Int J Adv Manuf Technol (1999) 15:785–790

The International Journal of Advanced Manufacturing Technology

Cutting Force Analysis in Transient State Milling Processes

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This paper proposes a method for predicting transient state forces in milling processes as the cutter engages with and disengages from a workpiece. Analysis of this nature can contribute to the fundamental understanding of forced vibrations, deflections, and dynamic stability of multi-flute milling systems at the start and end of a cutting pass, thereby facilitating process planning, tool geometry optimisation, and on-line diagnostics. The calculation of transient cutting forces is commonly performed with the determination of local forces and numerical integration along cutting edges. In an effort to provide an estimation of transient cutting forces without resorting to numerical integration, this paper uses the result of a frequency domain model obtained from convolution integration as a basis for examining the temporal discretisation of a transient state cutting into steady-state conditions with various engagement and disengagement positions. The resulting milling forces in axial, feed, and cross-feed directions are expressed explicitly in terms of workpiece material properties, tool geometry, cutting parameters and process configuration. The process of end milling is presented to illustrate the applicability of the proposed method. End-milling experiments were performed and results compared to the force predictions for the verification of the analytical models.

Keywords: Cutting parameters; End milling; Process planning; Tool geometry

1. Introduction

Cutting force is one of the important physical variables that embodies relevant process information in machining. Such information can be used to assist in understanding critical machining attributes such as machinability, cutter wear/fracture, machine tool chatter, machining accuracy and surface finish [1–3]. The capability of modelling cutting forces therefore provides an analytical basis for machining process planning, machine tool design, cutter geometry optimisation, and on-line monitoring/control.

The modelling of cutting forces is often made difficult by the complexity of the tool/workpiece geometry and cutting configuration. In milling processes involving multiple helical cutter flutes, the modelling of forces generally resorts in the determination of local cutting forces at one cutting point followed by the integration of the local cutting forces along cutting edges. Koenigsberger and Sabberwal [4] treated the local tangential force as the product of a specific cutting pressure and an instantaneous chip load. Tlusty and MacNeil [1] further considered the radial local force to be proportional to the tangential local force. These basic relationships have been widely used by other researchers [2,3,5–9] to model the effects of cutter runout, spindle tilt, and system deflections in order to predict cutting forces more accurately. While these models had been developed for specific milling configurations such as face or end milling, Zheng et al [10] presented a general model that provides the cutting forces for any milling configuration based on a given geometry of cutting flutes.

A typical cutting process involves both steady state cutting and transient cutting. In steady-state cutting, a cutter has fully entered the workpiece and so the radial depth of cut remains constant. This premise does not hold in the transient state, as for the cases of entry, exit, or gap cutting for which the radial depth of cut varies with time, causing cutting forces to exhibit a transient state behaviour. The transient-state cutting forces are quite different from those in the steady state, and the time-variation of cutting forces has a significant effect on the surface texture and dimensional accuracy of the machined parts. This aspect is of particular importance for small workpieces in which the majority of cutting time is spent in transient cutting.

Currently developed cutting force models, which are based on local cutting forces and integrating the forces along the cutting edge to obtain the resultant forces, are applicable to transient cutting [11,12]. These models allow the use of sophisticated local force determination methods that were developed by also considering tool deflection and surface texture [13] or by direct force measurement in slot cutting [14]. These models must resort to numerical integration in the time domain for complex tool geometry such as helical cutting edges. In order to obtain an analytical solution for cutting forces, based on the assumption that local forces are pro-

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portional to average uncut chip thickness, a convolution integration approach has been developed [10,15]. The approach does not rely on numerical integration but provides explicit force solutions, thereby offering advantages for the planning, optimisation, monitoring and control applications. The convolution integration approach has been extended to the analysis of tool runout, tool/work compliance, and chatter stability for both straight and ball-end cutter milling [9,16–18].

