into account the optimisation of the process.

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Linear and circular interpolators are the most common interpolators for CNC systems. A well-known approach for developing these interpolators is the use of a digital differential analyser (DDA) approach which is based on mathematical integration [1,2]. Qin and Bin [3] also developed a DDA algorithm for the interpolation of 3D circular paths. Another interpolation approach is based on pattern recognition [4,5]. In this approach, a set of rules is employed to determine the next step from a possible set of moves.

Both of these approaches were originally conceived with the assumption of hardware implementation of the interpolator. The advent of computing technologies has been driving a migration of the implementation of interpolators from hardware to software. It was then recognised that those interpolation approaches were insufficient for the machining of complex surfaces. Their drawbacks include limitations in feedrate fidelity, part precision and communications load [6].

There were attempts to develop more sophisticated interpolators for parametric curves [7–10]. Chou and Yang [7,8] developed a formulation for the command generation of parametric curve machining which takes into consideration the dynamics of the machine tool and the machining process. Wang and Wright [9] developed a real-time quintic spline interpolator for machine tool controllers with an open architecture. Zhang and Greenway [10] implemented a real-time non-uniform rational B-spline (NURBS) interpolator for an articulated robot.

For better regulation of the feedrate, Lo [11] proposed feedback interpolators for both 2D implicit curves and parametric curves. Yeh and Hsu [12] also presented compensatory interpolators for parametric curves. Lo [13] further proposed an approach to CNC tool-path generation so as to achieve constant feedrate for the cutter contact (CC) point path. Feedrate is an important parameter of a machining process. It affects the dynamics of the machine tool and the quality of the machined product. There were efforts to derive feedrate profiles based on modelling and optimisation of the machining processes [14–16]. The objective is to shorten the processing time, subject to surface finishing requirements and machine tool constraints. Chu et al. [14] carried out experimental studies on ball-end milling of free-form surfaces and considered local shape features and then developed a feature-based feedrate optimisation program. Yang and Sim [15] proposed the determination of a rational feedrate which could satisfy the part tolerance requirements based on modelling the surface error. The generation of smooth piecewise constant feedrate profiles was considered for multi-axis machines [16].

Variable feedrate interpolation is required for improving the efficiency of machining. Work on variable feedrate interpolation can be found in [17–20]. Farouki et al. [17–19] proposed variable feedrate interpolators for Pythagorean-hodograph curves which are a family of freeform parametric curves. Variable feedrate for general parametric curves was considered by Huang and Yang [20], based on the work of Chou and Yang [7]. However, there is insufficient work in the domain of implicitly defined curves.

Currently, linear and circular interpolation are common for commercial CNC systems. Some systems have conics (i.e. ellipse and parabola) interpolators. More advanced CNC systems have incorporated interpolators for NURBS and cubic spline curves [21]. Acceleration/deceleration can be programmed for producing smooth starting and stopping of the machine axes [22]; otherwise, interpolation is carried out with a constant feedrate. It is noted that there is no interpolator for general implicit curves in commercial systems.

This paper focuses on variable feedrate interpolation for implicitly defined planar curves, and will relate analytically geometric properties of such an implicit curve (i.e. tangent and curvature) to point motion along the curve (i.e. velocity and acceleration). Interpolation schemes for the cases of constant feedrate, path-geometry-varying feedrate and time-varying feedrate will be presented for general implicit curves. Detailed schemes are provided for the cases of varying feedrate for constant contour error and for the initial acceleration/final deceleration. Examples are provided to illustrate the use of the interpolation schemes and the switching between these schemes for shortening the machining time without compromising the quality of the machined surface under realistic machine tool constraints. In particular, machining efficiency can be increased by allowing the feedrate to increase when the curvature is decreased and vice versa, while satisfying a predefined tolerance of the contour error of the curve.

2. Planar Implicit Curves

This section relates geometric entities of an implicit curve to point motion along the curve. Tangent and curvature vectors for a general planar implicit curve are summarised. Making use of these vectors, the velocity and acceleration vectors of a point moving along the curve are related to the feedrate and feed-acceleration of motion.

2.1 Curve Tangent and Curvature

For a planar curve that follows the implicit equation

$$g(x, y) = 0 \tag{1}$$

the tangent and curvature vectors along the curve can be derived. Let **T** be the unit tangent vector and s be the arc length parameter, then

$$\mathbf{T} = [t_x\ t_y] \equiv [x'\ y'] \tag{2}$$

where $(\ )'$ represents the derivative with respect to s.