A limiting factor for the closed-form solution provided by [10] is its lack of mathematical basis for describing the transient cutting situation. The nature of its frequency domain expressions assumes that the cutting forces behaviour does not vary between successive time intervals, therefore, the solution is applicable to steady-state cutting only. In addressing the issue, this paper examined the modelling of transient-state milling forces based on the steady-state cutting force models developed for multi-flute operations in [10]. A temporal discretisation approach is devised to represent a transient cutting process by a series of steady-state cutting processes, each with different cutting configurations. Therefore, cutting forces during transient cutting can be computed in terms of part material property, cutting condition, tool geometry, machining configuration, as well as the instantaneous position of the tool relative to the part. End milling experiments have been performed and results presented for verification of the modelling methodology.

2. Analytical Methodology

In the work of [10], the steady-state resultant forces in a multi-flute milling process using zero cutter entry angle has been expressed as follows:

$$\boldsymbol{F}(\phi) = \begin{bmatrix} F_x(\phi) \\ F_y(\phi) \\ F_z(\phi) \end{bmatrix} = \sum_{k=-\infty}^{\infty} \boldsymbol{A}_k e^{jkN\phi} = \sum_{k=-\infty}^{\infty} \begin{bmatrix} A_{xk} \\ A_{yk} \\ A_{zk} \end{bmatrix} e^{jkN\phi} \quad (1)$$

where

$$\boldsymbol{A}_k = 2\pi f_0 \tilde{K}[FFT\{\boldsymbol{p}(\theta)w(\theta,T_0)\}FFT\{h'_\beta(\tau)*Tth(\tau)$$
$$-\sum_{i=1}^{\infty}\left(\left(\frac{y_{max}}{2\pi f_0}\right)\right)^2$$
$$\frac{1}{i!}FFT\{\boldsymbol{p}(\theta)\delta^{(i-1)}(\theta,T_0)\}FFT\left\{\left(\frac{1}{r(\tau_0)}\right.\right. \quad (2)$$
$$\left.\left.-\frac{1}{r(\tau)}\right)^i (h'_\beta(\tau)*Tth(\tau))\right\}\Big)\Big]$$

$$\tilde{\boldsymbol{K}} = K_t t_x \begin{bmatrix} 1 & K_r & 0 \\ -K_r & 1 & 0 \\ 0 & 0 & K_a \end{bmatrix} \quad (3)$$

$$\boldsymbol{p}(\theta) = \begin{bmatrix} \dfrac{\sin 2\theta}{2} \\ \dfrac{1-\cos 2\theta}{2} \\ \sin\theta \end{bmatrix} \quad (4)$$

$$T(\tau) = \frac{\cos^{-1}(-y_{max}/r(\tau))}{2\pi f_0} \quad (5)$$

The terms A_{xk}, A_{yk} and A_{zk} are the complex coefficients of a Fourier series. They are the dynamic cutting force components representing the amplitudes and the phase angles associated with the kth harmonics of tooth passing frequency.

The cutting forces are expressed explicitly in terms of material properties (K_t,K_r,K_a), tool geometry (diameter and cutting angles), cutting parameters (feedrate, speed, radial and axial depth of cut) and process configuration. The model is based on a general cutting edge function that can be applied to either end milling, face milling, or any other multi-flute cutting processes with known tool geometry. The basic premises that were established for the model involve, first, the cutting is for a steady state with no process parameter variations, secondly, the cutter entry angle into the workpiece is zero.

Transient-state cutting takes place in the initial engagement and final disengagement phases of a cutting pass during which the radial depth of cut varies along the cutter path. The model given by Eq. (1) does not describe the effects of varying cutting conditions, therefore it is not directly applicable to transient-state cutting. To extend the model to transient-state cutting, a decomposition procedure is first applied to represent any milling configuration with a non-zero entry angle by the combination of milling operations with zero entry angles. The decomposition procedure is followed by short-term discretisation of the steady-state cutting condition into time-varying transient state conditions.