Differentiating Eq. (1) with respect to s, we obtain

$$g_x x' + g_y y' = 0 \tag{3}$$

and,

$$x'^2 + y'^2 = 1 \tag{4}$$

since **T** is defined as a unit vector. Solving Eqs (3) and (4), we obtain

$$\mathbf{T} = [g_y\ -g_x]/j \tag{5}$$

where $j = \sqrt{(g_x^2 + g_y^2)}$

Let **C** be the curvature vector, then

$$\mathbf{C} = [c_x\ c_y] \equiv [x''\ y''] \tag{6}$$

Differentiating Eqs (3) and (4) with respect to s, then

$$\left[\begin{array}{l} \mathbf{T}W\mathbf{T}^T + [g_x\ g_y]\mathbf{C}^T = 0 \\ \qquad \mathbf{T}\mathbf{C}^T = 0 \end{array}\right.$$

$$\text{where } W = \begin{bmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{bmatrix} \tag{7}$$

The above system of equations can be solved to obtain the curvature vector

$$\begin{aligned} \mathbf{C} &= [0\ -\mathbf{T}W\mathbf{T}^T]Q^{-T} \\ &\equiv \frac{-(g_{xx}g_y^2 - 2g_{xy}g_xg_y + g_{yy}g_x^2)}{j^4}[g_x\ g_y] \end{aligned} \tag{8}$$

where

$$Q = \begin{bmatrix} t_x & t_y \\ g_x & g_y \end{bmatrix} = \begin{bmatrix} g_y/j & -g_x/j \\ g_x & g_y \end{bmatrix} \tag{9}$$

$$Q^{-1} = \begin{bmatrix} g_y/j & g_x/j^2 \\ -g_x/j & g_y/j^2 \end{bmatrix} \text{ and } |Q| = j \tag{10}$$

The radius of curvature along the curve is

$$r = \frac{1}{\sqrt{(c_x^2 + c_y^2)}} = \frac{(g_x^2 + g_y^2)^{3/2}}{|g_{xx}g_y^2 - 2g_{xy}g_xg_y + g_{yy}g_x^2|} \tag{11}$$

2.2 Motion Along an Implicit Curve

Let **P** be a point that moves along a curve, the feedrate and feed-acceleration of the point along the curve are

$$V = \frac{\mathrm{d}s}{\mathrm{d}t} \quad \text{and } A = \frac{\mathrm{d}V}{\mathrm{d}t} \tag{12}$$

If the position of the point is given by $\mathbf{P} = [x\ y]$, we have the velocity of the point

$$\dot{\mathbf{P}} = \begin{bmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} & \frac{\mathrm{d}y}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} \frac{\mathrm{d}x}{\mathrm{d}s} & \frac{\mathrm{d}y}{\mathrm{d}s} \end{bmatrix} \frac{\mathrm{d}s}{\mathrm{d}t} = V\mathbf{T} \tag{13}$$

and the acceleration

$$\ddot{\mathbf{P}} = \frac{\mathrm{d}}{\mathrm{d}t}(V\mathbf{T}) = V\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t} + \frac{\mathrm{d}V}{\mathrm{d}t}\mathbf{T} = V^2\,\mathbf{C} + A\mathbf{T} \tag{14}$$

3. Interpolation

Path interpolation can be considered as recursively finding the next point along the curve given the current point on the curve. For a CNC controller with a fixed sampling cycle time, the distance between the current point and the next point depends on the feedrate. In this section, we consider interpolation for the cases of constant feedrate, geometry-dependent feedrate and time-varying feedrate. Assume the current interpolation point is $\mathbf{P}_k(x_k, y_k)$ and the next interpolation point is $\mathbf{P}_{k+1}(x_{k+1}, y_{k+1})$.

By a Taylor series expansion, we obtain

$$\mathbf{P}_{k+1} = \mathbf{P}_k + \dot{\mathbf{P}}_k(t_{k+1} - t_k) + \tfrac{1}{2}\ddot{\mathbf{P}}_k(t_{k+1} - t_k)^2 + \text{higher order terms} \quad (15)$$

Where Δ, the sampling cycle time of the CNC controller, is assumed to be constant, and neglecting all terms higher than second order, we have the approximation

$$\mathbf{P}_{k+1} = \mathbf{P}_k + \dot{\mathbf{P}}_k\Delta + \tfrac{1}{2}\ddot{\mathbf{P}}_k\Delta^2 \quad (16)$$

This last equation must be evaluated in real-time for real-time path interpolation.