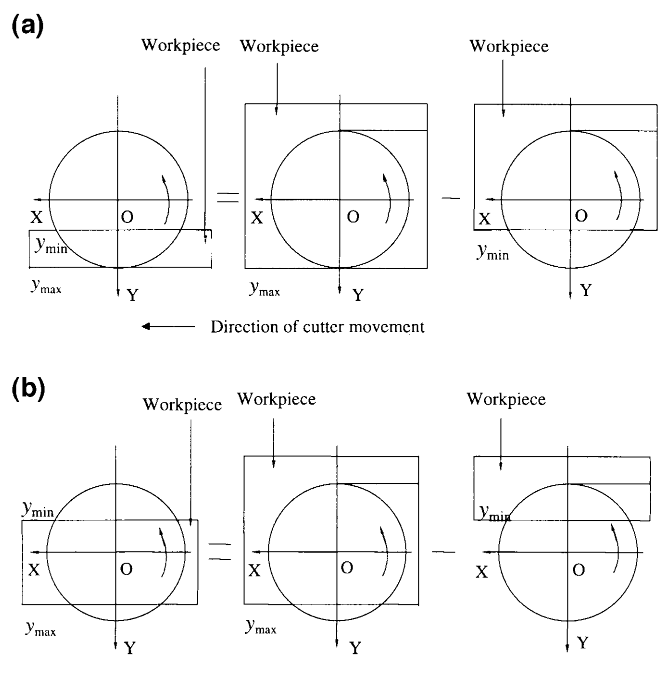

Fig. 1. The decomposition of a milling configuration with a non-zero entry angle into two basic cutting configurations with zero entry angles. (*a*) The decomposition of an up cutting. (*b*) The decomposition of a hybrid cutting.

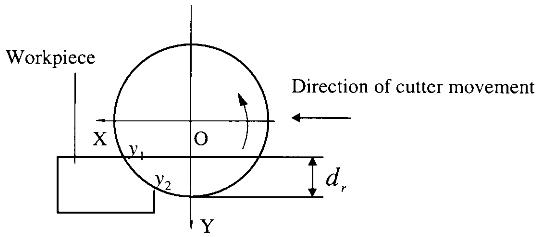

Fig. 2. Down milling at the point of entering a workpiece.

The decomposition procedure is graphically illustrated in Fig. 1. In Fig. 1(*a*), a down milling configuration with a non-zero entry angle is decomposed into two zero-entry angle configurations with different radial depths of cut. In Fig. 1(*b*) a hybrid (up and down) cutting configuration with a non-zero entry angle is also decomposed into two zero-entry angle configurations. With this procedure, the cutter entry angle for any general configuration can be represented by the linear combination of two cutter exit angles in models governed by Eq. (1).

During a very short time interval in cutting, the changes in radial depth of cut and in the cutting forces are small, and the cutting forces during a short time interval can be deemed to be in the steady state. Therefore, by segmenting a continuous transient cutting process into a series of steady-state cutting processes, each with different radial depths of cut, the transient cutting forces can be represented by using the model in steady-state cutting conditions. Denote the cutting forces generated by the milling operation with a radial depth of cut, y_{min} during time interval $t \in [m\Delta t,(m+1)\Delta t]$ as

$$\boldsymbol{F} = G(y_{max}(m\Delta t)) \quad (6)$$

where Δt is the discretisation time interval and m is an integer. Based on this definition and the decomposition concept shown in Fig. 1, the cutting forces for any transient state can be computed from the steady-state models. For example, in the case of a down cutting configuration at the start of a pass as shown in Fig. 2, cutting forces can be calculated as:

$$\boldsymbol{F} = G(y_2(\Delta tm)) - G(y_1(\Delta tm)) \quad (7)$$

In the case of a down cut configuration at the end of a cut, two portions of workpiece are cut simultaneously when the tool begins to disengage, as shown in Fig. 3(*a*). The cutting forces can be calculated at this moment as:

$$\boldsymbol{F} = G(y_2(\Delta tm)) - G(y_1(\Delta tm)) + G(y_4(\Delta tm)) - G(y_3(\Delta tm)) \quad (8)$$

When the material on the top of the workpiece is completely removed, the cutting forces can be calculated as shown in Fig. 3(*b*) as:

$$\boldsymbol{F} = G(y_4(\Delta tm)) - G(y_3(\Delta tm)) \quad (9)$$

The above discretisation procedure provides a practical way to calculate transient cutting forces in milling operations. Using the basic cutting process with a zero tool entry angle, the cutting forces for any milling configuration can be readily computed.