3.1 Constant Feedrate

Constant feedrate interpolation has been presented by Shiptalni et al. [6] and is included here for completeness. For constant feedrate interpolation, $A = 0$, Eq. (14) can be rewritten as

$$\ddot{\mathbf{P}} = V^2\,\mathbf{C} \quad (17)$$

and $\mathbf{P}_{k+1}$ may be approximated as

$$\mathbf{P}_{k+1} = \mathbf{P}_k + V\Delta\mathbf{T}_k + \tfrac{1}{2}(V\Delta)^2\mathbf{C}_k \quad (18)$$

or

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} + V\Delta \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \tfrac{1}{2}(V\Delta)^2 \begin{bmatrix} c_x \\ c_y \end{bmatrix}_k \quad (19)$$

3.2 Geometry-Dependent Feedrate

For a geometry-dependent feedrate $V(x, y)$,

$$A(t) = \frac{\mathrm{d}V}{\mathrm{d}t} = V_x\frac{\mathrm{d}x}{\mathrm{d}t} + V_y\frac{\mathrm{d}y}{\mathrm{d}t} = V\mathbf{T}\begin{bmatrix} V_x \\ V_y \end{bmatrix} \quad (20)$$

The interpolation equation can be written as

$$\mathbf{P}_{k+1} = \mathbf{P}_k + V_k\Delta\mathbf{T}_k + \tfrac{1}{2}V_k\mathbf{T}_k\begin{bmatrix} V_x \\ V_y \end{bmatrix}_k \Delta^2\mathbf{T}_k + \tfrac{1}{2}V_k^2\Delta^2\mathbf{C}_k \quad (21)$$

or

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \left(V_k\Delta + \tfrac{1}{2}V_k(t_xV_x + t_yV_y)_k\Delta^2 \right) \begin{bmatrix} t_x \\ t_y \end{bmatrix}_k + \tfrac{1}{2}V_k^2\Delta^2 \begin{bmatrix} c_x \\ c_y \end{bmatrix}_k \quad (22)$$

Interpolation for Constant Contour Error

Contour error shows how closely the CC points follow the path. Suppose the path is derived from connecting adjacent interpolation points by straight line segments. Contour error is the deviation of the resulting path from the original path.

Figure 1 shows a curve and two adjacent CC points **P** and **Q** on the curve. The figure also shows point **R** on the curve such that the unit tangent vector at **R** is parallel to the chord vector **PQ**. The maximum contour error between **P** and **Q** is e which is the distance between **R** and **PQ**. If the curve segment $\overset{\frown}{\mathbf{PQ}}$ can be approximated by a circular arc with a radius equal to the radius of curvature of the curve at **P**, then

$$e = r - \sqrt{\left(r^2 - \frac{|\mathbf{PQ}|^2}{4} \right)} \quad (23)$$

where r is the radius of curvature. For path interpolation $|\mathbf{PQ}| \approx V\Delta$. Thus, the contour error can be further approximated as

$$e = r - \sqrt{\left(r^2 - \frac{(V\Delta)^2}{4} \right)} \quad (24)$$

Alternatively, the feedrate is

$$V = \frac{2}{\Delta}\sqrt{(2re - e^2)} \approx \frac{\sqrt{(8e)}}{\Delta}\sqrt{r} \quad (25)$$

for $e \ll r$. Thus, for constant contour error interpolation, the feedrate profile may be specified as

$$V = \alpha\frac{\sqrt{(8e)}}{\Delta}\sqrt{r} \quad (26)$$

where e is the maximum allowed contour error and α is a constant between zero and one. For this feedrate profile, the feedrate is varied according to the local radius of curvature and the contour error remains nearly constant. This enables the machining time to be shortened by using higher feedrates for locations of large radius of curvature and lower feedrates for locations of small curvature, so as not to exceed a certain contour error constraint. There could be many ways for the