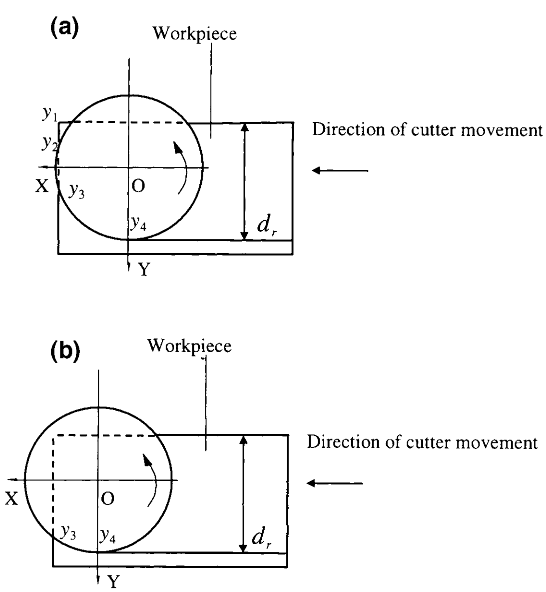

Fig. 3. Down milling (*a*) at the beginning of disengagement from a workpiece, and (*b*) towards the end of disengagement.

Table 1. Used K_t,K_r and K_a with respect to average cut thickness.

$K_t = 8.965(\bar{t}_c)^{-0.115} \times 10^4$ lb in^{-2}
$K_r = 0.310(\bar{t}_c)^{-0.080}$
$K_a = 0.702(\bar{t}_c)^{0.217}$

3. Experimental Verification

To verify the decomposition and segmentation principles involved in the modelling of transient-state cutting, a series of end milling experiments for various cutting configurations and process parameters were performed using a vertical milling machine. Three-dimensional cutting forces were measured with a platform piezoelectric dynamometer mounted between the workpiece and the machining table. The force measurements

Table 2. Cutting conditions in end milling experiment.

Cutting number	Cutting speed (r.p.m.)	Type of cutting	Engaging in or out	d_a (in)	d_r (in)	Feedrate (in min^{-1})
1	210	up	in	0.18	0.25	3.5
2	210	down	out	0.18	0.35	4
3	210	up	in	0.18	0.35	4
4	210	slot	in	0.18	7/16	3
5	210	slot	out	0.18	7/16	3
6	210	down	in	0.18	0.3	3.5
7	420	up	in	0.18	0.3	6
8	420	down	out	0.18	0.3	6
9	420	up	in	0.2	0.25	6.5
10	210	up	in	0.2	0.15	4
11	210	down	in	0.2	0.2	4
12	210	down	out	0.2	0.2	4
13	420	up	in	0.2	0.4	6.5
14	420	up	out	0.2	0.4	6.5