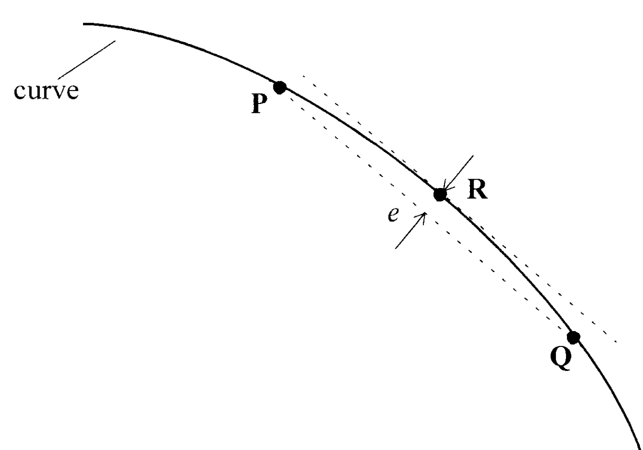


Fig. 1. Contour error along a curve owing to interpolation.

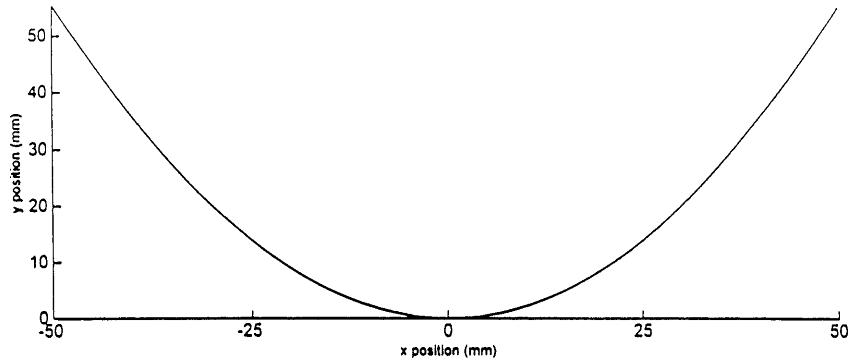


Fig. 2. The impicit curve $0 = x^2 - py$.

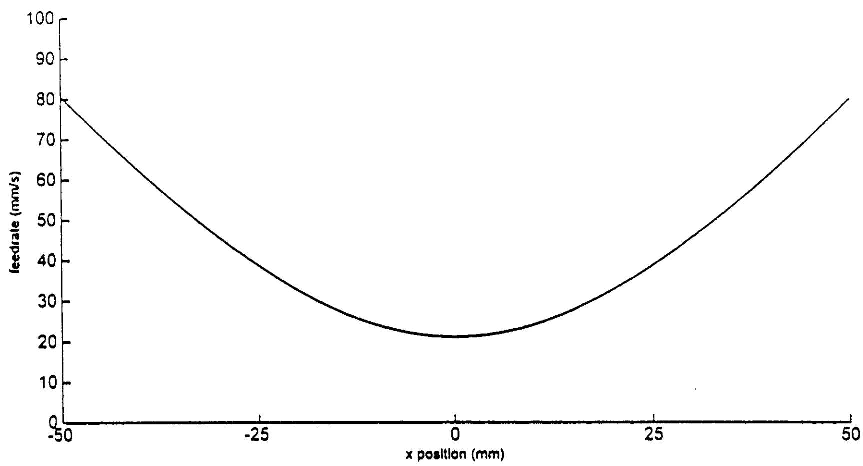


Fig. 3. Feedrate for constant contour error.

selection of α. An intuitive way is to select α based on a user selected nominal V and r for a path.

This implies the use of

$$V_k = \alpha \frac{\sqrt{(8e)}}{\Delta} \sqrt{(r_k)} \tag{27}$$

and

$$(t_x V_x + t_y V_y)_k = - \frac{\alpha \sqrt{(2e)}}{\Delta} \left\{ \frac{r^{5/2}}{j^3} \mathbf{T} W \mathbf{T}^T \left[3j \mathbf{T} W \mathbf{C}^T + \mathbf{T} \left(g_y \frac{\partial W}{\partial x} - g_x \frac{\partial W}{\partial y} \right) \mathbf{T}^T \right] \right\}_k \tag{28}$$

for substitution into Eq. (22) for realising the interpolation scheme.

3.3 Time-Varying Feedrate

For a given feedrate $V(t)$, $A(t) = \mathrm{d}V(t)/\mathrm{d}t$ which can be determined at each interpolation point. The interpolation equation can be written as

$$\mathbf{P}_{k+1} = \mathbf{P}_k + V_k \Delta \mathbf{T}_k + \tfrac{1}{2} A_k \Delta^2 \mathbf{T}_k + \tfrac{1}{2} V_k^2 \Delta^2 \mathbf{C}_k \tag{29}$$

or

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \left(V_k \Delta + \tfrac{1}{2} A_k \Delta^2 \right) \begin{bmatrix} t_x \\ t_y \end{bmatrix}_k + \tfrac{1}{2} V_k^2 \Delta^2 \begin{bmatrix} c_x \\ c_y \end{bmatrix}_k \tag{30}$$

Interpolation for Initial Acceleration and Final Deceleration

A continuous feedrate profile is desirable for changing from one feedrate to another. This allows the cutter to follow the path more accurately, improves the finishing of the resulting product, and avoids vibration of the CNC machine. Constant feed-acceleration is typically used in a trapezoid feedrate profile. Other examples of feedrate profiles include parabolic and exponential profiles.

For a constant feed-acceleration, the feedrate is a time-varying function and assumes the form of

$$V(t) = v_0 + A(t - t_0) \tag{31}$$

where A is the constant acceleration, and v_0 is the velocity at time t_0 when interpolation with constant acceleration starts.

This implies the use of

$$V_k = v_0 + Aj\Delta \quad \text{and } A_k = A \tag{32}$$

for substitution into Eq. (30) for constant acceleration interpolation where $\mathbf{P}_k$ is the jth interpolation point since t_0.

4. Examples

According to the previous section, an upper bound for the contour error implies that the feedrate should not be too high with respect to the local curvature of a curve. Practical restrictions on machine tools may also require the feedrate and the feed-acceleration to be within certain limits.

Here, a series of examples are used to demonstrate the use of the various interpolation schemes developed in Section 3. They also illustrate how these schemes can be used in combination to enable the shortening of machining time without compromising the accuracy of the machined surface or violating machine tool constraints.

The interpolation examples are based on an implicit curve that assumes the parabolic form of

$$g(x, y) = x^2 - py = 0 \tag{33}$$

where $p = 45$ and the units of x and y are in mm. The curve is shown in Fig. 2 in which the tool moves from $x = 50$ mm to -50 mm. The sampling cycle time Δ is assumed to be 0.01 s, which is typical for CNC machines. It is noted that the radius of curvature of the curve is smallest in the middle. In consideration of the contour error constraint, if a constant feedrate were used for machining the curve, the value of the feedrate would have to be very small, as dictated by the curvature of the middle of the curve. Thus, constant feedrate is not an efficient way to machine the curve.

The first example simulates interpolation feed for constant contour error for which α is set at 0.5 and the maximum allowed contour error e is 0.001 mm. Equation (22) is used for the interpolation in which the velocity terms are calculated according to Eqs (27) and (28). Figure 3 shows how the resulting feedrate varies along the path. The feedrate is minimum when $x = 0$ mm where the radius of curvature is smallest and increases outward as the radius of curvature of the curve is also increased outward from $x = 0$ mm. Throughout the path, the contour error remains approximately constant at 0.00025 mm, which is smaller than 0.001 mm, the maximum contour error allowed. This illustrates how machining efficiency may be gained by using the constant contour error interpolation scheme.

(*a*)

(*b*)