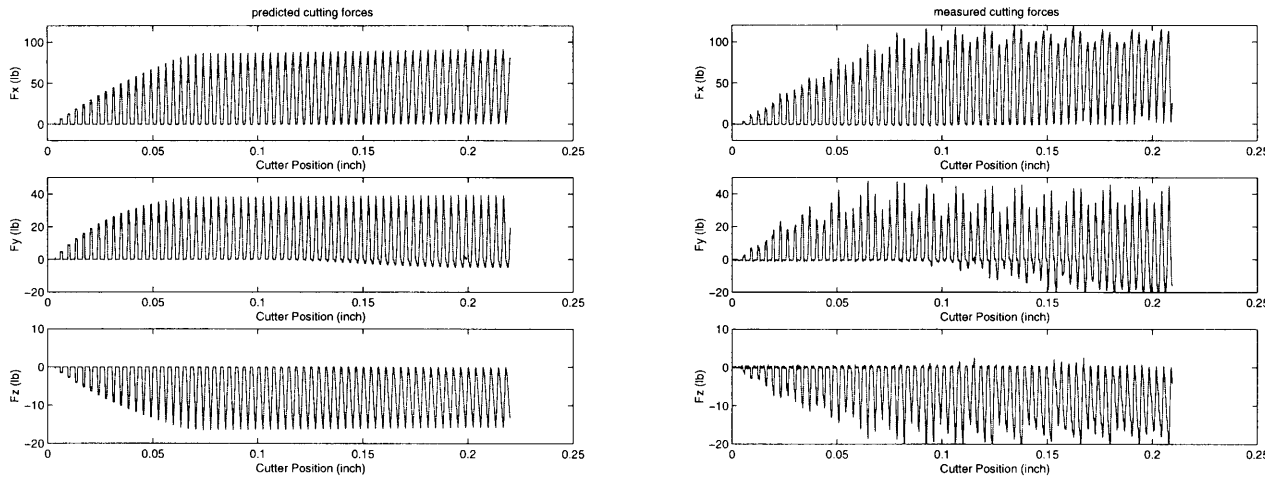

Fig. 4. Predicted and measured forces for cutting number 7.

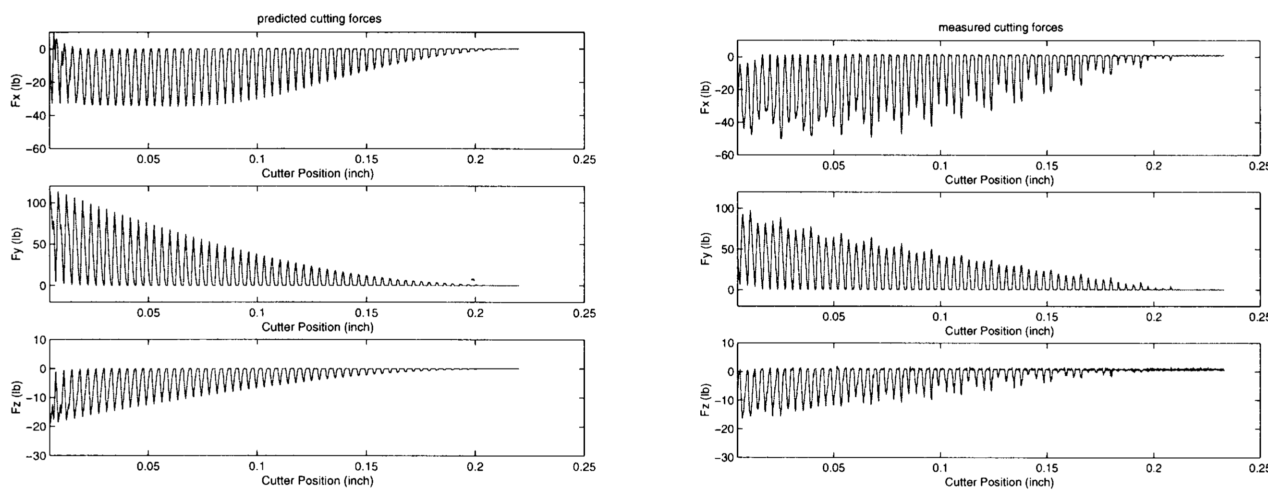

Fig. 5. Predicted and measured forces for cutting number 8.