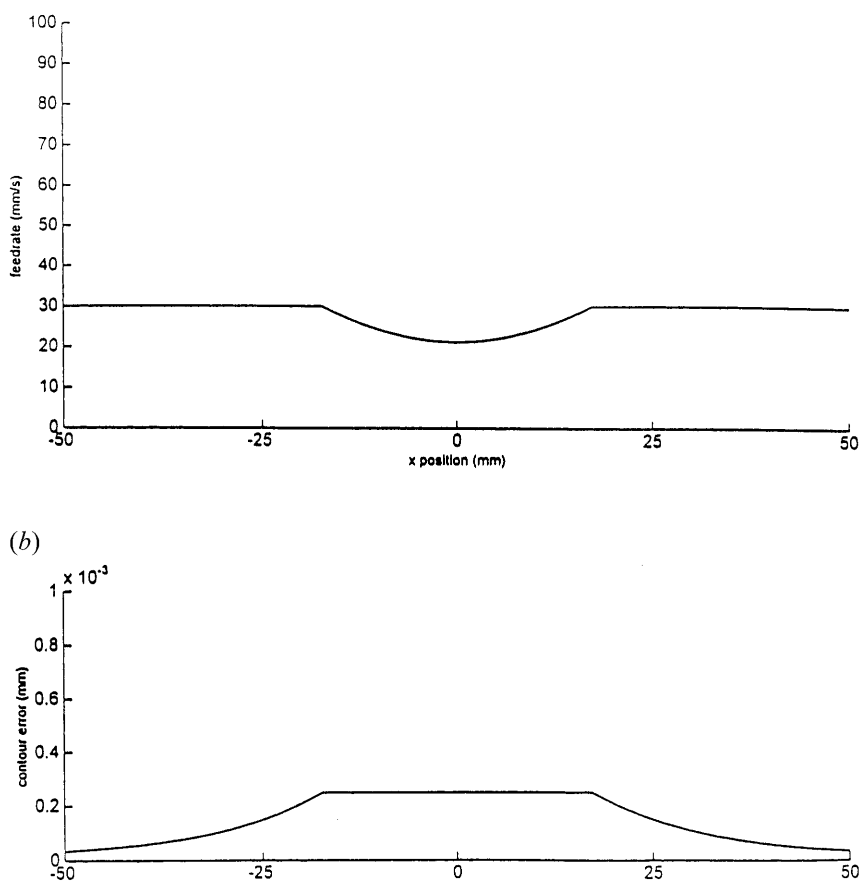


Fig. 4. Hybrid interpolation: (*a*) feedrate; (*b*) contour error.

We note that feedrate for constant contour error as in Eq. (26) may suggest using a very large feedrate at locations of gentle curvature, which would not be practical. For instance, the feedrate of about 80 mm s^{-1} (at $x = 50$ mm in Fig. 3) is suggested which may be high for typical machining operations. We expect a better interpolation scheme would be a hybrid scheme in which constant feedrate is used for path segments of large radius of curvature and constant contour error feedrate is used for segments of small curvature. A preset maximum feedrate V_{max} is used for path segments of constant feedrate. The condition for switching from constant feed to constant contour error feed is $\alpha\,(\sqrt{(8e)/\Delta})\sqrt{r_k} < V_{max}$ and that for switching the other way is $\alpha(\sqrt{(8e)/\Delta})\,\sqrt{r_k} > V_{max}$.

The second example simulates such a hybrid interpolation scheme. In this example, $V_{max} = 30$ mm s^{-1} which has been adopted for segments of constant feedrate. The scheme as stated in the previous example has been adopted for segments of constant contour error feedrate. The resulting feedrate interpolation profile is shown in Fig. 4(*a*). The feedrate profile is similar to the constant contour error feedrate profile of Fig. 3 in the middle segment near $x = 0$ mm. Constant feedrate of 30 mm s^{-1} has been used for the initial and final segments where the radius of curvature would suggest using a feedrate larger than V_{max} according to constant contour error feed. The contour error along the path has also been calculated using Eq. (24) and is shown in Fig. 4(*b*). Within the segment of constant contour error feed, the contour error remains unchanged as before. Within the segments of constant feedrate, the contour error becomes smaller, owing to the feedrate being smaller than allowed, according to the condition in Eq. (26).

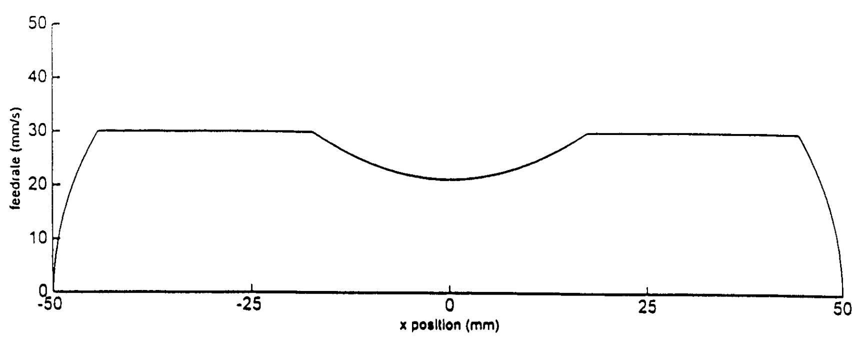
Finally, the inclusion of an initial segment of acceleration and a final segment of deceleration to the feedrate profile has also been considered to eliminate the need for demanding infinite acceleration at the beginning and at the end of the profile. This is similar in spirit to the use of trapezoid feedrate profiles. However, attention has to be paid to the switching between the various type of interpolation schemes, including the scheme for constant contour error interpolation.