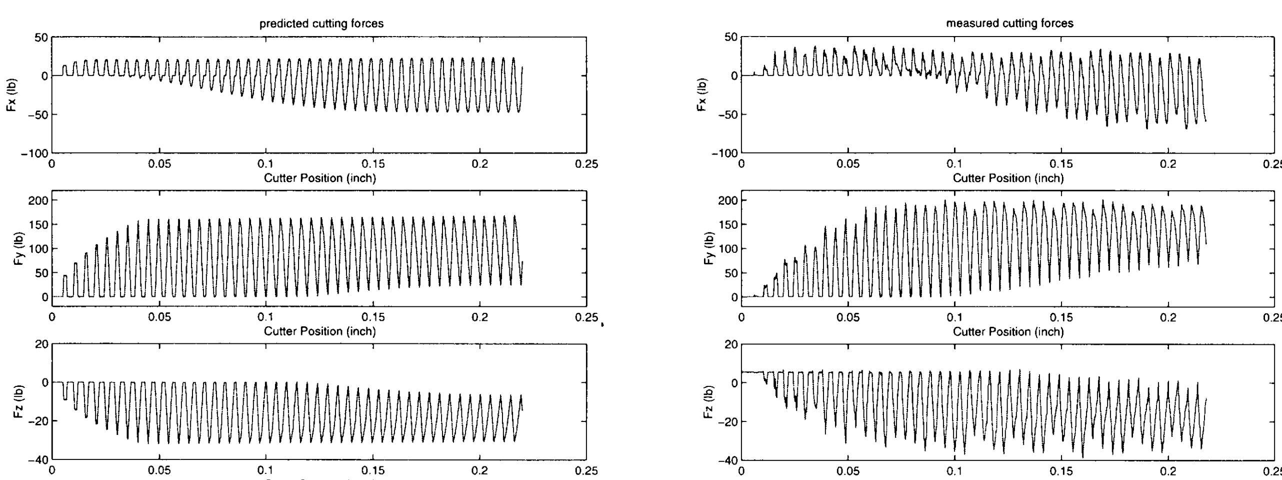

Fig. 6. Predicted and measured forces for cutting number 11.

Table 3. Cutting force predictions, measurements and prediction errors in experiments.

Cutting number	Direction	Mean of absolute measured force (lb)	Mean of absolute predicted force (lb)	Absolute error (lb)	Percentage error (%)
1	*X*	78.4	58.4	20.0	25.5
	Y	56.3	52.7	3.7	6.5
	Z	16.4	15.4	1	6.1
2	*X*	13.9	11.1	2.7	19.7
	Y	19.0	22.5	3.6	18.8
	Z	5.4	4.4	1.0	19.2
3	*X*	90.9	68.0	22.9	25.2
	Y	120.7	101.4	19.3	16.0
	Z	26.0	26.0	0	0.1
4	*X*	61.8	47.2	14.6	23.6
	Y	76.7	95.1	18.4	24.1
	Z	18.0	22.0	4.0	22.3
5	*X*	15.6	18.6	3.0	19.2
	Y	19.5	20.7	1.2	6.0
	Z	7.1	5.9	1.2	17.5
6	*X*	37.3	30.0	7.3	19.6
	Y	85.5	93.2	7.7	9.0
	Z	14.9	19.2	4.3	29.0
7	*X*	39.3	31	9.3	21.0
	Y	11.3	11.0	0.3	2.6
	Z	6.2	8.3	2.0	9.3
8	*X*	10.4	17.3	0.1	19.0
	Y	17.2	20	0.5	0.4
	Z	2.6	3.1	1.3	18.1
9	*X*	71.8	61.0	10.8	15
	Y	54.1	54.8	0.7	1.2
	Z	14.8	15.8	1	6.9
10	*X*	64.0	51.3	12.6	20
	Y	26.7	24.9	1.7	6.5
	Z	9.6	10.7	1.1	11
11	*X*	17.7	16.6	1.0	5.7
	Y	98.6	75.9	22.7	23.0
	Z	13.4	14.9	1.5	11.1
12	*X*	15.8	12.4	3.4	21.7
	Y	23.3	23.3	0.0	0.1
	Z	5.0	4.5	0.54	10.8
13	*X*	71.5	59.2	12.3	17.2
	Y	107.7	108.0	0.3	0.3
	Z	26.5	25.7	0.8	3.0
14	*X*	23.8	19.8	4.0	16.9
	Y	19.4	16.3	3.1	15.7
	Z	5.9	5.6	0.3	5.6

were sampled at 20000 points/second, then digitally low-pass filtered at a cut-off frequency of 250 Hz to eliminate the high-frequency components resulting from the machine tool dynamics and electrical noise.

A high-speed steel end mill with a diameter of $\frac{7}{16}$ in, a helix angle of 30° and 4 flutes was used to machine an aluminium 6061 workpiece. Table 1 lists the specific cutting pressure coefficients (K_t, K_r and K_a) with respect to the average cut thickness for the tool and workpiece materials [10]. The test conditions are given in Table 2. Based on the specific cutting pressure coefficients and the process parameters, the transient state cutting forces are predicted with a discretisation time interval (Δt) of $\frac{1}{4}$ of a cutter rotation period. Figures 4, 5 and 6 show the comparison of the predicted forces and the measured forces for cutting tests numbers 7, 8 and 11, respectively.

The waveform of the predicted cutting forces is closely matched by that of the measured ones for the tested cases for amplitude and pulsation period. Table 3 lists the means of the absolute predicted and measured forces, and the errors and percentage errors, during transient cutting. Percentage errors are calculated from the percentage of the prediction errors divided by the mean of measured forces during transient cutting. The maximum percentage prediction error is found to be less than 30% for all the cases tested.

The effect of runout of the cutter is observed from the repeated tooth passing patterns in the measured forces. Since the analytical model does not account for the existence of cutter runout, it contributes to the slight discrepancies between the predicted and the measured cutting forces. Other possible sources for the discrepancies are likely to be the variances in the specific cutting pressure coefficients and the modelling error associated with the discretisation of the transient cutting process.

4. Conclusions

This paper proposes a temporal discretisation method for the prediction of milling forces in the transient cutting state. The method extends a general cutting force model for steady-state cutting conditions to situations involving a time-varying depth of cut. Through temporal discretisation, a transient cutting is represented by a series of steady-state cutting with different depths of cut. The model provides 3D milling forces in terms of material properties, tool geometry, cutting parameters, and process configuration. The method can be used to facilitate the planning of cutting parameters, optimisation of tool geometry and on-line diagnostics. End milling experiments under a wide range of cutting conditions were performed and results presented in verification of the analytical model.

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Notation

$*$	convolution operator in the cutter angle domain
β	angular position of any cutting point in the cylindrical coordinate system
ϕ	angular position of cutter with respect to the -*Y*-direction
θ	angular position of any cutting point with respect to the -*Y*-direction
θ_1,θ_2	entry and exit angles
$\boldsymbol{A}_k$	as defined in Eq. (2)
A_{xk},A_{yk},A_{zk}	*k*th harmonics of cutting forces in the *X*-, *Y*-, and *Z*-directions
d_a,d_r	axial and radial depth of cut
f_0	frequency of spindle
F_x,F_y,F_z	resultant cutting forces in the angle domain in the *X*-, *Y*-, and *Z*-directions
$\boldsymbol{F}$	as defined in Eq. (1)
$G(y)$	cutting forces generated by a milling operation of zero entry angle
h'_β	derivative of height function of cutting edge with respect to β
$h(\beta)$	height function of one cutting edge with respect to β
K_r,K_a	radial and axial to tangential cutting force ratios
K_t	tangential cutting pressure constant
$\tilde{K}$	as defined in Eq. (3)
$\mathbf{p}$	as defined in Eq. (4)
N	number of cutting edge
$r(\beta)$	radius function of one cutting edge with respect to β
T_0	cutting engagement time of the cutting point at $\beta = 0$
$Tth(\beta)$	tooth sequence function
$\bar{t}_c$	average cut thickness
Δt	time interval for segmentation
$y_{\max}$	maximum positions of workpiece in the *Y*-direction