Equation (30) is used for constant feed-acceleration interpolation in which the feedrate is determined according to Eq. (32). During the initial acceleration, $v_0 = 0$ mm s^{-1} and $A = A_{max}$ are used, in which A_{max} is the selected constant acceleration. During the final deceleration, $v_0 = v_c$ mm s^{-1} and $A = -A_{max}$ where v_c is the current feedrate when switching to the deceleration. Whether the initial segment of constant feed-acceleration is followed by a segment of constant contour error feed or by a segment of constant feedrate depends on whether V_k reaches $\alpha(\sqrt{(8e)/\Delta})\sqrt{r_k}$ or V_{max} earlier in the interpolation. The switch to the final segment of deceleration is initiated when

$$s_c \geq s_f - \frac{v_c^2}{2A_{max}} \tag{34}$$

is satisfied where s_c and s_f are the cumulated arclength and the total arclength, respectively. The cumulated arclength from

(*a*)



(*b*)

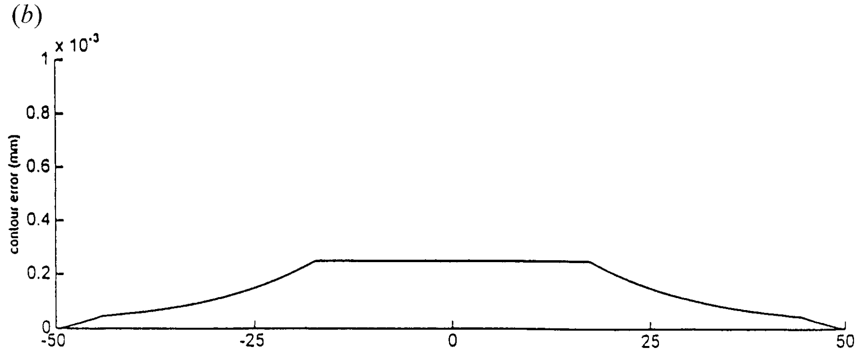


Fig. 5. Hybrid interpolation with acceleration/deceleration: (*a*) feedrate; (*b*) contour error.

the start point $\mathbf{P}_0$ to the current point $\mathbf{P}_k$ can be approximated by

$$s_c = \sum_{i=0}^{k-1} V_i \Delta \tag{35}$$

and it is to be computed in real-time. The total arclength can be found by solving the line integral

$$\int_g \sqrt{\left((\mathrm{d}x)^2 + (\mathrm{d}y)^2 \right)} \tag{36}$$

which can be computed before the cutting starts.

The final example demonstrates the inclusion of an initial segment of constant acceleration and a final segment of constant deceleration for the interpolation of the same implicit curve of the previous examples. The value for A_{max} is 35 mm s^2. The other conditions remain the same as in the second example of hybrid interpolation. The feedrate profile thus obtained is shown in Fig. 5(*a*). The profile contains five interpolation segments whereas the previous example has three segments. It can be seen that the feedrate increases initially from zero to V_{max} of 30 mm s^{-1} according to constant acceleration interpolation and then switches to constant feedrate interpolation. Toward the end, the interpolation also switches from constant feedrate to constant deceleration so that the feedrate is zero at the end of the curve interpolation. The contour error along the curve is also shown in Fig. 5(*b*). As in the previous examples, the contour error is always within the maximally allowed 0.001 mm. Furthermore, during the segments of constant feed-acceleration, the contour error is even smaller, in comparison to the same locations of the curve in the previous example. This is because, during the segments of constant feed-acceleration, an even smaller feedrate is used owing to the acceleration constraints.

5. Conclusion

In this paper, variable feedrate interpolation schemes for planar implicit curves have been presented. They enable interpolation points for general implicit curves to be generated in real-time for CNC machining. The schemes presented are interpolation with geometry-dependent feedrate and with time-varying feedrate. The cases of interpolation for constant contour error and for initial acceleration/final deceleration have been worked out, respectively. Examples have also been presented to illustrate the use of those schemes and how they can be potentially used in combination for real-time interpolation for improved machining efficiency, with consideration of realistic process and machine tool constraints. It is expected that the concept of varying the feedrate for efficiency, subject to process and machine tool constraints, can be further extended into other domains such as the machining of parametric curves and surfaces.

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